Regions of Inversion

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- Approximate Limits
  - For two terminal MOS onset of weak, moderate, and strong inversion by the surface potential values

TABLE 3.1
Approximate bounds between regions†

	Bound between depletion and weak inversion	Bound between weak and moderate inversion	Bound between moderate and strong inversion
In terms of surface potential ψ_s	$\phi_F + V_{CB}$	$2\phi_F + V_{CB}$	$2\phi_F + V_{CB} + \phi_Z$‡
In terms of V_{GB}, for a given V_{CB}	$V_{LB} = V_L + V_{CB}$	$V_{MB} = V_M + V_{CB}$	$V_{HB} = V_H + V_{CB}$
In terms of V_{GC}, for a given V_{CB}	$V_L = V_{FB} + \phi_F + \gamma\sqrt{\phi_F + V_{CB}}$	$V_M = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F + V_{CB}}$	$V_H = V_M + V_Z$§
In terms of V_{CB}, for a given V_{GB} (see Sec. 3.5)	$V_U = \left(-\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + V_{GB} - V_{FB}}\right)^2 - \phi_F$	$V_W = \left(-\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + V_{GB} - V_{FB}}\right)^2 - 2\phi_F$	$V_Q = \left(-\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + V_{GB} - V_{FB} - V_Z}\right)^2 - 2\phi_F$

Approximate Limits

- Voltages corresponding to region of inversion is with some changes can be find as before, for example for moderate inversion voltage (neglecting exp term):

$$V_{MB} = V_{FB} + (2\phi_F + \boldsymbol{V_{CB}}) + \gamma\sqrt{(2\phi_F + \boldsymbol{V_{CB}})}$$

- The corresponding values in terms of V_{GC} can be found by subtracting $\boldsymbol{V_{CB}}$, as suggested before. Thus, for example:

$$V_M = V_{MB} - \boldsymbol{V_{CB}} = V_{FB} + (2\phi_F) + \gamma\sqrt{(2\phi_F + \boldsymbol{V_{CB}})}$$

- The fact that the quantities in Fig. increase with increasing $\boldsymbol{V_{CB}}$ is a consequence of the body effect discussed before.

- As is apparent from the expressions in Table 3.1, how much V_L, V_M, and V_H will increase for a given increase in $\boldsymbol{V_{CB}}$ is determined by the value of the coefficient γ; hence, the name *body effect coefficient* for this quantity.

$$\gamma = \frac{\sqrt{2q\epsilon_s N_A}}{C'_{ox}}$$

- The *body effect* is **stronger** for heavier substrate dopings and/or thicker oxides.

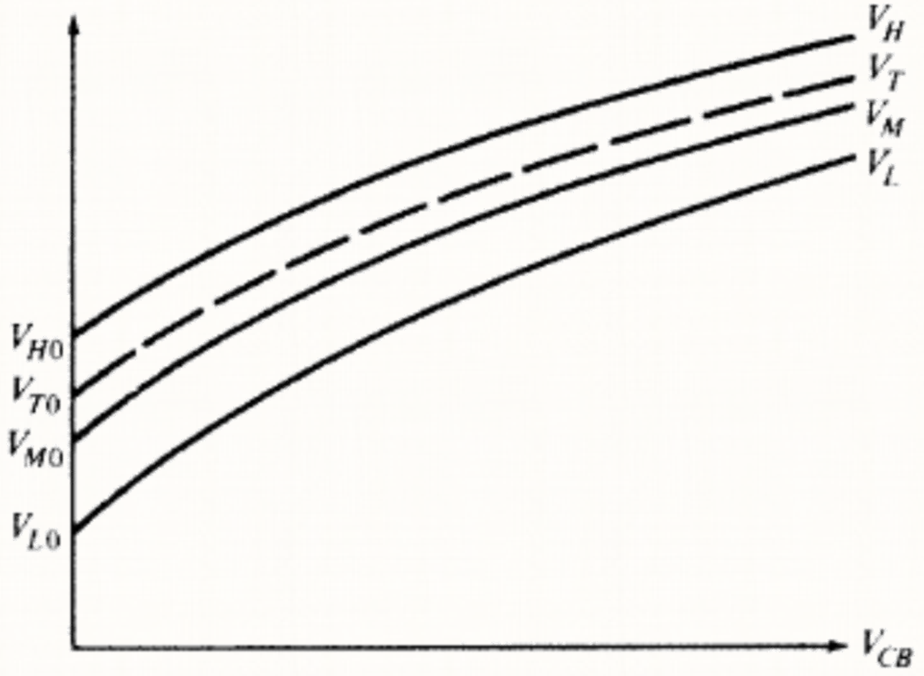


TABLE 3.2
Regions of inversion and properties (three-terminal MOS structure)

