

STATICS



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- Vector Mechanics for Engineers: Statics, 9th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Statics, 5th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self“Lectures notes on Statics”

Equilibrium of Rigid Bodies

By: Kaveh Karami

Associate Prof. of Structural Engineering

<https://prof.uok.ac.ir/Ka.Karami>

Equilibrium of Rigid Bodies

□ Introduction

- For a rigid body in static equilibrium, the external forces and moments are balanced and will impart no translational or rotational motion to the body.
- The *necessary* and *sufficient* condition for the static equilibrium of a body are that the *resultant force and couple from all external forces form a system equivalent to zero*,

$$\sum \vec{F} = 0$$

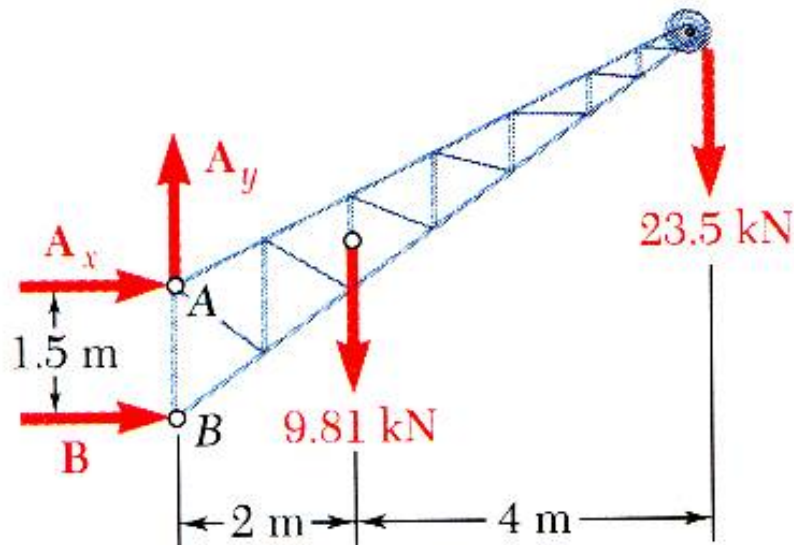
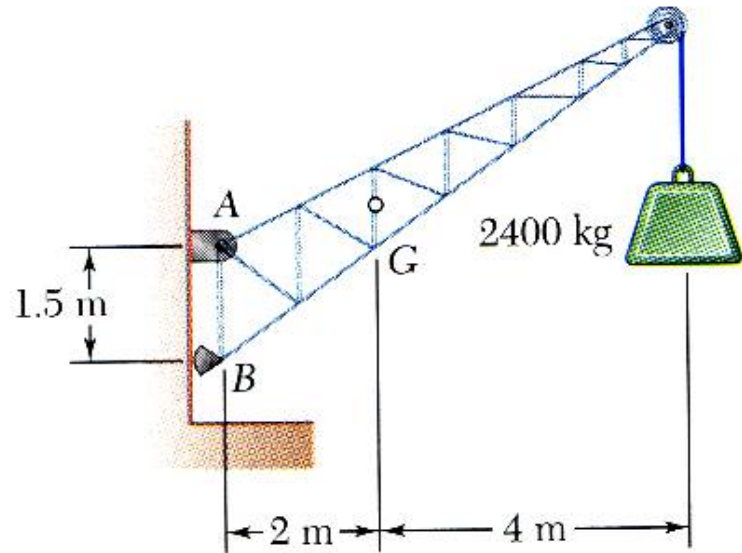
$$\sum \vec{M}_O = \sum (\vec{r} \times \vec{F}) = 0$$

- Resolving each force and moment into its rectangular components leads to 6 scalar equations which also express the conditions for static equilibrium,

$$\begin{array}{lll} \sum F_x = 0 & \sum F_y = 0 & \sum F_z = 0 \\ \sum M_x = 0 & \sum M_y = 0 & \sum M_z = 0 \end{array}$$

Equilibrium of Rigid Bodies

□ Free-Body Diagram



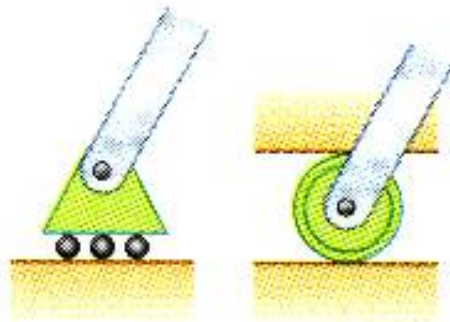
First step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free-body* diagram.

- Select the extent of the free-body and detach it from the ground and all other bodies.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown applied forces. These usually **consist of reactions** through which the ground and other bodies oppose the possible motion of the rigid body.
- Include the dimensions necessary to compute the moments of the forces.

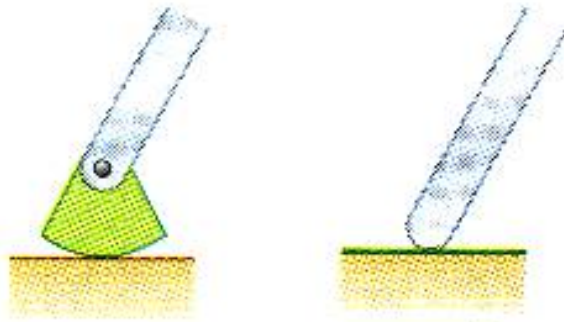
Equilibrium of Rigid Bodies

□ Reactions at Supports and Connections for a Two-Dimensional Structure

- Reactions equivalent to a force with known line of action.



