Inversion

Inversion

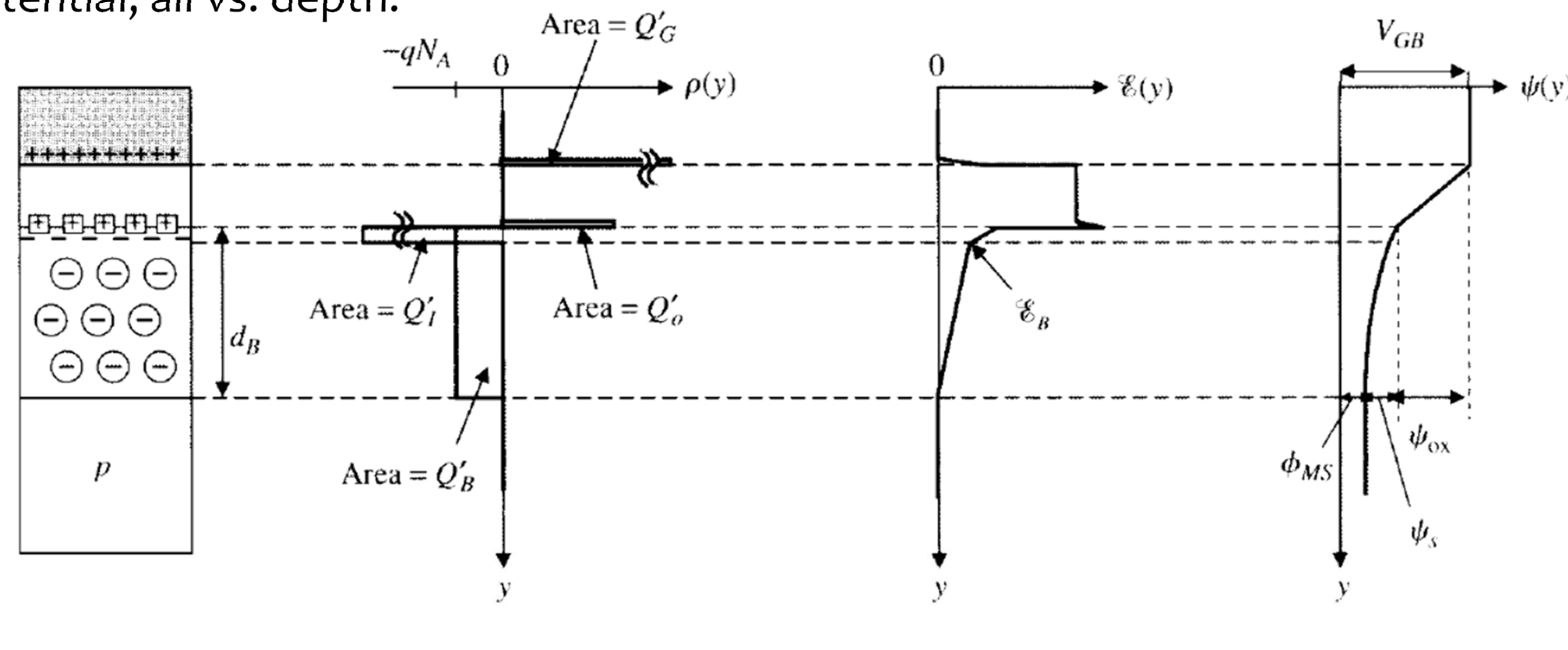
- In inversion, the hole contribution is negligible.
- We only need to consider the contributions from the acceptor atoms and the mobile electrons.

$$Q_c = Q_I + Q_B$$

- Calculating Q_I: As one goes away from the surface *n(y)* decreases rapidly owing to its exponential dependence on ψ(y) (figure & figure).
  - One can choose a point, below which the electron concentration will be negligible.
- Calculating Q_B: We will assume that the depletion region contains only acceptor atoms and is defined by a sharp boundary at a depth dB below the surface, with the semiconductor being neutral below it

Inversion

- *(a)* Two-terminal MOS structure with p-type substrate; *(b)* charge density; *(c)* electric field; *(d)* potential, all vs. depth.



