# STATICS

دانشگاه کردستان
University of Kurdistan
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- Vector Mechanics for Engineers: Statics, 9th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Statics, 5th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Statics"

## Forces in Beams and Cables

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# Forces in Beams and Cables

## ❑ Introduction

- Preceding chapters dealt with:
  a) determining external forces acting on a structure and
  b) determining forces which hold together the various members of a structure.

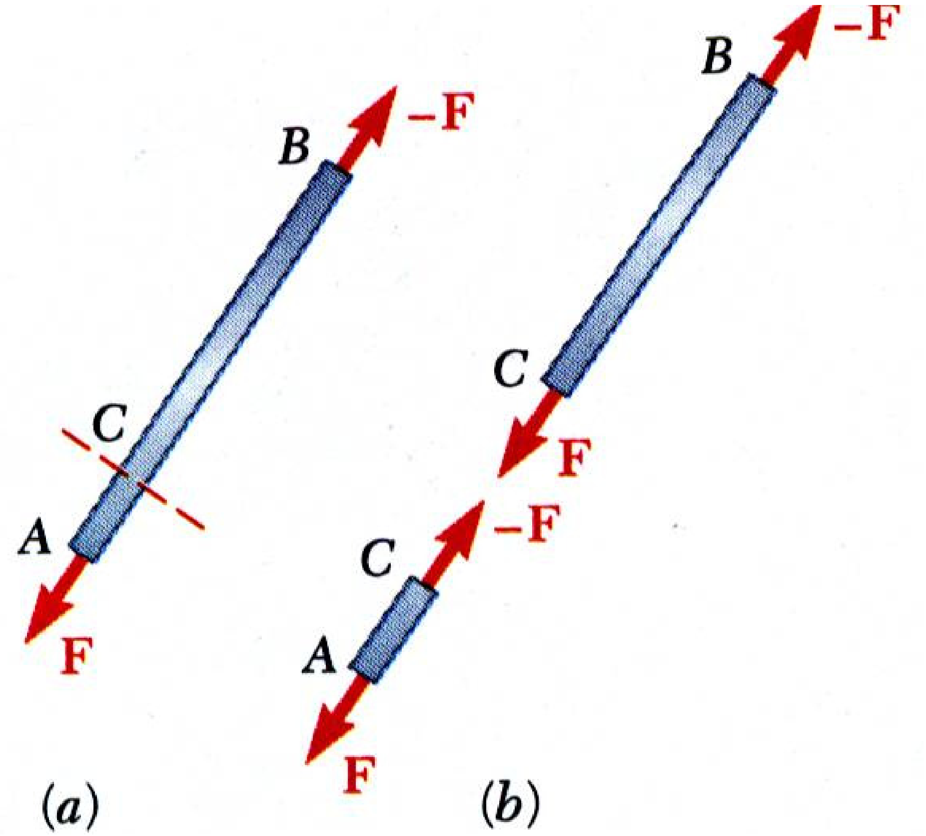
- The current chapter is concerned with determining the *internal forces* (i.e., ***tension/compression, shear, and bending***) which hold together the various parts of a given member.

- **Focus is on two important types of engineering structures:**
  ***a) Beams*** - usually long, straight, prismatic members designed to support loads applied at various points along the member.
  ***b) Cables*** - flexible members capable of withstanding only tension, designed to support concentrated or distributed loads.

# Forces in Beams and Cables

## ❑ Internal Forces in Members

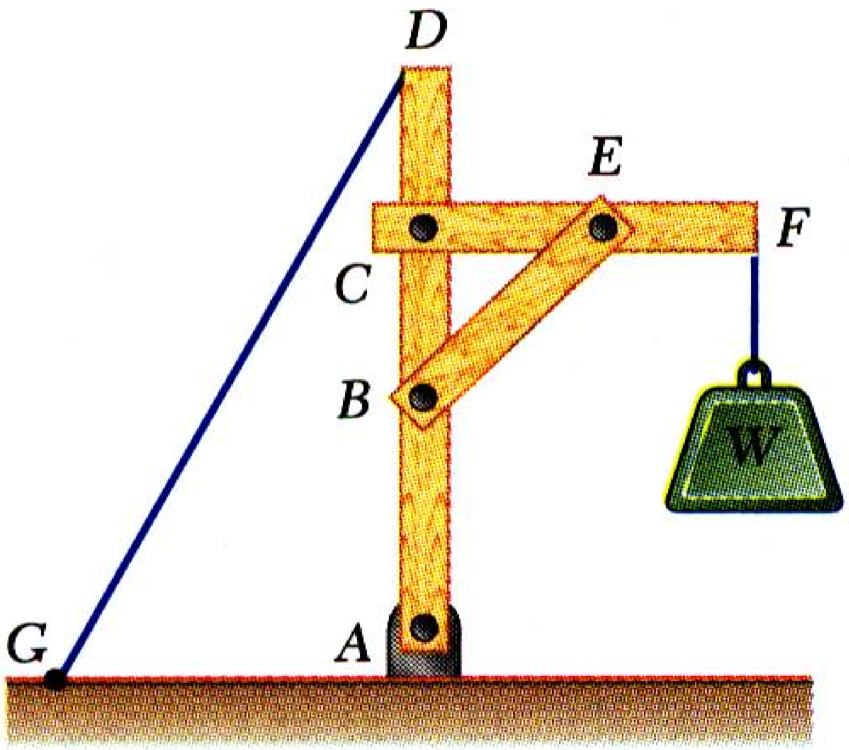
- Straight two-force member *AB* is in equilibrium under application of *F* and *-F*.
- *Internal forces* equivalent to *F* and *-F* are required for equilibrium of free-bodies *AC* and *CB*.

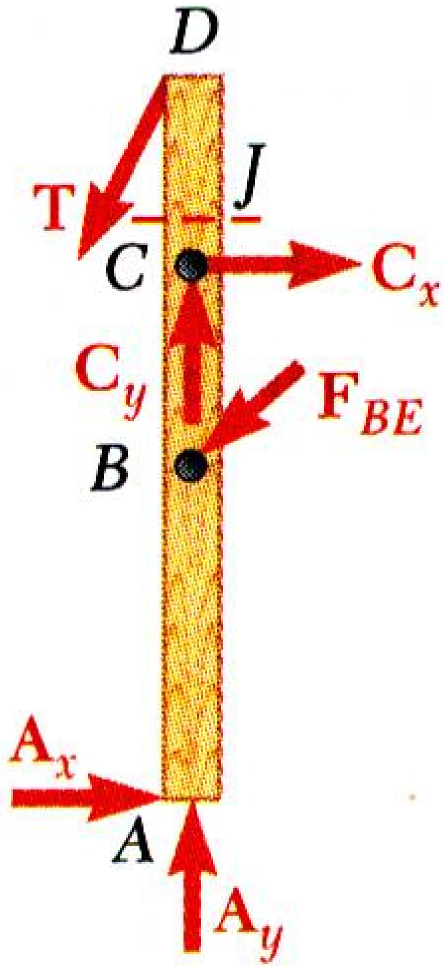


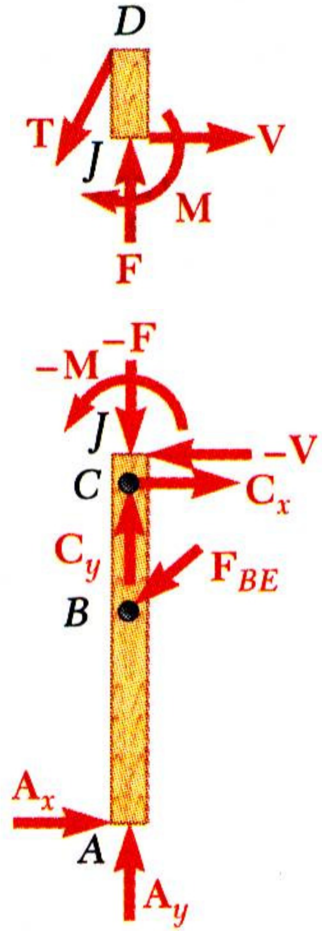
# Forces in Beams and Cables

## ❑ Internal Forces in Members

- Multiforce member *ABCD* is in equilibrium under application of cable and member contact forces.
- Internal forces equivalent to a force-couple system are necessary for equilibrium of free-bodies *JD* and *ABCJ*.



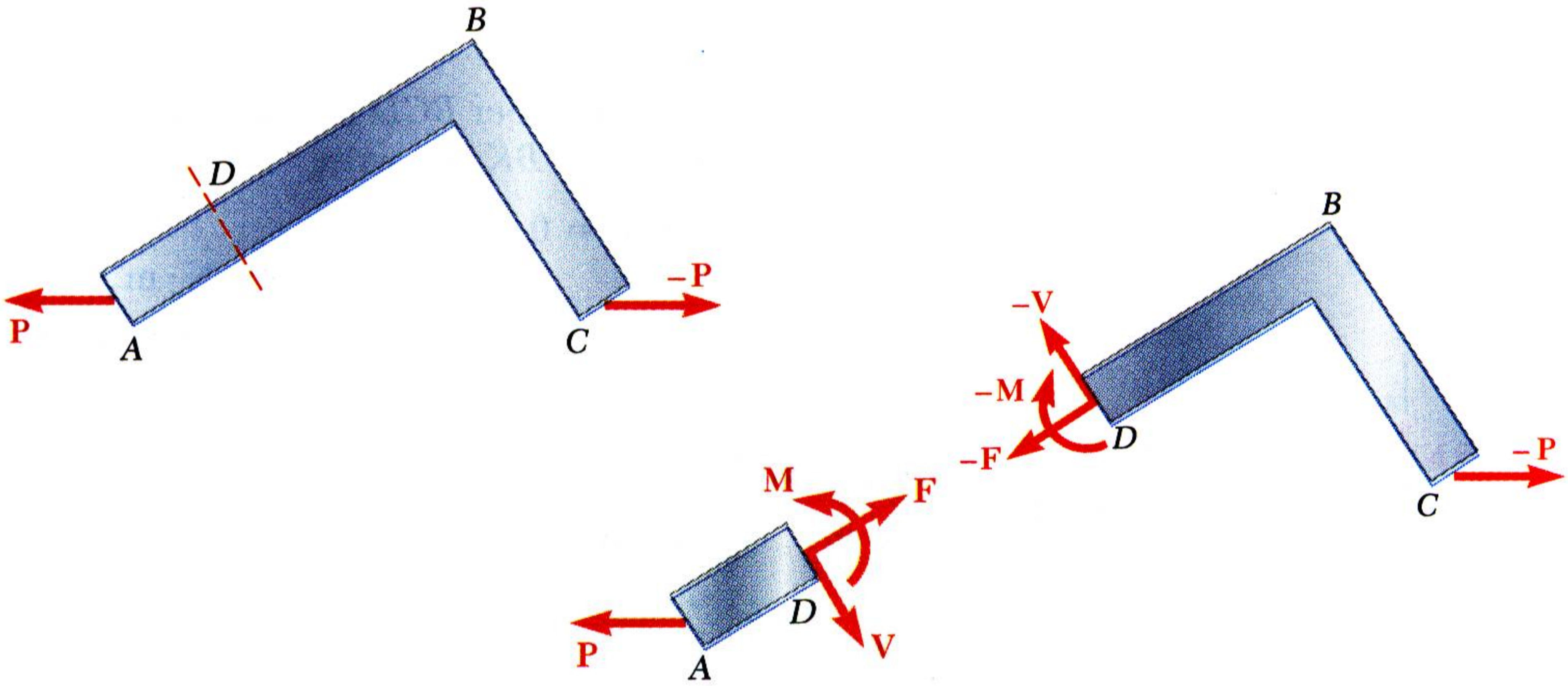



# Forces in Beams and Cables

## ❑ Internal Forces in Members

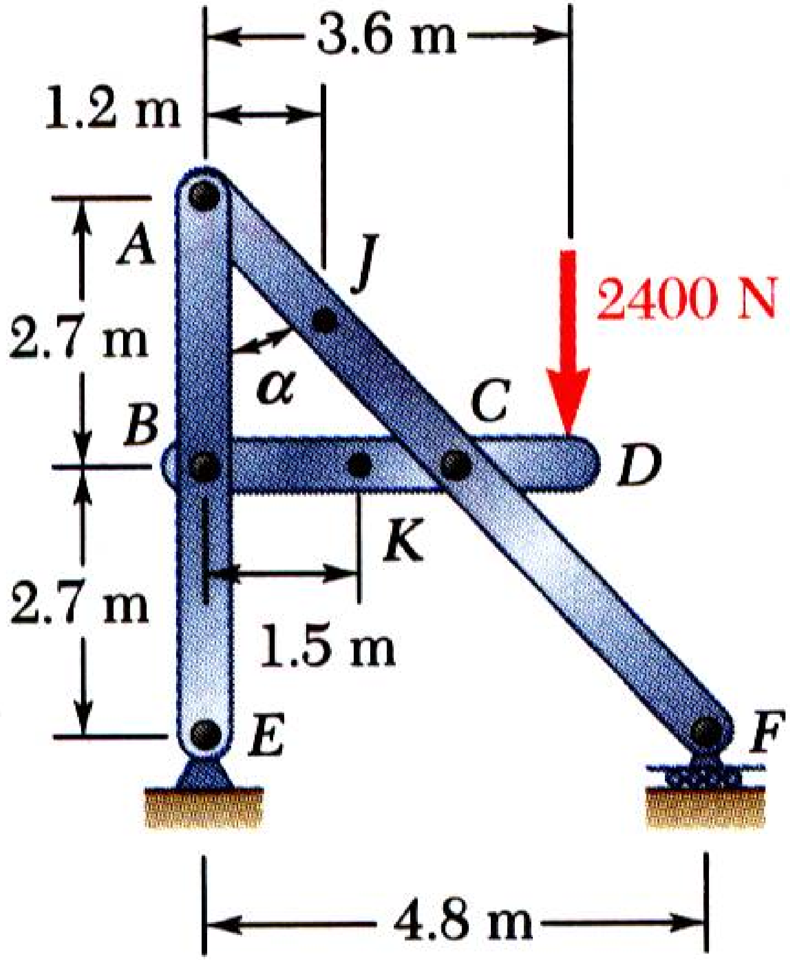
- An internal force-couple system is required for equilibrium of two-force members which ***are not straight***.



# Forces in Beams and Cables

## ❑ Sample Problem 01

Determine the internal forces *(a)* in member *ACF* at point *J* and *(b)* in member *BCD* at *K*.



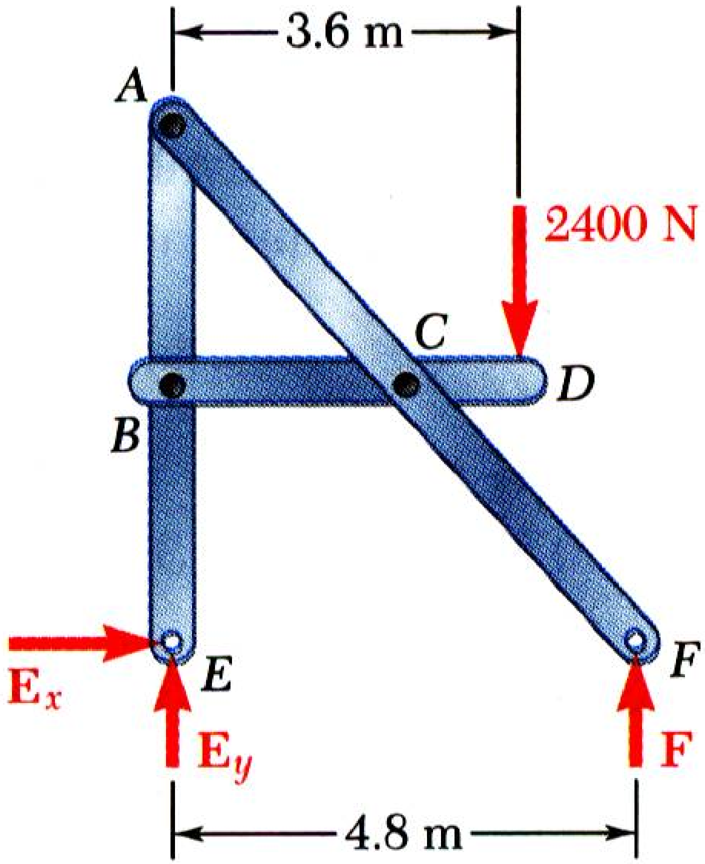
# Forces in Beams and Cables

## Sample Problem 01

SOLUTION:

- Compute reactions and connection forces.

Consider entire frame as a free-body:




$$\Rightarrow \boxed{F = 1800\ (\text{N})}$$

$$\Rightarrow \boxed{E_y = 600\ (N)}$$

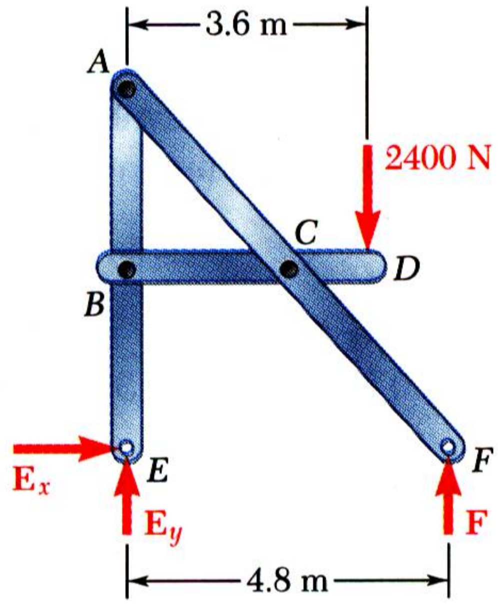
$$\boxed{E_x = 0}$$

# Forces in Beams and Cables

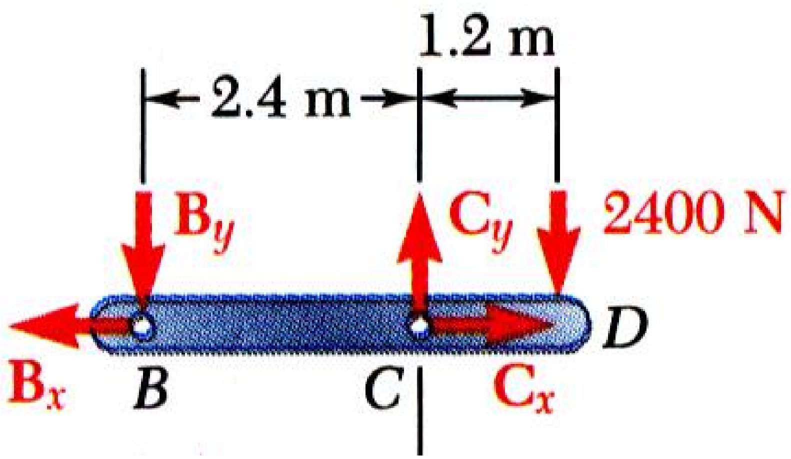
## ❑ Sample Problem 01

SOLUTION:

Consider member ***BCD*** as free-body:




$$\Rightarrow \boxed{C_y = 3600\ (\mathrm{N})}$$




$$\Rightarrow \boxed{B_y = 1200\,\mathrm{N}}$$

$$\boxed{-B_x + C_x = 0}$$

# Forces in Beams and Cables

## ❑ Sample Problem 01

SOLUTION:

Consider member *ABE* as free-body:

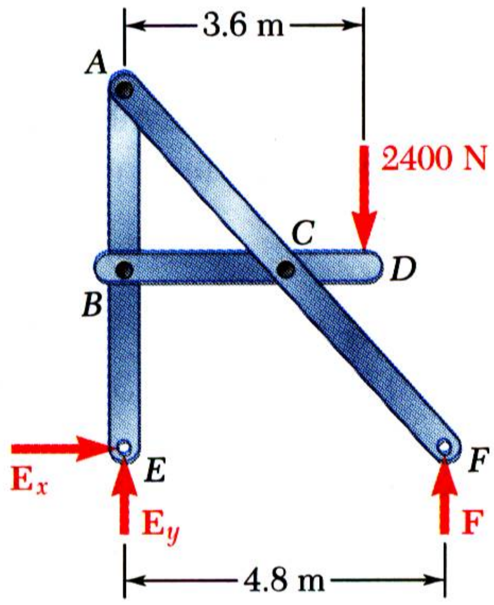
$$B_x = 0$$

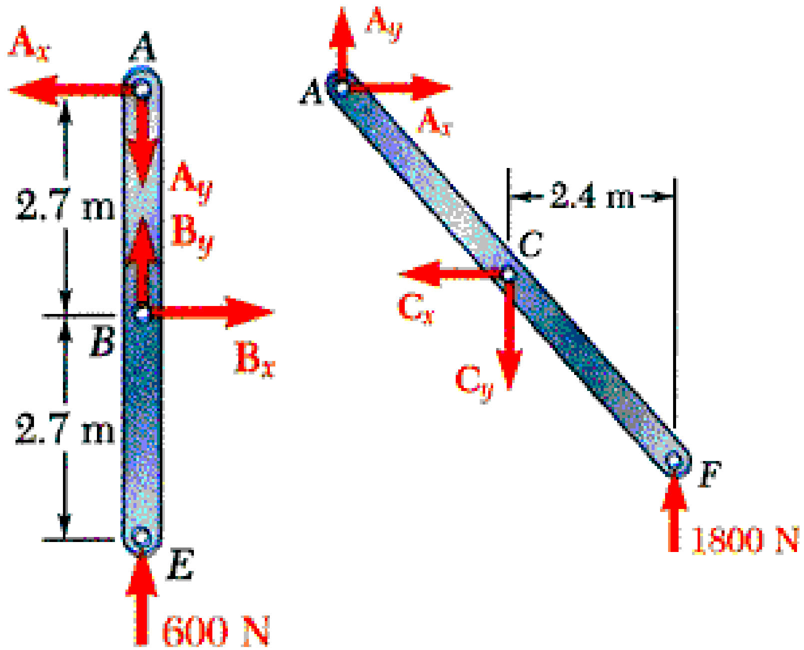
$$A_x = 0$$

$$A_y = 1800\ (N)$$

From member *BCD*,

$$C_x = 0$$

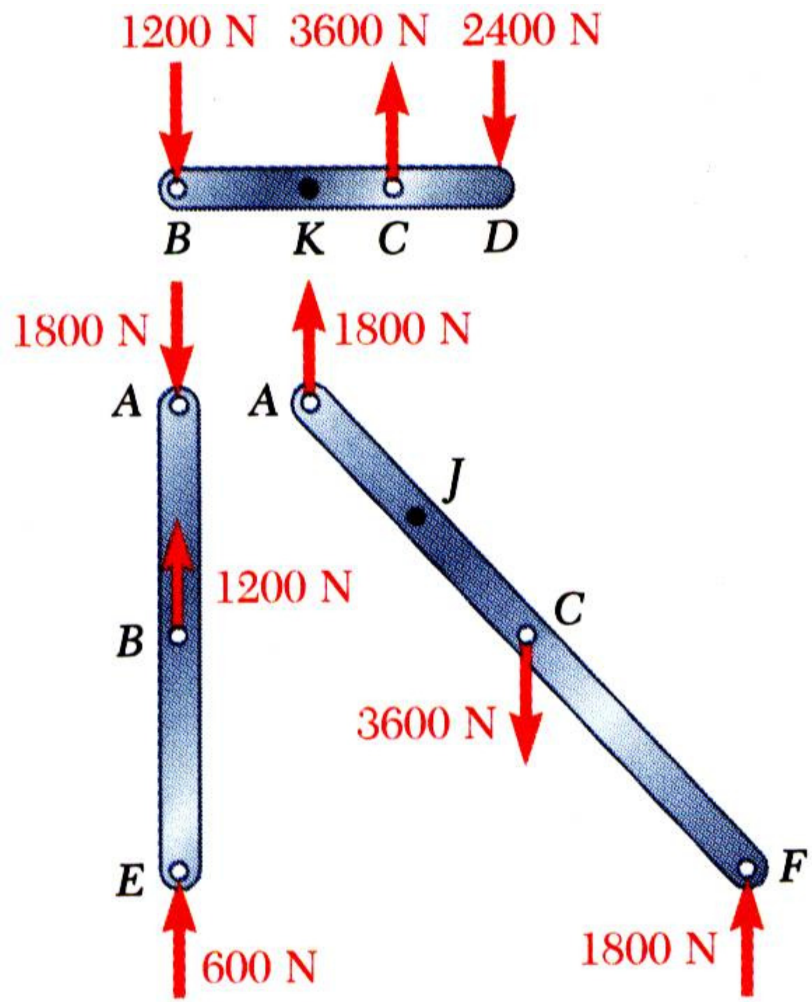


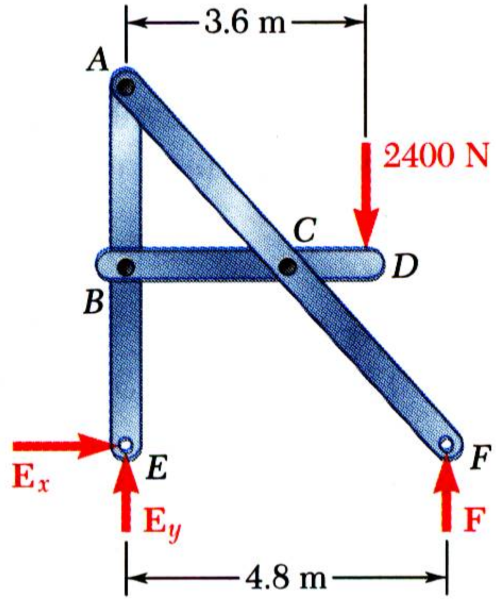
# Forces in Beams and Cables

## ❑ Sample Problem 01

SOLUTION:




Free body diagrams of members



# Forces in Beams and Cables

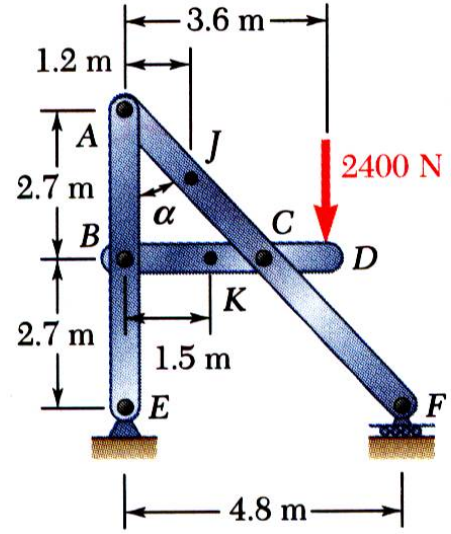
## ❑ Sample Problem 01

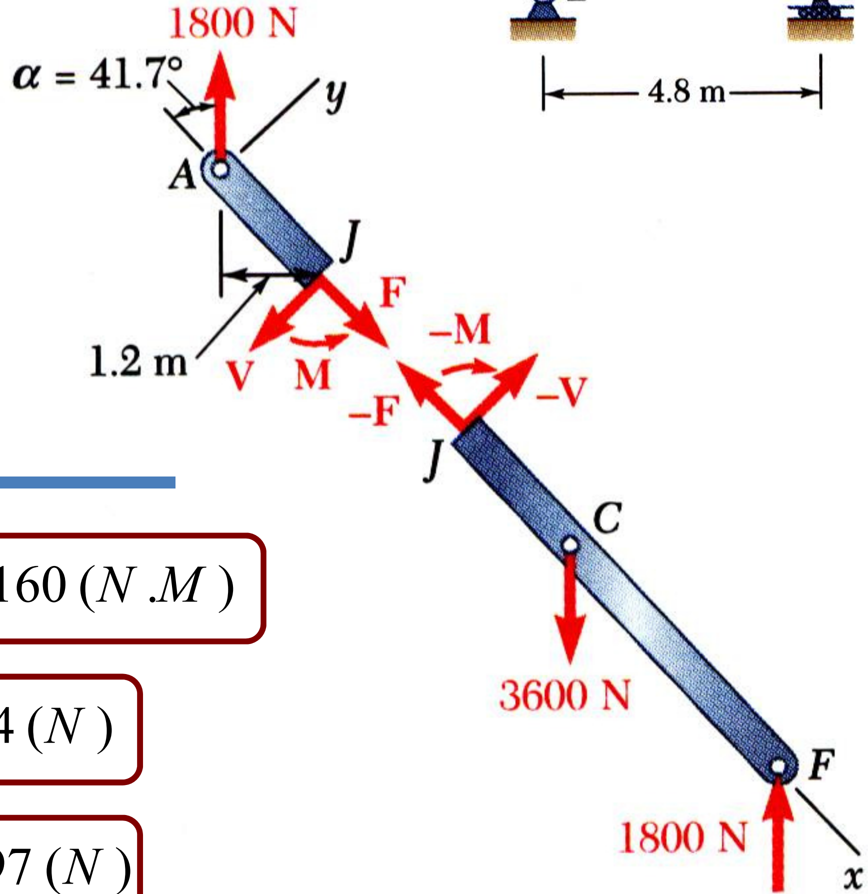
SOLUTION:

- Cut member *ACF* at *J*. The internal forces at *J* are represented by equivalent force-couple system.