	Weak inversion	Moderate inversion	Strong inversion
Definition in terms of surface potential ψ_s (see Fig. 3.6*a*)	$\phi_F + V_{CB} \le \psi_s < 2\phi_F + V_{CB}$	$2\phi_F + V_{CB} \le \psi_s < 2\phi_F + V_{CB} + \phi_Z$	$2\phi_F + V_{CB} + \phi_Z \le \psi_s$
Definition in terms of V_{GB} for a given V_{CB} (see Figs. 3.8*b* and 3.6)	$V_{LB} \le V_{GB} < V_{MB}$	$V_{MB} \le V_{GB} < V_{HB}$	$V_{HB} \le V_{GB}$
Definition in terms of V_{GC} for a given V_{CB} (see Figs. 3.8*b* and 3.6)	$V_L \le V_{GC} < V_M$	$V_M \le V_{GC} < V_H$	$V_H \le V_{GC}$
Definition in terms of V_{CB} for a given V_{GB} (see Fig. 3.1*c* and Sec. 3.5)[†]	$V_U \ge V_{CB} > V_W$	$V_W \ge V_{CB} > V_Q$	$V_Q \ge V_{CB}$
$\frac{\lvert Q_I' \rvert}{\lvert Q_B' \rvert}$	$\ll 1$	Varies	$\gg 1$ deep in strong inversion; not necessarily so near the bottom of the region
$\frac{C_i'}{C_b'}$	$\ll 1$ deep in weak inversion; not necessarily so near the top of the region	Varies	$\gg 1$
$\frac{d\psi_s}{dV_{GB}}$	Approximately constant; attains its maximum value in this region	Varies	Small
$\frac{d\psi_s}{dV_{CB}}$	Very small	Varies	Close to 1
Dependence of Q_I' on V_{GB} or V_{GC} for V_{CB} constant	Approximately exponential	—	Approximately first-degree polynomial
$\frac{d \ln \lvert Q_I' \rvert}{d\psi_s}$	$\frac{1}{\phi_t}$	Varies	$\frac{1}{2\phi_t}$

Strong Inversion

- In deep inversion : $|Q_I'| \gg |Q_B'| \rightarrow C_i' \gg C_b'$
- As seen in Fig., ψ_s changes only slightly with V_{GB} in strong inversion and can be assumed "pinned" to a fixed value.
- This value for three terminal MOS increases by $\boldsymbol{V_{CB}}$in compare to two terminal MOS

$$\psi_s \approx \phi_0 + \boldsymbol{V_{CB}}$$
$$\phi_0 \approx 2\phi_F + \Delta\phi$$
$$\Delta\phi \text{ is several } \phi_T$$

- The depletion region width can then also be assumed pinned at a value d_{Bm}

$$d_{Bm} = \sqrt{\frac{2\epsilon_s}{qN_A}}\sqrt{\phi_0 + \boldsymbol{V_{CB}}}$$

- Similarly, the depletion region charge is assumed to have reached a maximum value:

$$Q_B' = -\sqrt{2q\epsilon_s N_A}\sqrt{\phi_0 + \boldsymbol{V_{CB}}} = -\gamma C_{ox}'\sqrt{\phi_0 + \boldsymbol{V_{CB}}}$$

Strong Inversion…

- *For the case of strong inversion only,* $\boldsymbol{V_{CB}}$ can be interpreted as the ***effective reverse bias*** of the field-induced junction formed by the inversion layer and substrate. $(d\psi_s/d\boldsymbol{V_{CB}} \approx 1)$
- The inversion layer charge can be obtained by:

$$Q_I' = -C_{ox}'(V_{GB} - V_{FB} - \psi_s) - Q_B'$$

$$Q_I' = -C_{ox}'(V_{GB} - V_{TB})\ where\ V_{TB} = V_T + \boldsymbol{V_{CB}}$$

$$V_T = \phi_{MS} - \frac{Q_0'}{C_{ox}'} + \phi_0 - \frac{Q_B'}{C_{ox}'} = V_{FB} + \phi_0 + \gamma\sqrt{\phi_0 + \boldsymbol{V_{CB}}}$$