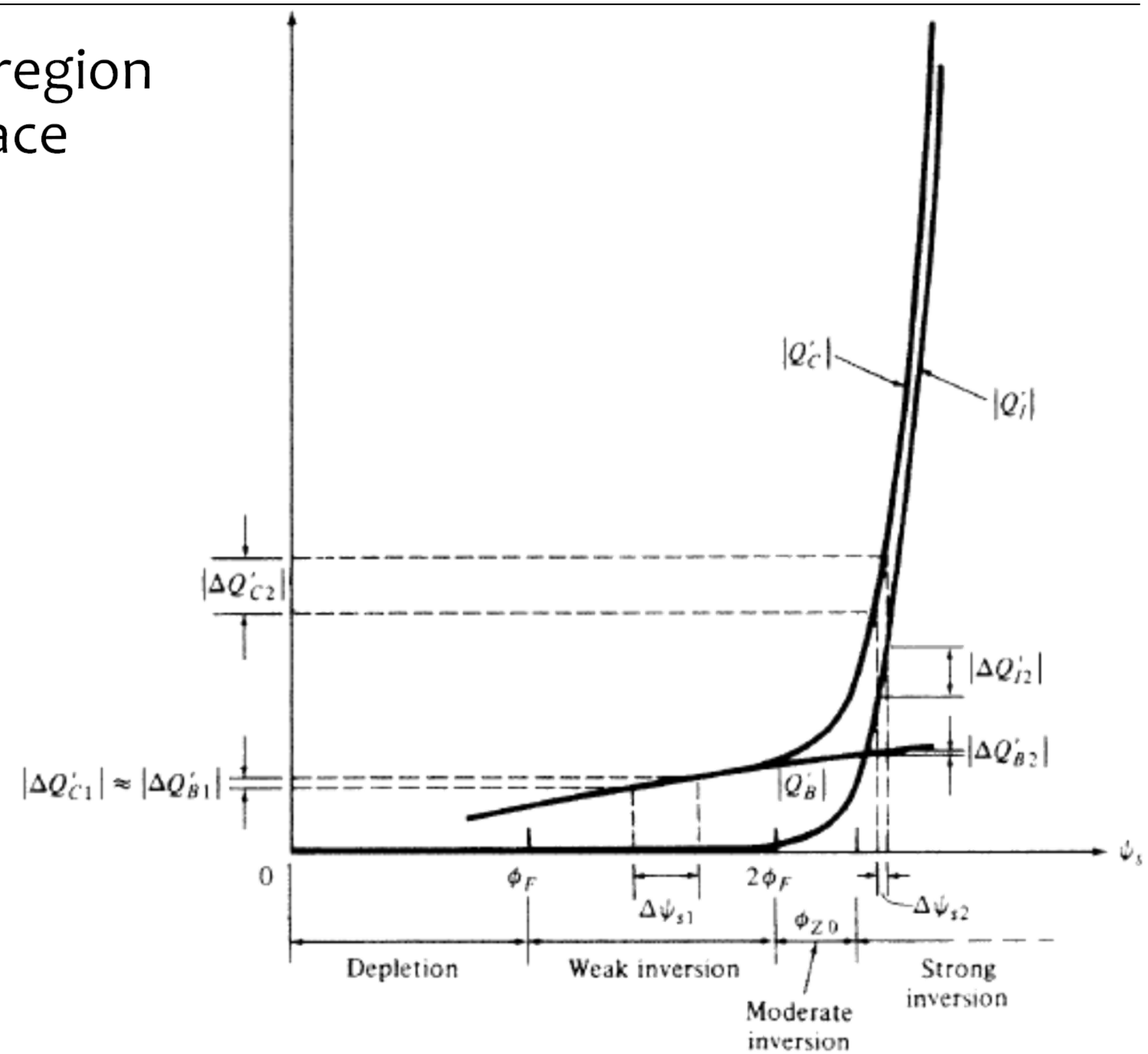
(a) (b) (c) (d)

Inversion

- Charge Sheet Approximation
  - Inversion layer thickness → 0 → The potential drop across the inversion layer, which is the area of the corresponding part in the field plot, approaches zero.
  - We can assume that all of the surface potential ψ_s is dropped across the depletion region.
  - The surface potential drop ψ_s is then the area under the corresponding triangle in the field plot.