Consider free-body *AJ*:

$$\alpha = 41.7^o$$




$$M = 2160\ (N.M)$$

$$F = 1344\ (N)$$

$$V = 1197\ (N)$$

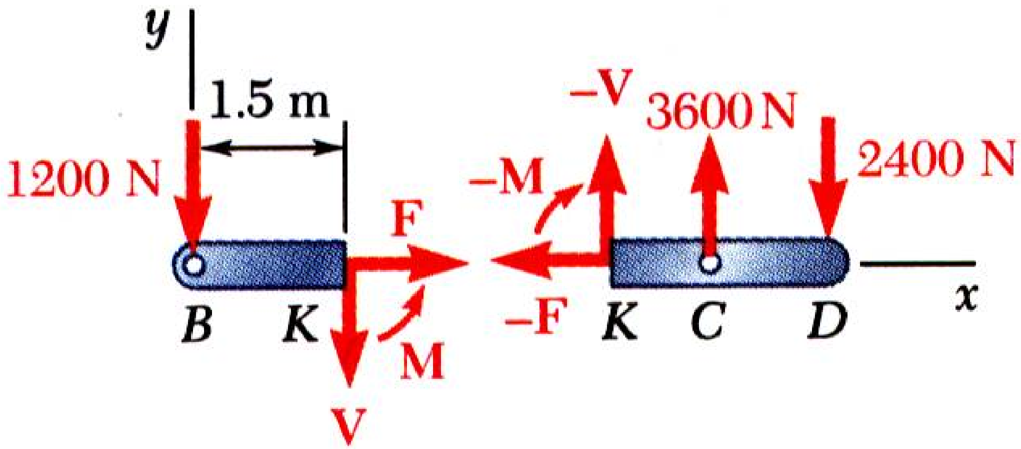
# Forces in Beams and Cables

## ❑ Sample Problem 01

SOLUTION:

- Cut member *BCD* at *K*. Determine a force-couple system equivalent to internal forces at *K*.

Consider free-body *BK*:




$$M = -1800\ (\mathrm{N \cdot m})$$

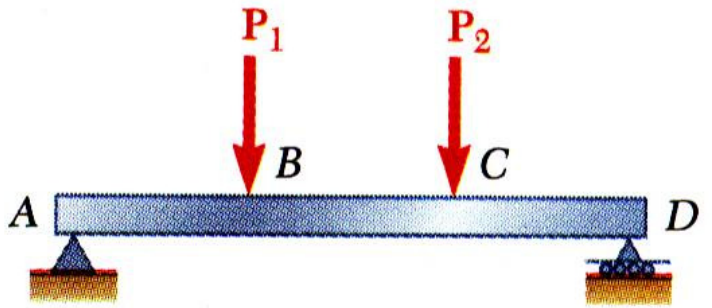
$$F = 0$$

$$V = -1200\ (\mathrm{N})$$

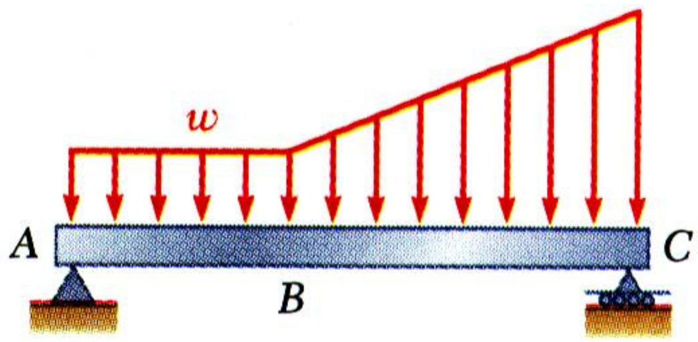
# Forces in Beams and Cables

## ❑ Various Types of Beam Loading and Support

- *Beam* - structural member designed to support loads applied at various points along its length.
- Beam can be subjected to ***concentrated*** loads or ***distributed*** loads or combination of both.
- *Beam design* is two-step process:
  1) determine shearing forces and bending moments produced by applied loads
  2) select cross-section best suited to resist shearing forces and bending moments
- Beams are classified according to way in which they are supported.
- ***Reactions at beam supports are determinate if they involve only three unknowns. Otherwise, they are statically indeterminate.***



(*a*) Concentrated loads

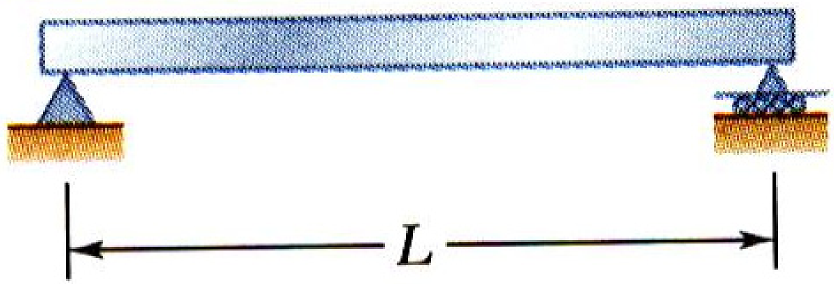


(*b*) Distributed load

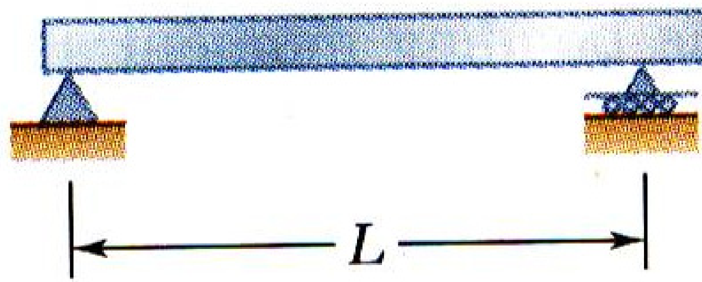
# Forces in Beams and Cables

## ❑ Various Types of Beam Loading and Support

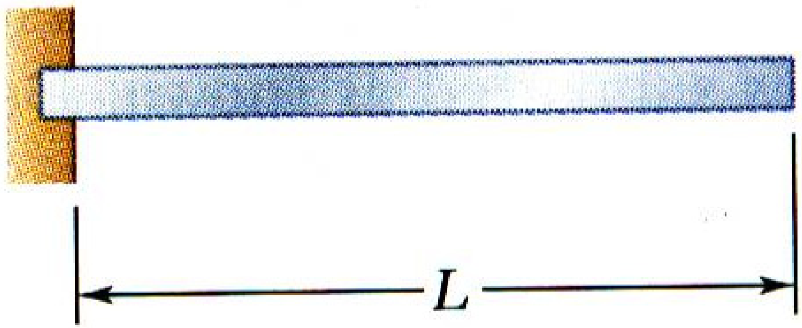
Statically determinate Beams.




(*a*) Simply supported beam




(*b*) Overhanging beam

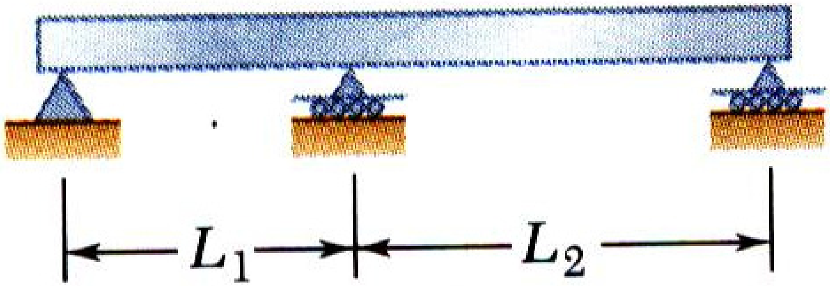



(*c*) Cantilever beam

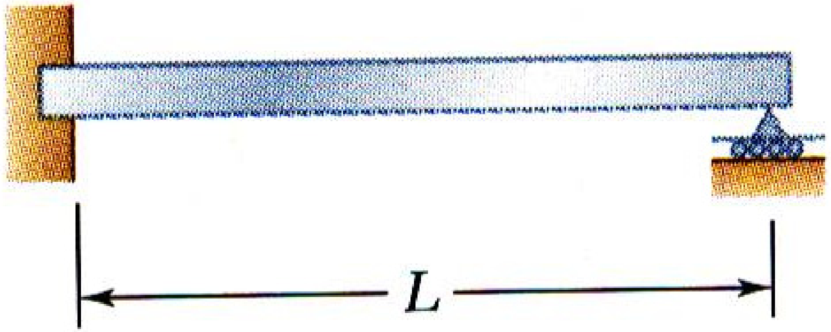
# Forces in Beams and Cables

## ❑ Various Types of Beam Loading and Support

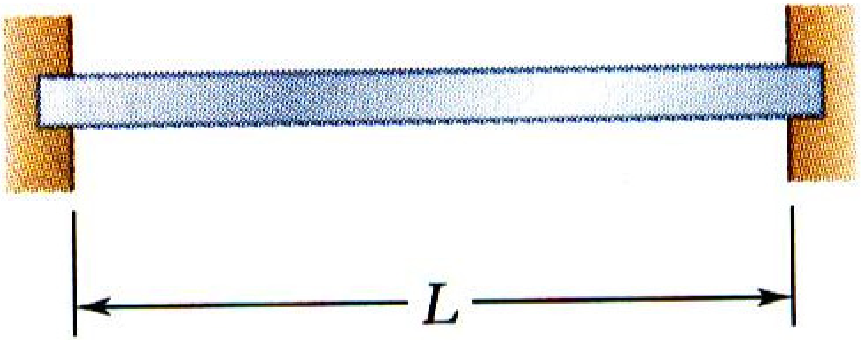
Statically indeterminate Beams.




($d$) Continuous beam




($e$) Beam fixed at one end and simply supported at the other end

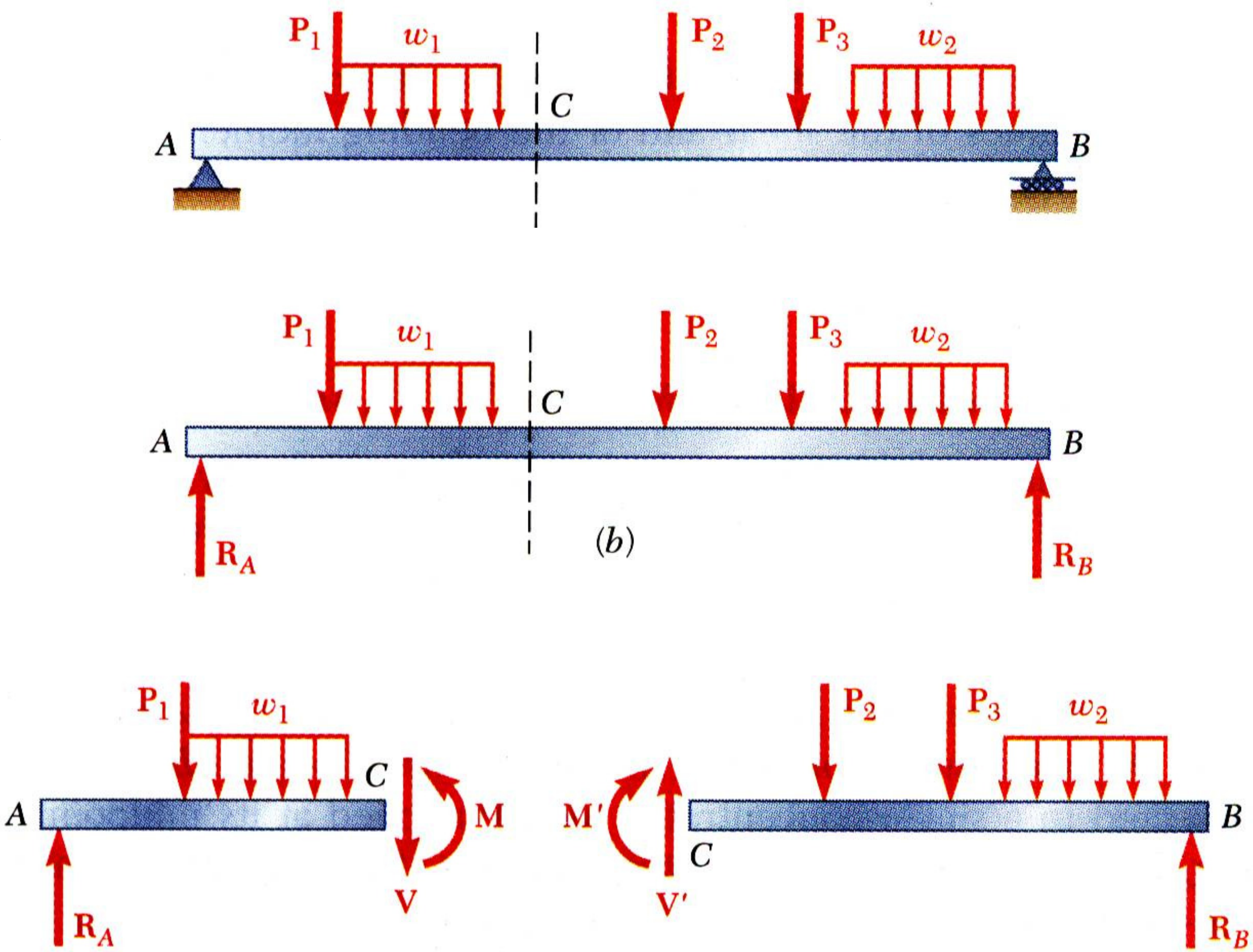



($f$) Fixed beam

# Forces in Beams and Cables

## ❑ Shear and Bending Moment in a Beam

- Wish to determine bending moment and shearing force at any point in a beam subjected to concentrated and distributed loads.
- Determine reactions at supports by treating whole beam as free-body.
- Cut beam at *C* and draw free-body diagrams for *AC* and *CB*. By definition, positive sense for internal force-couple systems are as shown.
- From equilibrium considerations, determine *M* and *V* or *M'* and *V'*.



# Forces in Beams and Cables

## ❑ Shear and Bending Moment in a Beam

- ***Variation of shear and bending moment*** along beam may be plotted.

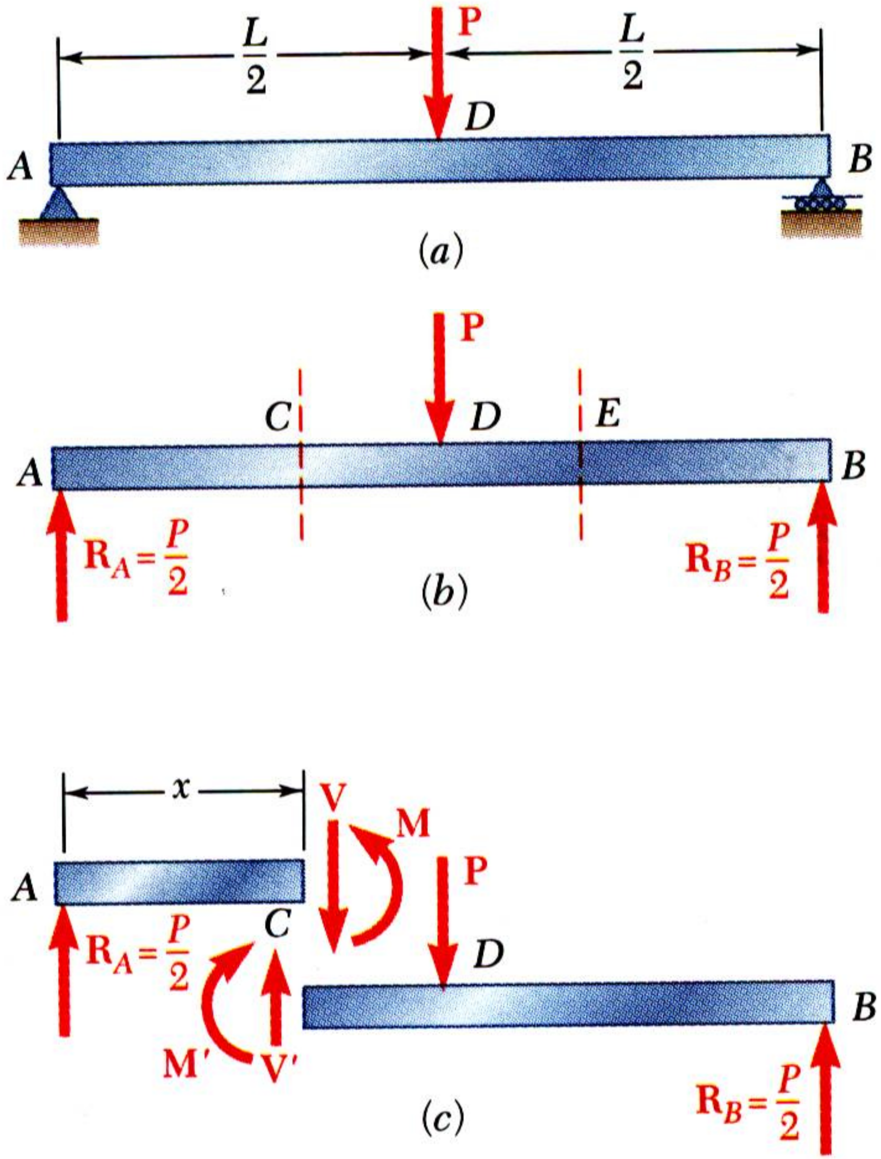
- Determine reactions at supports.

- Cut beam at $C$ and consider member $AC$,

$$x: \quad 0 \rightarrow \frac{L}{2}$$

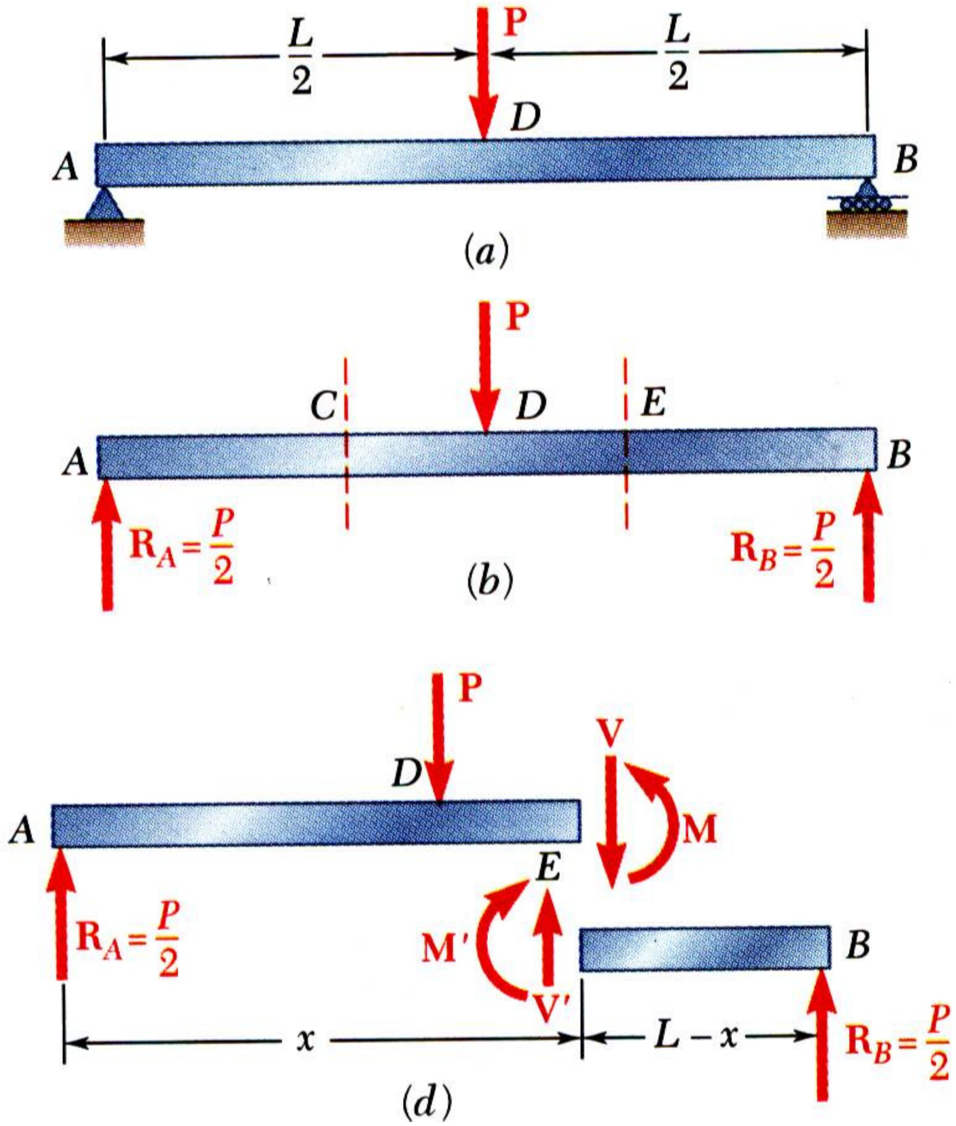
$$\sum M_{/C} = 0 \quad \Rightarrow \quad M - \frac{P}{2}x = 0 \quad \Rightarrow \quad \boxed{M = \frac{P}{2}\ x}$$

$$\sum F_y = 0 \quad \Rightarrow \quad \frac{P}{2} - V = 0 \quad \Rightarrow \quad \boxed{V = \frac{P}{2}}$$



# Forces in Beams and Cables

## ❑ Shear and Bending Moment in a Beam




- Cut beam at *E* and consider member *EB*,

$$x: \quad \frac{L}{2} \rightarrow L$$

$$\sum M_{/E} = 0 \quad \Rightarrow \quad M - \frac{P}{2}x + P(x - \frac{L}{2}) = 0$$