Rollers



Rocker

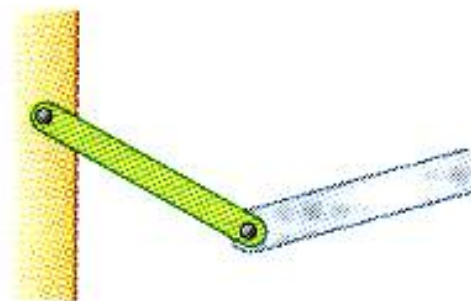
Frictionless surface



Force with known line of action



Short cable



Short link

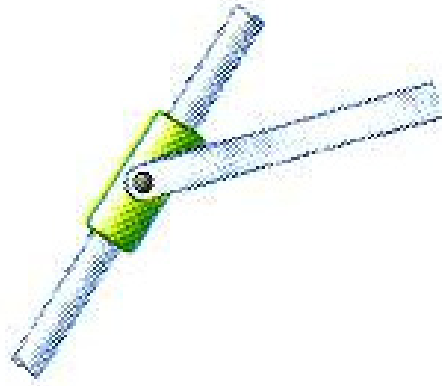


Force with known line of action

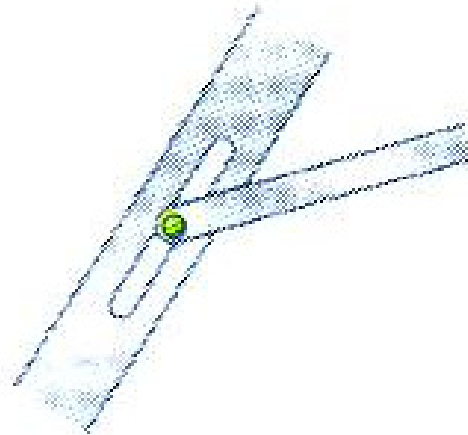
Equilibrium of Rigid Bodies

□ Reactions at Supports and Connections for a Two-Dimensional Structure

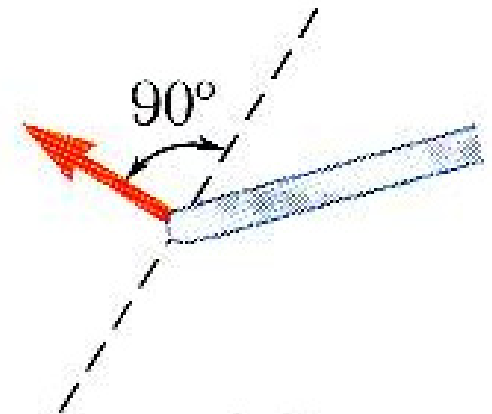
- Reactions equivalent to a force with known line of action.



Collar on
frictionless rod



Frictionless pin in slot

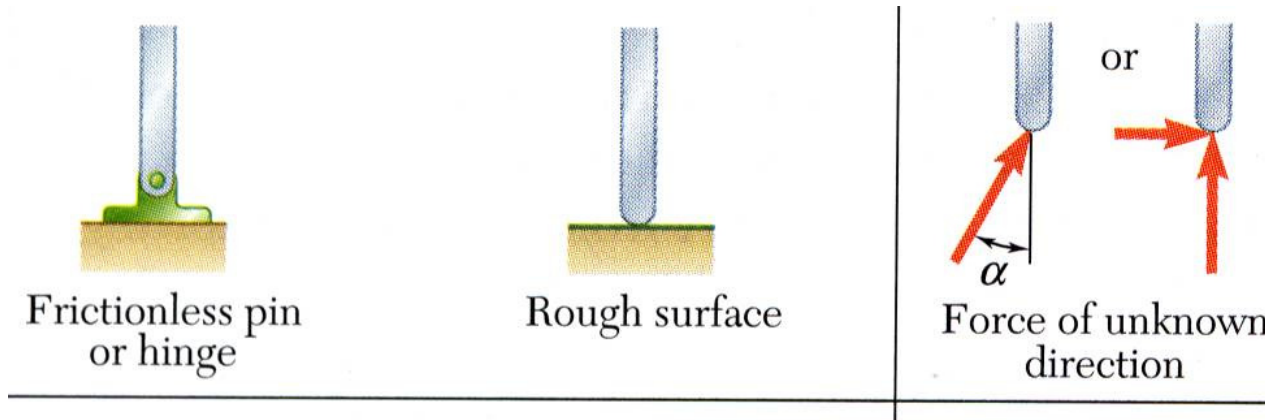


Force with known
line of action

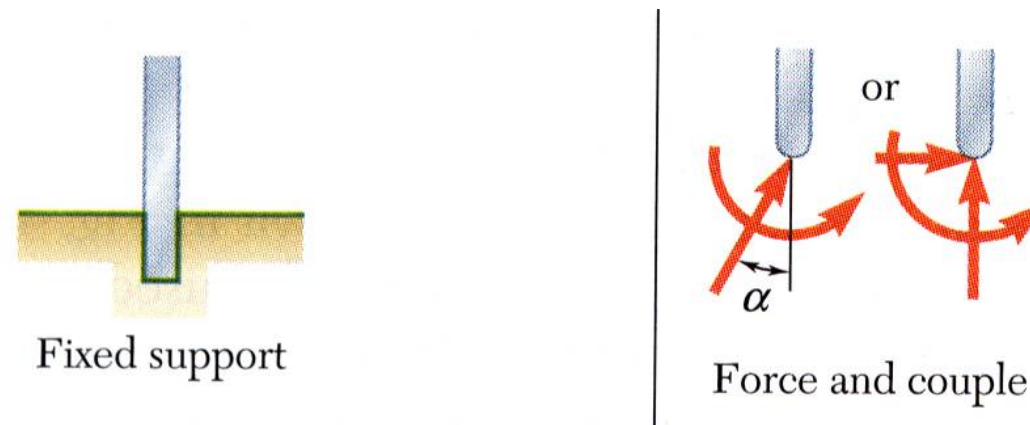
Equilibrium of Rigid Bodies

□ Reactions at Supports and Connections for a Two-Dimensional Structure

- Reactions equivalent to a force of unknown direction and magnitude.



- Reactions equivalent to a force of unknown direction and magnitude and a couple of unknown magnitude



Equilibrium of Rigid Bodies

□ Equilibrium of a Rigid Body in Two Dimensions

- For all forces and moments acting on a two-dimensional structure,

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

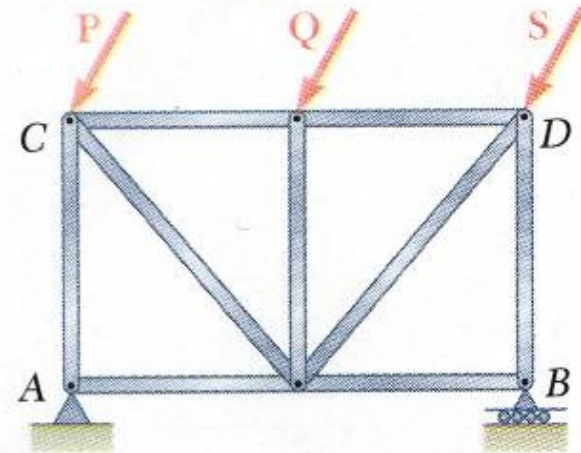
- Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

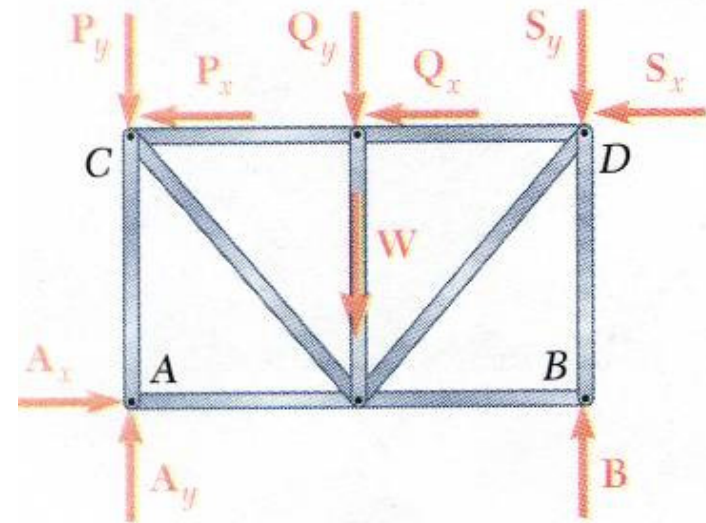
where A is any point in the plane of the structure.