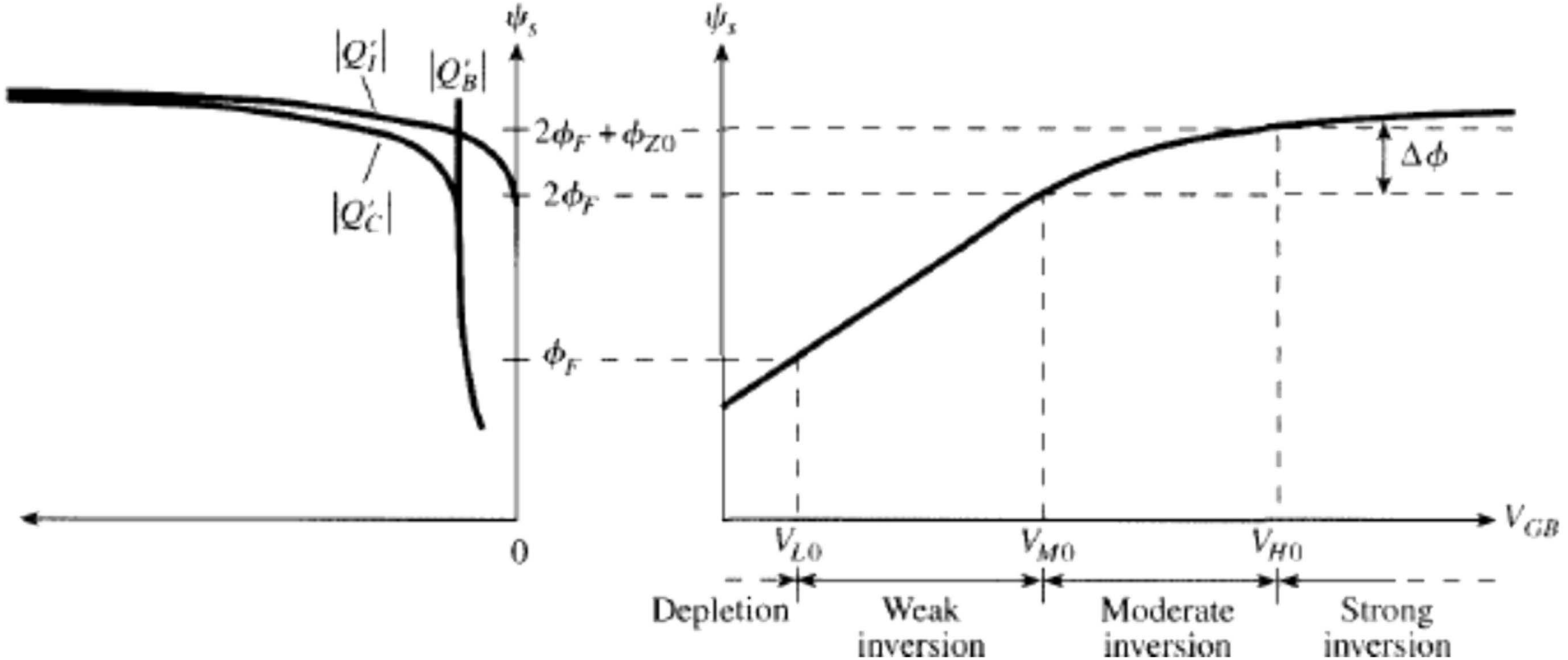
Inversion

- Magnitude of inversion layer charge, depletion region charge, and their sum (all per unit area) vs. surface potential.