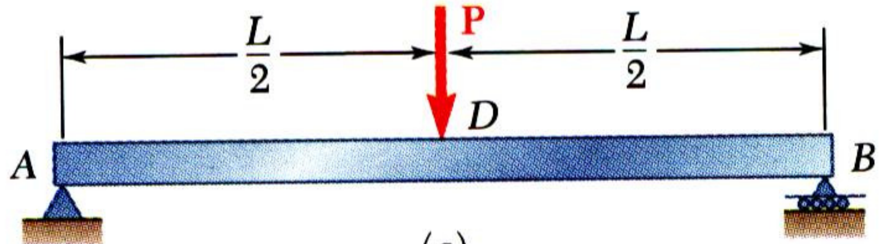
$$\Rightarrow \quad M = \frac{P}{2}(L - x)$$

$$\sum F_y = 0 \quad \Rightarrow \quad \frac{P}{2} - P - V = 0 \quad \Rightarrow \quad V = -\frac{P}{2}$$

# Forces in Beams and Cables

## ❑ Shear and Bending Moment in a Beam

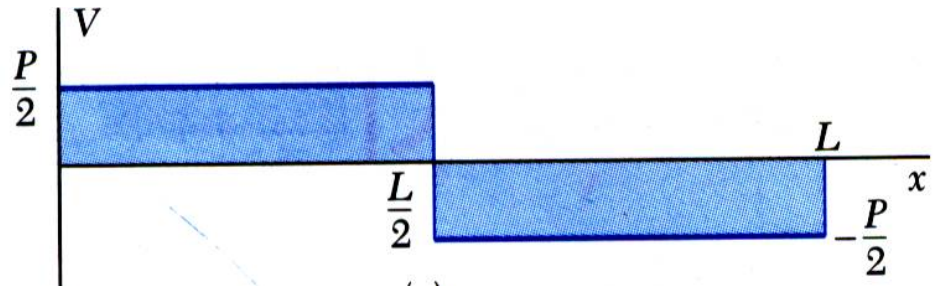
- ***For a beam subjected to <u>concentrated loads</u>, shear is constant between loading points and moment varies linearly.***



(*a*)

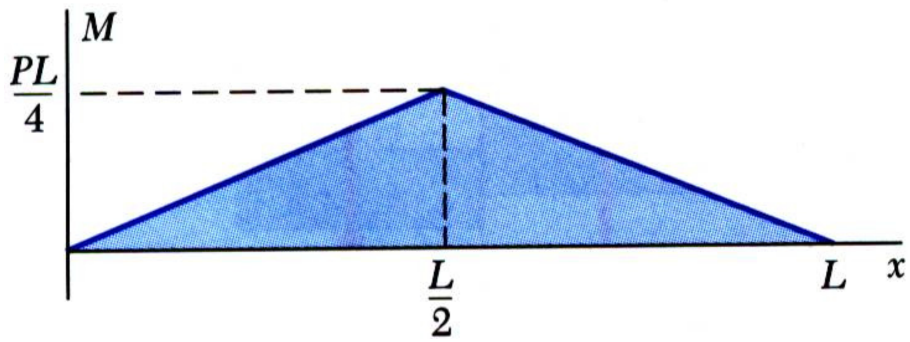
$$x: \quad 0 \rightarrow \frac{L}{2}, \qquad V = \frac{P}{2}$$

$$x: \quad \frac{L}{2} \rightarrow L, \qquad V = -\frac{P}{2}$$



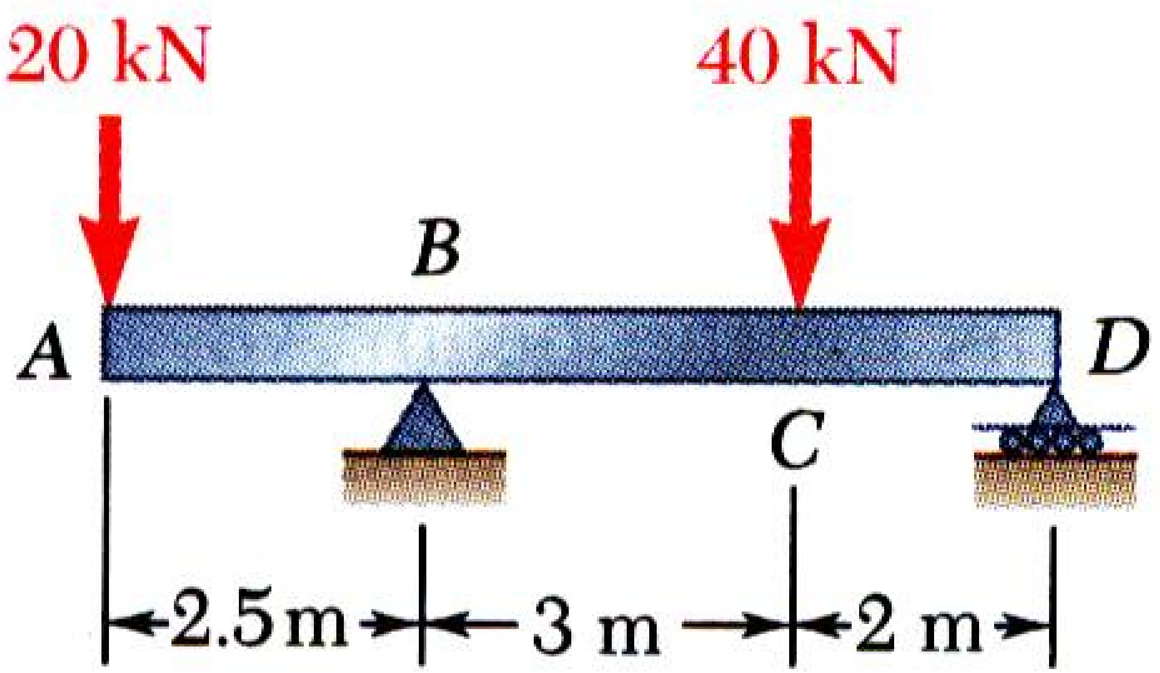

$$x: \quad 0 \rightarrow \frac{L}{2}, \qquad M = \frac{P}{2}x$$

$$x: \quad \frac{L}{2} \rightarrow L, \qquad M = \frac{P}{2}(L - x)$$



## ❑ Sample Problem 02

Draw the shear and bending moment diagrams for the beam and loading shown.

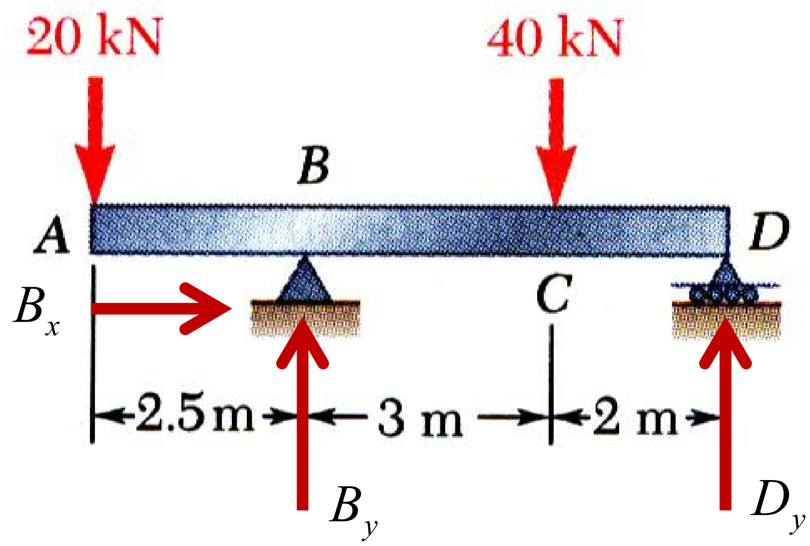


# Forces in Beams and Cables

## ❑ Sample Problem 02

SOLUTION:

- Determine reactions at supports.




$B_x = 0$

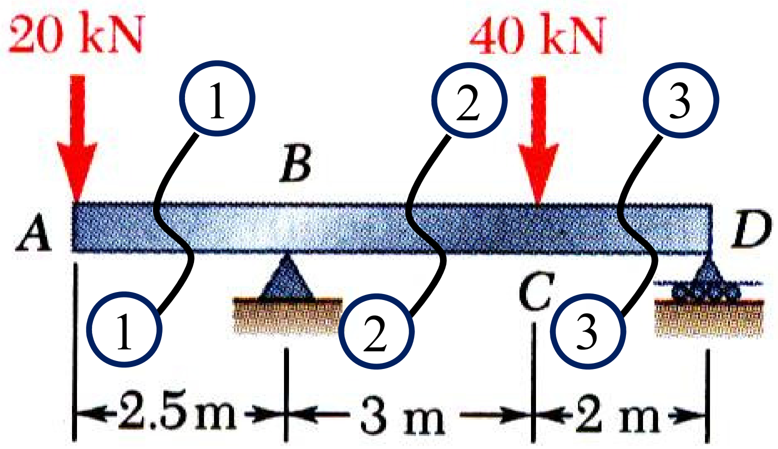
$D_y = 14\ (kN)$

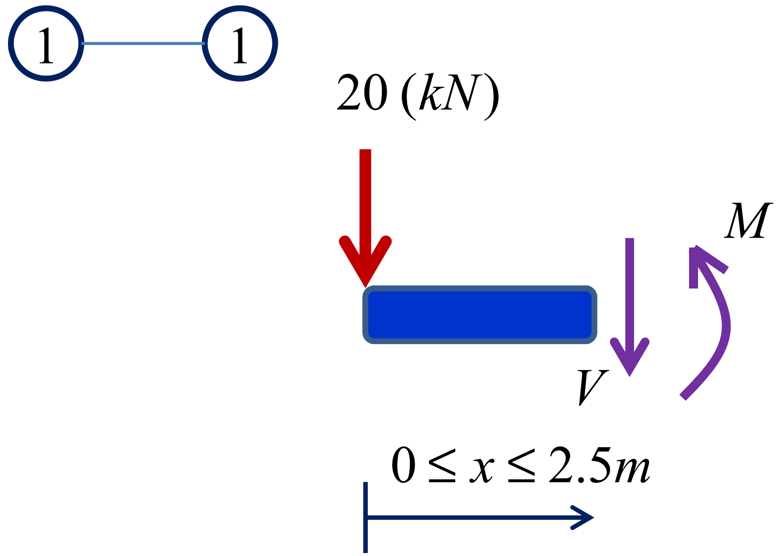
$B_y = 46\ (kN)$

# Forces in Beams and Cables

## ❑ Sample Problem 02

SOLUTION:



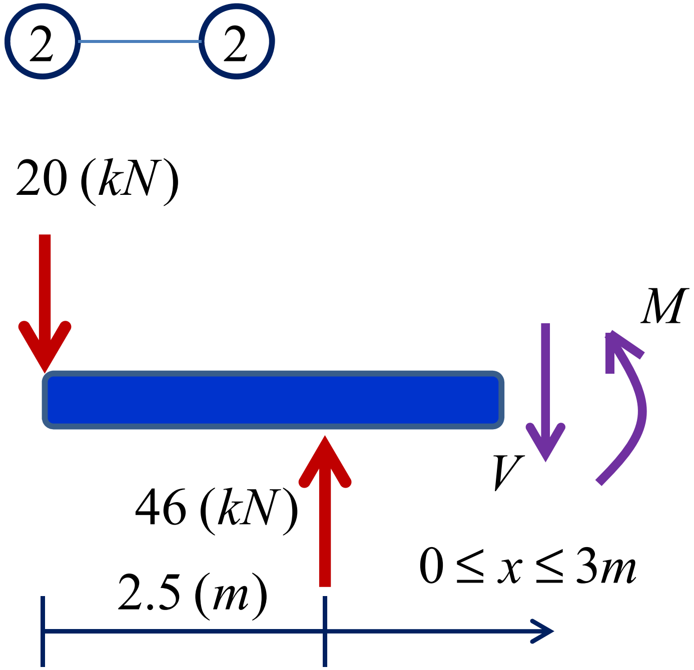

$$M = -20x$$

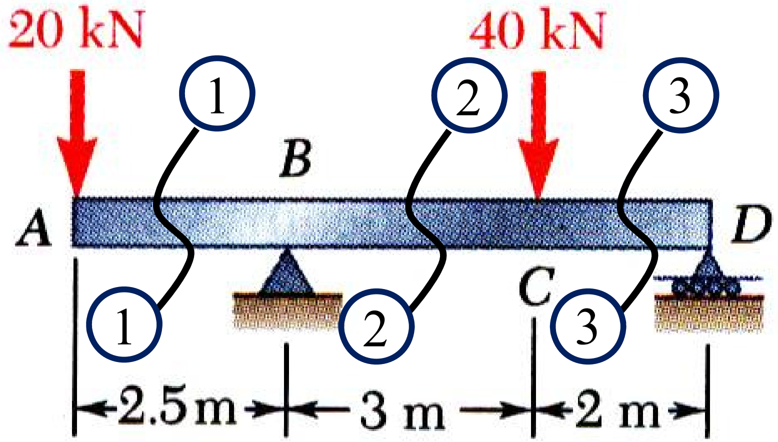
$$V = -20\ (kN)$$

# Forces in Beams and Cables

## ❑ Sample Problem 02

SOLUTION:



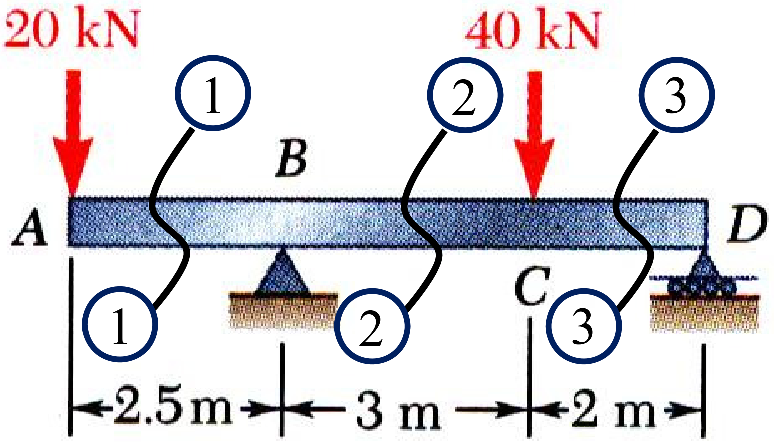

$$M = 26x - 50$$

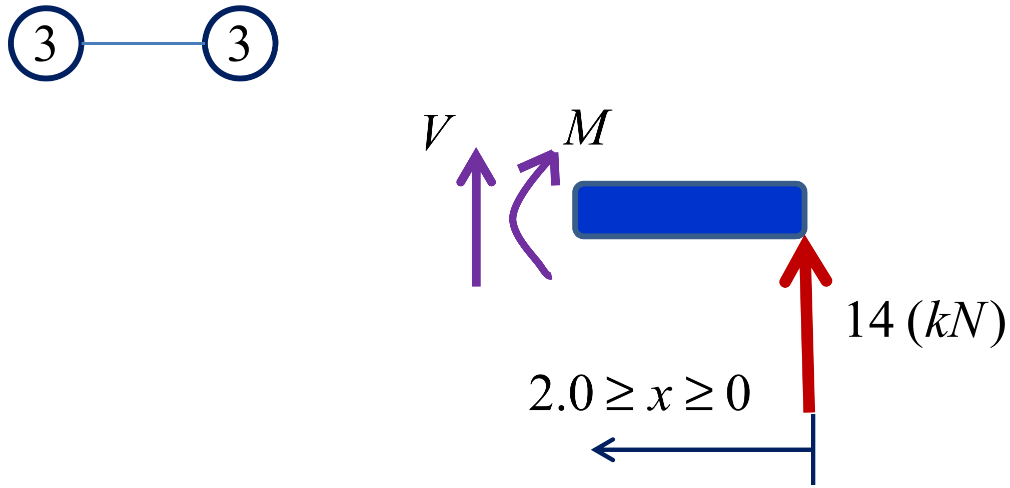
$$V = 26\ (kN)$$

# Forces in Beams and Cables

## ❑ Sample Problem 02

SOLUTION:




$$M = 14x$$

$$V = -14\ (kN)$$

## ❑ Sample Problem 02

SOLUTION:

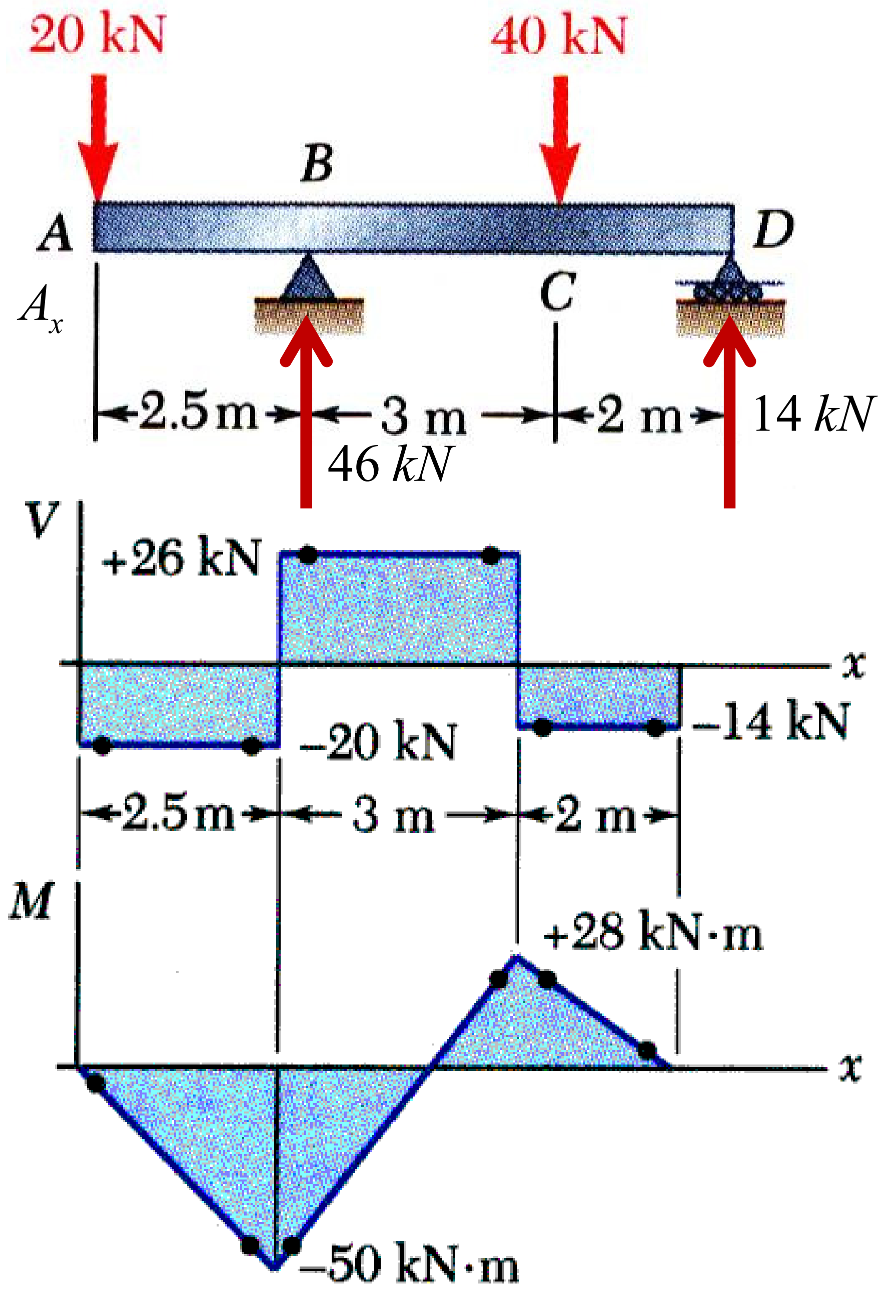
- Plot results.

  Note that shear is of constant value between concentrated loads and bending moment varies linearly.

$$x: \quad 0 \rightarrow 2.5\,(m)$$
$$M = -20x$$
$$V = -20\,(kN)$$

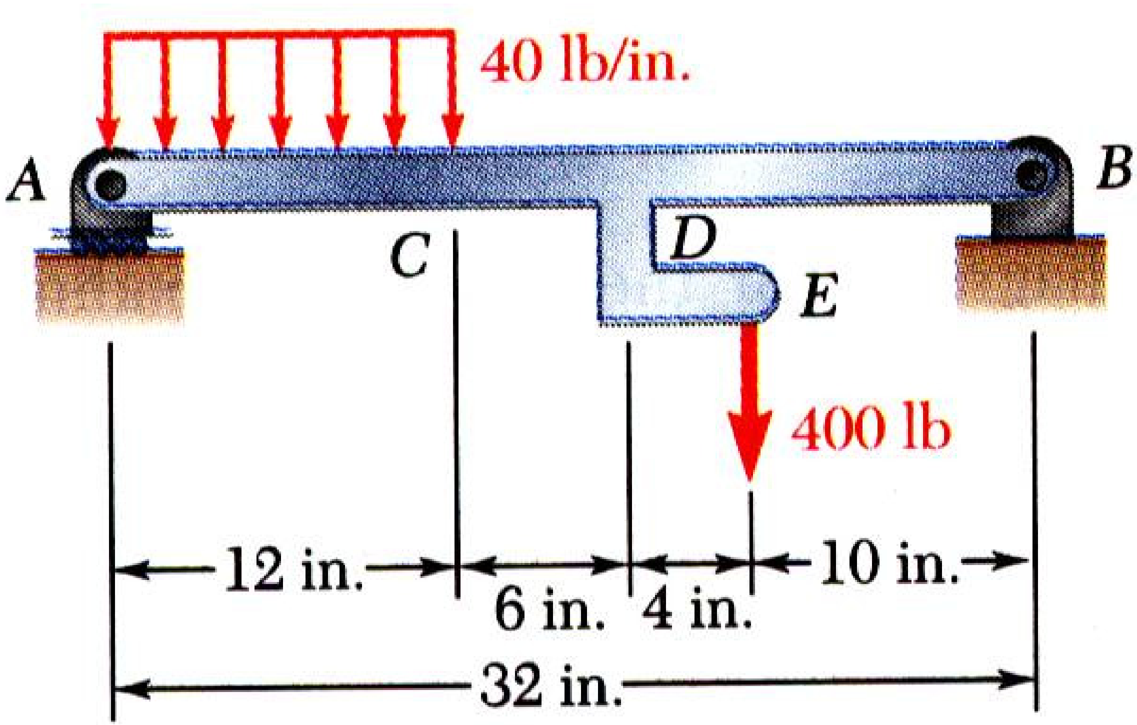
$$x: \quad 0 \rightarrow 3\,(m)$$
$$M = 26x - 50$$
$$V = 26\,(kN)$$

$$2.0\,(m) \leftarrow 0 : x$$
$$M = 14x$$
$$V = -14\,(kN)$$



## Sample Problem 03

Draw the shear and bending moment diagrams for the beam *AB*. The distributed load of 40 lb/in. extends over 12 in. of the beam, from *A* to *C*, and the 400 lb load is applied at *E*.

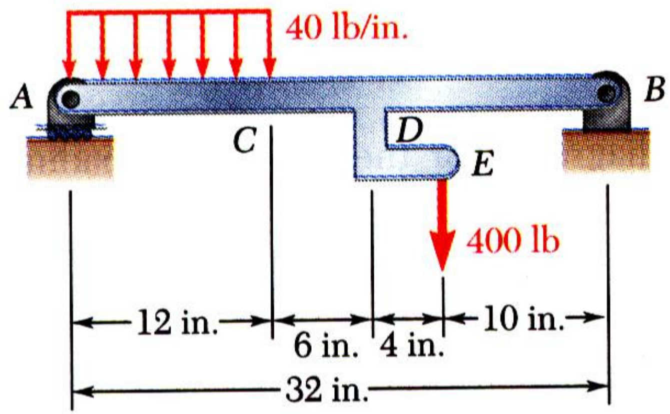


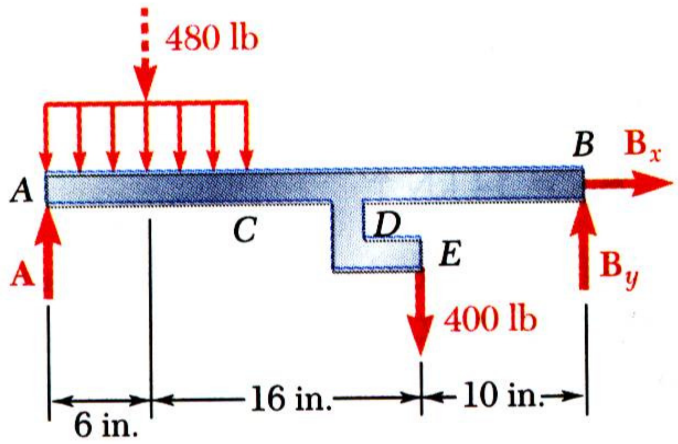
# Forces in Beams and Cables

## ❑ Sample Problem 03

SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *A* and *B*.




$$\Rightarrow \boxed{B_y = 365 \text{ (lb)}}$$

$$\Rightarrow \boxed{A = 515 \text{ (lb)}}$$

$$\boxed{B_x = 0}$$

- Note: The 400 lb load at *E* may be replaced by a 400 lb force and 1600 lb-in. couple at *D*.