- The 3 equations can be solved for no more than 3 unknowns.
- The 3 equations can not be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$



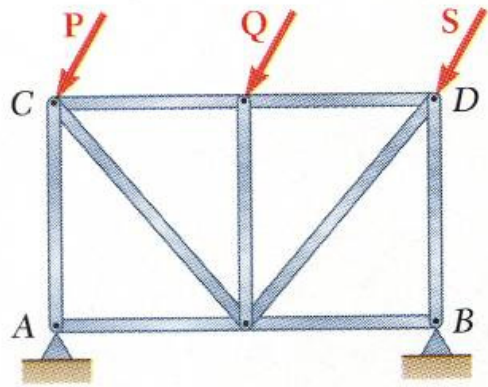
(a)



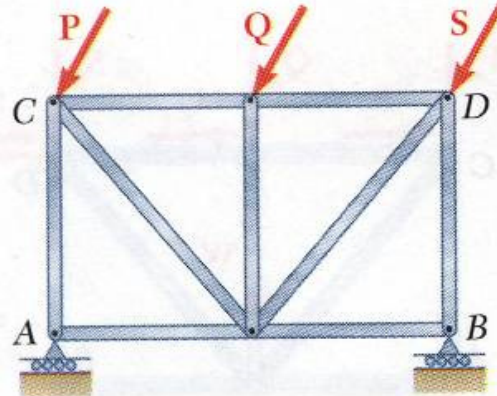
(b)

Equilibrium of Rigid Bodies

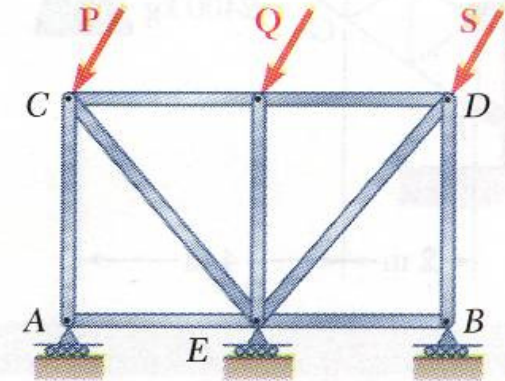
□ Statically Indeterminate Reactions



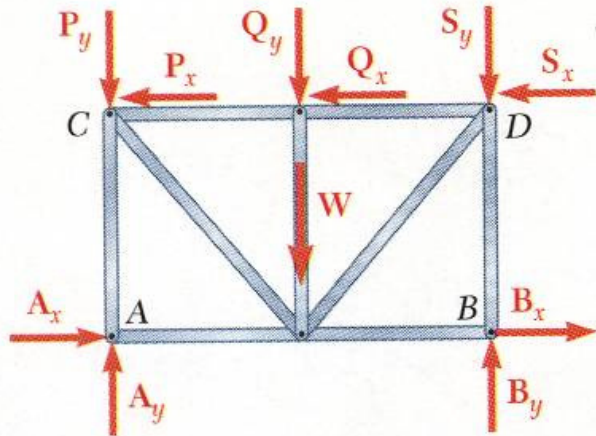
(a)



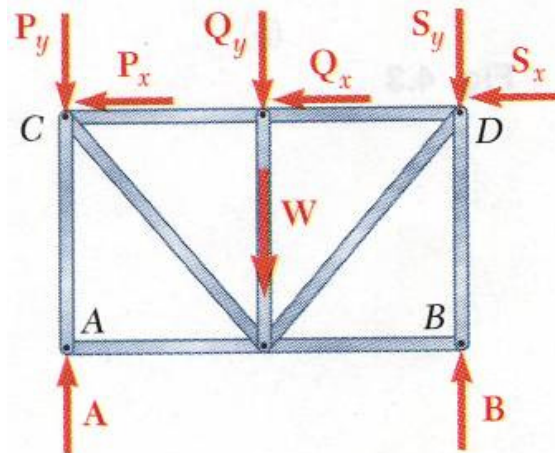
(a)



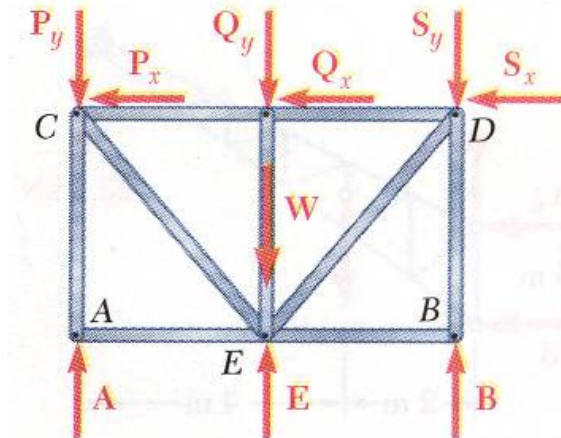
(a)



(b)



(b)



(b)

- More unknowns than equations

- Fewer unknowns than equations, partially constrained

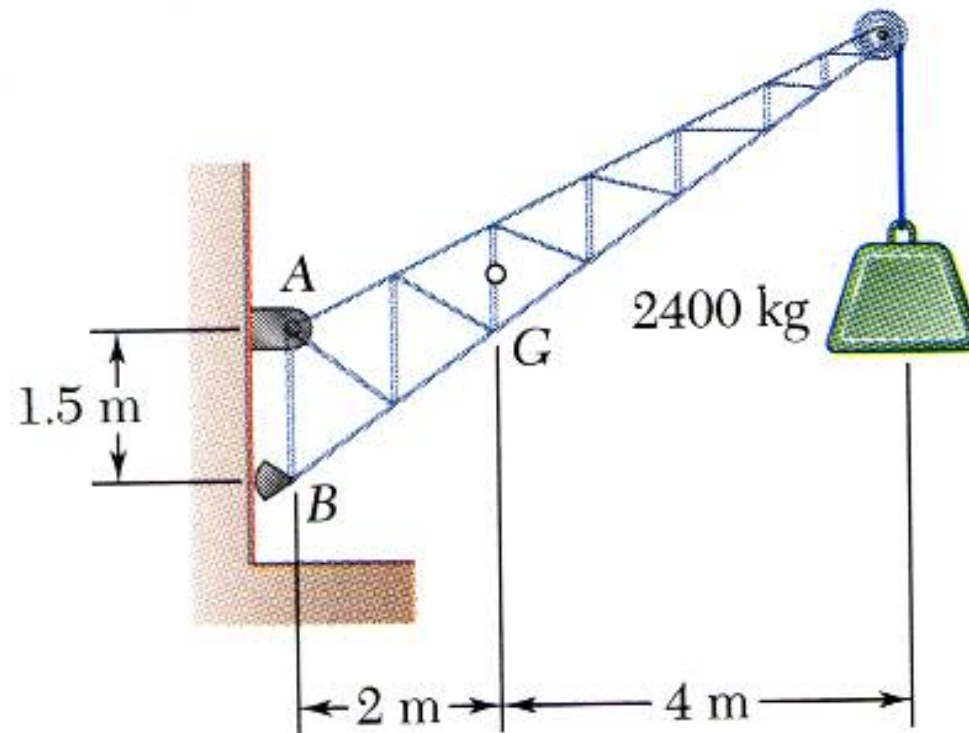
- Equal number unknowns and equations but improperly constrained

