Small-Signal Capacitance

Small-Signal Capacitance

- If V_{GB} is increased by a small amount ΔV_{GB} , a positive charge $\Delta Q'_G$ will flow into the gate terminal.
- For overall charge neutrality, a charge of equal value must flow out of the body terminal or, equivalently, a charge of value $-\Delta Q'_G$ must flow *into* the body terminal.
- An incremental (small-signal) capacitance per unit area, C'_{gb} can thus be defined to relate charge changes to voltage changes.

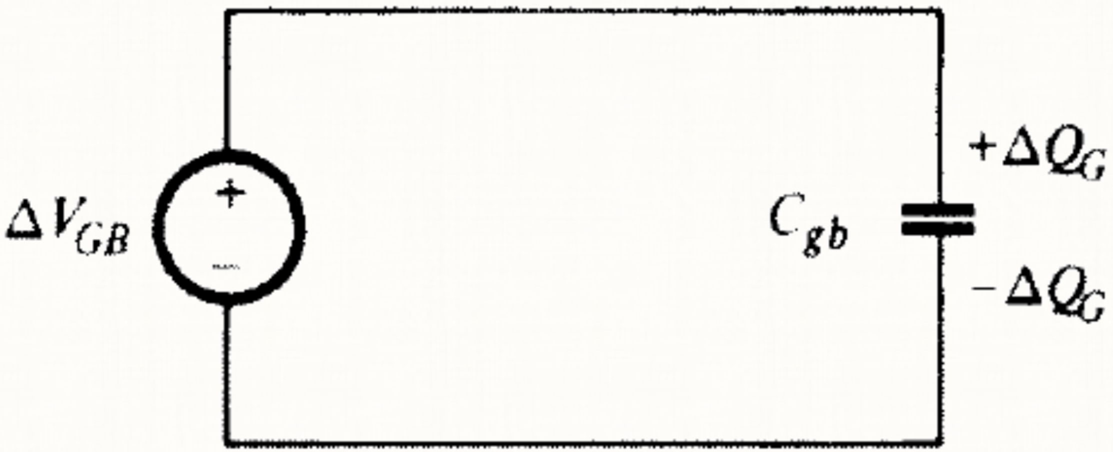
$$C'_{gb} = \frac{dQ'_G}{dV_{GB}}$$

- Charge neutrality:

$$\Delta Q'_C = -\Delta Q'_G$$

- On the other hand as discussed before:

$$\Delta V_{GB} = \Delta\psi_{ox} + \Delta\psi_s$$



Small-Signal Capacitance

- Therefore using past equations:

$$\frac{1}{C'_{gb}} = \frac{d\psi_{ox}}{dQ'_G} + \frac{d\psi_s}{dQ'_G} = \frac{1}{dQ'_G/d\psi_{ox}} + \frac{1}{-dQ'_C/d\psi_s}$$

- As discussed before: $Q'_G = C'_{ox}\psi_{ox}$, Therefore:

$$C'_{ox} = dQ'_G/d\psi_{ox}$$

- C'_c can be defined as small signal charge capacitance:

$$C'_C = -dQ'_C/d\psi_s$$

- Based on past equations:

$$Q'_C = -sgn(\psi_s)\sqrt{2q\epsilon_s N_A}\sqrt{\phi_T e^{-\psi_s/\phi_T} + \psi_s + e^{-2\phi_F/\phi_T}(\phi_T e^{\psi_s/\phi_T} - \psi_s - \phi_T)}$$

$$C'_C = sgn(\psi_s)\sqrt{2q\epsilon_s N_A}\frac{1 - \phi_T e^{-\frac{\psi_s}{\phi_T}} + e^{-\frac{2\phi_F}{\phi_T}}\left(\phi_T e^{\frac{\psi_s}{\phi_T}} - 1\right)}{2\sqrt{\phi_T e^{-\frac{\psi_s}{\phi_T}} + \psi_s + e^{-\frac{2\phi_F}{\phi_T}}\left(\phi_T e^{\frac{\psi_s}{\phi_T}} - \psi_s - \phi_T\right)}}, \psi_s \neq 0$$

