Complete All-Region Model

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- **Current Equations (no simplification):**
  - A key to the generality of the results we are about to develop is the observation that the current in the channel can be caused by both drift and diffusion:

$$\boldsymbol{I(x)} = \boldsymbol{I_{drift}(x)} + \boldsymbol{I_{diff}(x)}$$

  - The total current in the channel must be the same for all x and equal to the drain-to-source current.
  - Drift current: Using the shown figure and previous expressions:

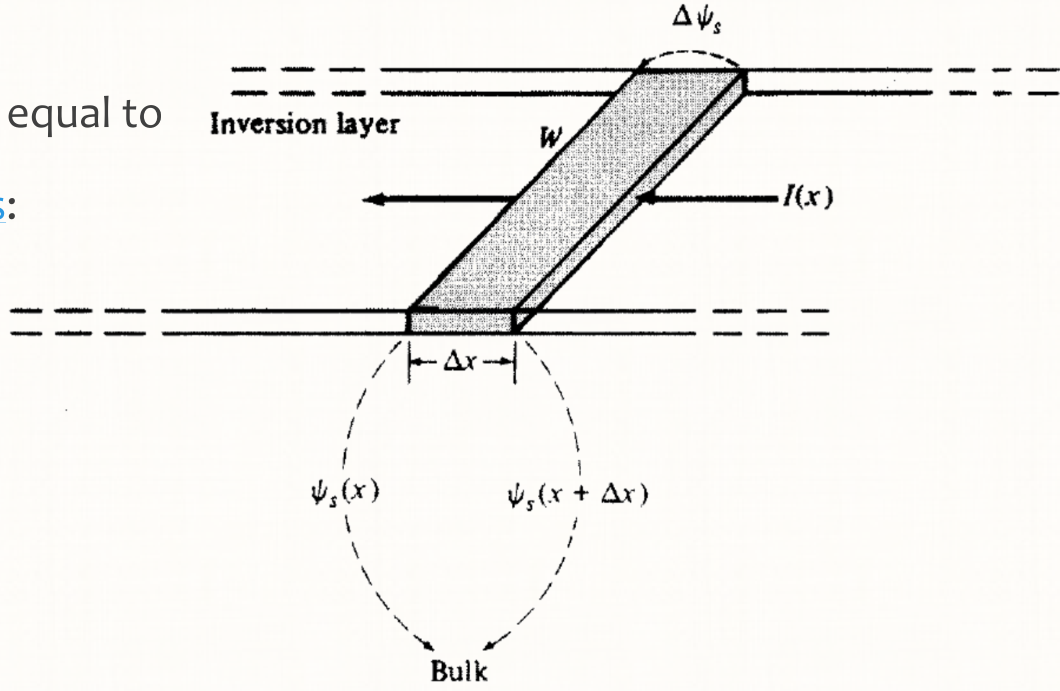
$$I_{drift}(x) = \mu(-Q_I')\frac{W}{\Delta x}\left(\psi_s(x+\Delta x) - \psi_s(x)\right)$$

  - If $\Delta x \to 0$

$$I_{drift}(x) = \mu W(-Q_I')\frac{d\psi_s(x)}{dx}$$

  - Diffusion current: using past expressions:

$$I_{diff}(x) = \mu W \phi_T \frac{dQ_I'}{dx}$$



Current Equations

- Total current:

$$I(x) = \mu W(-Q_I')\frac{d\psi_s(x)}{dx} + \mu W\phi_T\frac{dQ_I'}{dx}$$

- Parameters at the source end ($x = 0$): ψ_{s0} , Q_{I0}'
- Integrating the equation over the L , assuming *I* and μ are constant along the channel:

$$\int_0^L I_{DS}dx = \int_{\psi_{s0}}^{\psi_{sL}} \mu W(-Q_I')d\psi_s(x) + \int_{Q_{I0}'}^{Q_{IL}'} \mu W\phi_T dQ_I'$$

$$I_{DS} = \frac{W}{L}\mu\left(\int_{\psi_{s0}}^{\psi_{sL}} (-Q_I')d\psi_s(x) + \int_{Q_{I0}'}^{Q_{IL}'} \phi_T dQ_I'\right)$$