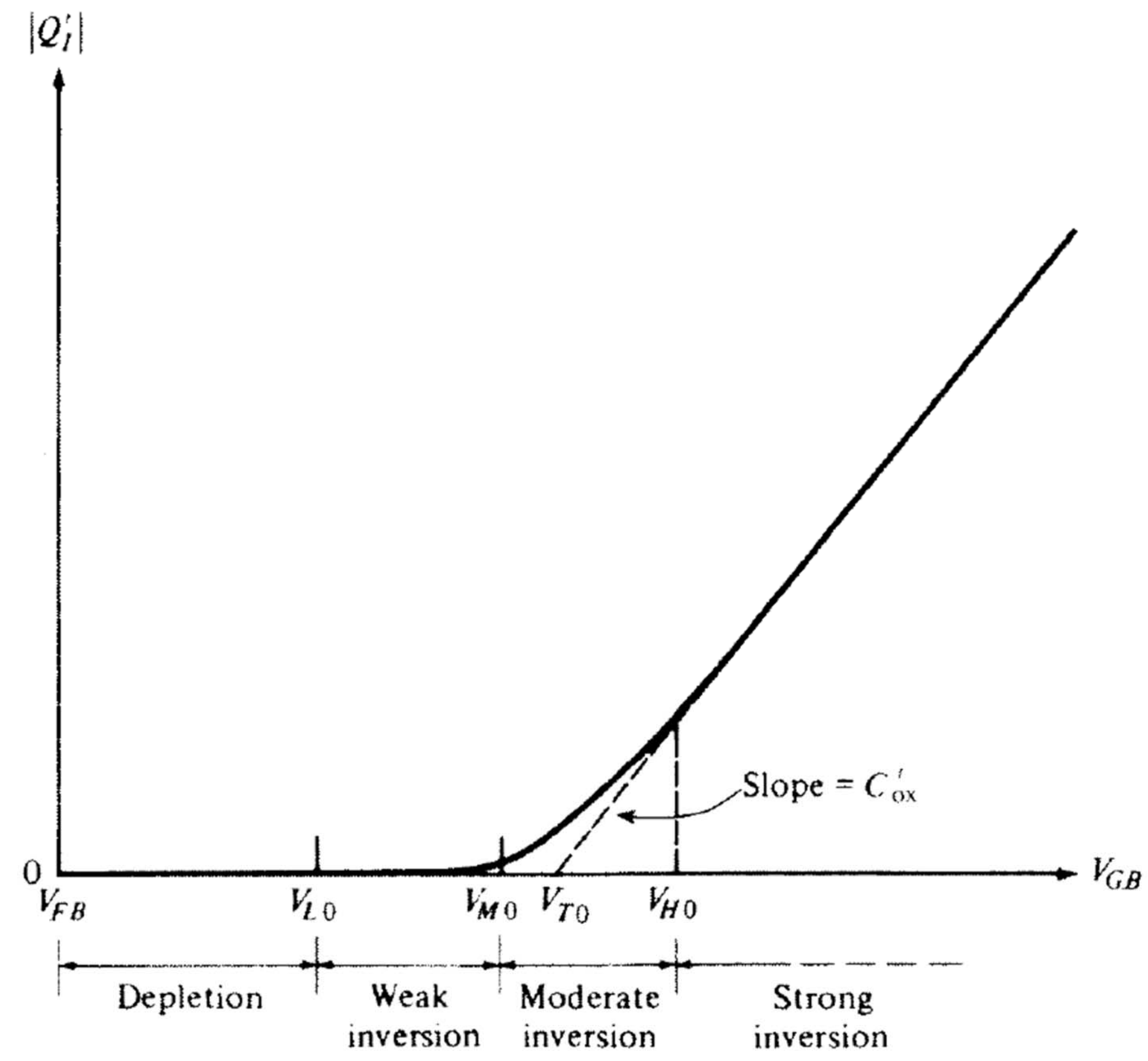
Inversion

- Surface potential vs. gate-substrate voltage (right) and charges vs. surface potential (left).



Inversion

- Magnitude of inversion layer charge per unit area vs. gate-substrate voltage.



Inversion

- Strong Inversion
  - In this regime as seen on figure, with changes of V_{GB} leads to small changes in ψ_s. It can be assumed constant in this case.

$$\psi_s = \phi_0 = 2\phi_F + \phi_{Z0}$$

  - Usually finding the exact value for ϕ_{Z0} is not easy. We can not ignore this value as it seen in figure although it was a bad choice historically!

$$\phi_0 = 2\phi_F + \Delta\phi \text{ ,} \Delta\phi \textit{ is several } \phi_T \text{ (}5\phi_T \textit{ to } 6\phi_T \textit{ is good chice)}$$

  - In this situation depletion region depth become constant and will not change further:

$$d_{Bm} = \sqrt{\frac{2\epsilon_s}{qN_A}}\sqrt{\phi_0}$$

  - Similarly, the depletion region charge is assumed to have reached a maximum value:

$$Q'_{B0} = -\sqrt{2q\epsilon_s N_A}\sqrt{\phi_0}$$

Inversion

- Calculating strong inversion charge:

$$Q'_I = -C'_{ox}(V_{GB} - V_{FB} - \psi_s) - Q'_B$$

$$Q'_I = -C'_{ox}(V_{GB} - V_{T0})\ where\ V_{T0} = \phi_{MS} - \frac{Q'_0}{C'_{ox}} + \phi_0 - \frac{Q'_{B0}}{C'_{ox}} = V_{FB} + \phi_0 + \gamma\sqrt{\phi_0}$$

