Strong Inversion

Strong inversion

- A transistor is said to operate in strong inversion if at least one of the two channel ends is strongly inverted.
- Several models have been developed for the strong-inversion region.
- **Complete Strong-Inversion Model**
  - **Nonsaturation:** With strong inversion guaranteed at the source end of the channel, if $V_{DB} = V_{SB}$ $(V_{DS} = 0)$, the drain end will also be strongly inverted.
  - If now the drain potential is raised, the level of inversion there will decrease and, eventually, strong inversion at that point will cease.
  - For the present, we assume that the drain potential is sufficiently low so that this does not occur.
  - With both channel ends strongly inverted , like before again we have:

$$\psi_{s0} \approx \phi_0 + V_{SB}$$
$$\psi_{sL} \approx \phi_0 + V_{DB}$$

Where

$$\phi_0 = 2\phi_F + \Delta\phi = 2\phi_F + 6\phi_t$$

Complete Strong-Inversion Model

- Strong inversion at both ends ensures strong inversion throughout the channel since the surface potential varies monotonically from ψ_{s0} at the source to ψ_{sL} at the drain (figure).
- As established in Sec. 4.3, in strong inversion, the current is almost totally due to drift.
- By substituting results to past equation for all region model we have:

$$\boldsymbol{I_{DS}} = \frac{W}{L}\mu C'_{ox}\left[(V_{GB} - V_{FB})(\psi_{SL} - \psi_{S0}) - \frac{1}{2}(\psi_{SL}^2 - \psi_{S0}^2) - \frac{2}{3}\gamma\left(\psi_{SL}^{3/2} - \psi_{S0}^{3/2}\right)\right]$$

$$= \frac{W}{L}\mu C'_{ox}\left[(V_{GB} - V_{FB})(V_{DB} - V_{SB}) - \frac{1}{2}[(\phi_0 + V_{DB})^2 - (\phi_0 + V_{SB})^2] - \frac{2}{3}\gamma\left((\phi_0 + V_{DB})^{3/2} - (\phi_0 + V_{SB})^{3/2}\right)\right]$$

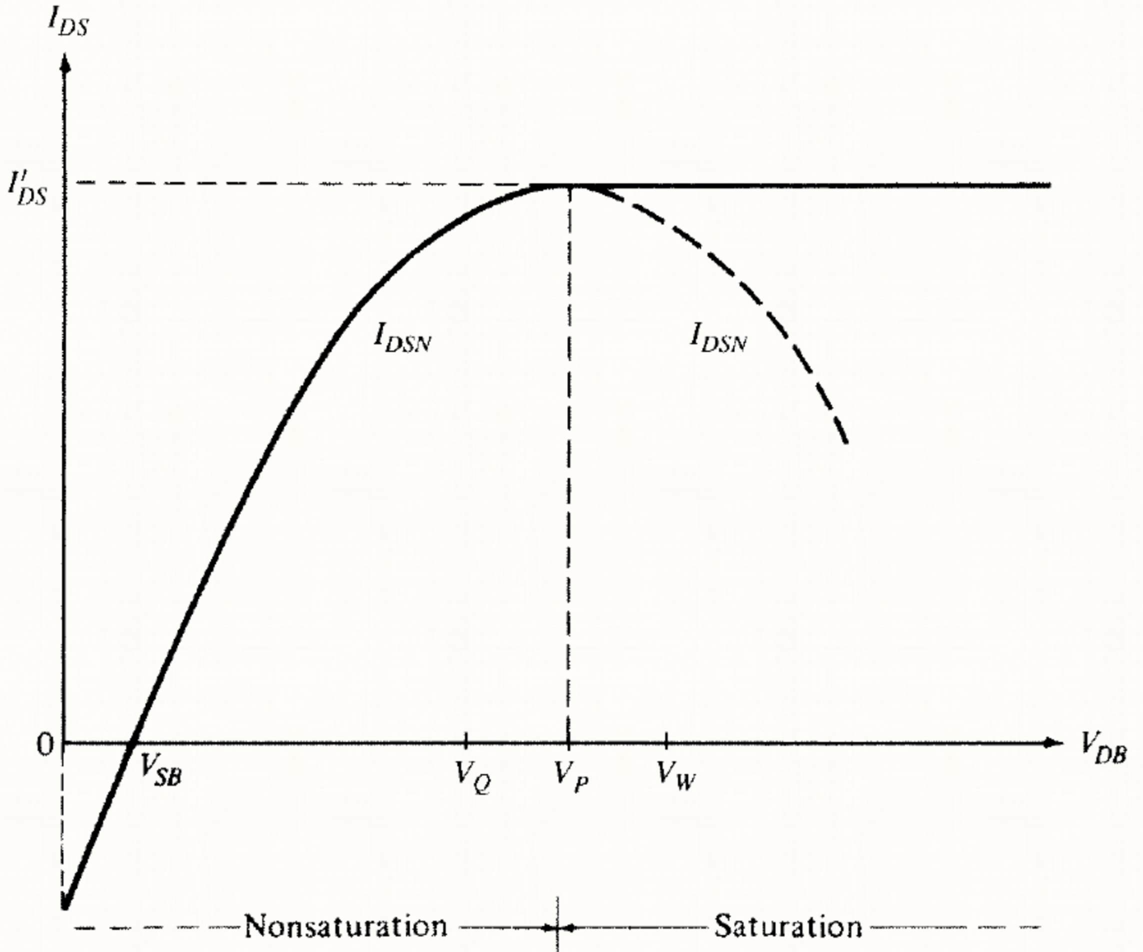
or, after some manipulations:

$$\boldsymbol{I_{DS}} = \frac{W}{L}\mu C'_{ox}\left[(V_{GB} - V_{FB} - \phi_0)(V_{DB} - V_{SB}) - \frac{1}{2}[(V_{DB})^2 - (V_{SB})^2] - \frac{2}{3}\gamma\left((\phi_0 + V_{DB})^{3/2} - (\phi_0 + V_{SB})^{3/2}\right)\right]$$

- These equations are the basis of the "level 2" model in the Berkeley SPICE simulator.

Complete Strong-Inversion Model

- I_{DSN} computed from last equation, and extension of its maximum value to the range $V_{DB} > V_P$.
- **Forward Saturation**
  - Assume now that $V_{DB} > V_{SB}$ and consider increasing values of V_{DB} on the horizontal axis in Fig.
  - Strictly speaking, last equation is valid only for $V_{DB} < V_Q$.
  - Above V_Q, the channel is not in strong inversion near the drain, and thus that equation is not valid.
  - Sometimes, nevertheless, last equation is used up to the point where the slope of the curve becomes zero; the resulting error may be tolerable in some applications.



Complete Strong-Inversion Model

- Finding V_P

$$\frac{dI_{DS}}{dV_{DB}} = 0 \rightarrow V_p = \left(-\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + \boldsymbol{V_{GB}} - V_{FB}}\right)^2 - \phi_F$$

$$I'_{DS} = I_{DSN}\Big|_{V_{DB}=V_P}$$

- According to the derivation and discussion there, it is easy to show that V_p is simply the value of V_{DB} that makes the gate-substrate threshold voltage at the *drain* end of the channel.
- It is clear that the value of V_P depends on $\boldsymbol{V_{GB}}$ but has *nothing* to do with V_{SB} as confirmed above.
- The value of V_P is close to the quantity V_W.

Complete Strong-Inversion Model

- When V_{DB} reach V_P consequently $Q_I' = 0$. In this situation past equation for drain current is not valid.
- Note that I_{DSN} should *not* be used for $V_{DB} > V_P$, because then a completely meaningless behavior is obtained, as shown by the broken curve in the figure.
- As mentioned before for $V_{DB} > V_P$ drift and diffusion currents remain constant and therefore for this case:

$$I_{DS}' = I_{DSN}\Big|_{V_{DB}=V_P} \quad for\ V_{DB} > V_P\ saturation\ region$$

- It should be noted although we assumed near the drain (because of increasing of depletion region) there is no electron but physically this not possible as the carriers would have to travel with infinite drift velocity in order for a nonzero current to be possible (eq). Therefore there are some finite carrier still in depletion region.
- Furthermore we neglect the channel length shrinkage due to V_{DB} which should have pronounced effect in short channel devices.