$$I_{DS} = I_{DS1} + I_{DS2}$$

$$I_{DS1} = \frac{W}{L}\mu\left(\int_{\psi_{s0}}^{\psi_{sL}} (-Q_I')d\psi_s(x)\right)$$

$$I_{DS2} = \frac{W}{L}\mu\left(\int_{Q_{I0}'}^{Q_{IL}'} \phi_T dQ_I'\right)$$

Current Equations …

- …

$$I_{DS2} = \frac{W}{L}\mu\left(\int_{Q'_{I0}}^{Q'_{IL}} \phi_T dQ'_I\right) = \frac{W}{L}\mu\phi_T(Q'_{IL} - Q'_{I0})$$

- Q'_I is a function of ψ_s:

$$Q'_I = -C'_{ox}(V_{GB} - V_{FB} - \psi_s) - Q'_B$$

$$I_{DS1} = \frac{W}{L}\mu\left(\int_{\psi_{s0}}^{\psi_{sL}} (-Q'_I) d\psi_S(x)\right)$$

$$= \frac{W}{L}\mu C'_{ox}\left[(V_{GB} - V_{FB})(\psi_{SL} - \psi_{S0}) - \frac{1}{2}(\psi_{SL}^2 - \psi_{S0}^2)\right] + \frac{W}{L}\mu\left(\int_{\psi_{s0}}^{\psi_{sL}} (Q'_B) d\psi_S(x)\right)$$

$$I_{DS2} = \frac{W}{L}\mu C'_{ox}\phi_T(\psi_{SL} - \psi_{S0}) - \frac{W}{L}\mu\phi_T(Q'_B(\psi_{SL}) - Q'_B\ (\psi_{S0}))$$

Current Equations …

- But we know:

$$Q'_B = -\sqrt{2q\epsilon_s N_A}\sqrt{\psi_S} = -\gamma C'_{ox}\sqrt{\psi_S}$$

- Substituting to prev. Eq.:

$$\boldsymbol{I_{DS1}} = \frac{W}{L}\mu C'_{ox}\left[(V_{GB} - V_{FB})(\psi_{SL} - \psi_{S0}) - \frac{1}{2}(\psi_{SL}^2 - \psi_{S0}^2) - \frac{2}{3}\gamma\left(\psi_{SL}^{3/2} - \psi_{S0}^{3/2}\right)\right]$$

$$\boldsymbol{I_{DS2}} = \frac{W}{L}\mu C'_{ox}\left[\phi_T(\psi_{SL} - \psi_{S0}) - \phi_T\gamma\left(\psi_{SL}^{1/2} - \psi_{S0}^{1/2}\right)\right]$$

- Now we have the current in term of surface potential ψ_S, therefore if we can evaluate it we can find current.

Evaluating ψ_{S0} and ψ_{SL}

- By substituting V_{CB} to V_{SB} and V_{DB} for each end of channel we have:

$$\psi_{S0} = V_{GB} - V_{FB} - \gamma\sqrt{\psi_{S0} + e^{-(2\phi_F+\boldsymbol{V_{SB}})/\phi_T}\left(\phi_T e^{\psi_{S0}/\phi_T}\right)}$$