- V_{T0} is called extrapolated threshold voltage of the MOS two terminal structure.
- Ex. Calculate the V_{T0} for the process of past example:
  - As it is calculated we have:

$$V_{FB} = 1.0412\ V\ and\ \gamma = 0.337\ V^{1/2}$$

  - If $\phi_0 = 2\phi_F + 6\phi_T = 1.107\ V$

$$V_{T0} = -1.042 + 1.107 + 0.337\sqrt{1.107} = 0.419\ V$$

- It should be emphasized that, whereas V_{T0} is a quantity that appears in the strong inversion expression, the MOS structure is not in strong inversion for $V_{GB} = V_{T0}$ (figure)

Inversion

- Weak inversion
  - As we calculated:

$$Q_I' = -\sqrt{2q\epsilon_s N_A}\left(\sqrt{\psi_s + \phi_T e^{(\psi_s - 2\phi_F)/\phi_T}} - \sqrt{\psi_s}\right)$$

$$\xi = \phi_T e^{(\psi_s - 2\phi_F)/\phi_T}$$

  - In weak inversion $\psi_s < 2\phi_F$, therefore $\xi \ll \psi_s$
  - Taylor expansion of $\sqrt{\psi_s + \xi}$ around $\xi = 0$:

$$\sqrt{\psi_s + \xi} = \sqrt{\psi_s} + \frac{1}{2\sqrt{\psi_s}}\xi$$

  - By substituting to Q_I':

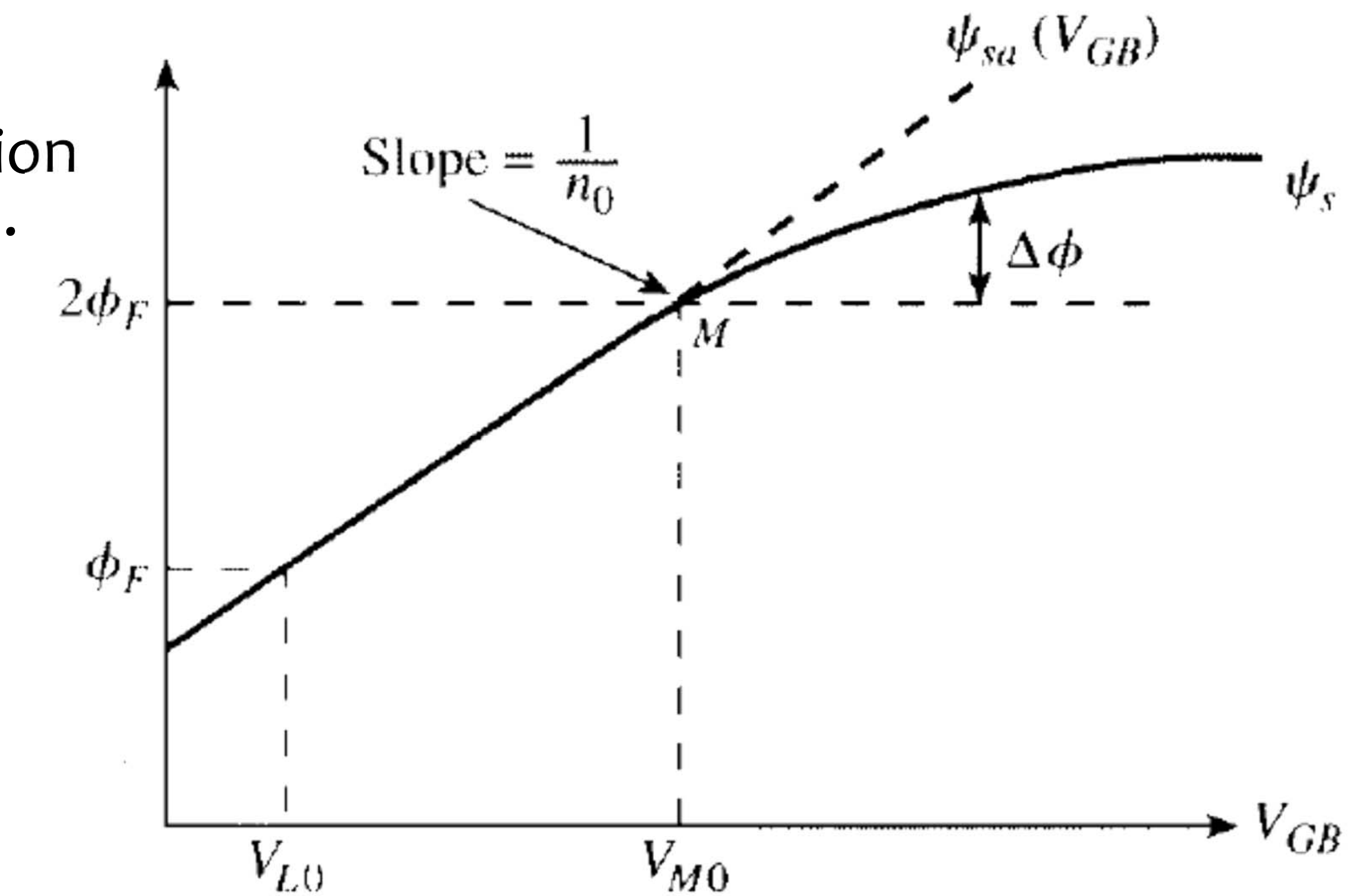
$$Q_I' = -\sqrt{2q\epsilon_s N_A}\frac{1}{2\sqrt{\psi_s}}\xi = \frac{-\sqrt{2q\epsilon_s N_A}}{2\sqrt{\psi_s}}\phi_T e^{(\psi_s - 2\phi_F)/\phi_T}$$