# Forces in Beams and Cables

## ❑ Sample Problem 03

SOLUTION:

- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From A to $C$:

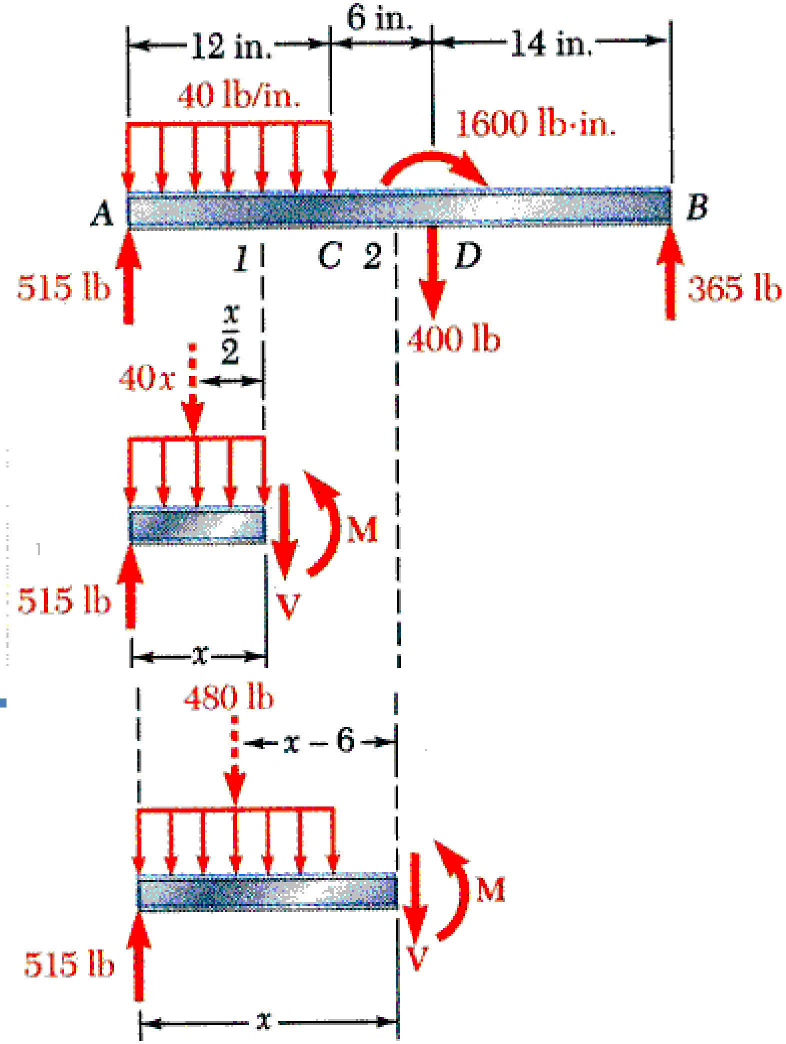
$$M = 515x - 20x^2$$

$$V = 515 - 40x$$

From $C$ to $D$:

$$M = 2880 + 35x$$

$$V = 35 \text{ (lb)}$$



# Forces in Beams and Cables

## ❑ Sample Problem 03

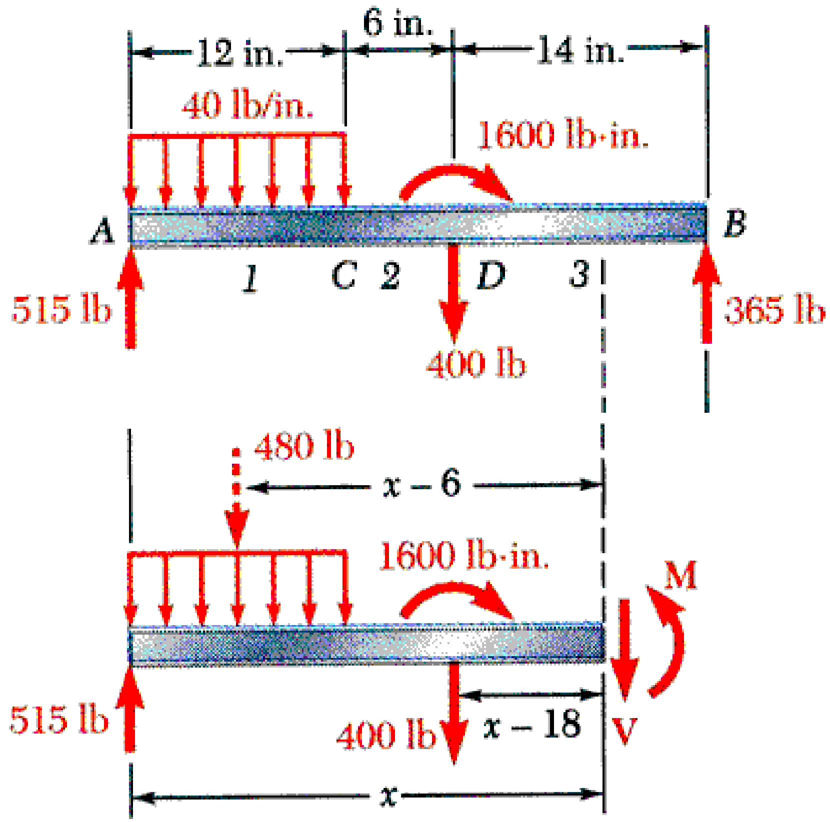
SOLUTION:

- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From $D$ to $B$:

$$M = 11680 - 365x$$

$$V = -365 \text{ (lb)}$$



# Forces in Beams and Cables

## ❑ Sample Problem 03

SOLUTION:

From A to $C$: $x: \quad 0 \rightarrow 12\ (in.)$

$$M = 515x - 20x^2$$

$$V = 515 - 40x$$

From $C$ to $D$: $x: \quad 12 \rightarrow 18\ (in.)$

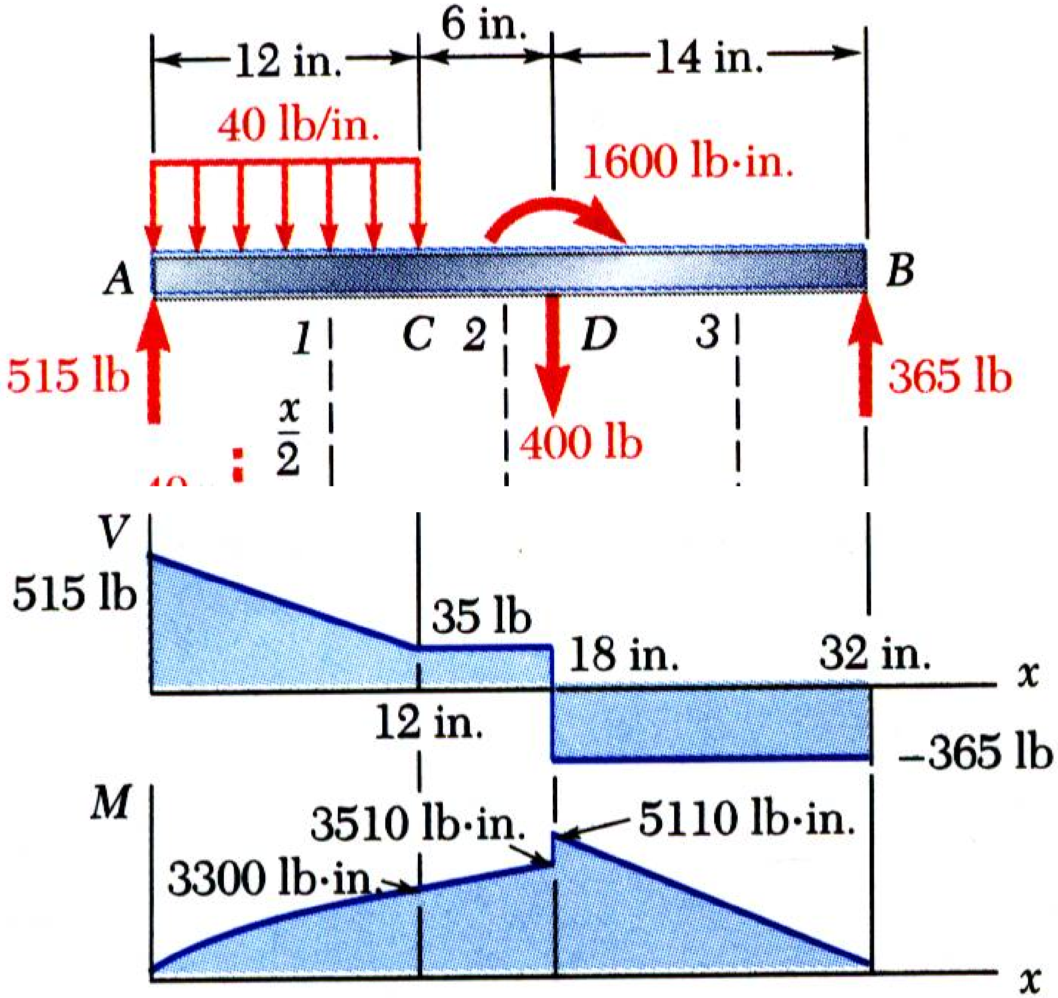
$$M = 2880 + 35x$$

$$V = 35\ \text{(lb)}$$

From $D$ to $B$: $x: \quad 18 \rightarrow 32\ (in.)$

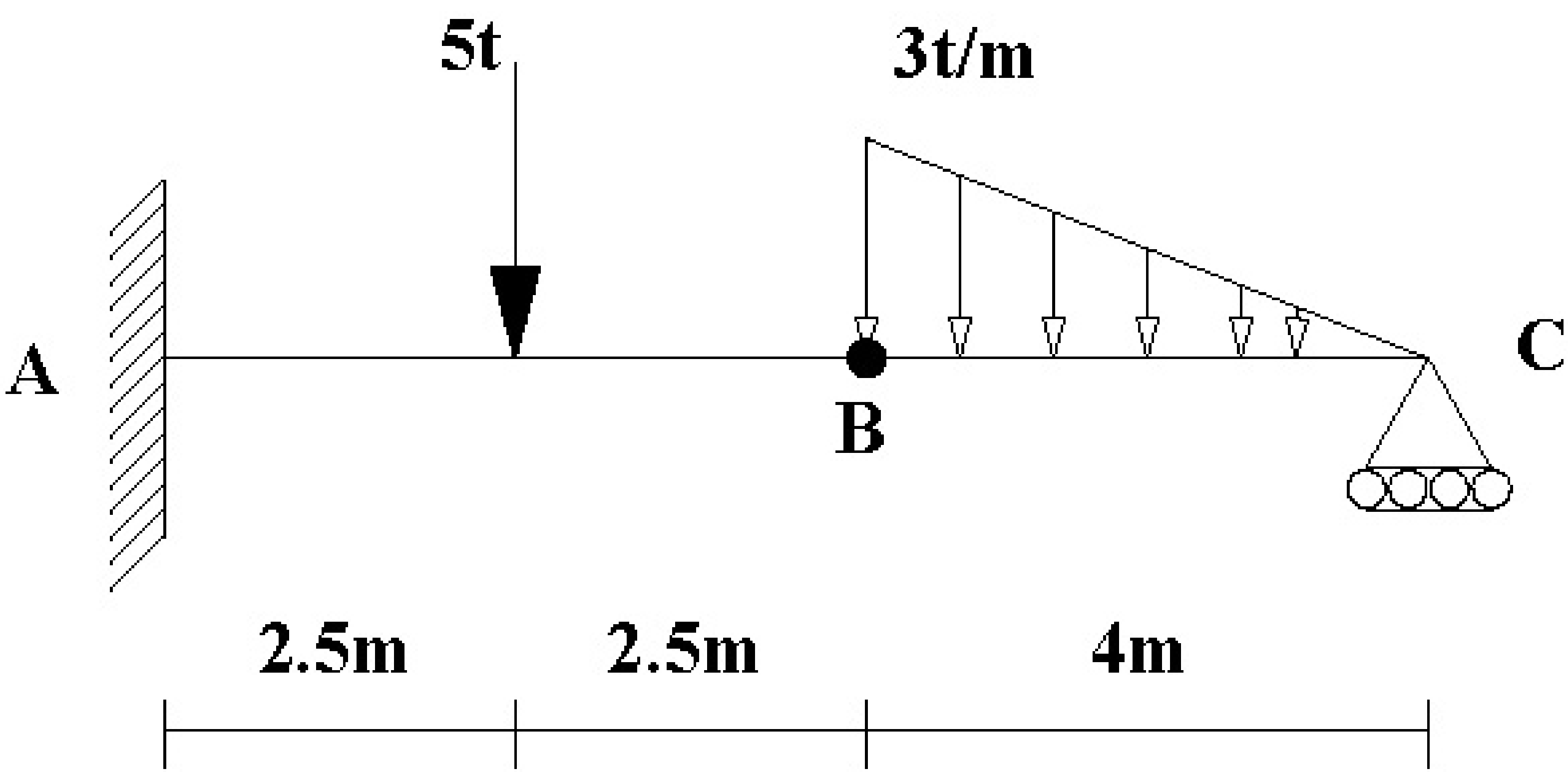
$$M = 11680 - 365x$$

$$V = -365\ \text{(lb)}$$



## ❑ Sample Problem 04

Sketch the shear and bending-moment diagrams for the beam and loading shown.



# Forces in Beams and Cables

## ❑ Sample Problem 04

SOLUTION:

- Determine the unknown reactions

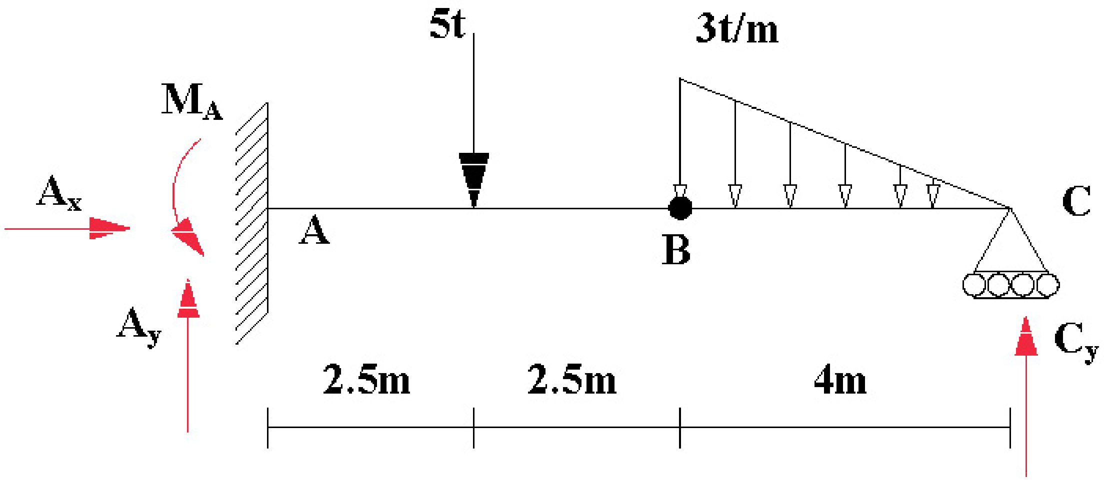



Fig.1

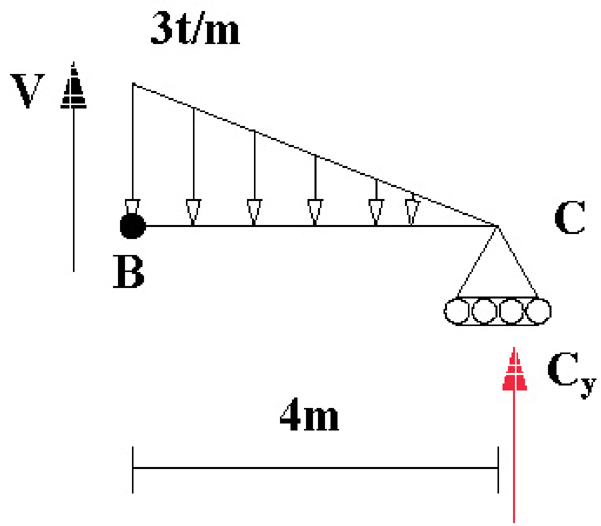



Fig.2

$$C_y = 2t$$

$$M_A = 32.5\ (t.m)$$

# Forces in Beams and Cables

## ❑ Sample Problem 04

SOLUTION:

- Determine the unknown reactions

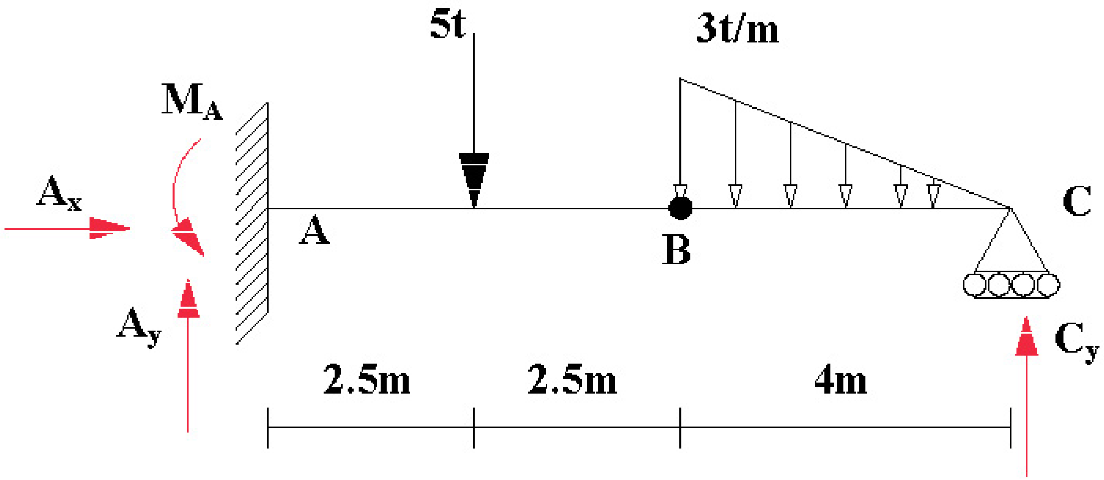



Fig.1

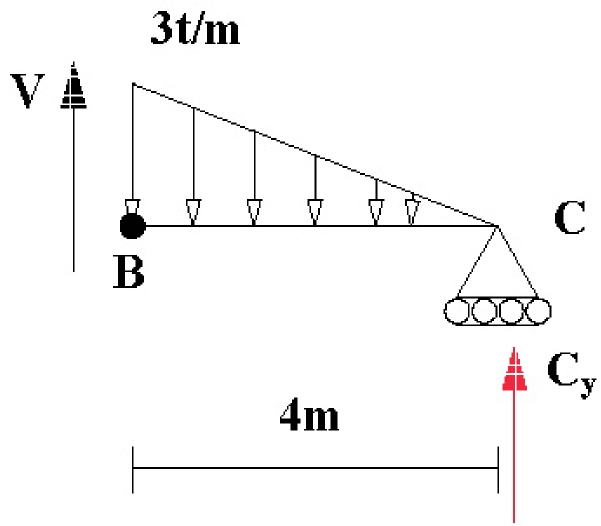


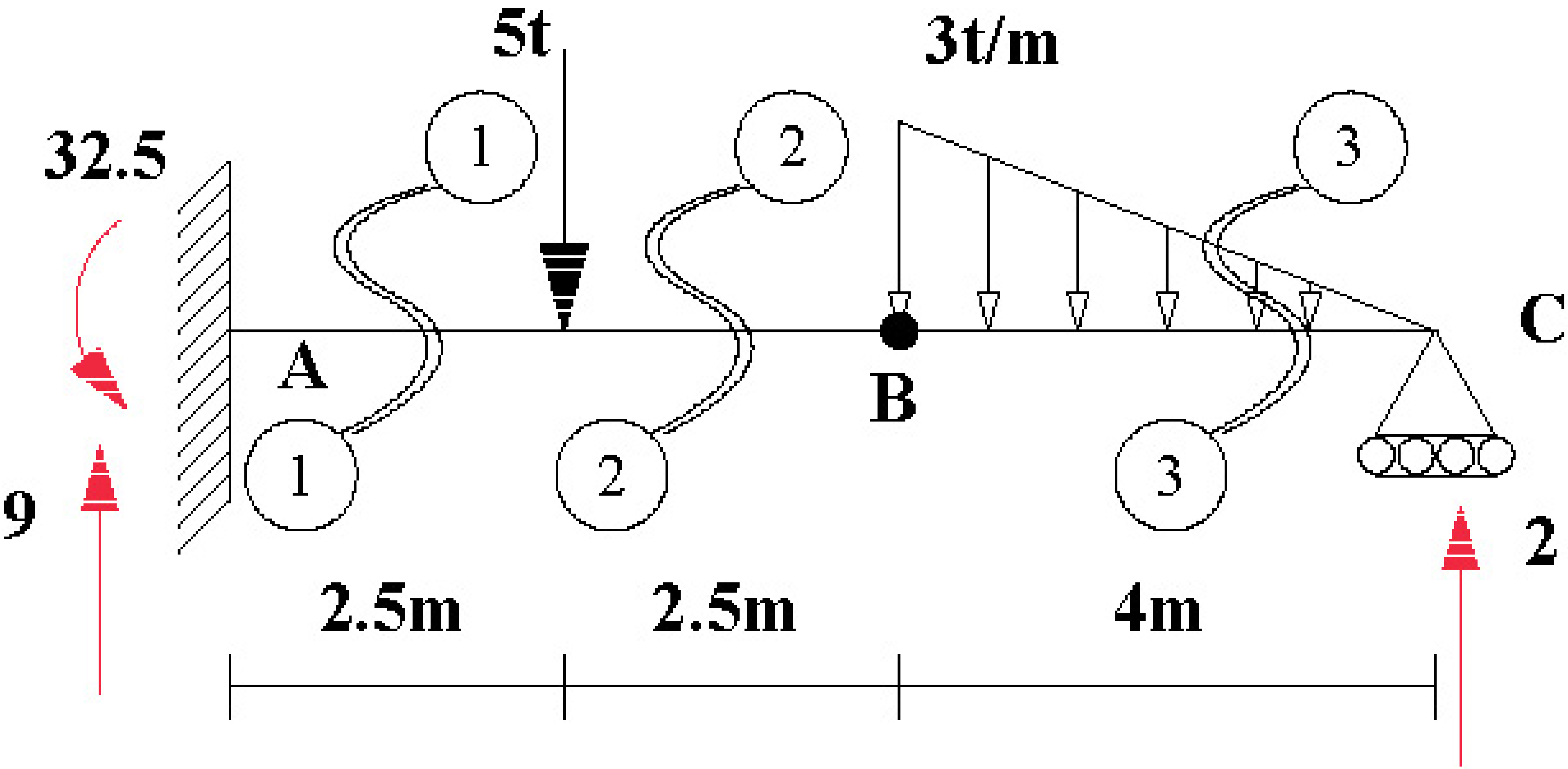

Fig.2

$$A_y = 9\,t$$

$$A_x = 0$$

## ❑ Sample Problem 04

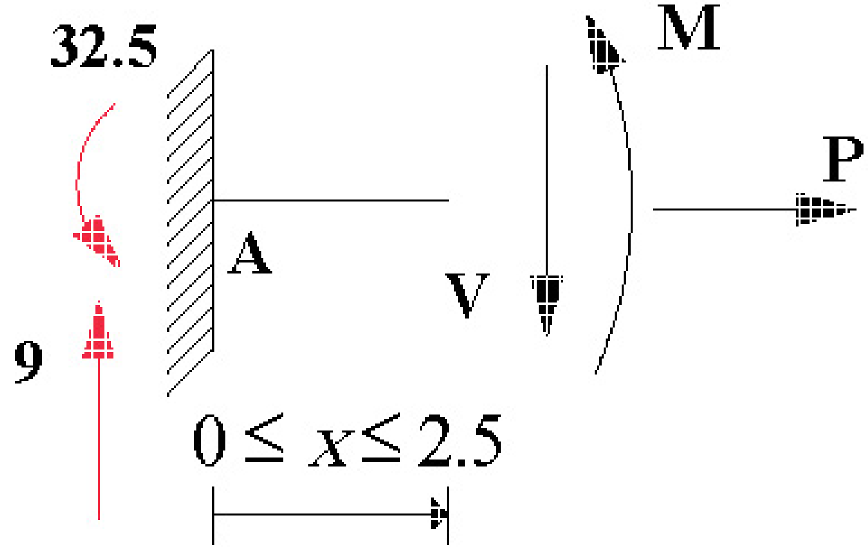
SOLUTION:



# Forces in Beams and Cables

## Sample Problem 04

SOLUTION:



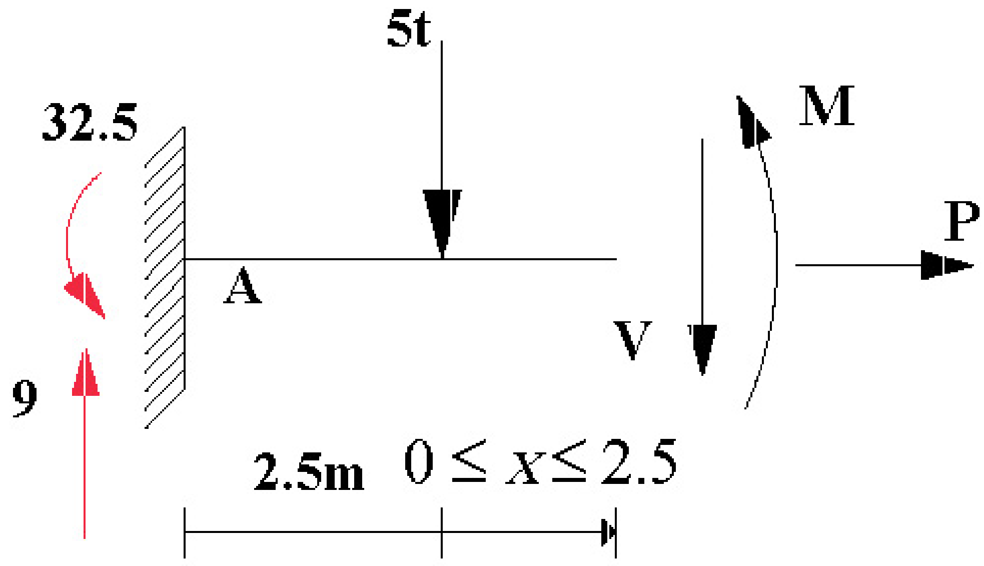

1—1 $0 \le x \le 2.5$

$$M = 9x - 32.5$$

$$V = 9\,t$$

## ❑ Sample Problem 04

SOLUTION:




(2)—(2) $0 \leq x \leq 2.5$

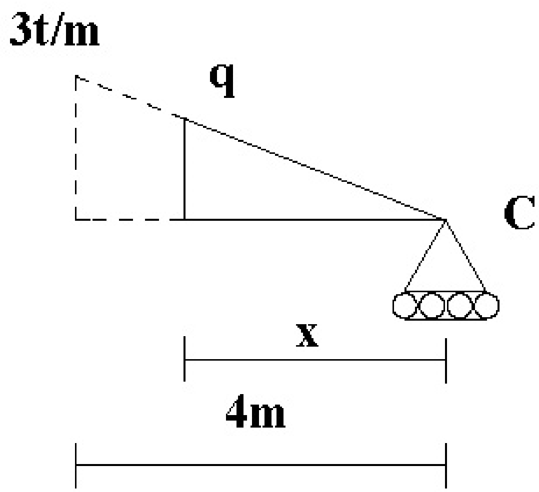
$$M = 4x - 10$$

$$V = 4t$$

# Forces in Beams and Cables

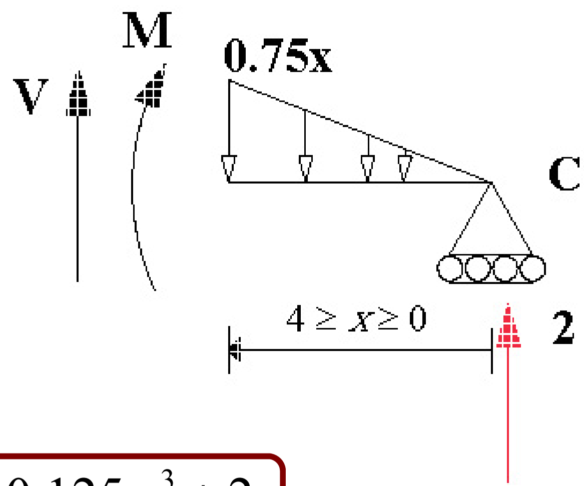
## ❑ Sample Problem 04

SOLUTION:




$$q = 0.75x$$

(3)—(3) $4 \geq x \geq 0$

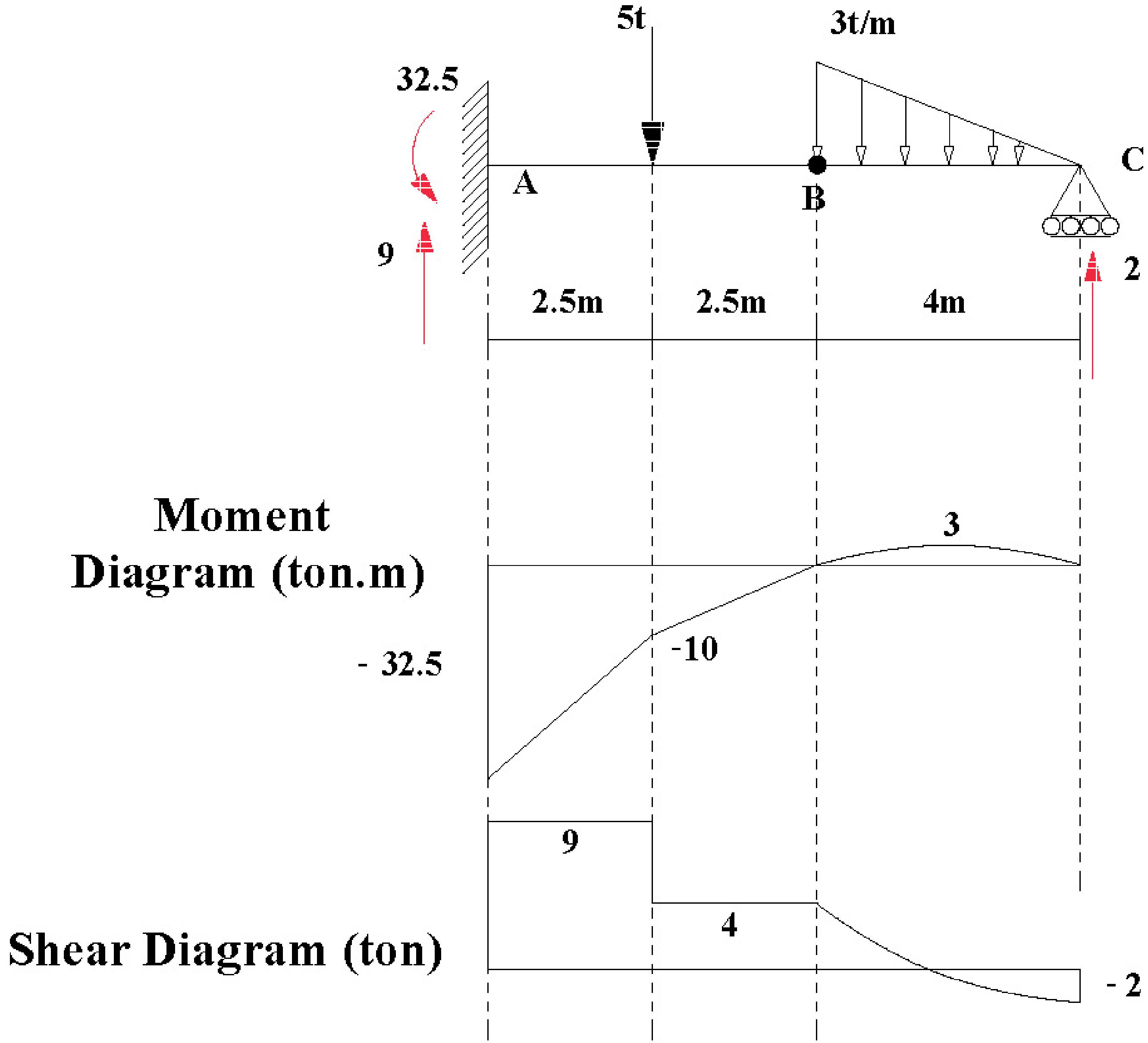



$$M = -0.125x^3 + 2x$$

$$V = 0.375x^2 - 2$$

## ❑ Sample Problem 04

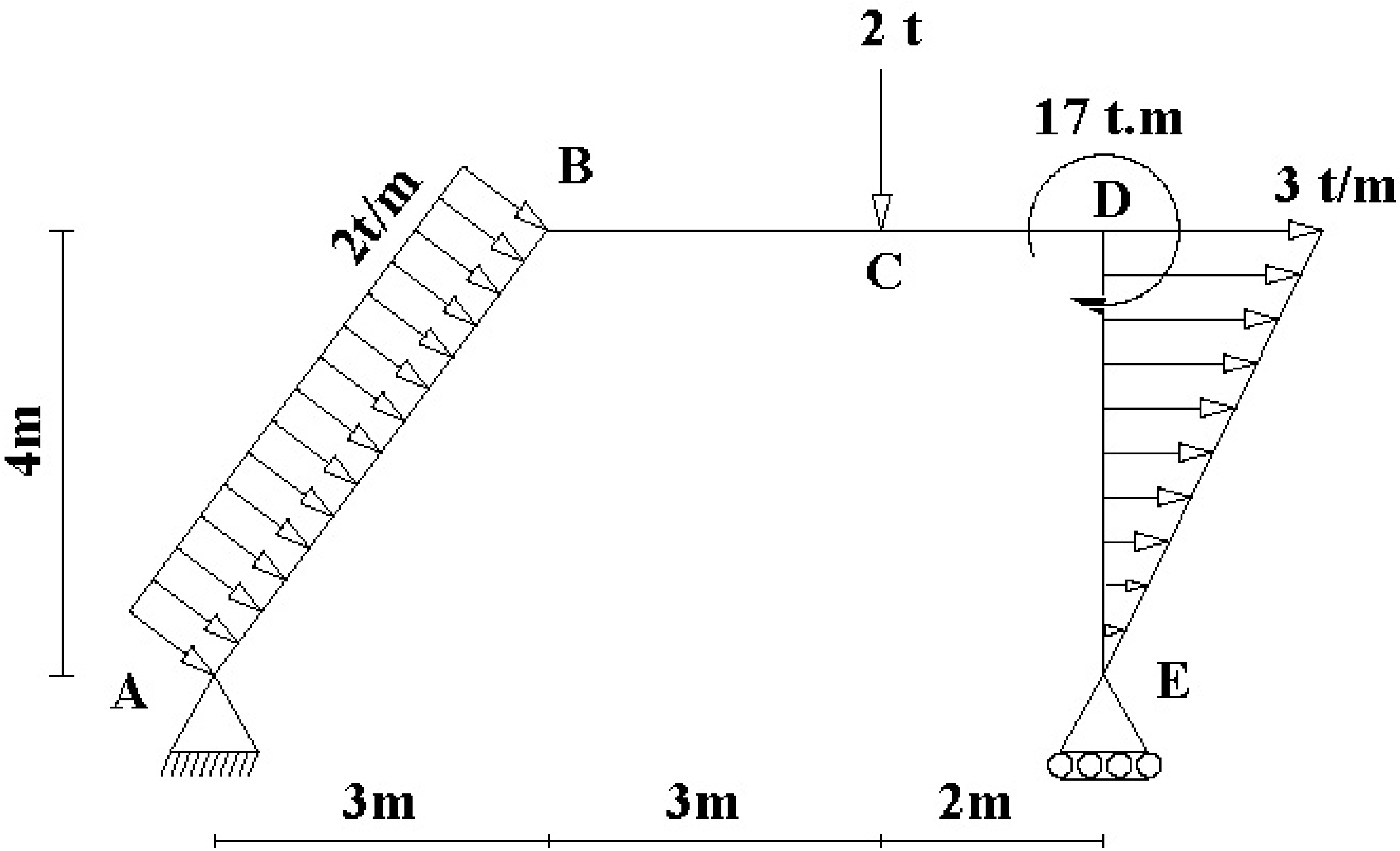
SOLUTION:



# Forces in Beams and Cables

## ❑ Sample Problem 05

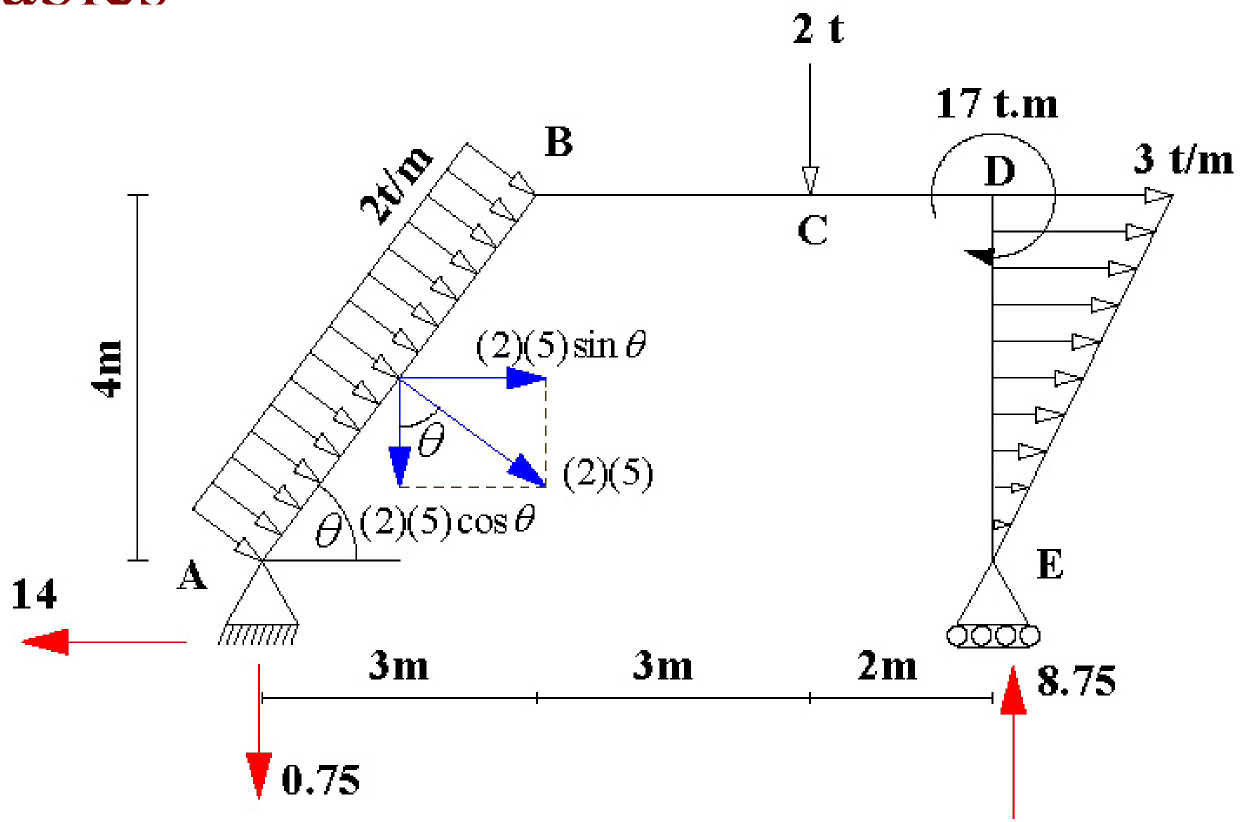
Sketch the axial, shear and bending-moment diagrams for the Frame and loading shown.



# Forces in Beams and Cables

## ❑ Sample Problem 05

SOLUTION:




$$E_y = 8.75t$$

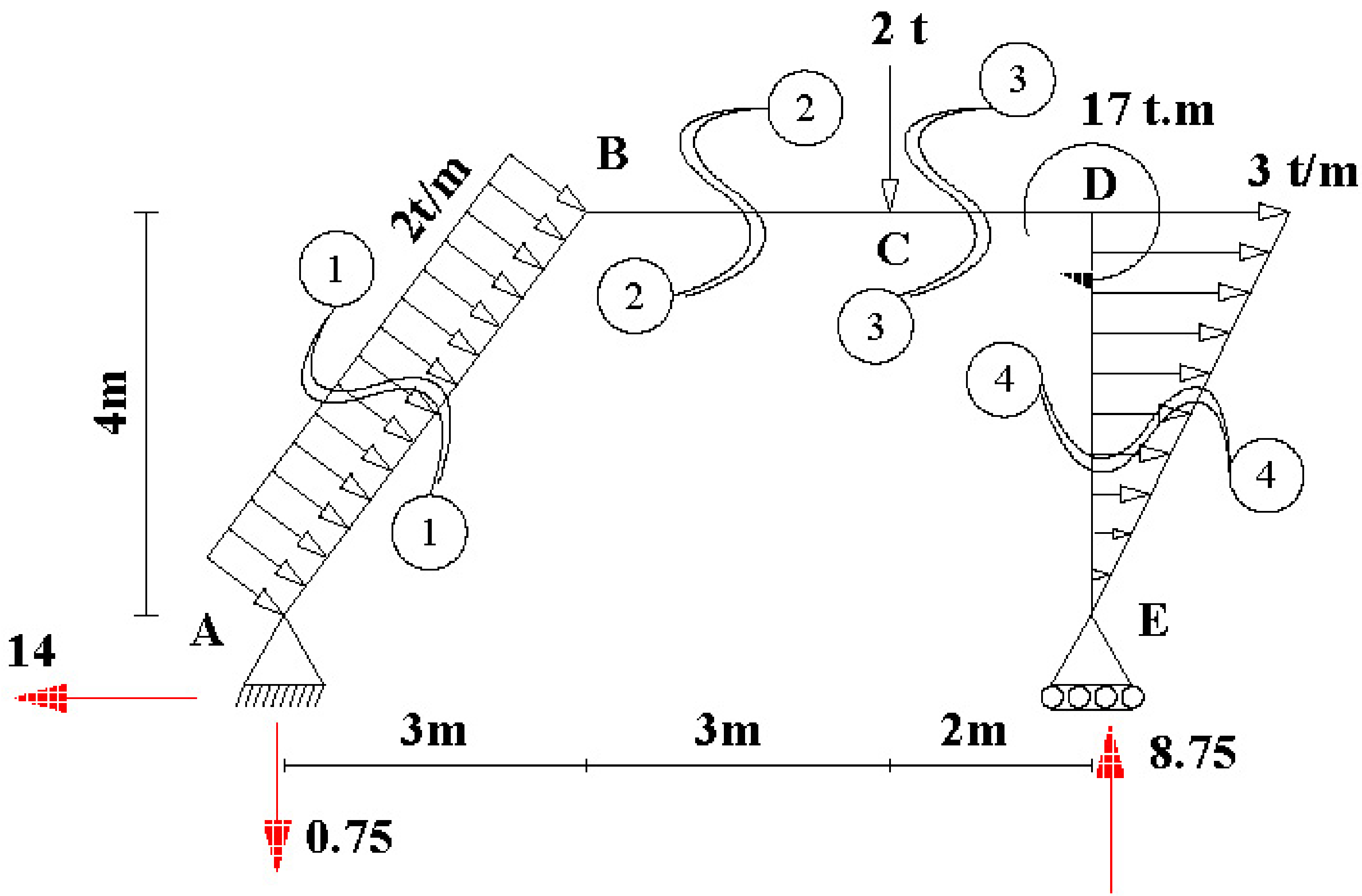
$$A_y = 0.75t$$

$$A_x = 14t$$

# Forces in Beams and Cables

## ❑ Sample Problem 05

SOLUTION:



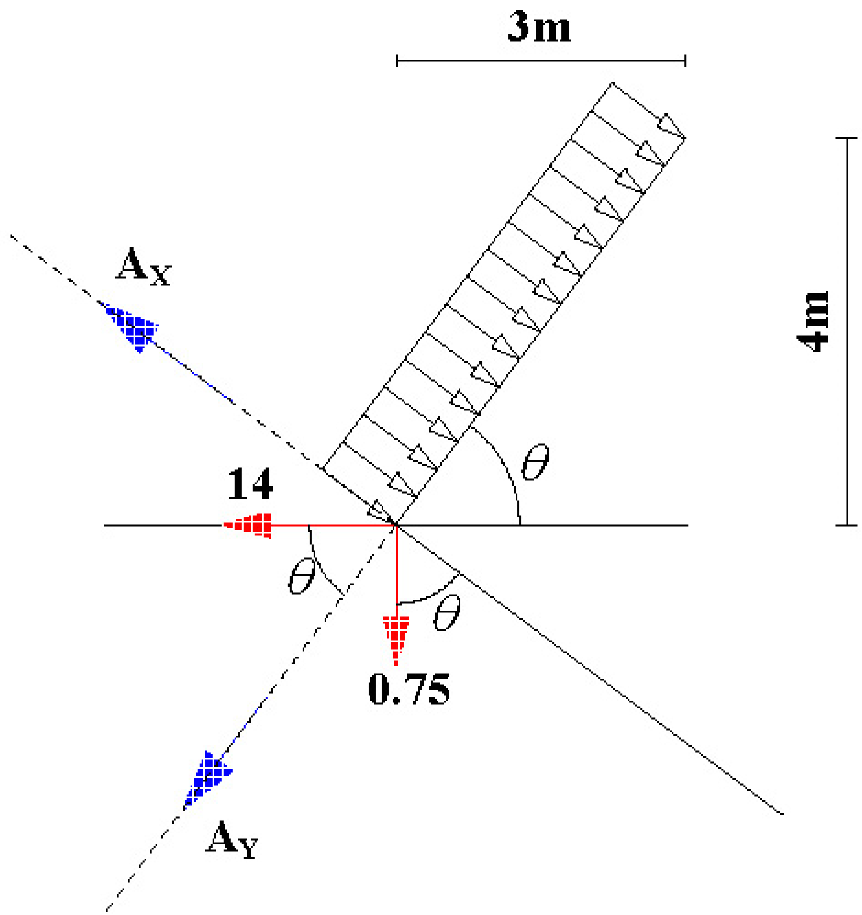
# Forces in Beams and Cables

## ❑ Sample Problem 05

SOLUTION:

$$A_X = 10.75t$$

$$A_Y = 9t$$

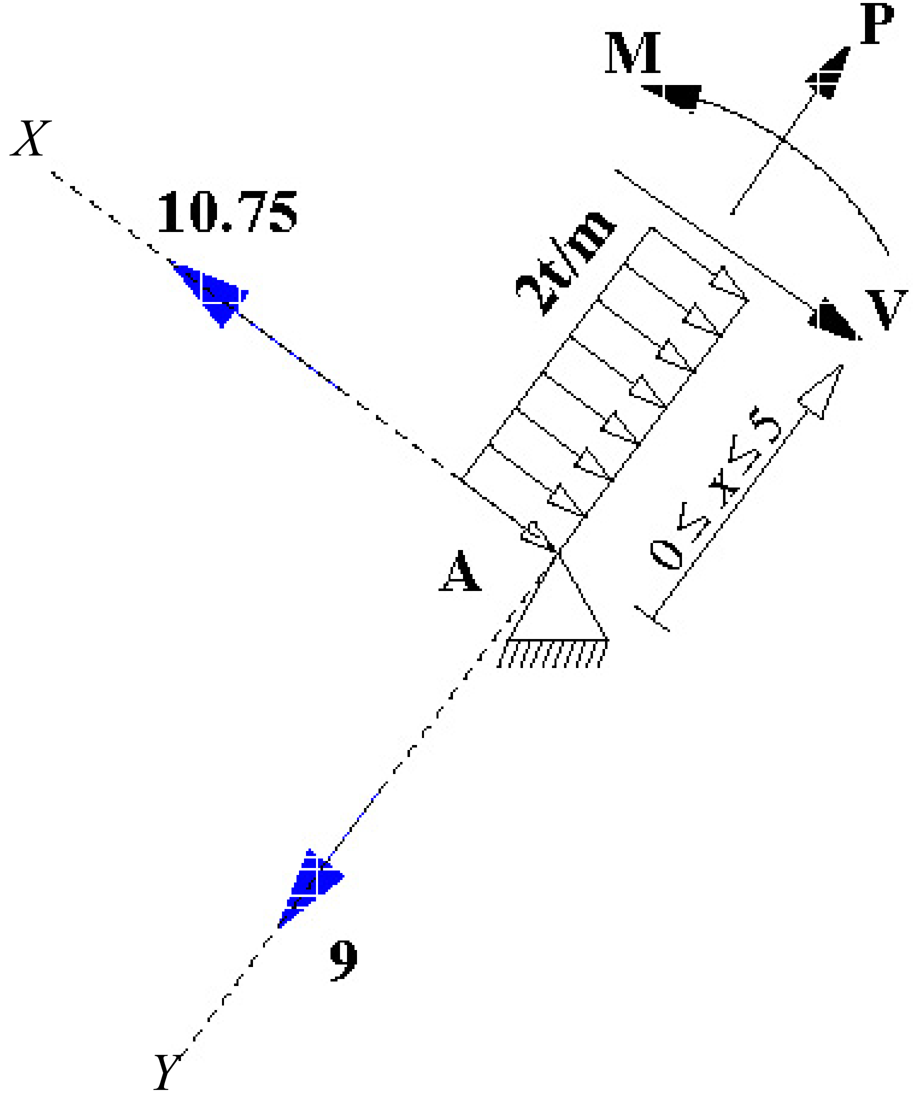


# Forces in Beams and Cables

## ❑ Sample Problem 05

SOLUTION:

(1)—(1) $0 \le x \le 5$




$$M = -x^2 + 10.75x$$

$$V = -2x + 10.75$$

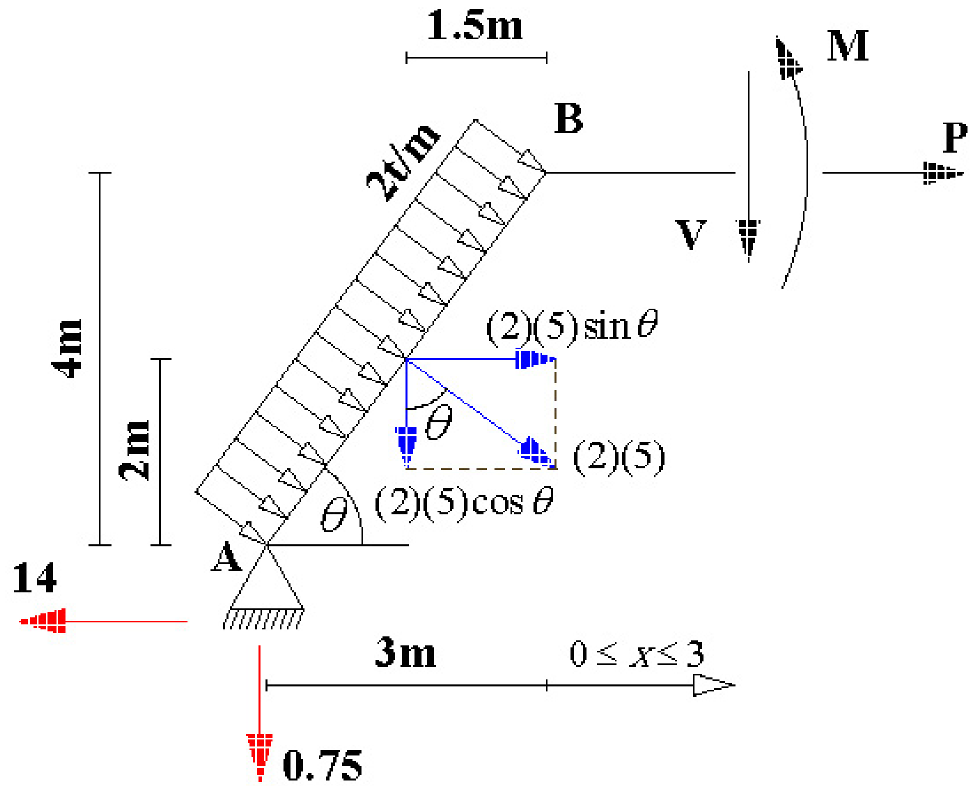
$$P = 9\,t$$

# Forces in Beams and Cables

## ❑ Sample Problem 05

SOLUTION:

(2)—(2) $0 \le x \le 3$




$$\Rightarrow \quad M = -6.75x + 28.75$$

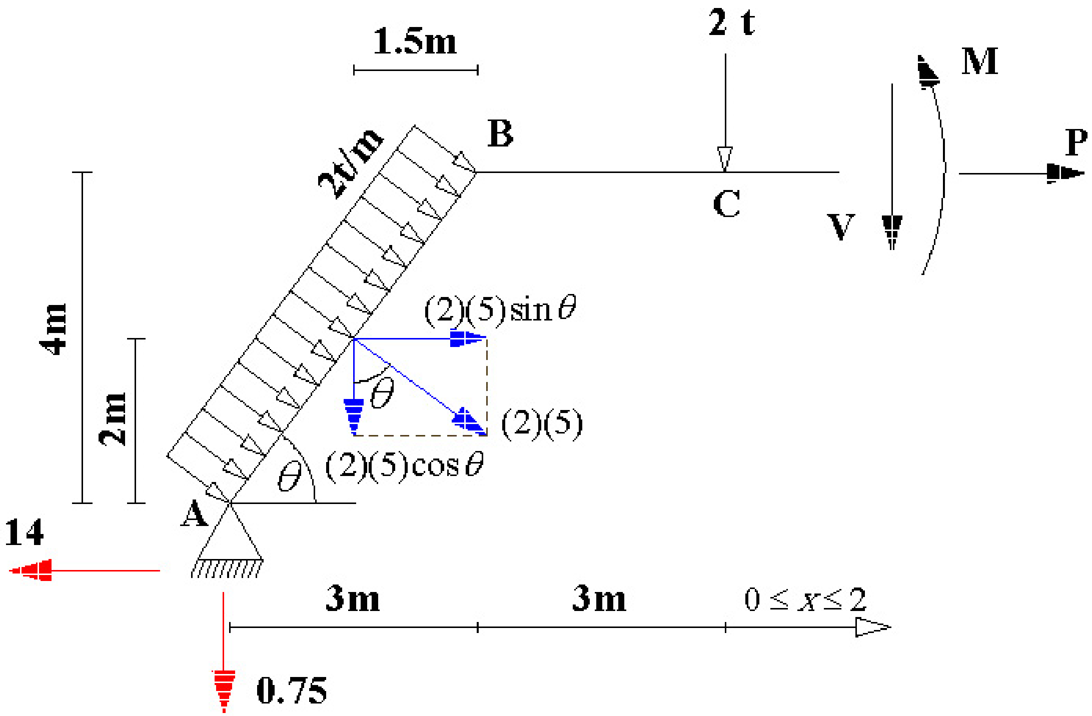
$$V = -6.75\, t$$

$$P = 6\, t$$

# Forces in Beams and Cables

## ❑ Sample Problem 05

SOLUTION:




3 — 3 $\quad 0 \le x \le 2$

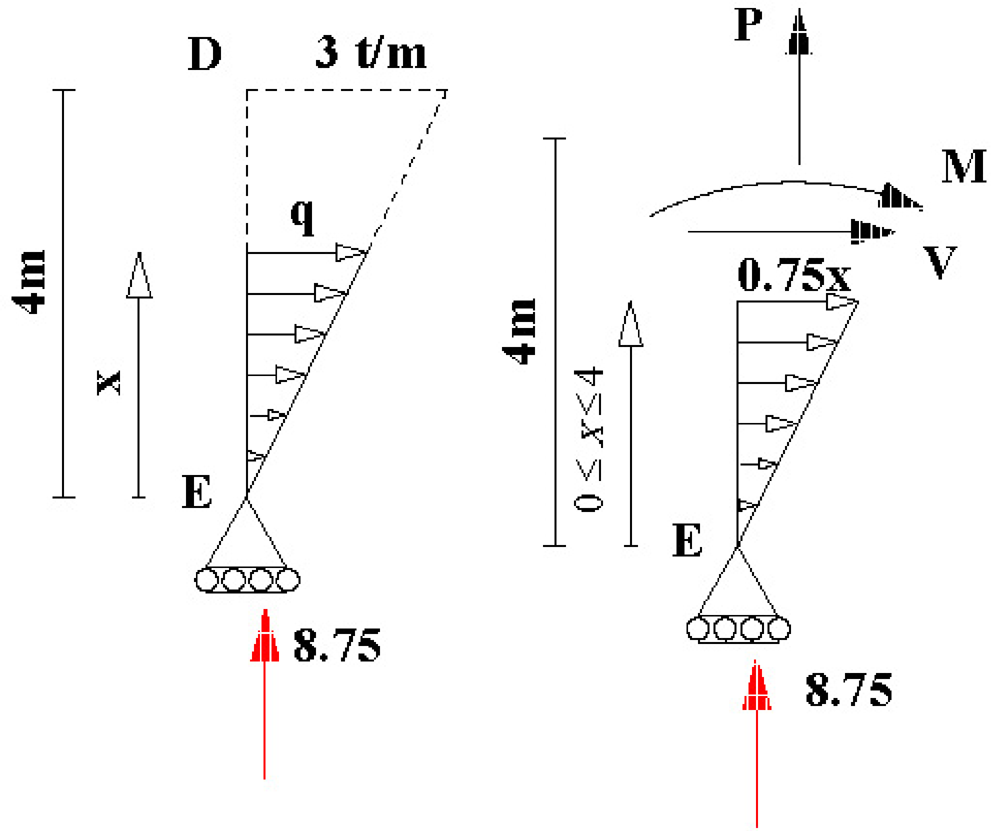
$$\Rightarrow \boxed{M = -8.75x + 8.5}$$

$$\boxed{V = -8.75\,t}$$

$$\boxed{P = 6\,t}$$

## ❑ Sample Problem 05

SOLUTION:




(4)—(4) $0 \le x \le 4$

$$q = 0.75x$$

$$M = 0.125x^3$$

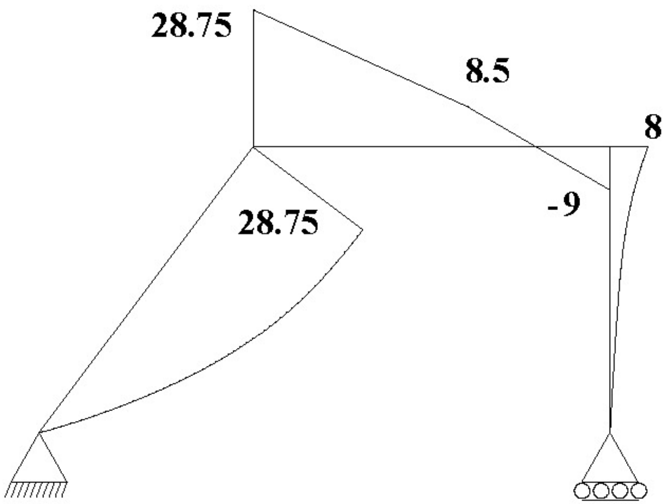
$$V = -0.375x^2$$

$$P = -8.75\,t$$

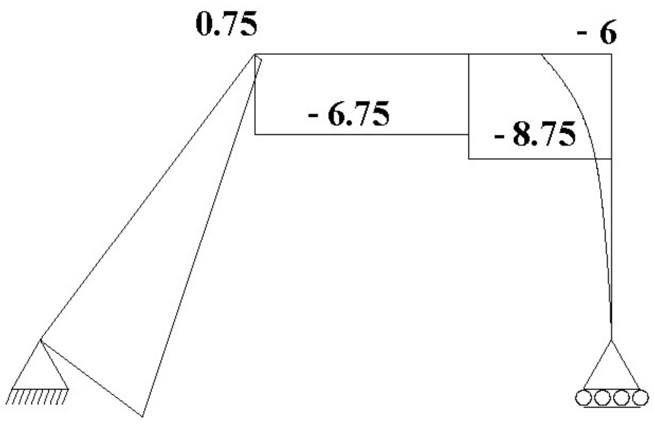
# Forces in Beams and Cables

## ❑ Sample Problem 05

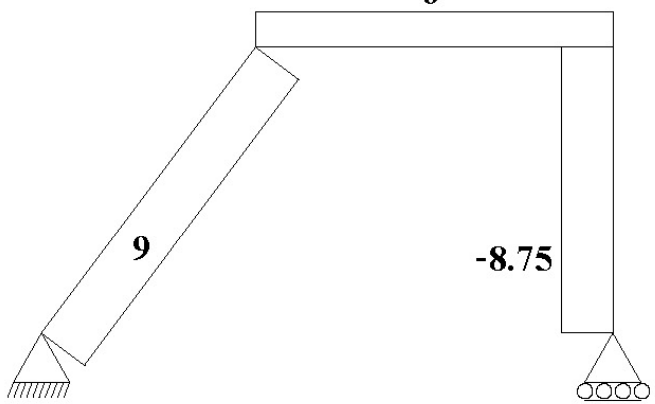
SOLUTION:




Moment Diagram (ton.m)




Shear Diagram (ton)

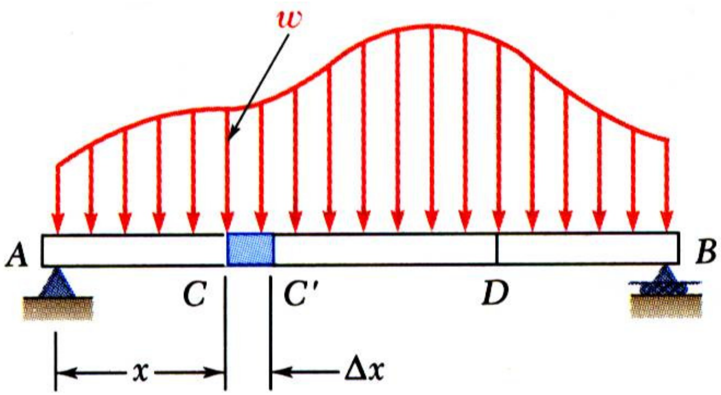



Axial Diagram (ton)

# Forces in Beams and Cables

## ❑ Relations Among Load, Shear, and Bending Moment

- Relations between load and shear:

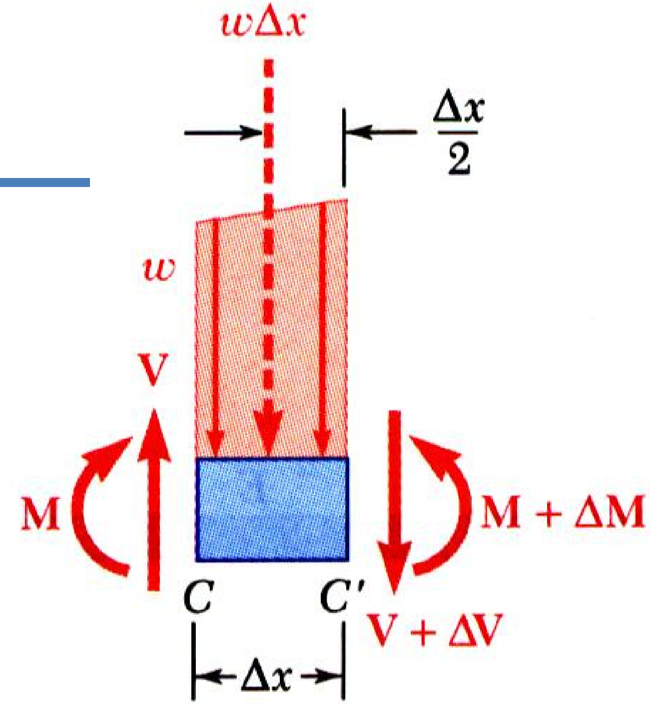



$$V-(V+\Delta V)-w\Delta x=0 \quad\Rightarrow\quad \frac{\Delta V}{\Delta x}=-w$$

$$\Rightarrow\quad \lim_{\Delta x\to 0}\frac{\Delta V}{\Delta x}=-w \quad\Rightarrow\quad \boxed{\frac{dV}{dx}=-w}$$

$$\boxed{V_D-V_C=-\int_{x_C}^{x_D} w\,dx=-(\text{area under load curve})}$$

- Relations between shear and bending moment:



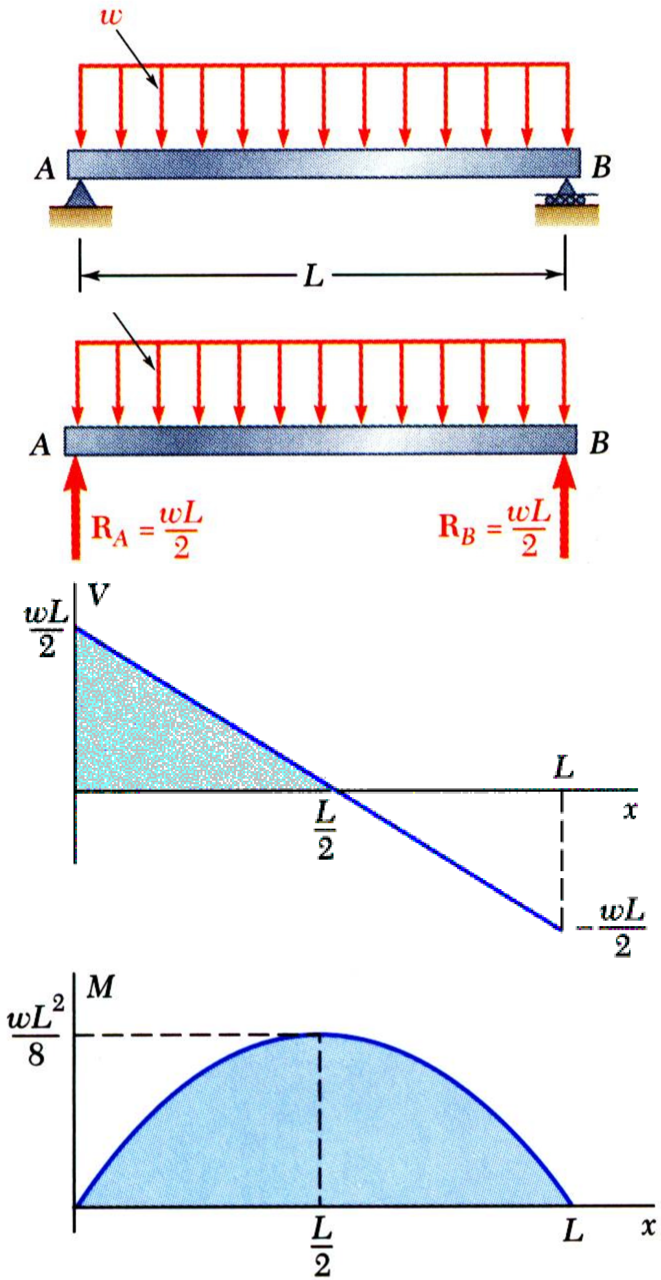

$$(M+\Delta M)-M-V\Delta x+w\Delta x\frac{\Delta x}{2}=0 \quad\Rightarrow\quad \frac{\Delta M}{\Delta x}=V-\frac{1}{2}w\Delta x$$

$$\lim_{\Delta x\to 0}\frac{\Delta M}{\Delta x}=\lim_{\Delta x\to 0}(V-\frac{1}{2}w\Delta x)=V \quad\Rightarrow\quad \boxed{\frac{dM}{dx}=V}$$

$$\boxed{M_D-M_C=\int_{x_C}^{x_D} V\,dx=(\text{area under shear curve})}$$

# Forces in Beams and Cables

## ❑ Relations Among Load, Shear, and Bending Moment




- Reactions at supports, $R_A = R_B = \frac{wL}{2}$
- Shear curve,

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

- Moment curve,

$$M - M_A = \int_0^x V dx$$

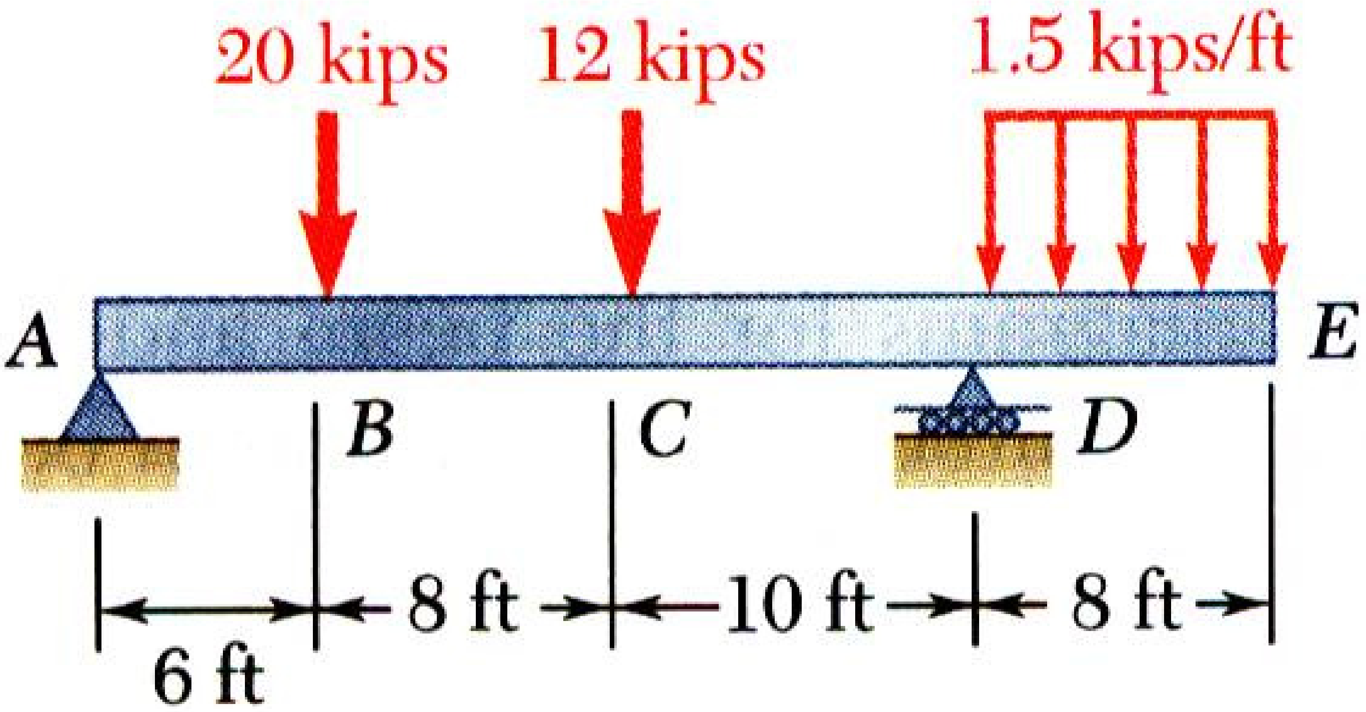
$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}(Lx - x^2)$$

$$M_{\max} = \frac{wL^2}{8} \quad \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

# Forces in Beams and Cables

## ❑ Sample Problem 06

Draw the shear and bending-moment diagrams for the beam and loading shown.

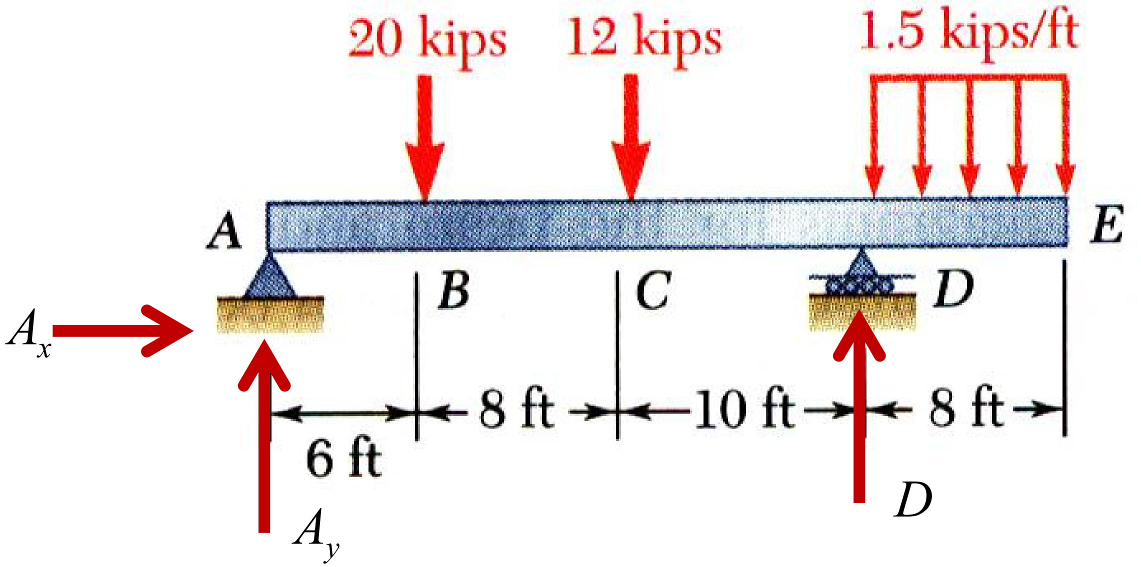


# Forces in Beams and Cables

## ❑ Sample Problem 06

SOLUTION:

- Taking entire beam as a free-body, determine reactions at supports.




$$\Rightarrow \boxed{D = 26 \text{ (kips)}}$$

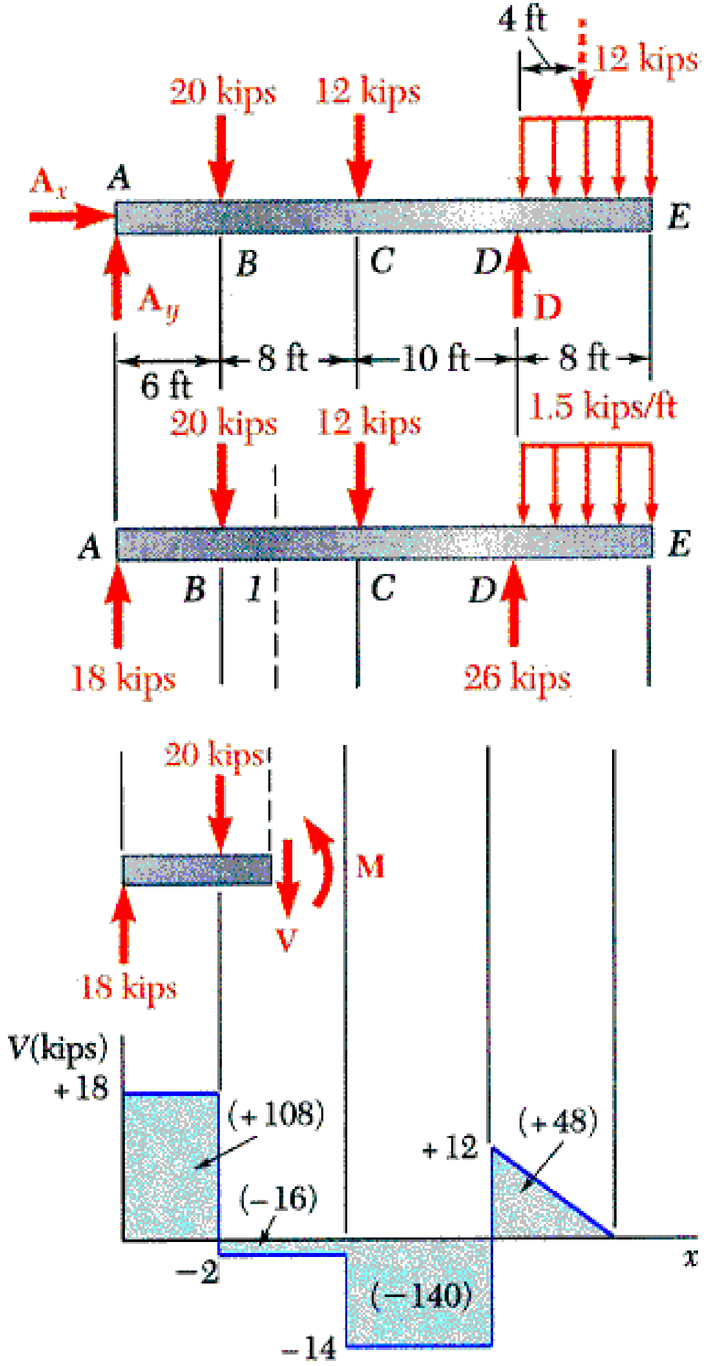
$$\Rightarrow \boxed{A_y = 18 \text{ (kips)}}$$

# Forces in Beams and Cables

## ❑ Sample Problem 06

SOLUTION:

- Between concentrated load application points, $dV/dx = -w = 0$ and shear is constant.
- With uniform loading between *D* and *E*, the shear variation is linear.