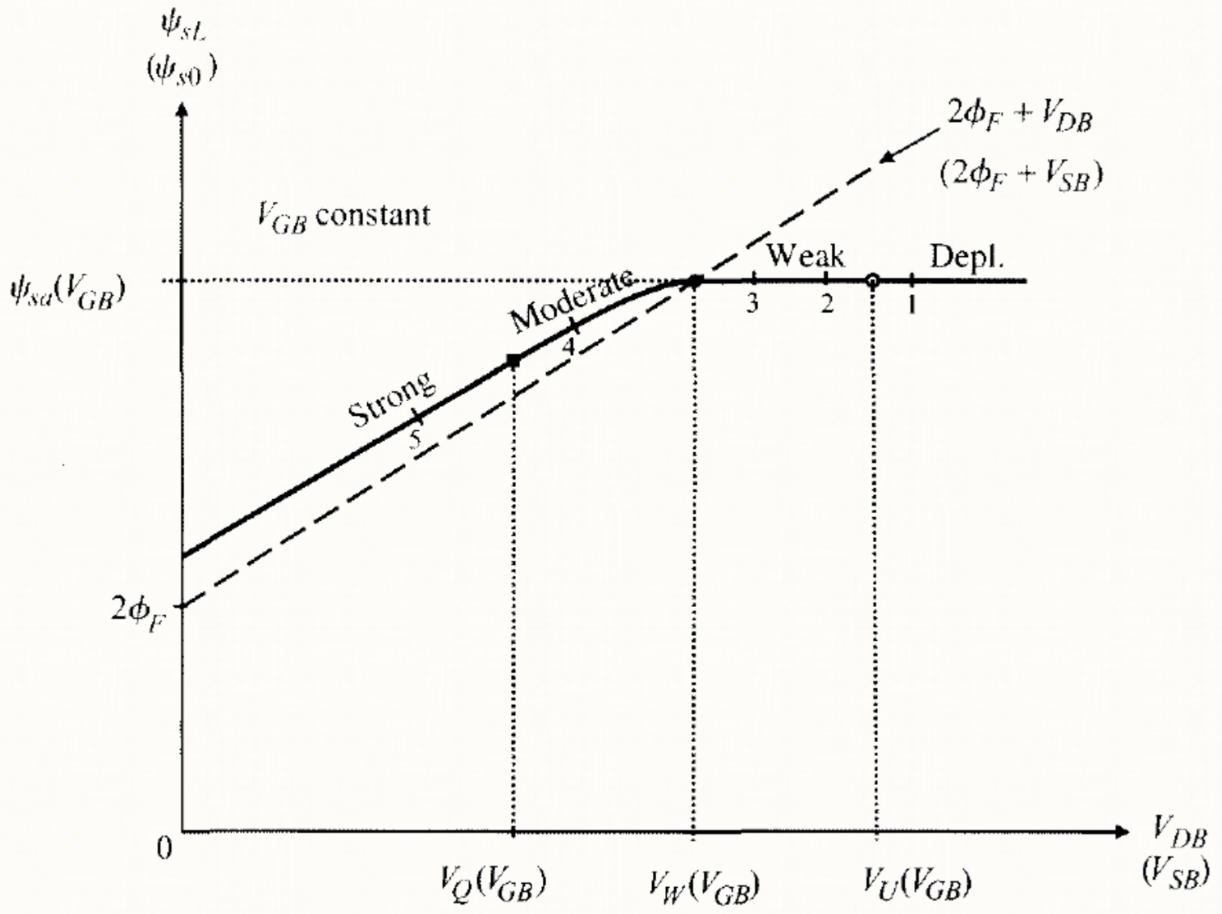
$$\psi_{SL} = V_{GB} - V_{FB} - \gamma\sqrt{\psi_{SL} + e^{-(2\phi_F+\boldsymbol{V_{DB}})/\phi_T}\left(\phi_T e^{\psi_{SL}/\phi_T}\right)}$$

- These equations can be solved for ψ_{S0} and ψ_{SL} by iteration. This can easily be done with a computer.

- Two techniques have emerged as the preferred approaches to compute ψ_S .
  - First- or second-order Newton-Raphson scheme : uses an iterative numerical procedure, in which a good initial estimate is made of ψ_S . Convergence to tight tolerances can be achieved in two to three iterations.
  - Analytical approximations: These start from very good approximations to ψ_S for specific regions of operation, and then use either smoothing functions or fixed refinement steps to produce highly accurate, explicit approximations to ψ_S with errors compared to accurate numerical solutions of less than 1 nV.