Equilibrium of Rigid Bodies

□ Sample Problem 01

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B . The center of gravity of the crane is located at G .

Determine the components of the reactions at A and B .



Equilibrium of Rigid Bodies

□ Sample Problem 01

SOLUTION:

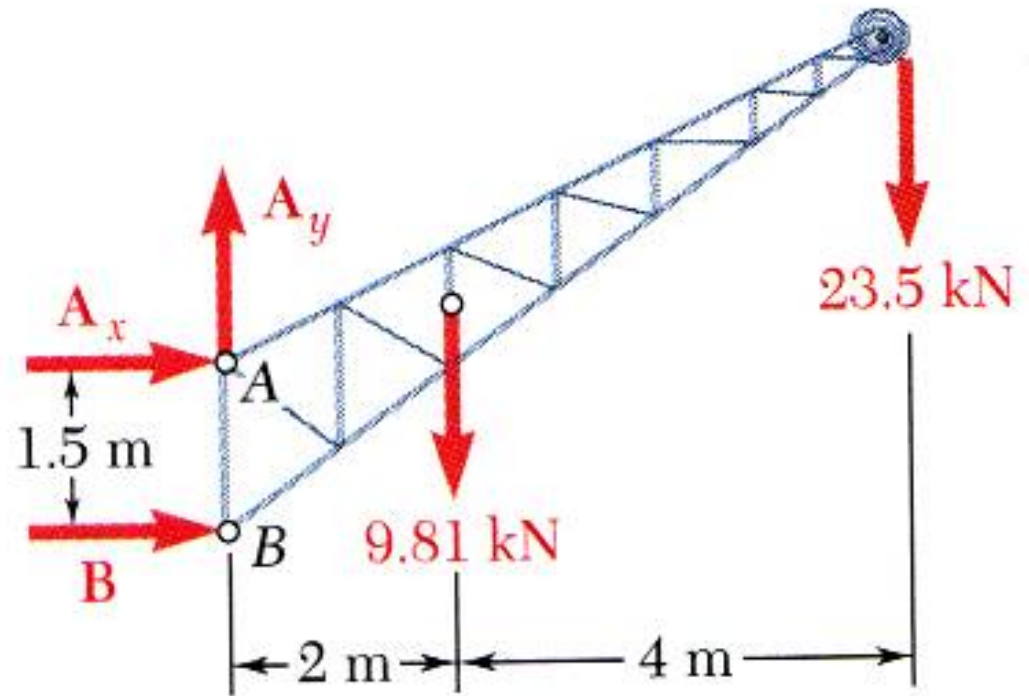
- Create the free-body diagram.
- Determine B by solving the equation for the sum of the moments of all forces about A .

$$\Rightarrow B = +107.1 \text{ (kN)}$$

- Determine the reactions at A by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\Rightarrow A_x = -107.1 \text{ (kN)}$$

$$\Rightarrow A_y = +33.3 \text{ (kN)}$$

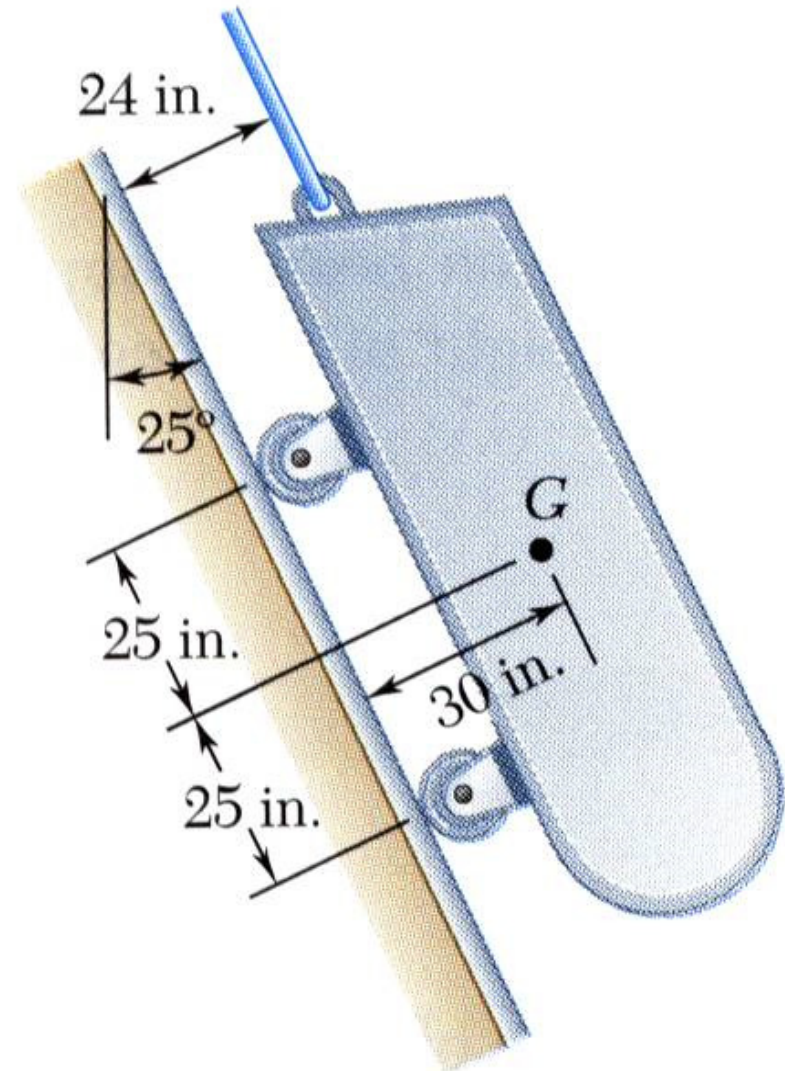


Equilibrium of Rigid Bodies

□ Sample Problem 02

A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb, and it is applied at G . The cart is held in position by the cable.