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- If $\psi_s \neq 0$, both numerator and denominator become zero therefore from Hopital rule:

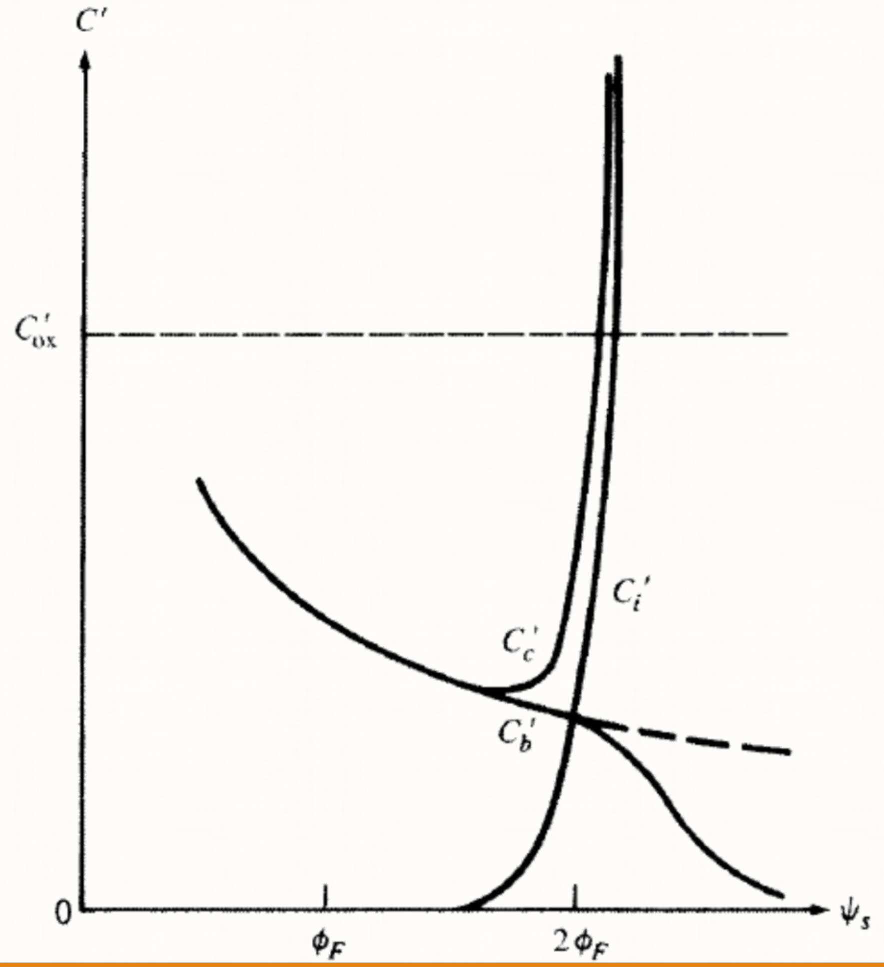
$$C_C' = \sqrt{2q\epsilon_s N_A \left(1 + e^{-\frac{2\phi_F}{\phi_T}}\right) / \phi_T}$$

- Deep in accumulation: (ψ_s below zero by several ϕ_T)

$$C_C' = \sqrt{2q\epsilon_s N_A / 2\phi_T}\, e^{-\frac{\psi_s}{2\phi_T}}$$

- Deep in depletion and inversion :(ψ_s below zero by several ϕ_T)

$$C_C' = \sqrt{2q\epsilon_s N_A} \frac{1 + \phi_T e^{\frac{\psi_s - 2\phi_F}{\phi_T}}}{2\sqrt{\psi_s + \phi_T e^{\frac{\psi_s - 2\phi_F}{\phi_T}}}}$$