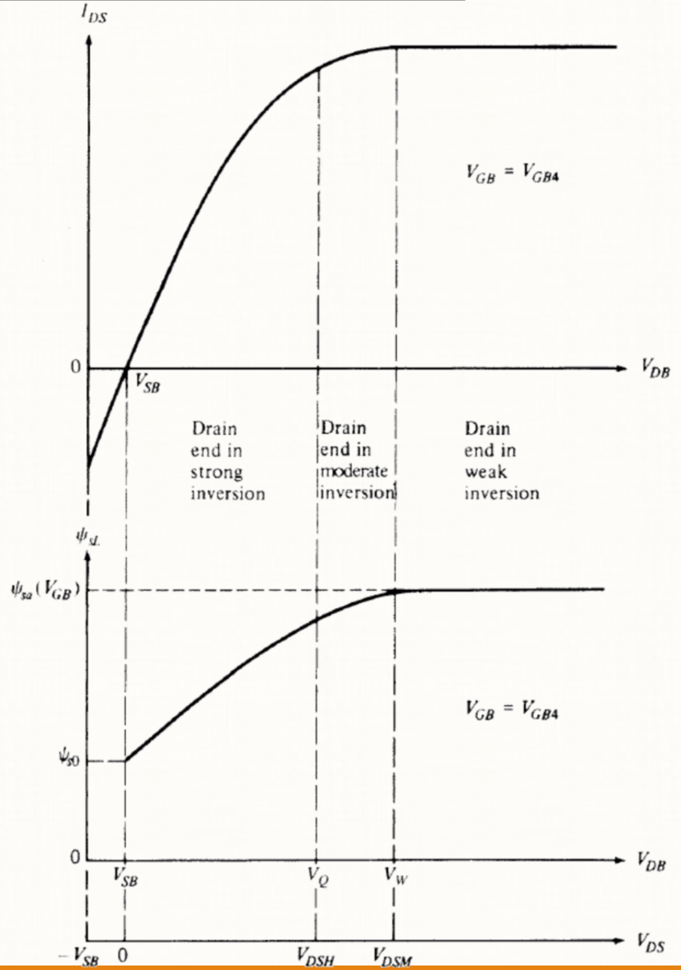
Evaluating ψ_{S0} and ψ_{SL}…

- A plot of $\boldsymbol{\psi_{SL}/\psi_{S0}}$vs. $\boldsymbol{V_{DB}/V_{SB}}$ is shown
- The plot saturates at a value dependent on V_{GB}, shown in Fig.
- Edge effect near drain and source are neglected here. (small dimension effect) .
- Now to plot fig. we raise the $\boldsymbol{V_{DB}}$ voltage (as a parameter) respect to $\boldsymbol{V_{SB}}$ (keep it fix).
- $\boldsymbol{\psi_{SL}}$ and $\boldsymbol{\psi_{S0}}$ can be calculated from previous equations respect to $\boldsymbol{V_{DB}}$ and $\boldsymbol{V_{SB}}$.
- Now with current equations we can find $\boldsymbol{I_{DS1}}$ and $\boldsymbol{I_{DS2}}$. Finally $\boldsymbol{I_{DS} = I_{DS1} + I_{DS2}}$
- Note: described equations predicts the current in all these regions. Experiments agree well with this expression