Determine the tension in the cable and the reaction at each pair of wheels.



Equilibrium of Rigid Bodies

□ Sample Problem 02

SOLUTION:

- Create a free-body diagram

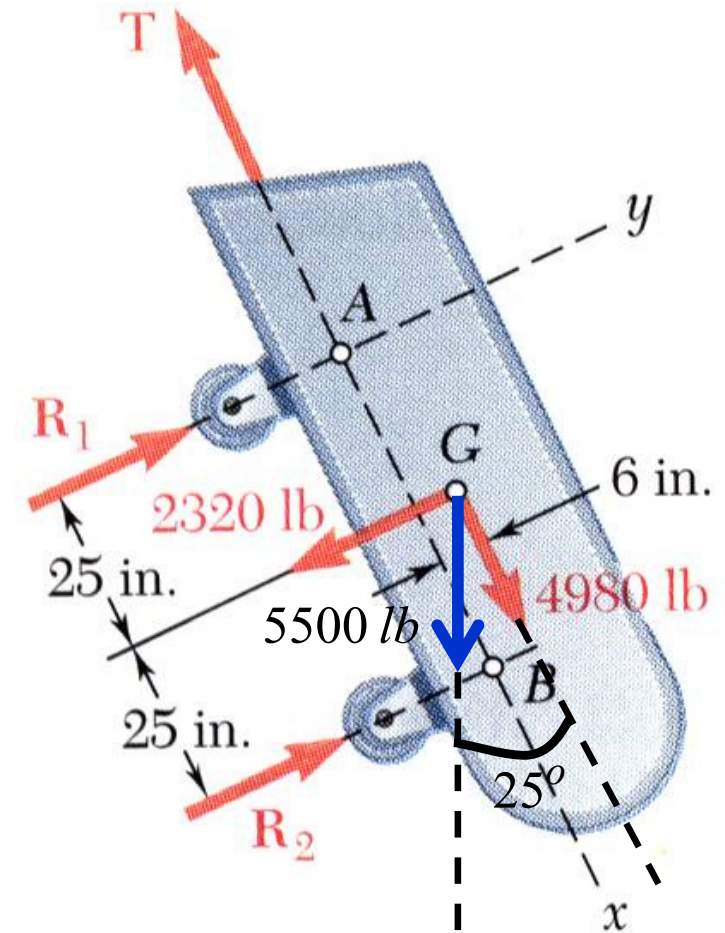
$$W_x = 4980 \text{ (lb)}$$

$$W_y = -2320 \text{ (lb)}$$

- Determine the reactions at the wheels.

$$\Rightarrow R_2 = 1758 \text{ (lb)}$$

$$\Rightarrow R_1 = 562 \text{ (lb)}$$



- Determine the cable tension.

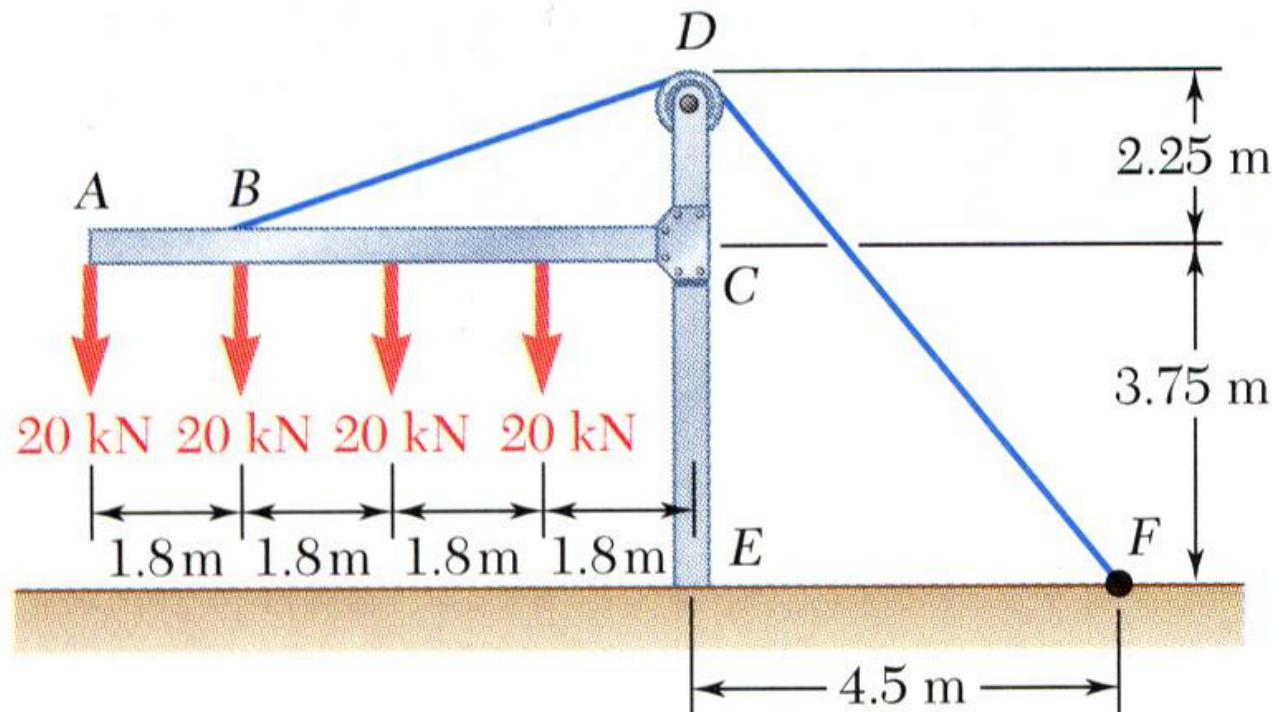
$$\Rightarrow T = 4980 \text{ (lb)}$$

Equilibrium of Rigid Bodies

□ Sample Problem 03

The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

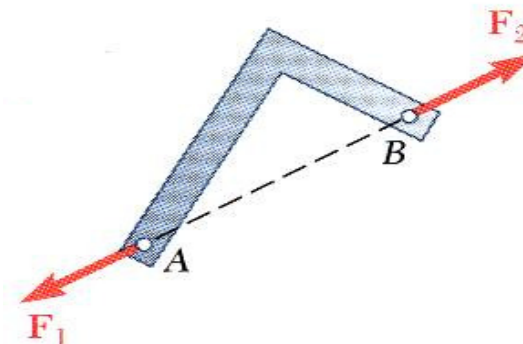
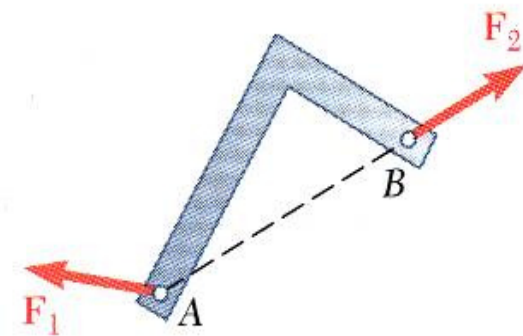
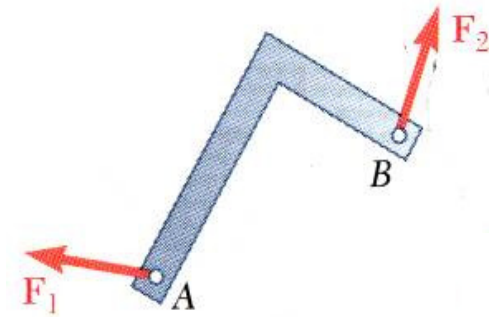
Determine the reaction at the fixed end E .