Small-Signal Capacitance

- Consideration of the individual contributions of the depletion region and inversion layer charges to C_C':

$$\Delta Q_C' = \Delta Q_B' + \Delta Q_I'$$

$$C_C' = -\frac{dQ_B'}{d\psi_S} + \frac{-dQ_I'}{d\psi_s}$$

- Depletion region incremental capacitance per unit area:

$$C_b' = -dQ_B'/d\psi_S$$

- This capacitance relates changes of the potential across the depletion region to the associated changes of the depletion charge.
- Capacitance per unit area associated with the inversion layer:

$$C_i' = -dQ_I'/d\psi_s$$

Small-Signal Capacitance

- As discussed before we can find these two small signal capacitances:

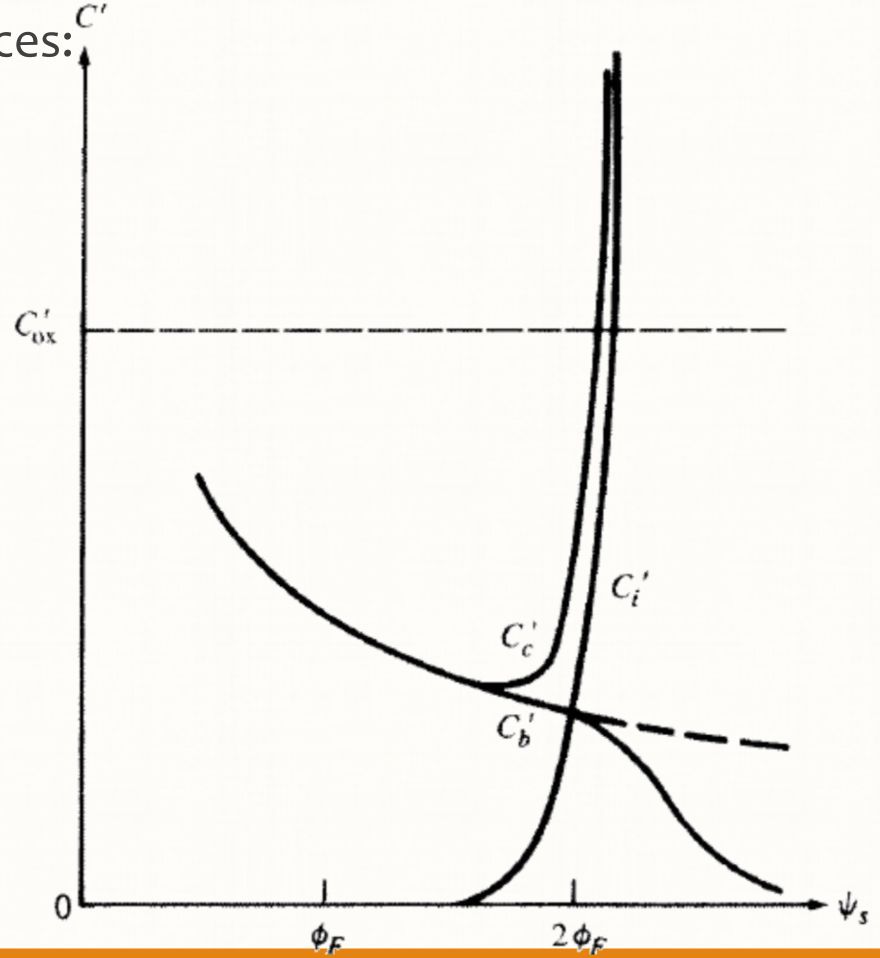
$$C_b' = \sqrt{2q\epsilon_s N_A}\,\frac{1}{2\sqrt{\psi_s + \phi_T e^{\frac{\psi_s - 2\phi_F}{\phi_T}}}}$$