- The quantity V_{TB} is the G-B *(gate-body) extrapolated threshold voltage.* Its meaning is illustrated in Fig (d). It is not the border voltage for strong inversion mechanism!
- Another way to express the equations using V_{GC}:

$$Q_I' = -C_{ox}'(V_{GC} - V_T)$$

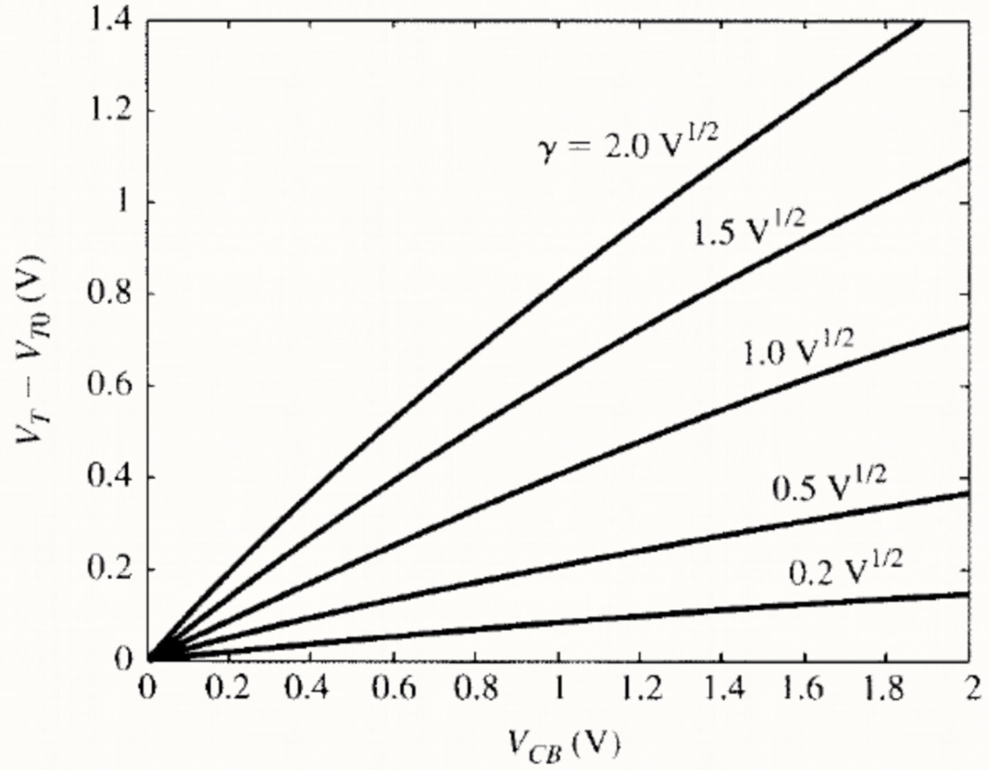
The quantity V_T is the G-C *extrapolated threshold voltage.*

$$V_T = V_{FB} + \phi_0 + \gamma\sqrt{\phi_0 + \boldsymbol{V_{CB}}} = V_{T0} + \left(\gamma\sqrt{\phi_0 + \boldsymbol{V_{CB}}} - \gamma\sqrt{\phi_0}\right)$$

- The plot of V_T vs. $\boldsymbol{V_{CB}}$ has been included in Fig.

Strong Inversion…

- The threshold increase $V_T - V_{T0}$ due to the body effect is shown vs. $\boldsymbol{V_{CB}}$ for various values of γ in Fig.
- Note that, although we have assumed $\boldsymbol{V_{CB} \geq 0}$, results are approximately valid even if $\boldsymbol{V_{CB}}$ is somewhat below zero (thus forward-biasing the source-body junction), as long as the $\boldsymbol{V_{CB}}$ value is not enough to cause appreciable junction current. This has been confirmed experimentally.