Equilibrium of Rigid Bodies

□ Equilibrium of a Two-Force Body

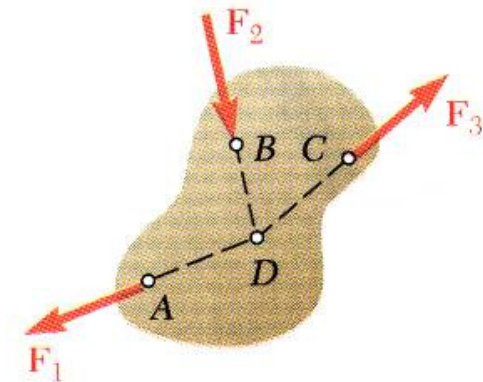
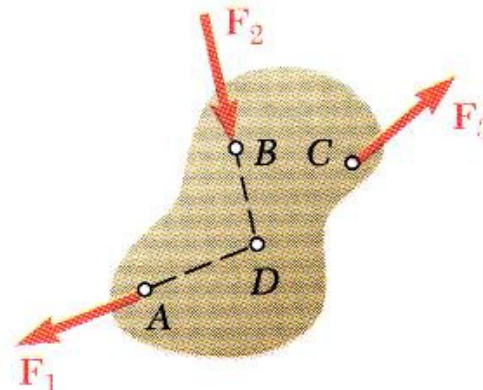
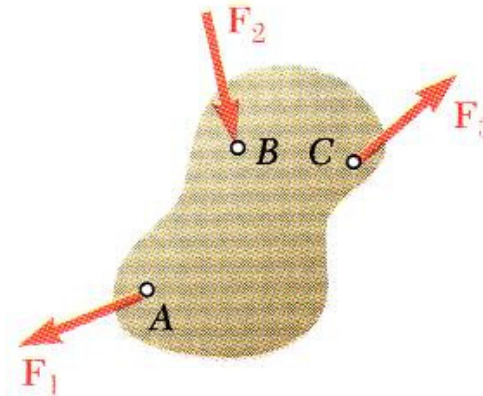
- Consider a plate subjected to two forces F_1 and F_2
- For static equilibrium, the sum of moments about A must be zero. The moment of F_2 must be zero. It follows that the line of action of F_2 must pass through A .
- Similarly, the line of action of F_1 must pass through B for the sum of moments about B to be zero.
- Requiring that the sum of forces in any direction be zero leads to the conclusion that F_1 and F_2 must have *equal magnitude but opposite sense*.



Equilibrium of Rigid Bodies

□ Equilibrium of a Three-Force Body

- Consider a rigid body subjected to forces acting at only 3 points.
- Assuming that their lines of action intersect, the moment of F_1 and F_2 about the point of intersection represented by D is zero.
- Since the rigid body is in equilibrium, the sum of the moments of F_1 , F_2 , and F_3 about any axis must be zero. It follows that the moment of F_3 about D must be zero as well and that the line of action of F_3 must pass through D .
- ***The lines of action of the three forces must be concurrent.***

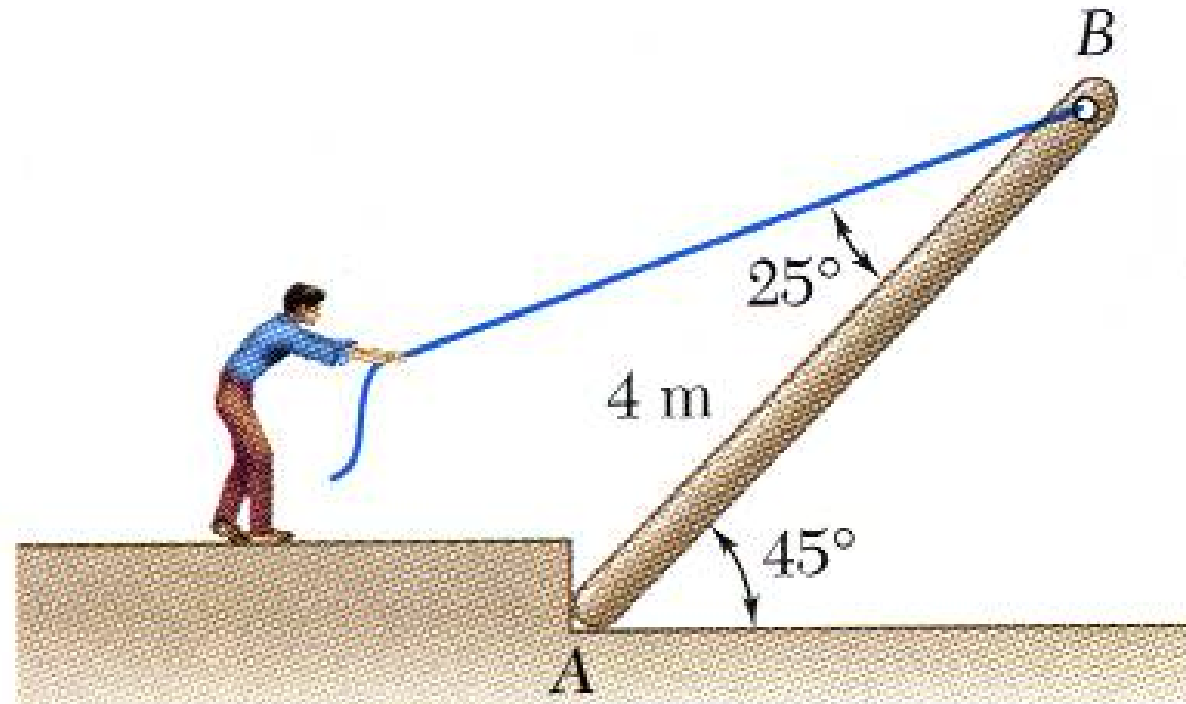


Equilibrium of Rigid Bodies

□ Sample Problem 04

A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at A .