$$C_i' = \sqrt{2q\epsilon_s N_A}\,\frac{\phi_T e^{\frac{\psi_s - 2\phi_F}{\phi_T}}}{2\sqrt{\psi_s + \phi_T e^{\frac{\psi_s - 2\phi_F}{\phi_T}}}}$$

$$C_C' = C_b' + C_i'$$

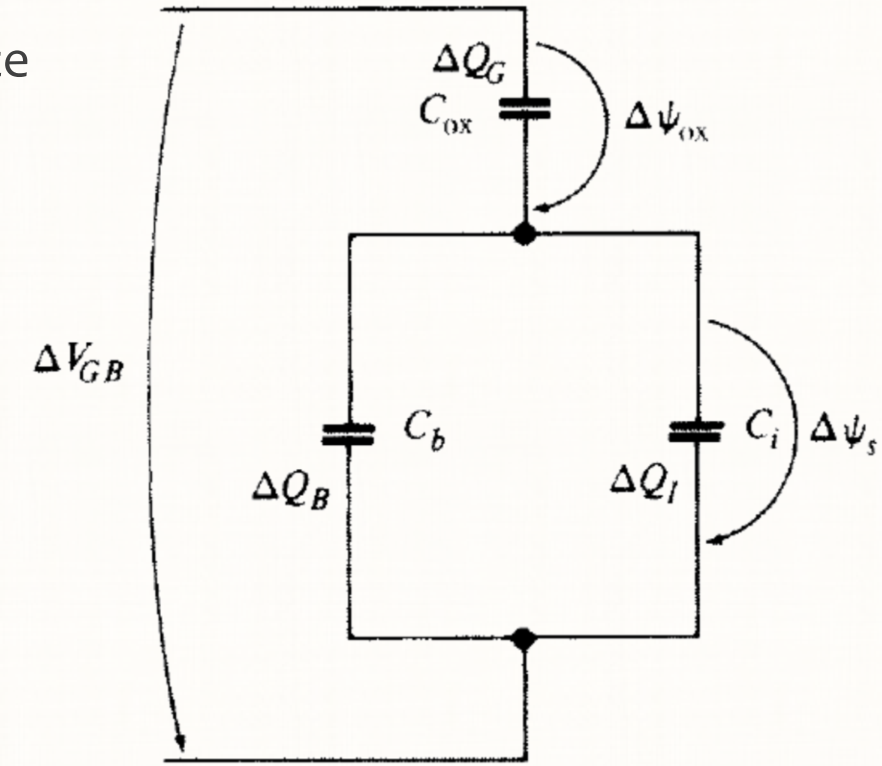
- Finally:

$$\frac{1}{C_{gb}'} = \frac{1}{C_{ox}'} + \frac{1}{C_i' + C_b'}$$



Small-Signal Capacitance

- Small-signal equivalent circuit, relating small *changes* of potentials and charges around a bias point. It does *not* relate total values of potentials and charges.
- Our ultimate goal in this development is to plot the total capacitance seen externally (C'_{gb}) vs. the total externally applied bias (V_{GB}).
  - First we should find ψ_s from V_{GB}.
  - Then C'_c can be calculated from corresponding ψ_s.