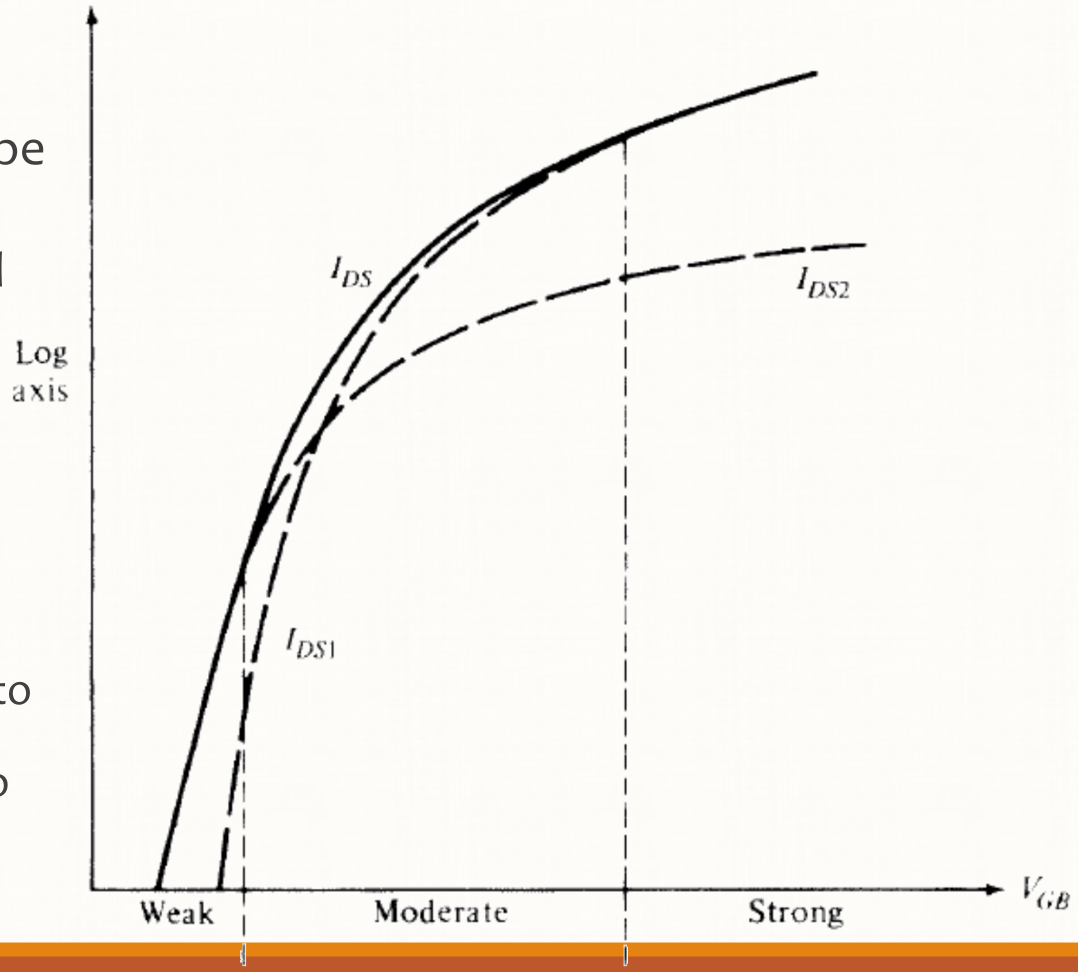
Evaluating ψ_{S0} and ψ_{SL}…

- Saturation always take place for large $\boldsymbol{V_{DB}}$.
- For following figure saturation effect is shown for specific $V_{GB}(=V_{GB4})$.
- Drain end of the channel:
  - Increasing $\boldsymbol{V_{DB}}$ eventually drives the drain end of the channel into weak inversion and then depletion, where $\boldsymbol{\psi_{SL}}$ becomes practically constant at a value that depends only on $V_{GB}(=V_{GB4})$.
  - Increasing $\boldsymbol{V_{DB}}$ further has little effect on $\boldsymbol{\psi_{SL}}$ *and* $|Q_I'|$ *becomes very small*. Thus $\boldsymbol{I_{DS1}}$ and $\boldsymbol{I_{DS2}}$ also become independent of $\boldsymbol{V_{DB}}$.
- Source end of the channel:
  - As the $\boldsymbol{V_{SB}}$ is constant and it selected so that the this part be at strong inversion therefore it is heavily inverted due to very large $|Q_I'|$.
  - $\boldsymbol{\psi_{S0}}$ is strongly dependent to $\boldsymbol{V_{SB}}$.
- The current at any point in the channel should be constant and it is not dependent only to $|Q_I'|$ as shown in previous equations. (drift and diffusion)