Equilibrium of Rigid Bodies

□ Sample Problem 04

SOLUTION:

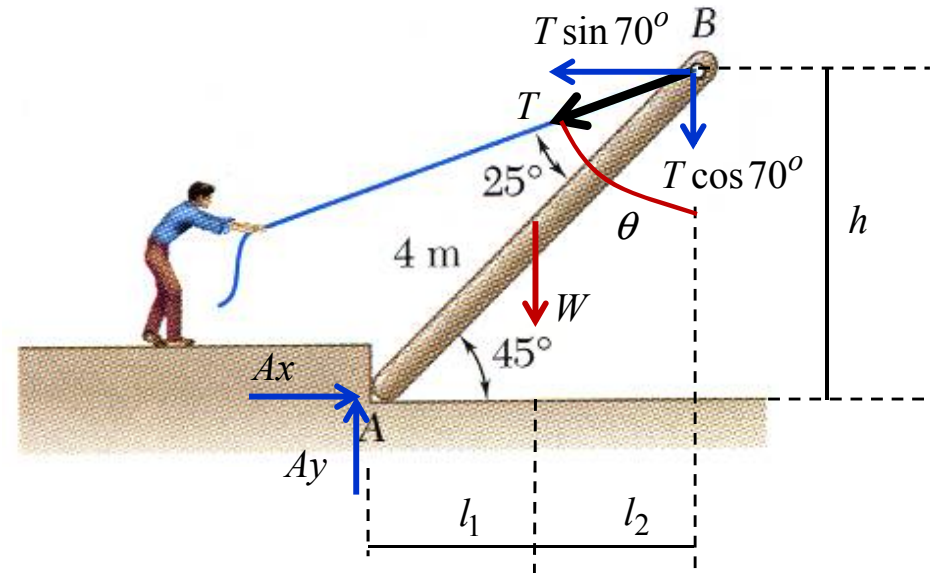
- Create a free-body diagram of the joist.

$$W = 98.1 \text{ (N)}$$

$$l_1 = l_2 = \sqrt{2} \text{ (m)}$$

$$h = 2\sqrt{2} \text{ (m)}$$

$$\theta = 70^\circ$$



$$\Rightarrow (T \sin 70^\circ)(2\sqrt{2}) - (98.1)(\sqrt{2}) - (T \cos 70^\circ)(\sqrt{2} + \sqrt{2}) = 0$$

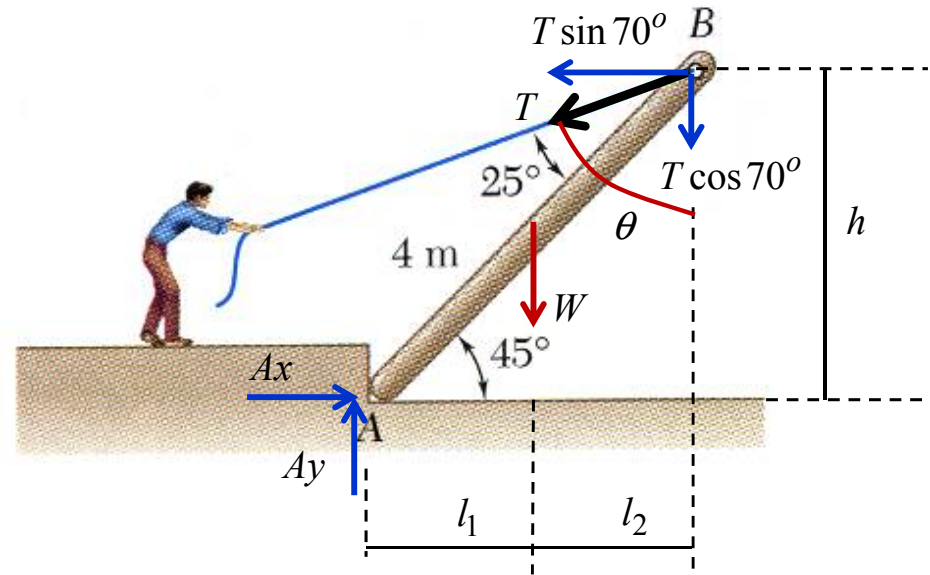
$$T = 82.07 \text{ (N)}$$

Equilibrium of Rigid Bodies

□ Sample Problem 04

SOLUTION:

- Create a free-body diagram of the joist.