Weak Inversion

- In weak inversion : $|Q_I'| \ll |Q_B'| \rightarrow C_i' \ll C_b'$
- In this case as the $\psi_s < 2\phi_F + \boldsymbol{V_{CB}}$, like before we can find:

$$Q_I' = \frac{-\sqrt{2q\epsilon_s N_A}}{2\sqrt{\psi_s}}\phi_T e^{(\psi_s-(2\phi_F+\boldsymbol{V_{CB}}))/\phi_T}$$

- In weak inversion, $\boldsymbol{V_{CB}}$ *cannot* be interpreted as an effective reverse bias, as explained before.
- In weak inversion, the surface potential is practically independent of $\boldsymbol{V_{CB}}$ and is practically equal to ψ_{sa}

$$\psi_s \approx \psi_{sa}(V_{GB}) = \left(-\frac{\gamma}{2}+\sqrt{\frac{\gamma^2}{4}+V_{GB}-V_{FB}}\right)^2$$

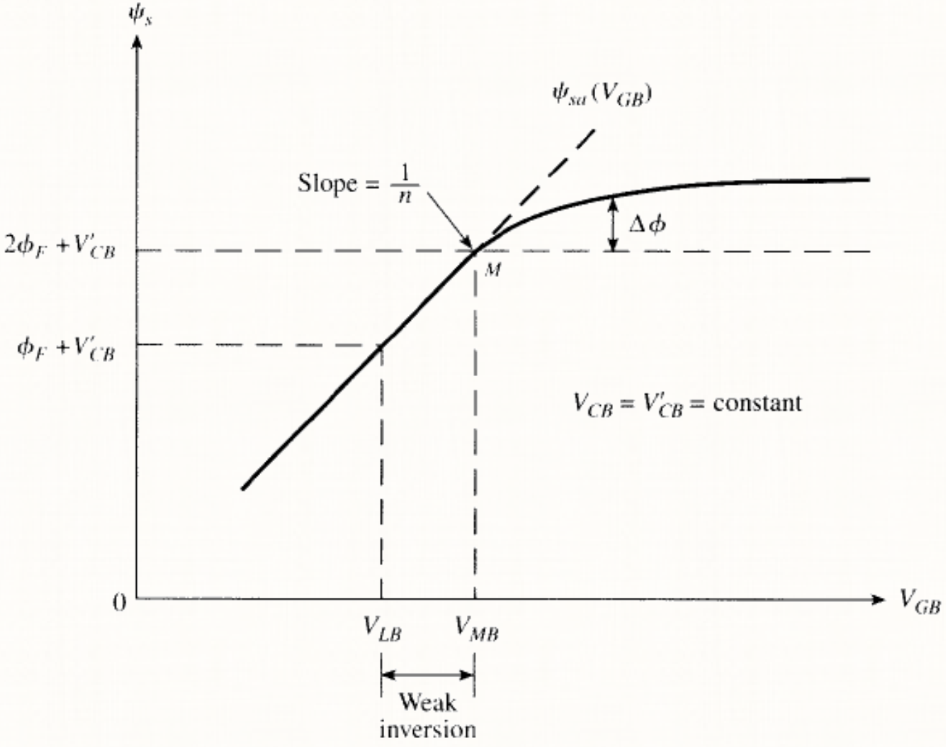
Therefore, in mentioned Eq. the *only* term dependent on $\boldsymbol{V_{CB}}$ is $e^{-(\boldsymbol{V_{CB}})/\phi_T}$

$$Q_I' = \frac{-\sqrt{2q\epsilon_s N_A}}{2\sqrt{\psi_{sa}(V_{GB})}}\phi_T e^{(\psi_{sa}(V_{GB})-2\phi_F)/\phi_T} e^{-(\boldsymbol{V_{CB}})/\phi_T}$$

Weak Inversion…

- For a fixed $\boldsymbol{V_{CB}}$, Q_I' turns out to be nearly exponentially dependent on $\boldsymbol{V_{GB}}$.
- Let V_{CB}' denote the constant value of $\boldsymbol{V_{CB}}$ for which this curve is obtained.
- As seen, the width of the region in terms of ψ_{sa} is only ϕ_F.
- As $\boldsymbol{V_{GB}}$ changes over the region, the corresponding variation of the term $\sqrt{\psi_{sa}(V_{GB})}$, is very small compared with the large variation of the exponential in that equation. Therefore:

$$\sqrt{\psi_{sa}(V_{GB})} \approx \sqrt{2\phi_F + V_{CB}'}$$



Weak Inversion…

- Thus:

$$Q_I' = \frac{-\sqrt{2q\epsilon_s N_A}}{2\sqrt{2\phi_F + V_{CB}'}} \phi_T e^{(\psi_{sa} - 2\phi_F)/\phi_T} e^{-(V_{CB}')/\phi_T}$$

- In analogy with the corresponding development for the two-terminal structure, a simplified, approximate expression can be developed based on the observation that the slope of $\psi_{sa}(V_{GB})$ is nearly constant in weak inversion.