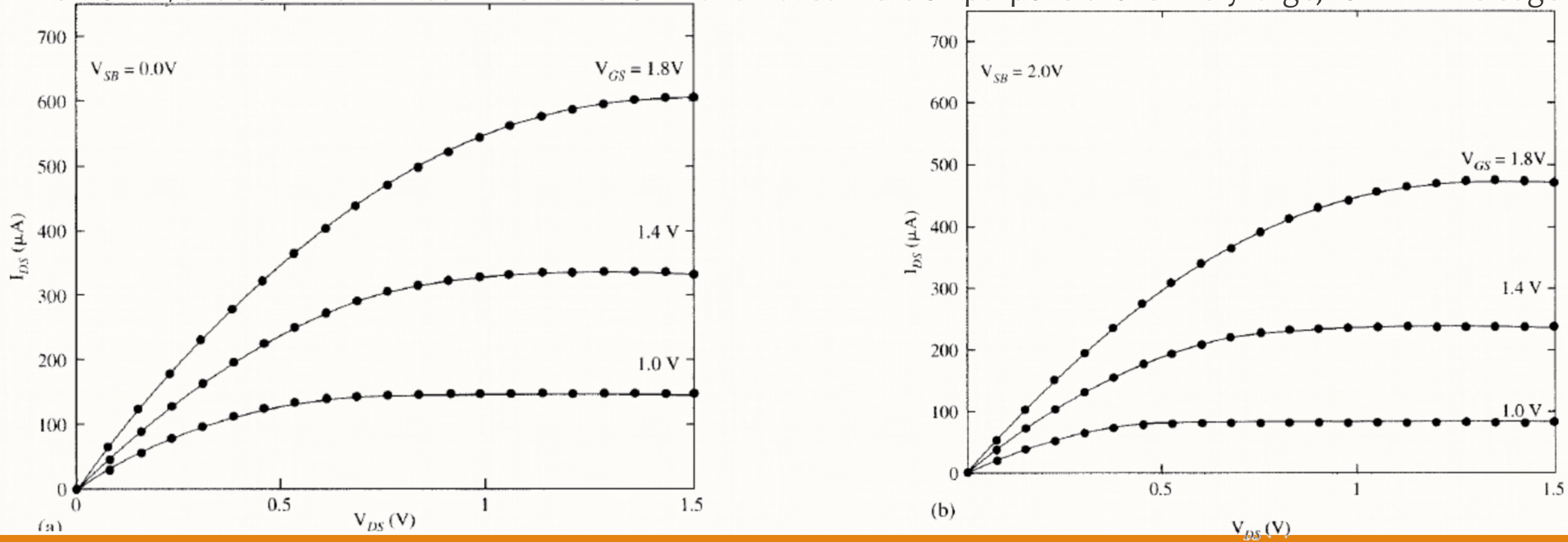
Evaluating ψ_{S0} and ψ_{SL}…

- In this figure consider that $\boldsymbol{V_{DB}}$ selected so that $\boldsymbol{I_{DS}}$ is in saturation region.
- By changing the V_{GB}, $\boldsymbol{I_{DS}}$ components $\boldsymbol{I_{DS1}}$ and $\boldsymbol{I_{DS2}}$ can be calculated.
- To include a large range of currents, a logarithmic vertical axis is used.
- Here $\boldsymbol{V_{DB}} > \boldsymbol{V_{SB}}$, so that the more heavily inverted channel end is the one next to the source.
- Everything below moderate inversion is called weak inversion.
- It is seen in Fig. that:
  - In **strong** inversion , $\boldsymbol{I_{DS}} \approx \boldsymbol{I_{DS1}}$, so the current is mainly due to the presence of drift.
  - In **weak** inversion , $\boldsymbol{I_{DS}} \approx \boldsymbol{I_{DS2}}$, so the current is mainly due to the presence of diffusion.
  - In **moderate** inversion *both* $\boldsymbol{I_{DS1}}$ and $\boldsymbol{I_{DS2}}$ are important.



Evaluating ψ_{S0} and ψ_{SL}…

- In following Fig., we show a comparison of the preceding model (lines) to full computer solution of the semiconductor equations (dots)·
- The simulation is a 2D numerical solution of the Poisson and drift-diffusion equations, allowing for the spreading of the inversion layer below the surface. The values of W and *L* used were on purpose chosen very large, to minimize edge effects.