Inversion

- To calculate the preceding equation we need to estimate ψ_s
- In weak inversion the inversion layer charge is extremely small compared with the depletion region charge, it does not influence the surface potential. Therefore The total semiconductor charge is practically equal to the depletion region charge. (figure)

$$\psi_s \approx \psi_{sa} = \left(-\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + V_{GB} - V_{FB}}\right)^2$$

$$Q_I' = \frac{-\sqrt{2q\epsilon_s N_A}}{2\sqrt{\psi_s}} \phi_T e^{(\psi_{sa}(V_{GB}) - 2\phi_F)/\phi_T}$$



Inversion

- It can be seen in Fig. 2.18 that the slope of ψ_{sa} (and thus of ψ_s, in weak inversion) vs. V_{GB} is almost constant
- The *inverse* of this slope is often denoted by *n.*

$$n = \left(\frac{d\psi_{sa}}{dV_{GB}}\right)^{-1} = 1 + \frac{\gamma}{2\sqrt{\psi_{sa}(V_{GB})}}$$

- Typically, *n* is between 1 and 1.5.
- Making Q'_I more simpler: ($Q'_I = \frac{-\sqrt{2q\epsilon_s N_A}}{2\sqrt{\psi_{sa}}}\phi_T e^{(\psi_{sa}(V_{GB})-2\phi_F)/\phi_T}$)

1. Constant value $\sqrt{\psi_{sa}}$: when V_{GB} is varied, the variation of $\sqrt{\psi_{sa}}$ is negligible when compared with the drastic variation of the exponential term. Therefore we can assume $\sqrt{\psi_{sa}} = \sqrt{\phi_x}$. ϕ_x is the value of the surface potential at any convenient point in weak inversion. But it is common to choose it $\phi_x = 2\phi_F$

Inversion

- …

2. Simplifying $\psi_{sa}(V_{GB}) - 2\phi_F$: as the slope in week inversion is constant therefore:

$$n_0 = n\Big|_{\psi_{sa}=2\phi_F} = 1 + \frac{\gamma}{2\sqrt{2\phi_F}}$$

Then:

$$\psi_{sa} - 2\phi_F = \frac{1}{n_0}(V_{GB} - V_{M0})$$

- Using these two assumption we obtain:

$$Q_I' = Q_{M0}' e^{(V_{GB}-V_{M0})/n\phi_T}$$

Where

$$Q_{M0}' = \frac{-\sqrt{2q\epsilon_s N_A}}{2\sqrt{2\phi_F}}\phi_T$$

- Q_{M0}', represents the value of Q_I' at the upper limit of weak inversion ($V_{GB} = V_{M0}$).

Inversion

- Comparing our assumption (b&c) with exact solution (a)
  - For weak and strong inversion regions our assumption works perfectly.
  - But for moderate region the exact equation should be solved numerically! (need time and efforts)