Small-Signal Capacitance

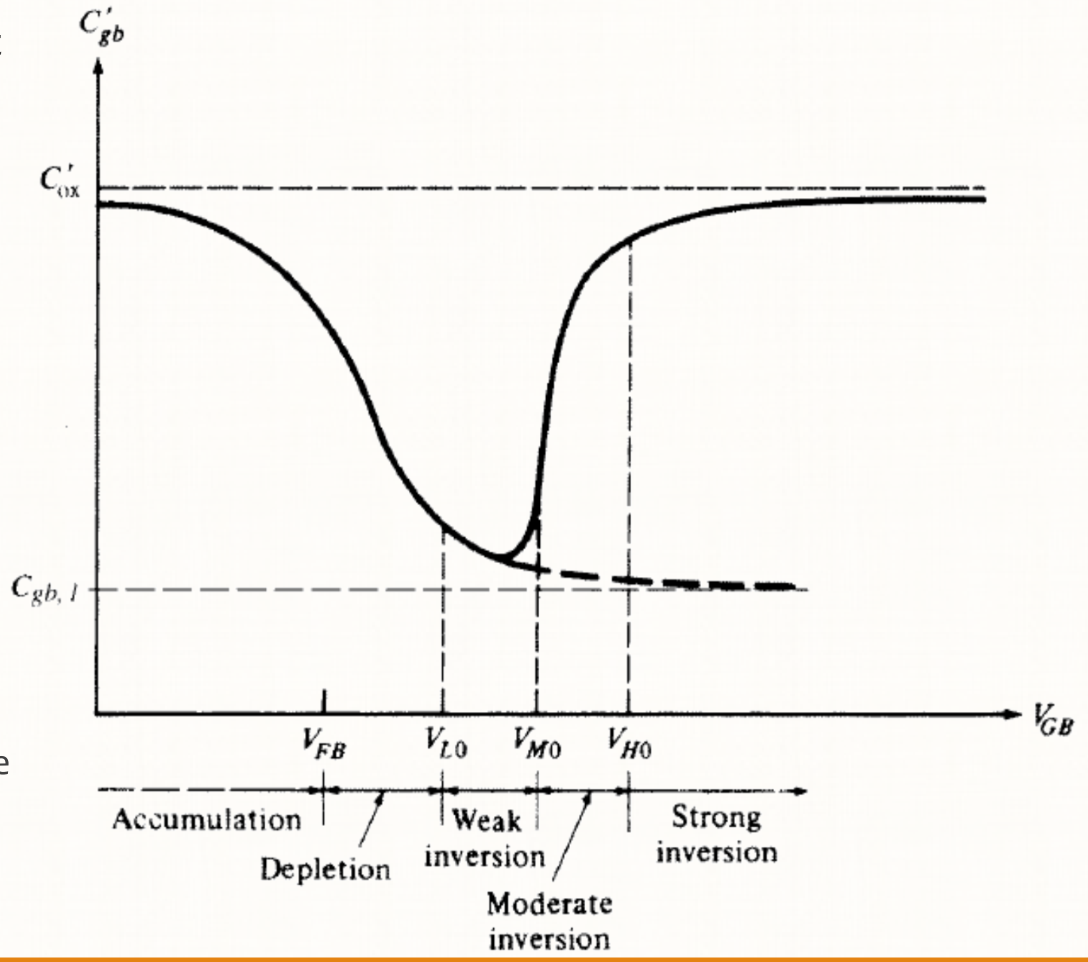
- Static changes → after V_{GB} is changed by a small amount ΔV_{GB}, it remains fixed at its new value. We then wait long enough for a new equilibrium to be reached.
- Deep in accumulation: C'_{gb} → C'_{ox}
  - When ψ_s become more negative → C'_C increases

$$C'_C = \sqrt{2q\epsilon_s N_A / 2\phi_T}\, e^{-\frac{\psi_s}{2\phi_T}}$$

  - As the C'_C and C'_{ox} are series capacitance, therefore:

$$\frac{1}{C'_{gb}} = \frac{1}{C'_{ox}} + \frac{1}{C'_C} \approx \frac{1}{C'_{ox}}$$

- From accumulation to neighboring region C'_{gb} change a lot!
  - Therefore it can be used a varactor (a bias-controlled small-signal capacitor) device for RF applications.
- In the weak inversion region, C'_{gb} is seen to reach a minimum.
  - In weak inversion (except for points close to the upper limit of the region), the inversion layer capacitance is negligible (figure). Therefore C'_{gb} → C'_C in this region. (it should be also noted that in this regime $C'_B > C'_i$)