A Graphical Interpretation

- In this figure, we show $-Q_I'$ vs. ψ_S, as obtained from below Eq., for a given V_{GB}.

$$Q_I' = -C_{ox}'\left(V_{GB} - V_{FB} - \psi_s - \gamma\sqrt{\psi_S}\right)$$

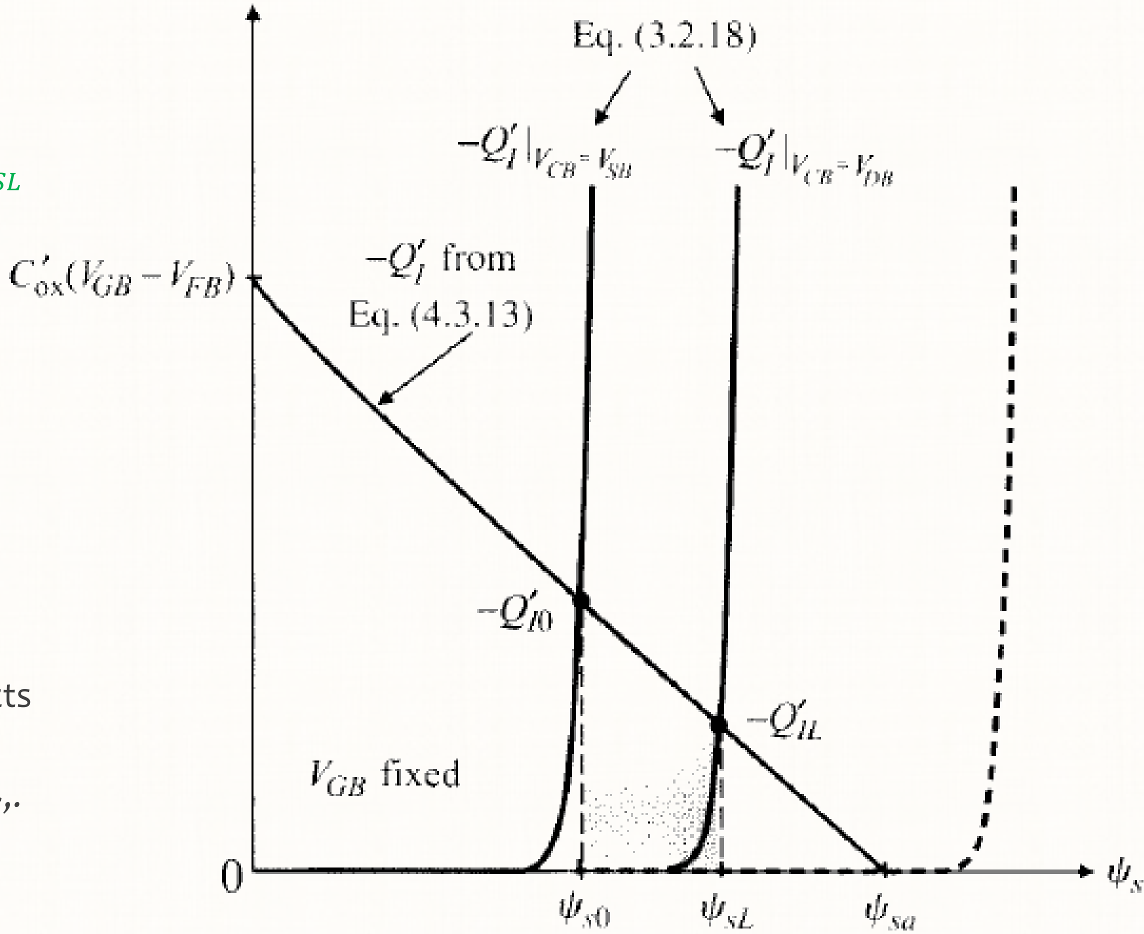
- The only part of this curve that is relevant to us is that between ψ_{S0} and ψ_{SL} the values of the surface potential at the source and the drain.
- This curve should also satisfy following Eq. (derived for 3 terminal device before) :

$$Q_I' = -\sqrt{2q\epsilon_s N_A}\left(\sqrt{\psi_s + \phi_T e^{(\psi_s-(2\phi_F+\mathbf{V_{CB}}))/\phi_T}} - \sqrt{\psi_s}\right)$$

- Q_{I0}' and Q_{IL}' corresponds to surface potential ψ_{S0} and ψ_{SL}.
- Shaded region is regarding to following integral:

$$\mathbf{I_{DS1}} = \frac{W}{L}\mu\left(\int_{\psi_{s0}}^{\psi_{sL}} (-Q_I')d\psi_S(x)\right)$$

- let us determine the point where the curve representing above Eq intersects the horizontal axis.
- This can be found by setting that equation equal to zero and solving for ψ_S,. The result is nothing but ψ_{Sa}. (first introduced in 2 terminal device)
- ψ_{Sa} is the potential which inversion layer is negligible.