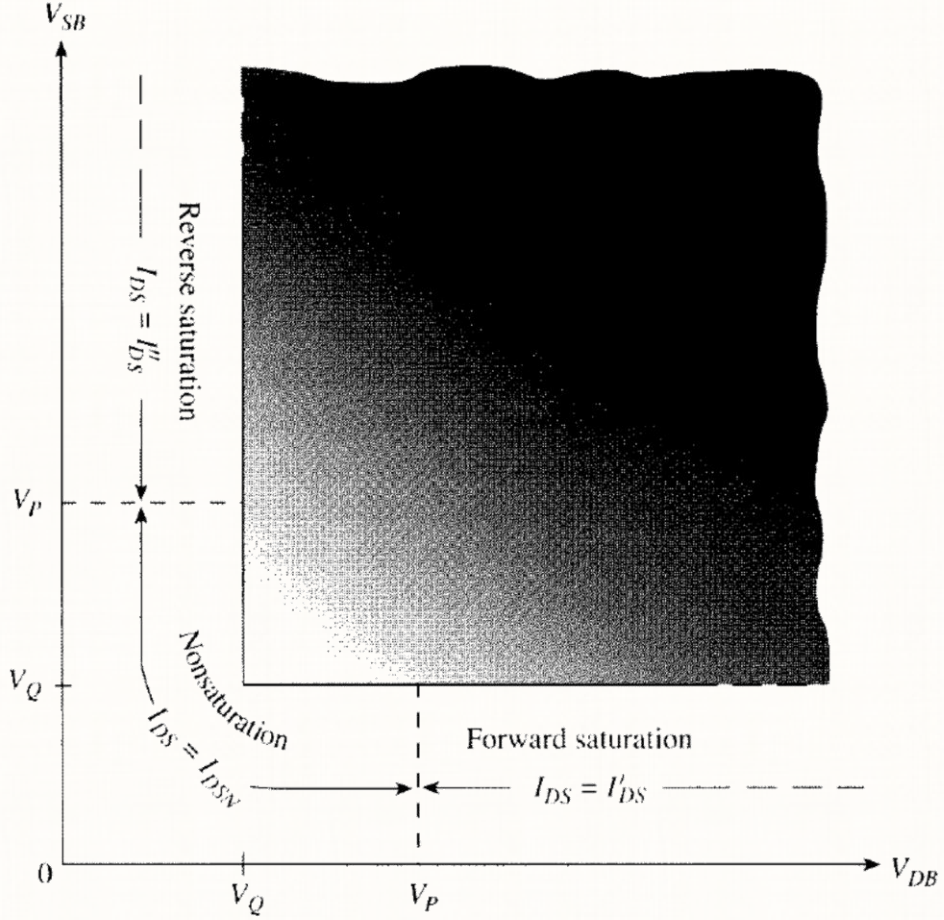
Complete Strong-Inversion Model

- Reverse Saturation:
  - If, instead, $V_{DB} < V_{SB}$ and V_{SB} is raised to V_P and beyond, the phenomena discussed previously will take place at the source rather than the drain.
  - In that case, what we have said previously will apply to the current entering the source terminal with the role of V_{DB} played by V_{SB}, and vice versa.
  - The reverse saturation current, in analogy with the previous results, will be given by:

$$I''_{DS} = I_{DSN}\Big|_{V_{SB}=V_P}$$

Complete Strong-Inversion Model

- Complete strong-inversion model. The quantities V_P and V_Q depend on V_{GS}.