Small-Signal Capacitance

- Moderate and deep inversion: $C'_{gb} \rightarrow C'_{ox}$
  - It is because C'_i drastically increases and although C'_b decreases as theses capacitors are parallel, C'_C increases and consequently $C'_{gb} \rightarrow C'_{ox}$ (same as deep accumulation which we have high level hole concentration)

- High frequency changes of $V_{GB} (f > 100\ KHz)$ the behavior of small signal capacitance will be changed!
  - If ΔV_{GB} is a small-signal sinusoidal voltage, the steady-state charge changes will also be sinusoidal.
  - For high frequency regime, electron density in inversion layer cannot change with ΔV_{GB} frequency. Why?
    - This is because electron generation in depletion region is slow mechanism (thermal generation and recombination) and therefore, due to this slow generation electron density in inversion layer approximately remain constant!
    - If there is communication with outside (Source and Drain), electron density change can be enhanced.

Small-Signal Capacitance

- High frequency regime…
  - Based on this discussion, C_i' should be remain constant and therefore C_b' will changed. This means that ΔV_{GB} only cover and uncover the ionic charges! From the past equation one can see small C_b' play an essential rule in calculation of C_{gb}'.
  - In deep inversion regime: $\psi_s = \phi_0 = 2\phi_F + \phi_{Z0}$

$$minimum\ C_{gb}' \rightarrow C_{gb,l}' = \frac{C_{ox}'}{1 + \frac{2}{\gamma}\sqrt{\phi_0}}\ why?$$

  - Back to small signal capacitance for low freq. regime
  - From the figure (problem):

$$\frac{d\psi_s}{dV_{GB}} = \frac{C_{ox}'}{C_{ox}' + C_i' + C_b'}$$

    - Each slope in figure corresponds to incremental small capacitance discussed here.

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- For strong inversion regime this slope corresponds to C_i'(becomes very large and the slope diminishes).

$$\frac{d|Q_I'|}{dV_{GB}} = \frac{d|Q_I'|}{d\psi_S}\frac{d\psi_S}{dV_{GB}} = \frac{C_{ox}'C_i'}{C_{ox}' + C_i' + C_b'} \approx C_{ox}'$$

- Generally:

$$\frac{d\ln|Q_I'|}{dV_{GB}} = \frac{C_{ox}'}{C_{ox}' + C_i' + C_b'}\frac{C_i'}{|Q_I'|}$$

In week inversion, C_i', can be neglected and the slope becomes:

$$n = 1 + \frac{C_b'}{C_{ox}'} = \frac{dV_{GB}}{d\psi_S}$$

TABLE 2.1
Regions of inversion and properties

	Weak inversion	Moderate inversion	Strong inversion
Definition in terms of surface potential ψ_s	$\phi_F \le \psi_s < 2\phi_F$	$2\phi_F \le \psi_s < 2\phi_F + \phi_{Z0}$	$2\phi_F + \phi_{Z0} \le \psi_s$
Definition in terms of gate-body voltage V_{GB}	$V_{L0} \le V_{GB} < V_{M0}$	$V_{M0} \le V_{GB} < V_{H0}$	$V_{H0} \le V_{GB}$
$\frac{\lvert Q'_I \rvert}{\lvert Q'_B \rvert}$	$\ll 1$	Varies	$\gg 1$ deep in strong inversion; not necessarily so near the bottom of the region
$\frac{C'_i}{C'_b}$	$\ll 1$ deep in weak inversion; not necessarily so near the top of the region	Varies	$\gg 1$
$\frac{d\psi_s}{dV_{GB}}$	Approximately constant; attains its maximum value in this region	Varies	Small
Dependence of Q'_I on V_{GB}	Approximately exponential	—	Approximately first-degree polynomial
$\frac{d \ln \lvert Q'_I \rvert}{d\psi_s}$	$\frac{1}{\phi_t}$	Varies	$\frac{1}{2\phi_t}$