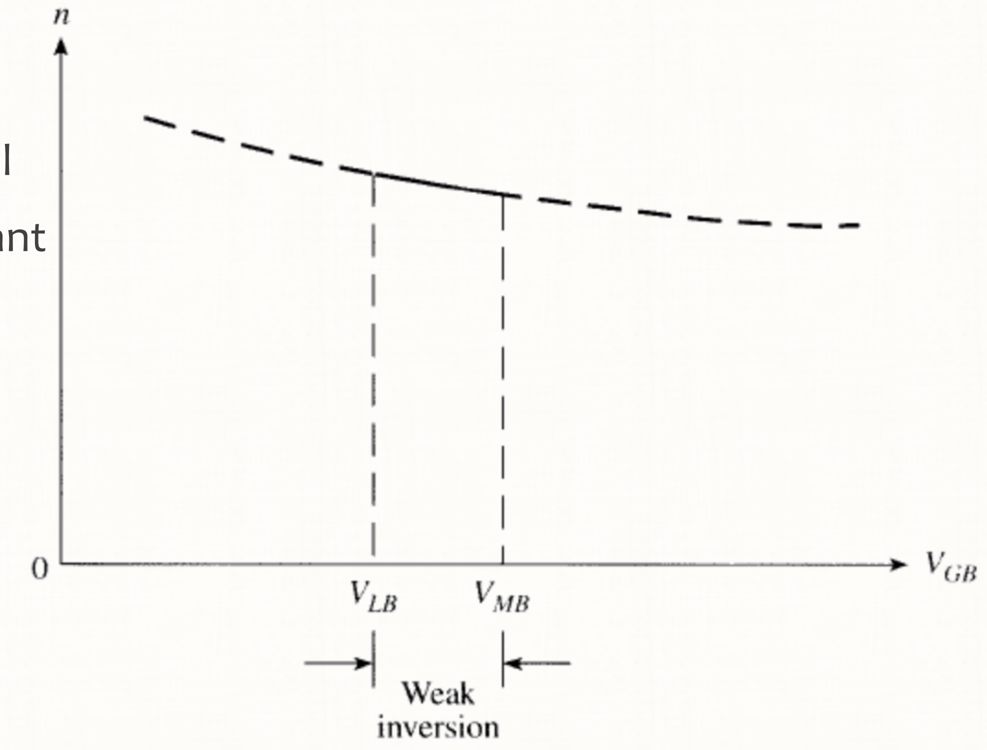
- hat the *inverse* of this slope was denoted by *n*

$$n = \left(\frac{d\psi_{sa}}{dV_{GB}}\right)^{-1} = 1 + \frac{\gamma}{2\sqrt{\psi_{sa}(V_{GB})}}$$

$$n = n\Big|_{\psi_{sa}=2\phi_F+V_{CB}'} = 1 + \frac{\gamma}{2\sqrt{2\phi_F + V_{CB}'}}$$

Then:

$$\psi_{sa} - (2\phi_F + V_{CB}') \approx \frac{1}{n}(V_{GB} - V_{MB}) = \frac{1}{n}(V_{GC} - V_M)$$



Weak Inversion…

- Substituting to previous equations:

$$Q_I' \approx Q_M' e^{(V_{GB}-V_{MB})/n\phi_T}$$

or

$$Q_I' \approx Q_M' e^{(V_{GC}-V_M)/n\phi_T}$$

Where

$$Q_M' = \frac{-\sqrt{2q\epsilon_s N_A}}{2\sqrt{2\phi_F + V_{CB}'}}\phi_T$$

- This result represents the value of Q_I' at the top of weak inversion ($V_{GB} = V_{MB}$).
- These equations predicts exponential behavior for Q_I' in weak inversion.
- As was the case with the two-terminal structure, this is only approximately true.
- If $\boldsymbol{V_{GB}}$ *is fixed and* $\boldsymbol{V_{CB}}$ *is varied instead above equations can be misleading*. The reason is that the dependence of Q_I' on $\boldsymbol{V_{CB}}$ is hidden in Q_M', V_{MB} and n, each of which depends on $\boldsymbol{V_{CB}}$ in a complicated manner.
- In this case previous Eq. is ideal for such cases, since it makes explicit the exponential dependence of Q_I' on $\boldsymbol{V_{CB}}$ in a simple manner.

Moderate Inversion

- For a given $\boldsymbol{V_{GB}}$ value in moderate inversion, one can numerically solve the implicit equation for ψ_s and substitute ψ_s into equation to find Q_I'.