Body-Referenced Simplified Strong-Inversion Model

- We can develop a strong-inversion model corresponding to the symmetric linearization model.
- We assume again that the diffusion current is negligible in strong inversion.
- Based our past equation we have:

$$\boldsymbol{I_{DSN}} = \frac{W}{L}\mu C'_{ox}\left[\left(V_{GB} - V_{FB} - \phi_0 - \frac{V_{SB} + V_{DB}}{2} - \gamma\sqrt{\phi_0 + \frac{V_{SB} + V_{DB}}{2}}\right)\right](V_{DB} - V_{SB})$$

- This model produces current values almost identical to past equation but is computationally simpler.

Source-Referenced Simplified Strong-Inversion Model

- If we use $\psi_{se} = \psi_{s0} = \phi_0 + V_{SB}$, and substituting to our general equation:

$$\boldsymbol{I_{DSN}} = \frac{W}{L}\mu C'_{ox}\left[\left(V_{GB} - V_{SB} - V_{FB} - \phi_0 - \gamma\sqrt{\phi_0 + V_{SB}}\right)(V_{DB} - V_{SB}) - \frac{\alpha}{2}(V_{DB} - V_{SB})^2\right]$$

Substituting following expressions in above equation:

$$V_{GS} = V_{GB} - V_{SB}$$
$$V_{DS} = V_{DB} - V_{SB}$$

and extrapolated threshold voltage

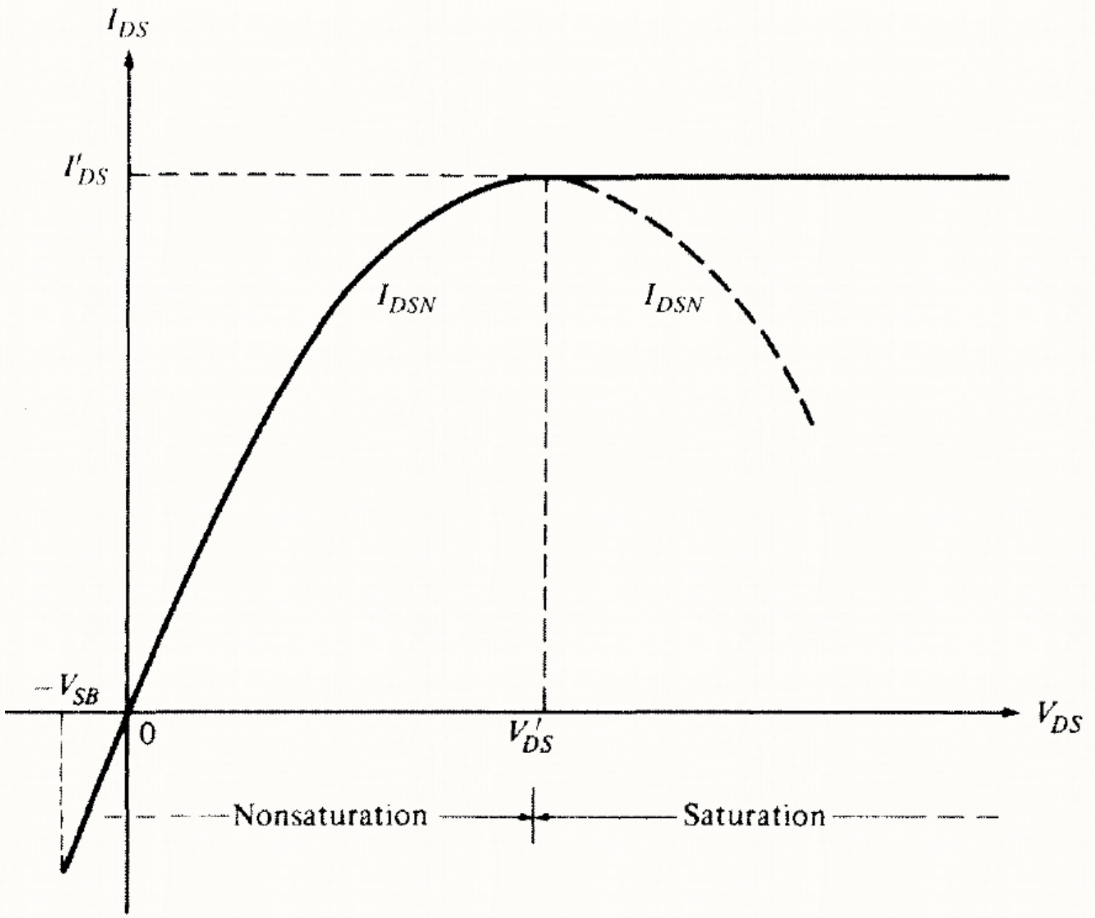
$$V_T = V_{FB} + \phi_0 + \gamma\sqrt{\phi_0 + \boldsymbol{V_{SB}}}$$