A Graphical Interpretation…

- The graphical construction in previous Fig. facilitates a feel for the behavior of $\boldsymbol{I_{DS1}}$ and $\boldsymbol{I_{DS2}}$.
- The particular operating point assumed in previous Fig. happens to be in the strong inversion nonsaturation region. Why ?
  - When $\boldsymbol{V_{DB}}$ is raised a little, the corresponding $-Q_I'$; curve will move to the right in the figure; the value of ψ_{SL} will increase, and so will the currents in Eq. thus $\boldsymbol{I_{DS}} = \boldsymbol{I_{DS1}} + \boldsymbol{I_{DS2}}$ is increase.
- Eventually, as $\boldsymbol{V_{DB}}$ is raised further, the corresponding Q_I'; plot will move far to the right, as illustrated by the broken-line curve; ψ_{SL} will asymptotically reach the value ψ_{Sa} and will be unable to rise further.
- Q_{IL}' now becomes essentially zero, and the trapezoid becomes a triangle; $\boldsymbol{I_{DS1}}$ and $\boldsymbol{I_{DS2}}$ stay fixed as $\boldsymbol{V_{DB}}$ is raised further.
- We have now reached ***saturation***, and the current flattens out at its maximum value.

Symmetry

- It is clear from previous Eqs. that $\boldsymbol{I_{DS}}$ can be written in the form:

$$I_{DS} = \frac{W}{L}\left(f(\psi_{SL}) - f(\psi_{S0})\right)$$

$$f(\psi_S) = \mu C'_{ox}\left[(V_{GB} - V_{FB} + \phi_T)\psi_S + \phi_T\,\gamma\psi_S^{1/2} - \frac{1}{2}\psi_S^2 - \frac{2}{3}\gamma\psi_S^{3/2}\right]$$

- This Eqs. is in a form that emphasizes the symmetry of the transistor.
- If the potentials at the source and drain are interchanged, the only difference will be that $\boldsymbol{I_{DS}}$ will change sign.

Surface Potential, Charge, and Currents vs. Position

- For visualizing transistor operation and for calculating certain quantities, it will be useful to relate the surface potential ψ_S to the position along the channel x.

$$I_{DS} = \frac{W}{L} F(\psi_{SL}, \psi_{S0})$$

where F is an appropriate function.

- Furthermore as the current is independent of x therefore:

$$I_{DS} = \frac{W}{x} F(\psi_S(x), \psi_{S0})$$

- Eliminating I_{DS} between the preceding two equations we obtain:

$$\frac{x}{L} = \frac{F(\psi_S(x), \psi_{S0})}{F(\psi_{SL}, \psi_{S0})}$$

- This equation gives the relation between x and $\psi_S(x)$.

Surface Potential, Charge, …

- The easy way to get results from it is to give values $\psi_S(x)$ between ψ_{S0} and ψ_{SL} and determine x. *The results is shown in following figure.*
- Regions of inversions:
  - In **moderate inversion**, the variation of ψ_S with x for $\boldsymbol{V_{DB}} > \boldsymbol{V_{SB}}$ is less pronounced.
  - In **weak inversion**, the two curves would practically coincide because, in that region, the variations of the surface potential along the channel is negligible even when $\boldsymbol{V_{DB}} > \boldsymbol{V_{SB}}$.
- $|Q_I'|$ as predicted by the all-region model decreases monotonically along the channel as we go from the source toward the drain, although we can see it here as well. As the potential increase…
- One can similarly evaluate $\frac{d\psi_S(x)}{dx}$ and $\frac{dQ_I'}{dx}$ as a function of x.
- It is then found that $\boldsymbol{I_{drift}(x)}$ decreases with x, whereas $\boldsymbol{I_{diff}(x)}$ increases.