# Forces in Beams and Cables

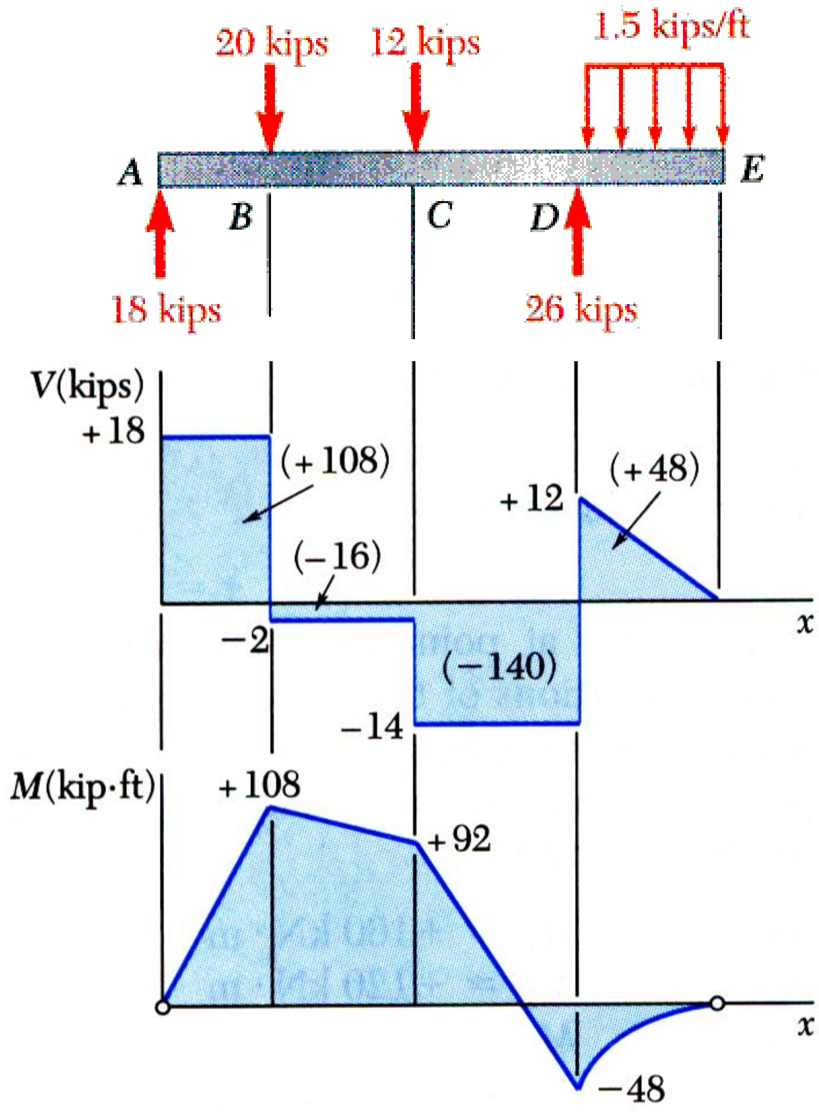
## ❑ Sample Problem 06

SOLUTION:

- Between concentrated load application points, $dM/dx = V = \text{constant}$. The change in moment between load application points is equal to area under the shear curve between points.

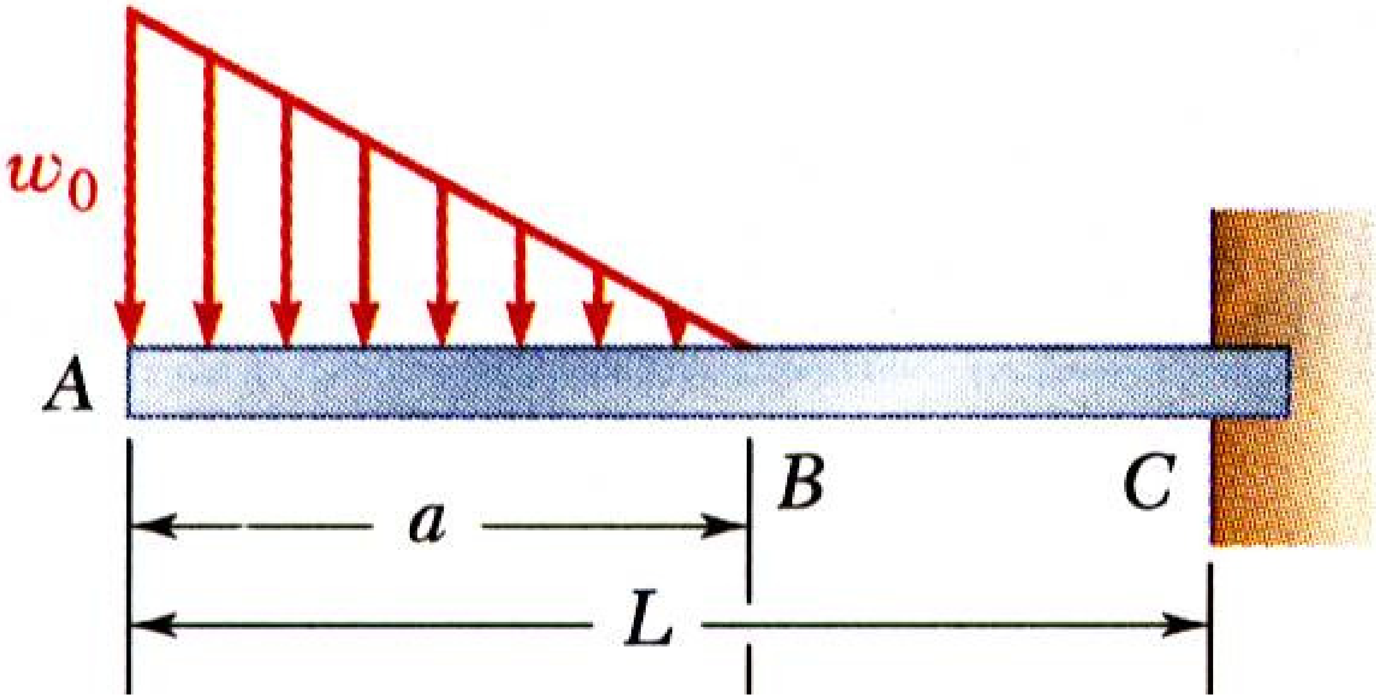
$$M_B = +108 \text{ kip} \cdot \text{ft}$$
$$M_C = +92 \text{ kip} \cdot \text{ft}$$
$$M_D = -48 \text{ kip} \cdot \text{ft}$$
$$M_E = 0$$

- With a linear shear variation between $D$ and $E$, the bending moment diagram is a parabola.



## ❑ Sample Problem 07

Sketch the shear and bending-moment diagrams for the cantilever beam and loading shown.

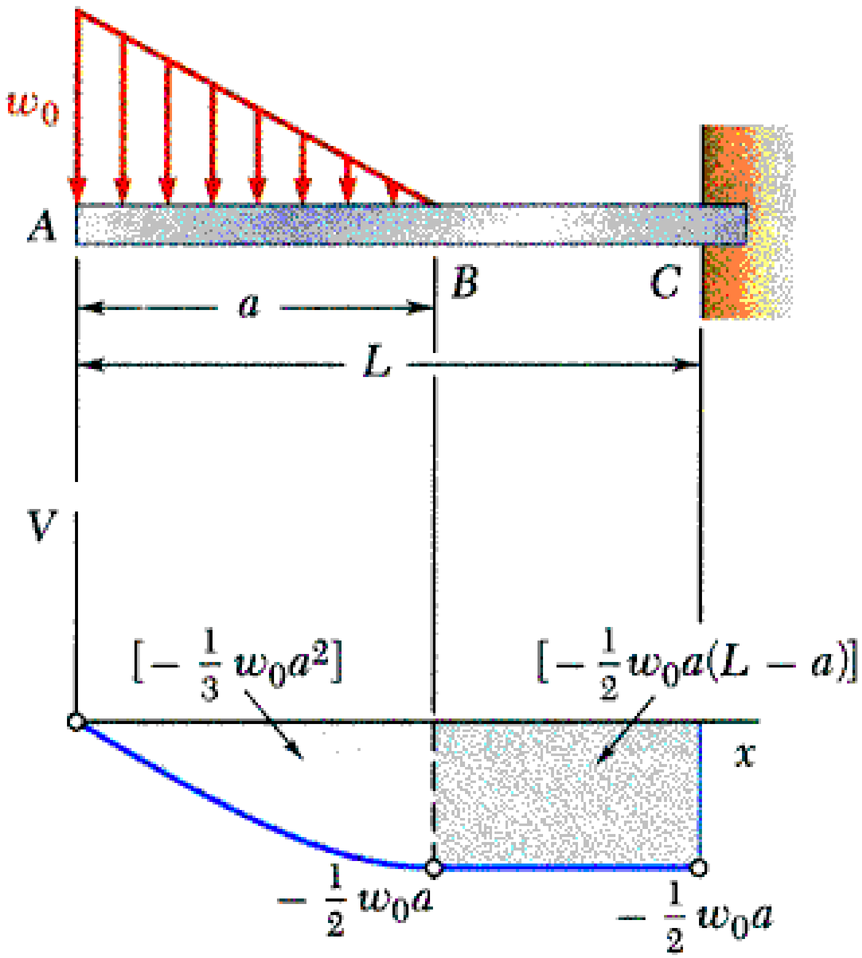


# Forces in Beams and Cables

## ❑ Sample Problem 07

SOLUTION:

- The change in shear between $A$ and $B$ is equal to negative of area under load curve between points. The linear load curve results in a parabolic shear curve.




$$V_B = -\tfrac{1}{2}w_0 a$$

- With zero load, change in shear between $B$ and $C$ is zero.

# Forces in Beams and Cables

## ❑ Sample Problem 07

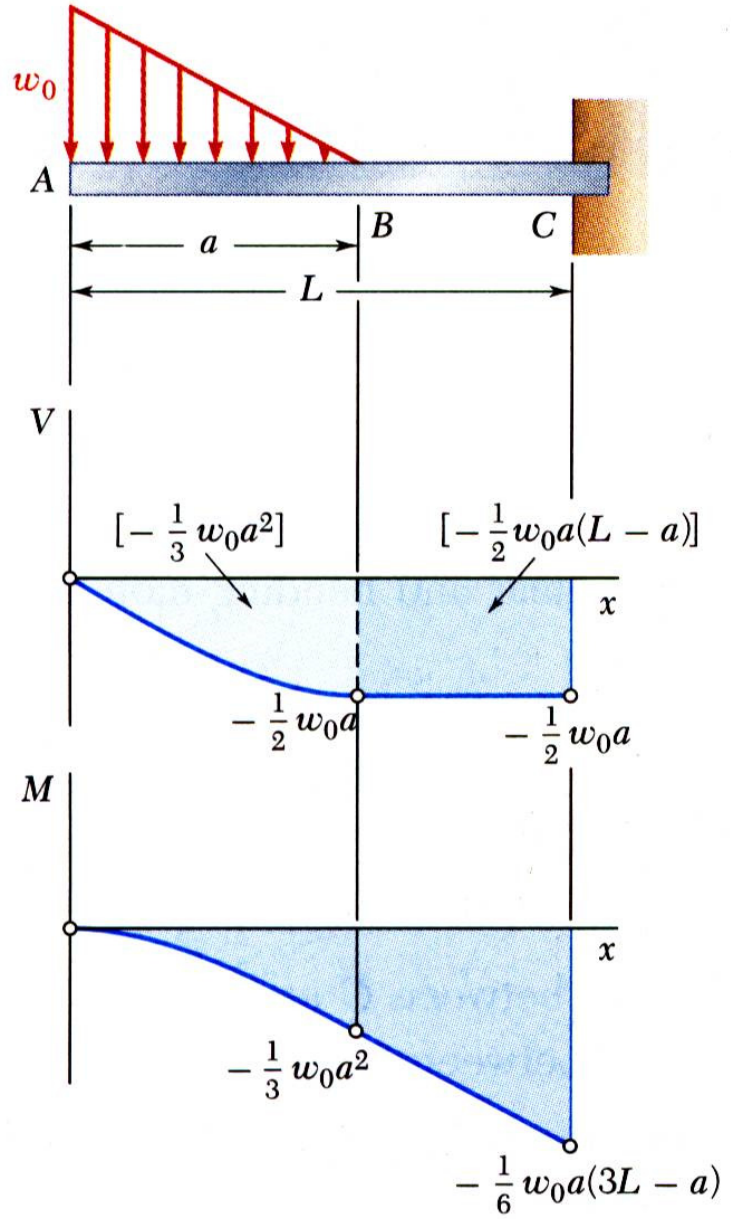
SOLUTION:

- The change in moment between $A$ and $B$ is equal to area under shear curve between the points. The parabolic shear curve results in a cubic moment curve.

$$M_B = -\frac{1}{3} w_0 a^2$$

$$M_C = -\frac{1}{6} w_0 a(3L - a)$$

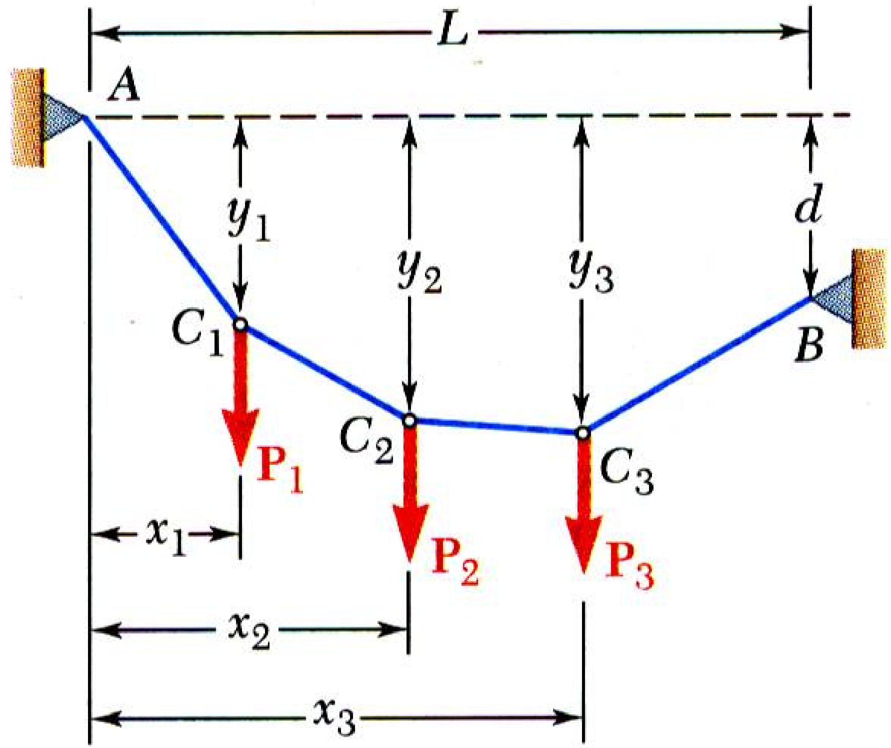
- The change in moment between $B$ and $C$ is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.



# Forces in Beams and Cables

## Cables With Concentrated Loads

- Cables are applied as structural elements in suspension bridges, transmission lines, aerial tramways, guy wires for high towers, etc.
- For analysis, assume:
  a) concentrated vertical loads on given vertical lines,
  b) weight of cable is negligible,
  c) cable is flexible, i.e., resistance to bending is small,
  d) portions of cable between successive loads may be treated as two force members

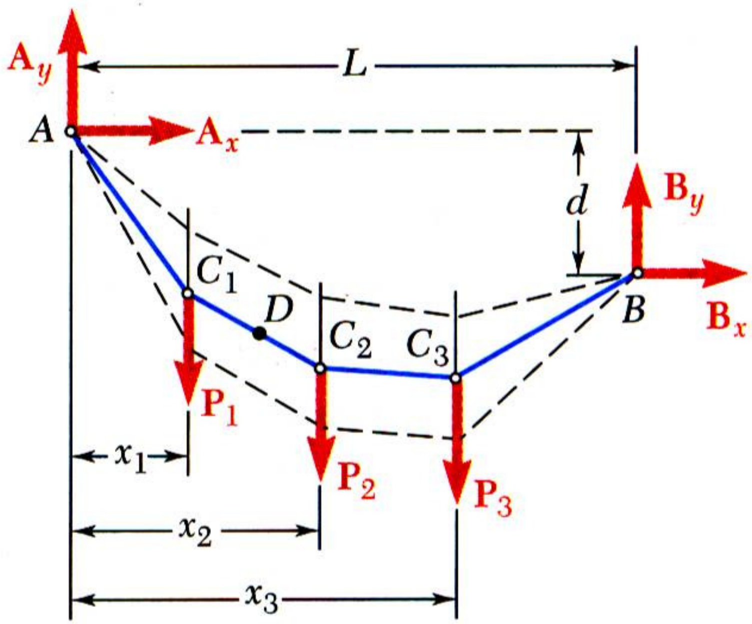



- Wish to determine ***shape of cable*** and ***vertical distance from support A*** to each load point.

# Forces in Beams and Cables

## ❑ Cables With Concentrated Loads

- Consider entire cable as free-body. Slopes of cable at $A$ and $B$ are not known - two reaction components required at each support.
- ***Four unknowns are involved and three equations of equilibrium are not sufficient to determine the reactions.***
- Additional equation is obtained by considering equilibrium of portion of cable $AD$ and assuming that coordinates of point $D$ on the cable are known. The additional equation is

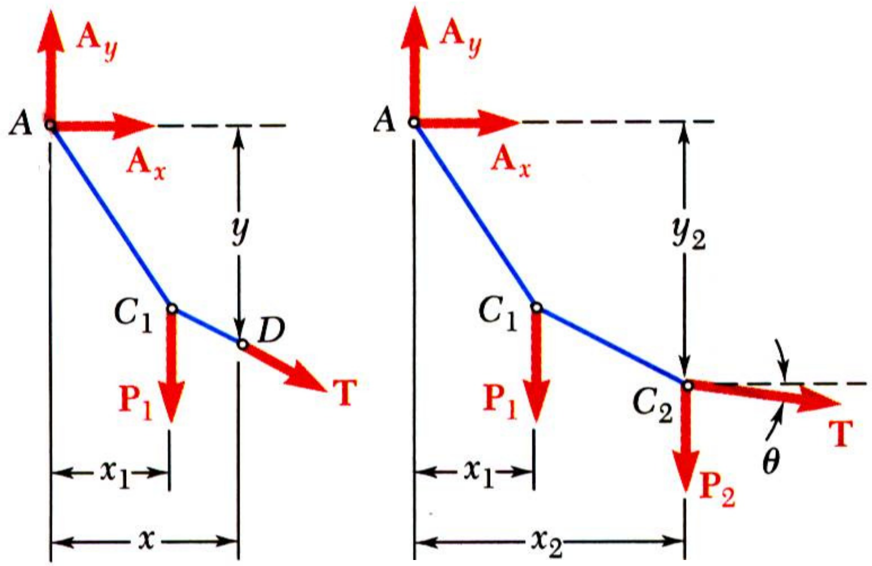



$$\sum M_D = 0 \quad \Rightarrow \quad \begin{matrix} A_x \\ A_y \end{matrix} \checkmark$$

- For other points on cable,

$$\sum M_{C_2} = 0 \quad \text{yields } y_2$$

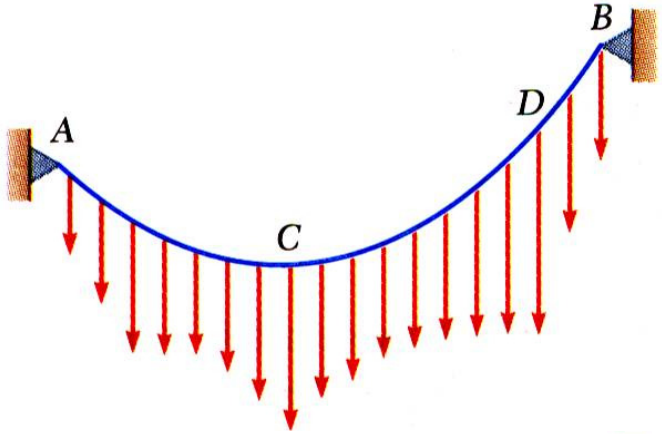
$$\sum F_x = 0, \sum F_y = 0 \;\; \text{yield } T_x, T_y$$



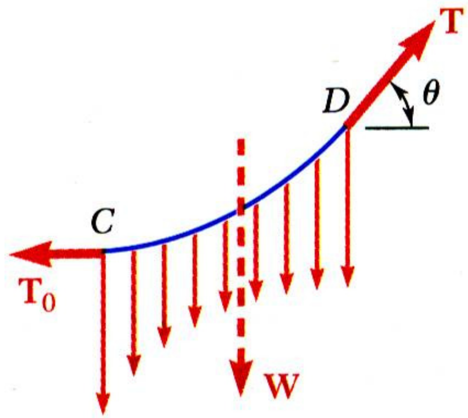

$$\boxed{T_x = T\cos\theta = A_x = \text{constant}}$$

# Forces in Beams and Cables

## ❑ Cables With Distributed Loads



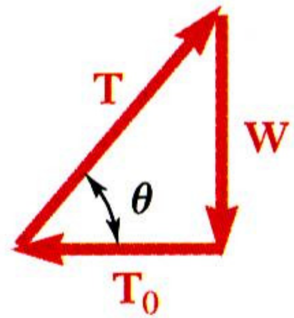

- For cable carrying a distributed load:
  - a) cable hangs in shape of a curve
  - b) internal force is a tension force directed along tangent to curve.
- Consider free-body for portion of cable extending from lowest point *C* to given point *D*. Forces are horizontal force $\boldsymbol{T_0}$ at C and tangential force $\boldsymbol{T}$ at *D*.




- From force triangle:

$$T\cos\theta = T_0 \qquad T\sin\theta = W$$

$$T = \sqrt{T_0^2 + W^2} \qquad \tan\theta = \frac{W}{T_0} \tag{1}$$



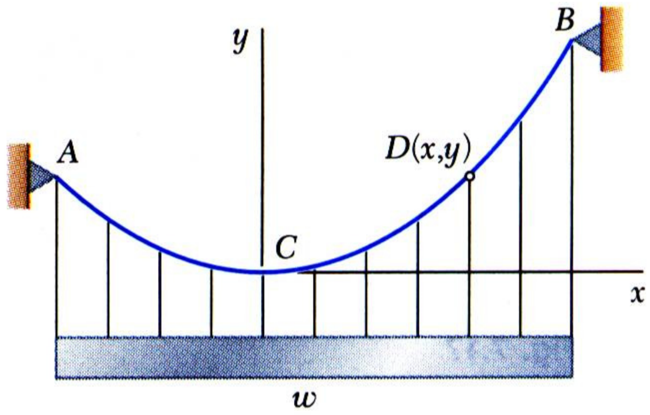

- Horizontal component of $\boldsymbol{T}$ is uniform over cable.
- Vertical component of $\boldsymbol{T}$ is equal to magnitude of $W$ measured from lowest point.
- **Tension is minimum at lowest point and maximum at A and B.**

$$\theta\uparrow \;\Rightarrow\; \cos\theta\downarrow \;\overset{T\cos\theta = T_0}{\Rightarrow}\; T\uparrow$$

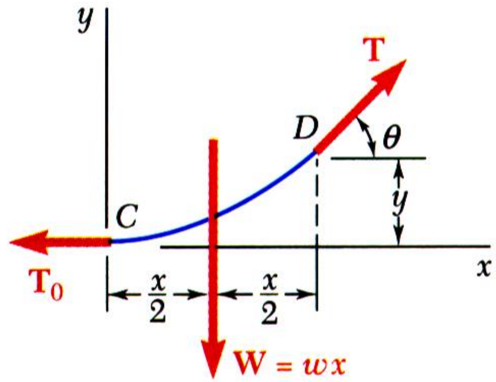
# Forces in Beams and Cables

## ❑ Parabolic Cable

- Consider a cable supporting a uniform, horizontally distributed load, e.g., support cables for a suspension bridge.




- With loading on cable from lowest point $C$ to a point $D$ given by $W = wx$, internal tension force magnitude and direction are




(1) $\overset{W=wx}{\Rightarrow}$ $$T = \sqrt{T_0^2 + w^2x^2} \qquad \tan\theta = \frac{wx}{T_0}$$ (2)

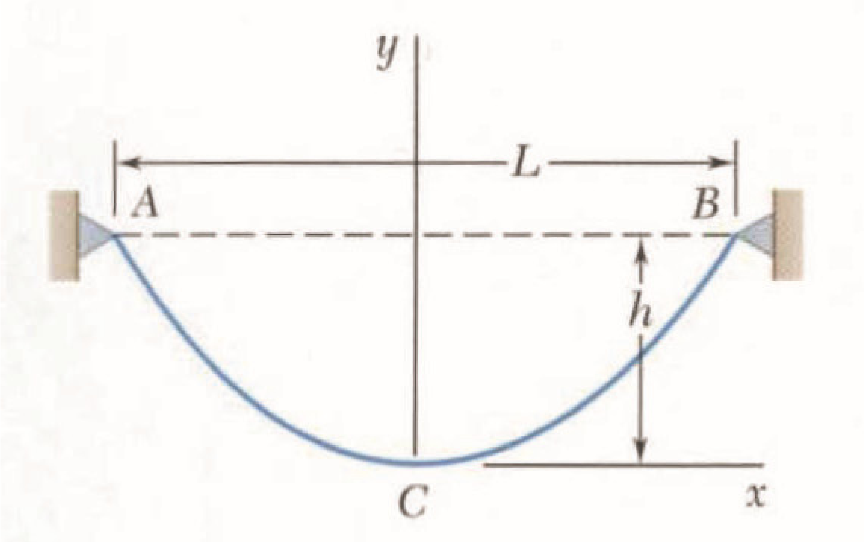
- Summing moments about $D$,

$$\sum M_D = 0 \;\Rightarrow\; wx\frac{x}{2} - T_0 y = 0 \;\Rightarrow\; y = \frac{wx^2}{2T_0}$$

The cable forms a parabolic curve.