$$\boldsymbol{I_{DSN}} = \frac{W}{L}\mu C'_{ox}\left[\left(V_{GS} - V_T\Big|_{V_{SB}}\right)V_{DS} - \frac{\alpha}{2}V_{DS}^2\right]$$

Source-Referenced Simplified …

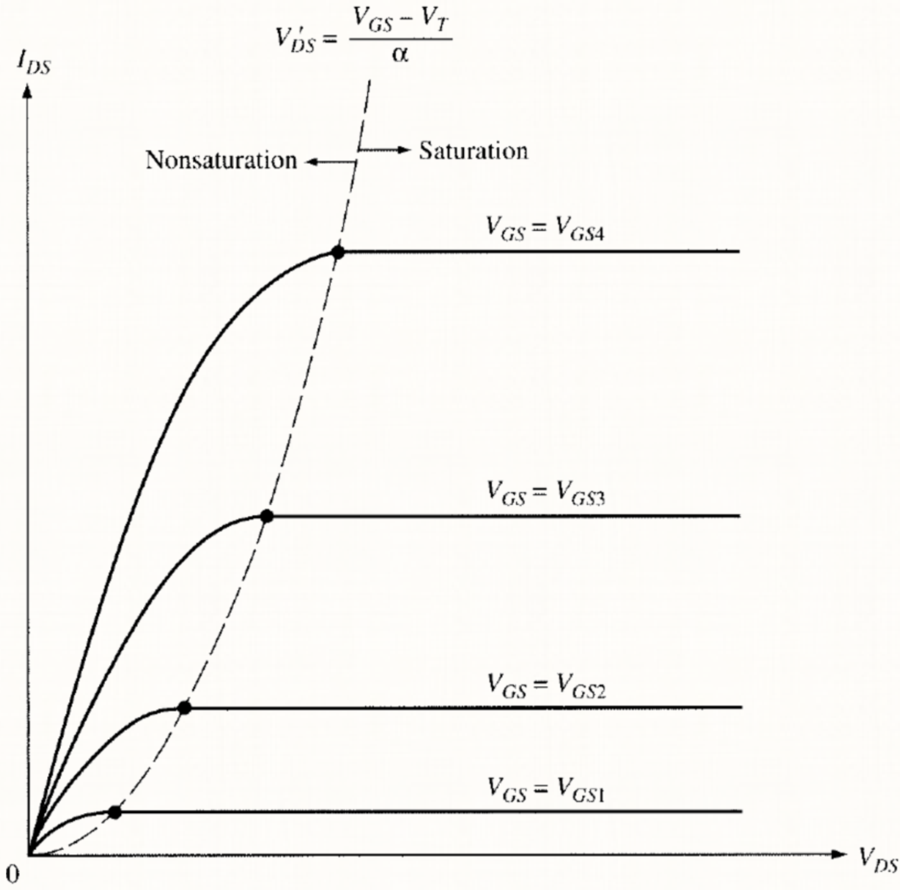
- The value of V_{SB} is normally zero or positive.
- Sometimes, in low-voltage circuits work, somewhat negative values for V_{SB} are used to reduce the value of V_T but it is made sure that V_{SB} is not too negative, so that no significant forward-bias current flows in the source-body junction. (Equation valid).
- For $\frac{dI_{DSN}}{dV_{DS}} = 0 \ \rightarrow \ V'_{DS} = \frac{V_{GS}-V_T}{\alpha}$
- The corresponding value of the drain current I'_{DS} is found by using $V_{DS}= V'_{DS}$, so we have:

$$I'_{DS} = \frac{W}{L}\mu C'_{ox}\frac{V_{GS}-V_T}{2\alpha} \text{ for } V_{DS} \geq V'_{DS}$$



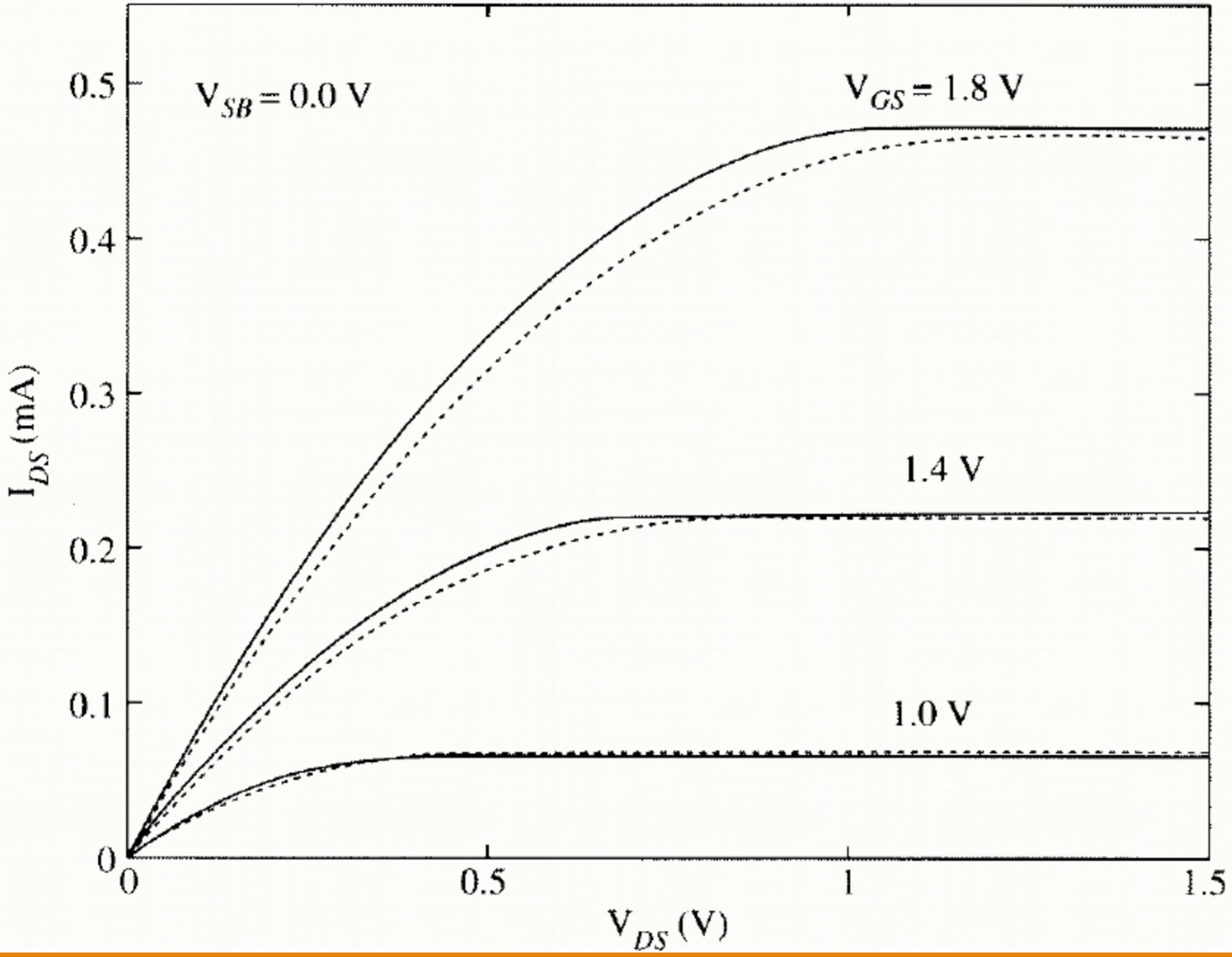
Source-Referenced Simplified …

- A set of I_{DS} - V_{DS} characteristics for various V_{GS} values is shown in Fig.