## ❑ Parabolic Cable

When the supports *A* and B of the cable have the same elevation, the distance L between the supports is called the ***span*** of the cable and the vertical distance *h* from the supports to the lowest point is called the ***sag*** of the cable




$$if \quad h \,\&\, L \quad is\ known \quad \Rightarrow \begin{cases} x = L/2 \\ y = h \end{cases} \Rightarrow$$

$$h = \frac{w(L/2)^2}{2T_0} \quad \Rightarrow \quad \boxed{T_0 = \frac{wL^2}{8h}}$$

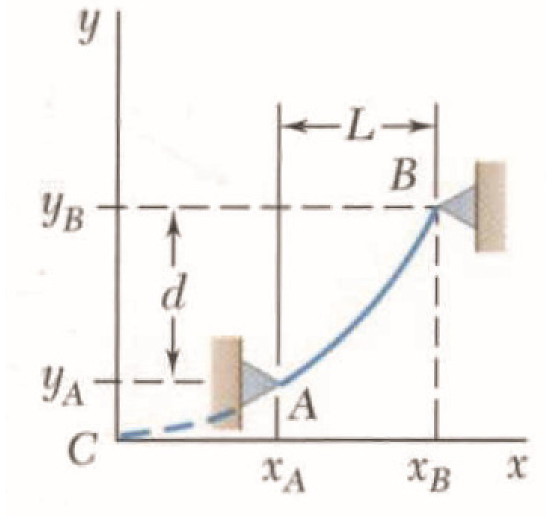
$$(2) \quad \Rightarrow \quad \boxed{T = \sqrt{\left(\frac{wL^2}{8h}\right)^2 + w^2x^2} \qquad \tan\theta = \frac{8h}{L^2}x}$$

$$y = \frac{wx^2}{2T_0} \quad \overset{T_0}{\Rightarrow} \quad y = \frac{wx^2}{2\left(\frac{wL^2}{8h}\right)} \quad \Rightarrow \quad \boxed{y = \frac{4h}{L^2}x^2}$$

# Forces in Beams and Cables

## ❑ Parabolic Cable

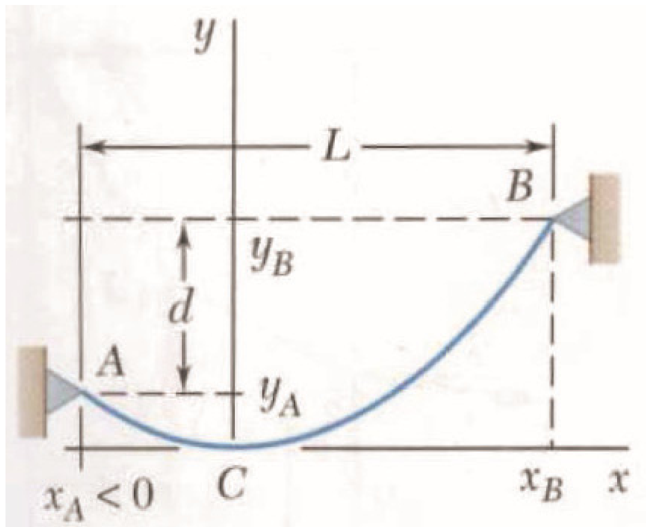
When the supports have different elevations, the position of the lowest point of the cable is not known and the coordinates Point A and B should be determined.




$A(x_A\,,\,y_A)$
$B(x_B\,,\,y_B)$ $\Rightarrow$ $y=\dfrac{wx^2}{2T_0}$ $\Rightarrow$ Should be satisfied then we have 2 equations

$x_B - x_A = L$
$y_B - y_A = d$ 2 equations

4 unknown 4 equations $\Rightarrow$ $x_A\,,\,y_A$ $x_B\,,\,y_B$ ✓



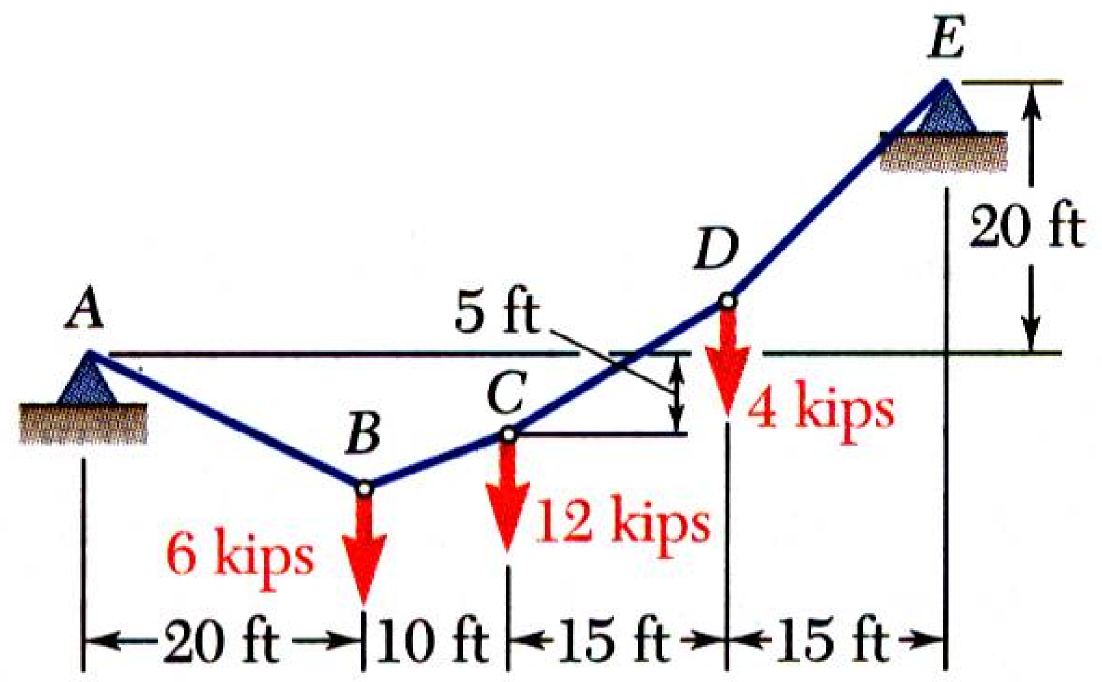

Length of the cable from C to B

$$S_B = x_B\left[1+\frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right] \qquad if \quad \frac{y_B}{x_B} < 0.5$$

# Forces in Beams and Cables

## ❑ Sample Problem 08

The cable *AE* supports three vertical loads from the points indicated. If point *C* is 5 ft below the left support, determine *(a)* the elevation of points *B* and *D*, and *(b)* the maximum slope and maximum tension in the cable.

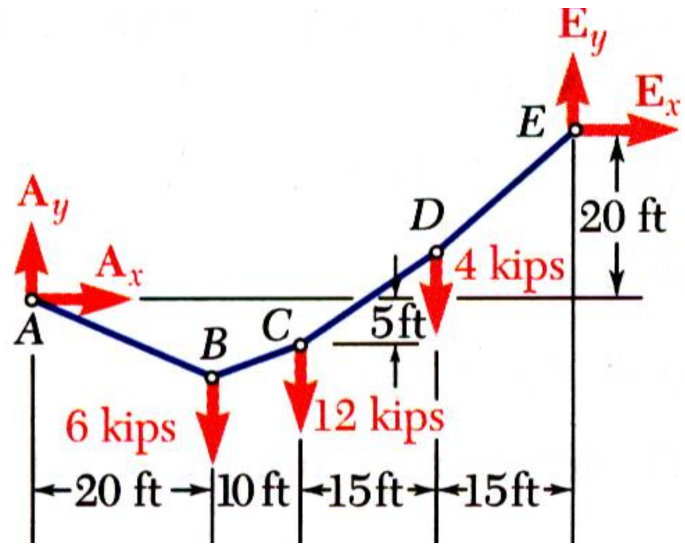


# Forces in Beams and Cables

## ❑ Sample Problem 08

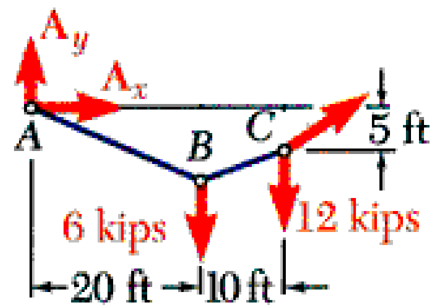
SOLUTION:

- Determine two reaction force components at $A$ from solution of two equations formed from taking entire cable as a free-body and summing moments about $E$,




$$\Rightarrow \quad A_x(20) - A_y(60) + 660 = 0 \qquad (I)$$

and from taking cable portion $ABC$ as a free-body and summing moments about $C$.




$$\qquad (II)$$

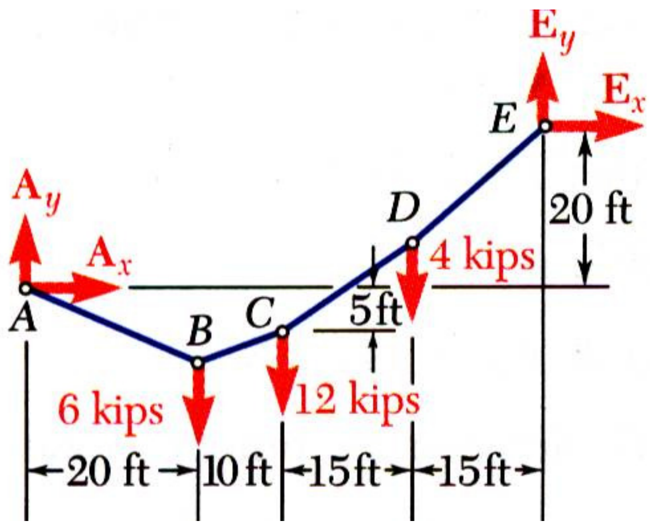
Solving simultaneously,

$$(I)\ \&\ (II) \quad \Rightarrow \quad A_x = -18\ \text{(kips)} \quad , \quad A_y = 5\ \text{(kips)}$$

# Forces in Beams and Cables

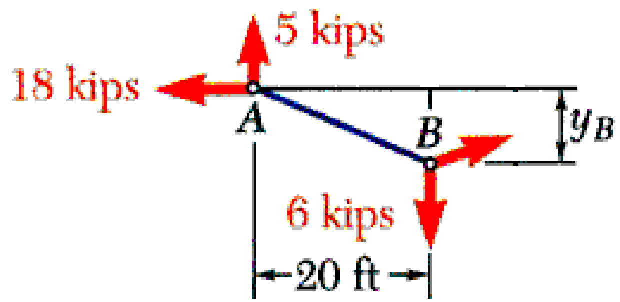
## ❑ Sample Problem 08

SOLUTION:

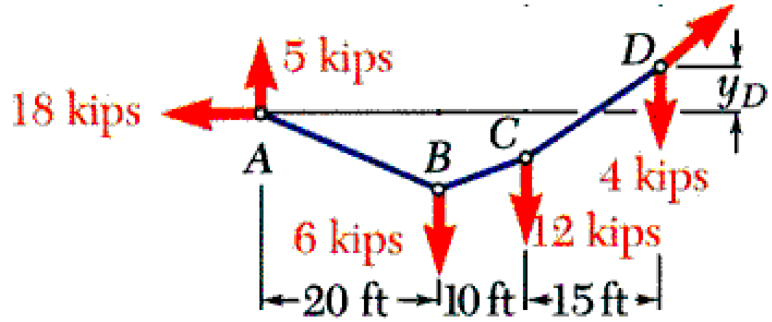



- Calculate elevation of $B$ by considering $AB$ as a free-body and summing moments $B$.

$$\Rightarrow \boxed{y_B = -5.56\ (\text{ft})}$$




Similarly, calculate elevation of $D$ using $ABCD$ as a free-body.



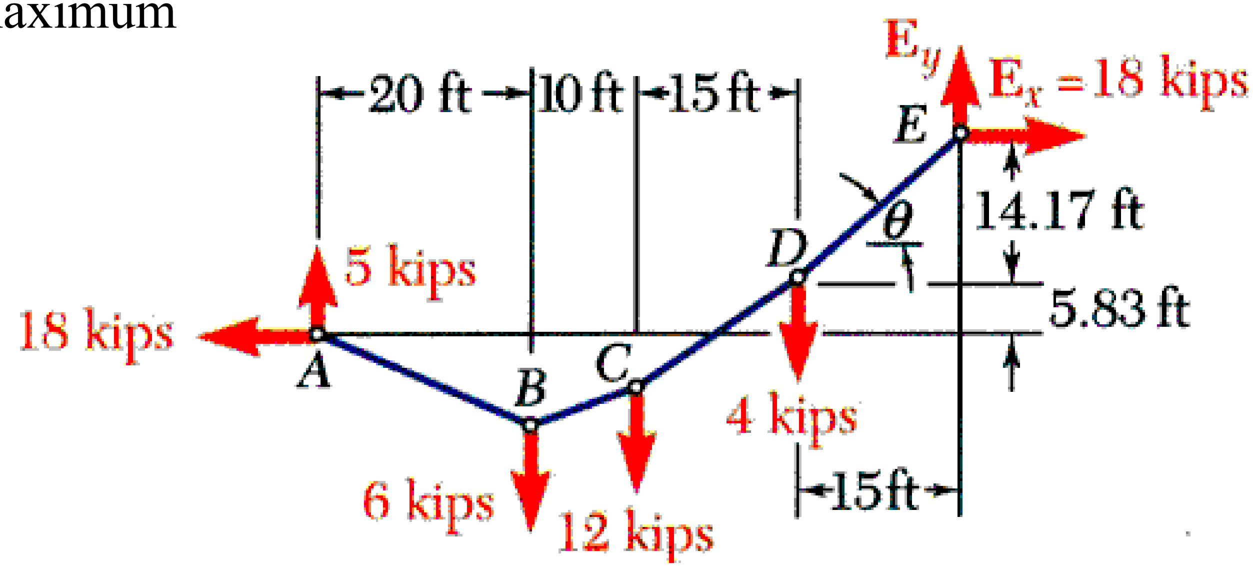

$$\Rightarrow \boxed{y_D = 5.83\ (\text{ft})}$$

# Forces in Beams and Cables

## ❑ Sample Problem 08

SOLUTION:

- Evaluate maximum slope and maximum tension which occur in *DE*.

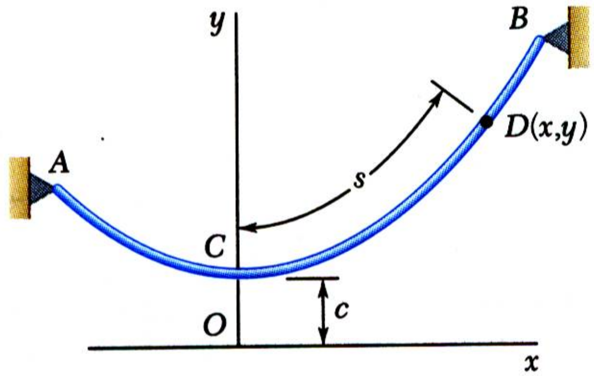



$$\theta = 43.4°$$

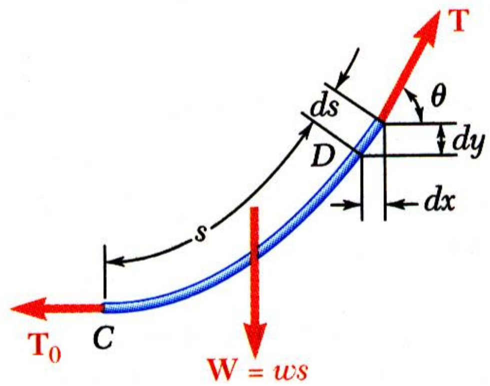
$$T_{max} = 24.8 \text{ kips}$$

# Forces in Beams and Cables

## ❑ Catenary




- Consider a cable uniformly loaded along the cable itself, e.g., cables hanging under their own weight.



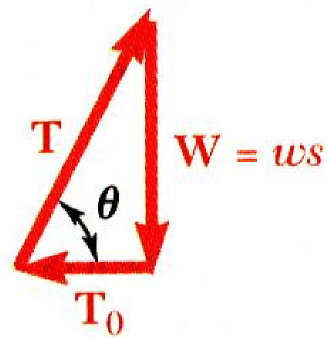

- With loading on the cable from lowest point $C$ to a point $D$ given by $W = ws$, the internal tension force magnitude is

$$T = \sqrt{T_0^2 + w^2 s^2} = w\sqrt{c^2 + s^2} \qquad c = T_0 / w \qquad (I)$$

- To relate horizontal distance $x$ to cable length $s$,

$$dx = ds \cos\theta = ds \frac{T_0}{T} = ds \frac{wc}{w\sqrt{c^2 + s^2}} \quad \Rightarrow \quad dx = \frac{ds}{\sqrt{1 + s^2/c^2}}$$

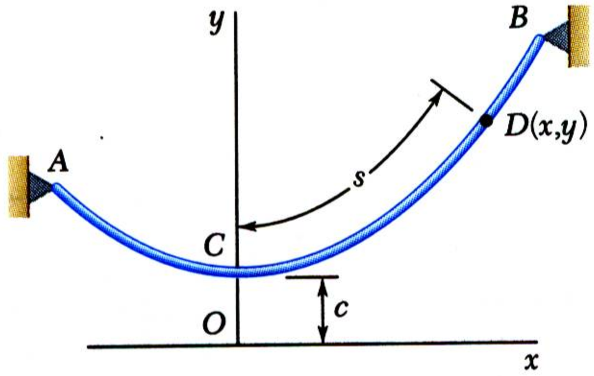
$$\Rightarrow \quad x = \int_0^s \frac{ds}{\sqrt{1 + s^2/c^2}} = c \sinh^{-1} \frac{s}{c} \quad \Rightarrow \quad s = c \sinh \frac{x}{c} \qquad (II)$$



# Forces in Beams and Cables

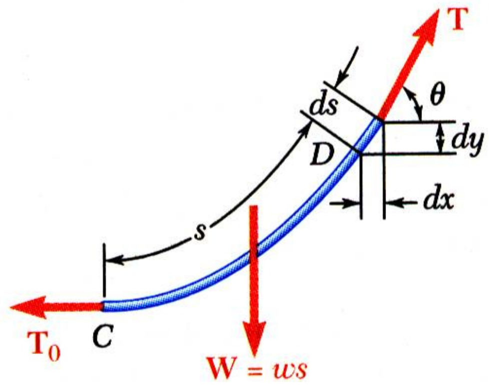
## ❑ Catenary

- To relate $x$ and $y$ cable coordinates,




$$dy = dx \tan\theta = dx\frac{W}{T_0} = dx\frac{ws}{wc} = dx\frac{s}{c} \quad \Rightarrow \quad dy = \sinh\frac{x}{c}\,dx$$

$$y - c = \int_0^x \sinh\frac{x}{c}\,dx = c\cosh\frac{x}{c} - c \quad \Rightarrow \quad \boxed{y = c\cosh\frac{x}{c}} \quad (III)$$




which is the equation of a catenary.

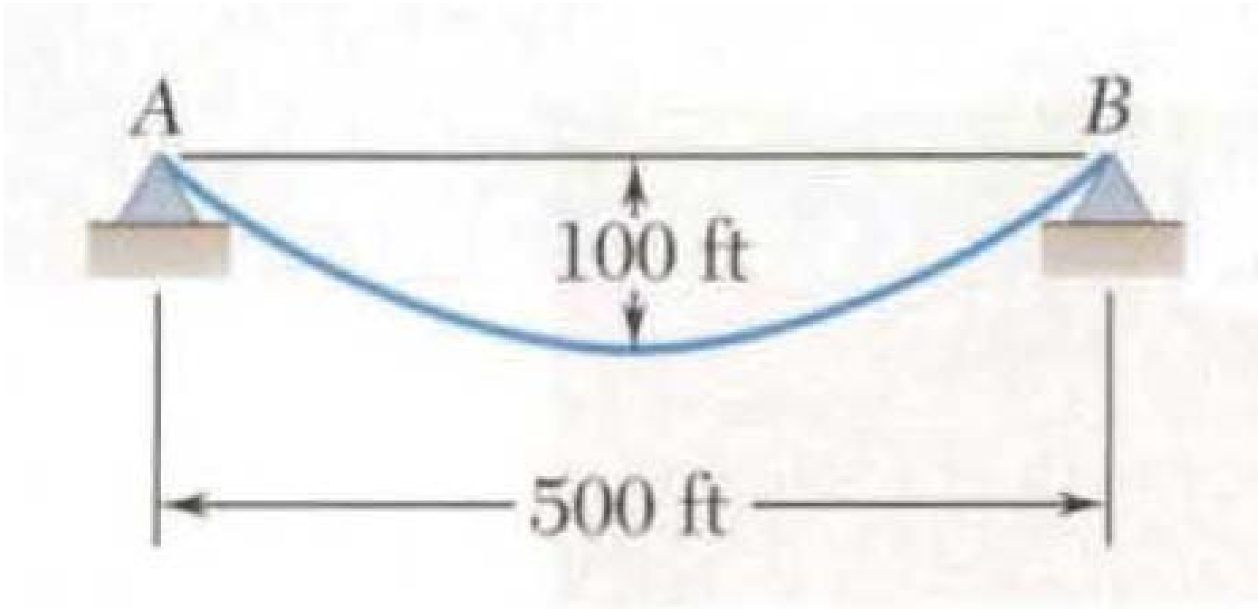
$$(II)\ \&\ (III) \quad \Rightarrow \quad \boxed{y^2 - s^2 = c^2} \quad (IV)$$

$$(I)\ \&\ (IV) \quad \Rightarrow \quad \boxed{T_0 = wc \quad , \quad W = ws \quad , \quad T = wy}$$

# Forces in Beams and Cables

## ❑ Sample Problem 09

A uniform cable weighing 3 lb/ft is suspended between two points A and B as shown. Determine (a) the maximum ,and minimum values of the tension in the cable, (b) the length of the cable.

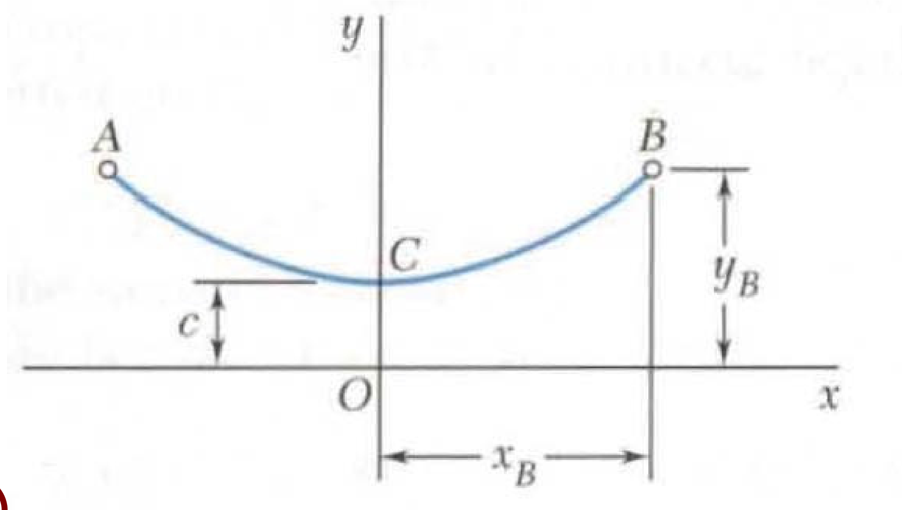


# Forces in Beams and Cables

## ❑ Sample Problem 09

SOLUTION:

**Equation of Cablc.** The origin of coordinates is placed at a distance $c$ below the lowest point of the cable. The equation of the cable is given by




The coordinates of point Bare

$$\frac{100}{c}+1=\cosh\frac{250}{c}$$

# Forces in Beams and Cables

## ❑ Sample Problem 09

SOLUTION:

The value of e is determined by assuming successive trial values, as shown in the following table:

$$\frac{100}{c} + 1 = \cosh\frac{250}{c}$$

| $c$ | $\frac{250}{c}$ | $\frac{100}{c}$ | $\frac{100}{c} + 1$ | $\cosh\frac{250}{c}$ |
|---|---|---|---|---|
| 300 | 0.833 | 0.333 | 1.333 | 1.367 |
| 350 | 0.714 | 0.286 | 1.286 | 1.266 |
| 330 | 0.758 | 0.303 | 1.303 | 1.301 |
| 328 | 0.762 | 0.305 | 1.305 | 1.305 |

# Forces in Beams and Cables

## Sample Problem 09

SOLUTION:

Maximum and minimum value of the Tension

$$T_{\min} = 984\ (lb)$$
$$T_{\max} = 1284\ (lb)$$

Length of Cable

$$s_{CB} = 275\ (ft)$$

$$s_{AB} = 550\ (ft)$$