Source-Referenced Simplified …

- A comparison of the source-referenced strong-inversion model to the symmetric linearization all-region model is shown in Fig.
- The above model24 is the basis of the "level 3" model in the Berkeley SPICE simulator.



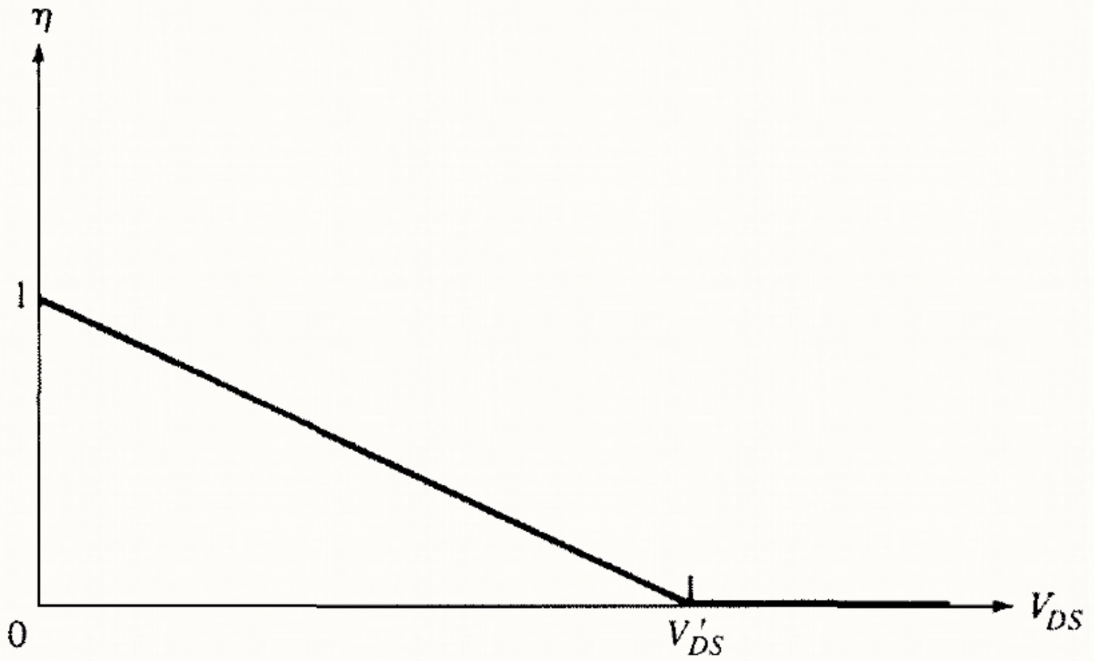
Source-Referenced Simplified …

- Last equation can be put in a very compact form by defining a convenient quantity η as follows:

$$\eta = \begin{cases} 1 - \dfrac{V_{DS}}{V'_{DS}}, & V_{DS} \leq V'_{DS} \\ 0, & V_{DS} > V'_{DS} \end{cases}$$

- The drain current can now be expressed as follows:

$$I_{DS} = I'_{DS}(1-\eta), \textit{both nonsaturation and saturation}$$

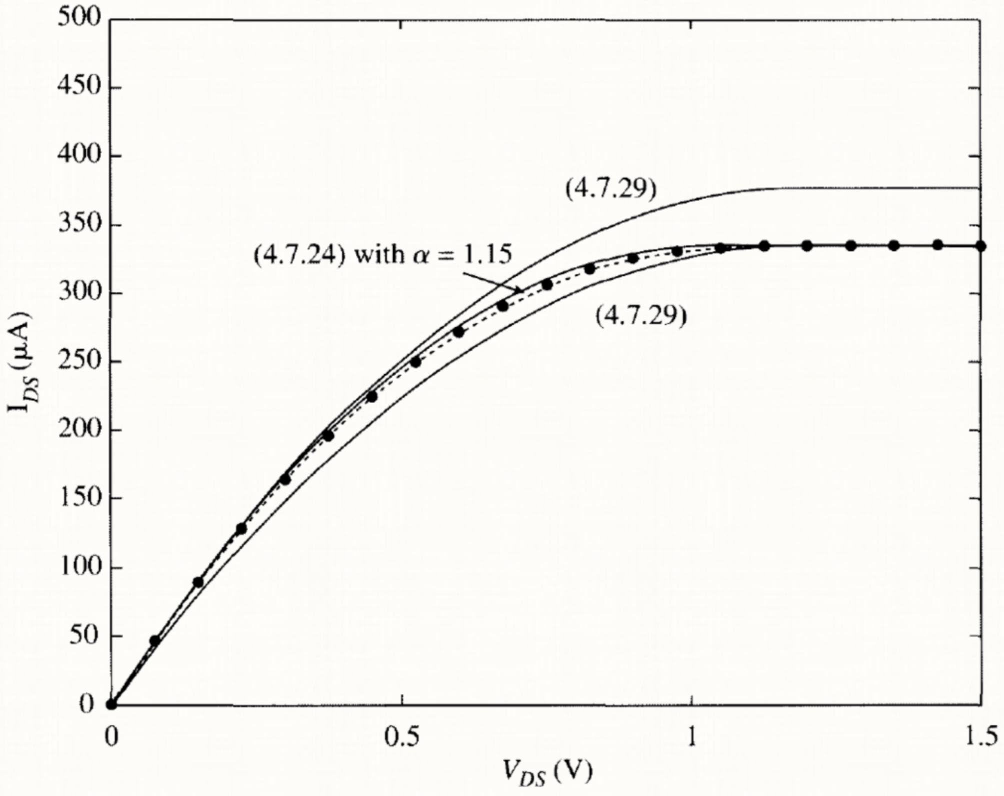


Source-Referenced Simplified …

- Now return to the problem of choosing an appropriate value for α.
- The derivation of MOSFET characteristics in the early days implicitly assumed α=1.
- This corresponds to line c in Fig., which is a very poor approximation to the solid line in the same figure. This approximation is equivalent to assuming that the depletion region depth is the same all along the channel and equal to its actual value at the source.

$$\alpha_I = 1 + \frac{\gamma}{2\sqrt{\phi_0 + V_{SB}}}$$

- Only for small V_{DS} , α_I is good approximation for α. Therefore a good choice for α is it should be $1 \le \alpha \le \alpha_I$



Source-Referenced Simplified …

- Such modifications are of the following general form:

$$\alpha = 1 + d_2 \frac{\gamma}{2\sqrt{\phi_0 + V_{SB} + \phi_3}}$$

- Where various values for the quantities d_2 and ϕ_3 are used.
  - Such as $0.5 \leq d_2 \leq 0.8$ and $\phi_3 = 0$.
- **Issues with a Popular Model for Circuit Design:**
- When γ is *small*, the value of α is close to 1, as seen from the previous equations. In this case:

$$I_{DS} = \begin{cases} \frac{W}{L}\mu C'_{ox}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right], & V_{DS} \leq V'_{DS}, small\ \gamma \\ \frac{W}{L}\mu C'_{ox}\frac{V_{GS} - V_T}{2}, & V_{DS} \geq V'_{DS}, small\ \gamma \end{cases}$$

Source-Referenced Simplified …

- These equations have been used widely for approximate hand calculations for circuit design, and even for quick computer calculations, and are the basis of the historical "level I" model in SPICE.
- The lower curve represents in fig. with the value for $\frac{W}{L}\mu C'_{ox}$ adjusted for good matching in saturation. The model clearly fails in nonsaturation.
- If $\frac{W}{L}\mu C'_{ox}$ is instead adjusted for a good fit in the low nonsaturation region, then the model is inaccurate in saturation, as shown by the upper curve.
- *The inadequacy of past equations is even more apparent in processes with large body effect.*
- If the model of source-referenced equation is used instead, we have the additional flexibility of choosing α; with $\alpha = 1.15$, we obtain the middle curve, which agrees well with the accurate calculations