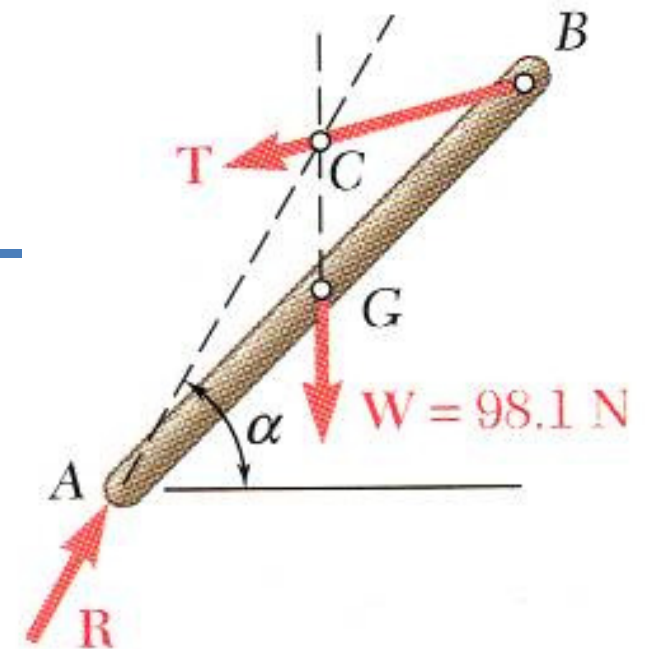


$$\Rightarrow A_x = 77.12 \text{ (N)}$$

$$\Rightarrow A_y = 125.83 \text{ (N)}$$

$$R = 147.58 \text{ (N)}$$

$$\alpha = 58.5^\circ$$



Equilibrium of Rigid Bodies

□ Equilibrium of a Rigid Body in Three Dimensions

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

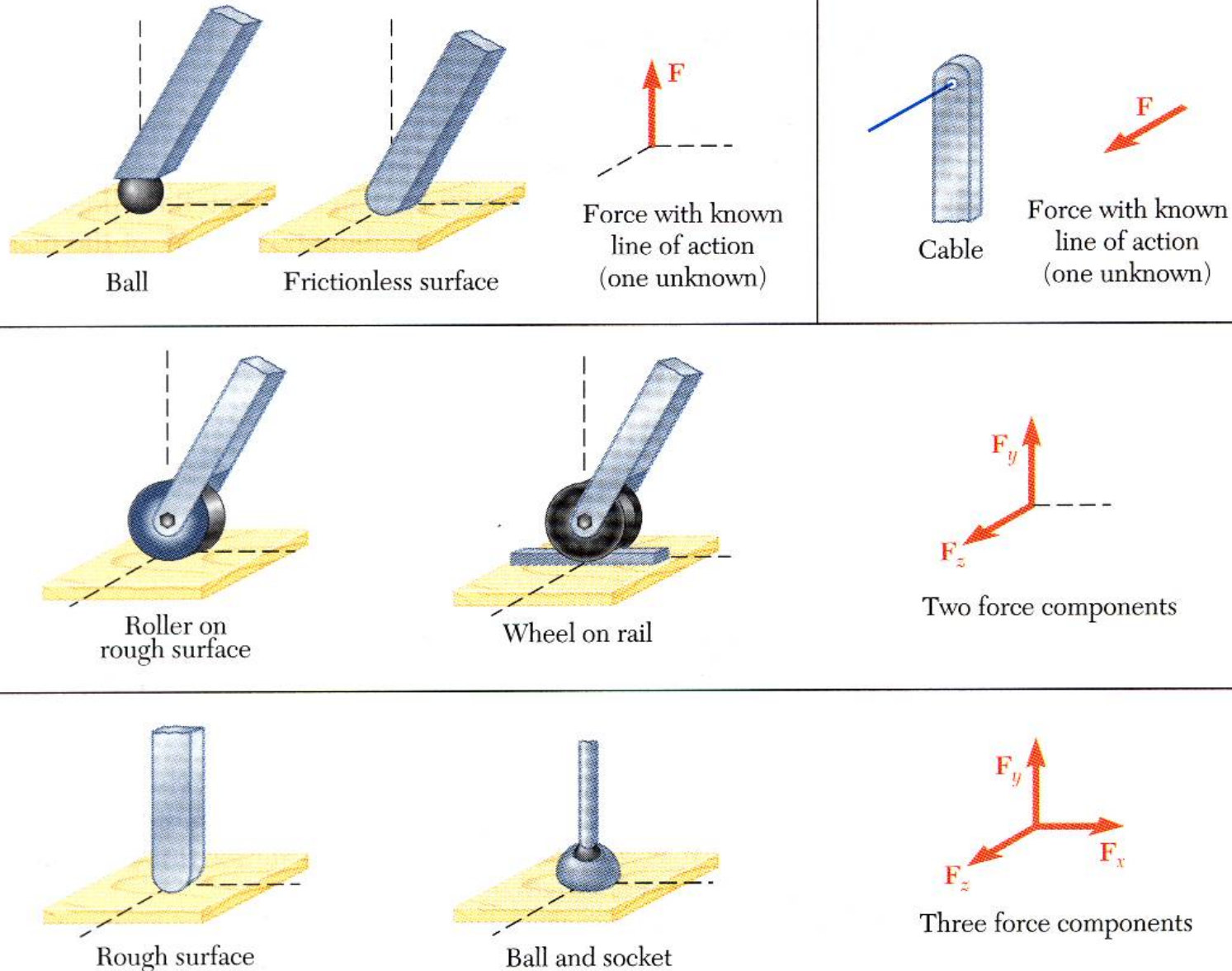
$$\begin{array}{lll} \sum F_x = 0 & \sum F_y = 0 & \sum F_z = 0 \\ \sum M_x = 0 & \sum M_y = 0 & \sum M_z = 0 \end{array}$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.
- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum (\vec{r} \times \vec{F}) = 0$$

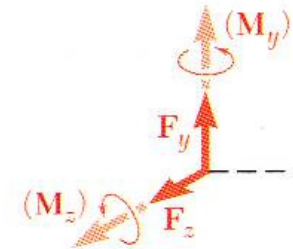
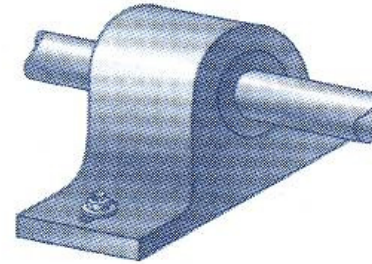
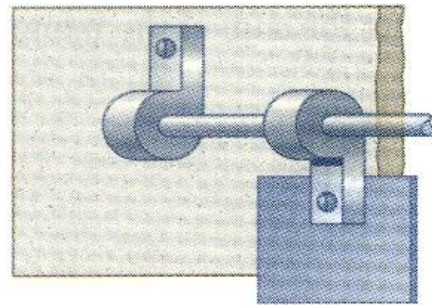
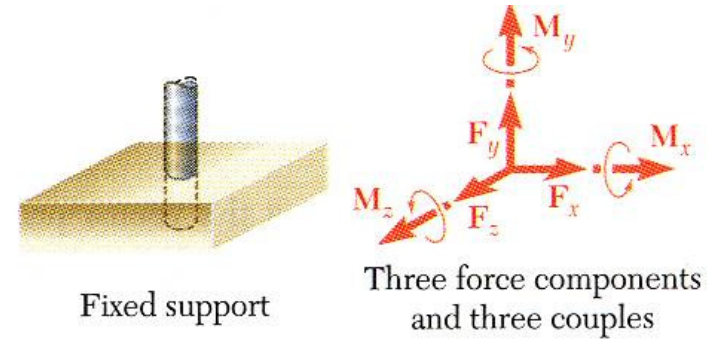
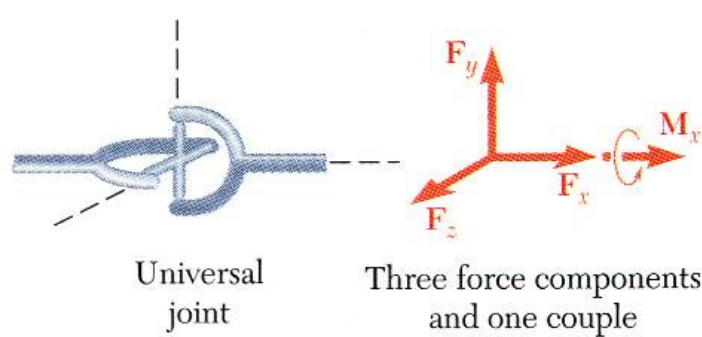
Equilibrium of Rigid Bodies

□ Reactions at Supports and Connections for a Three-Dimensional Structure

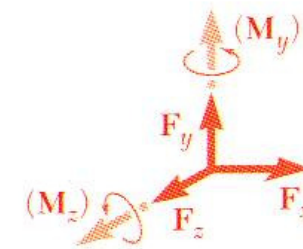
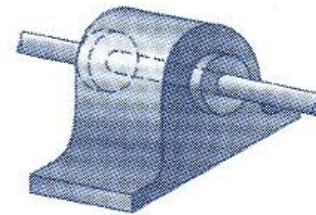
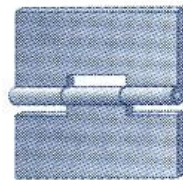
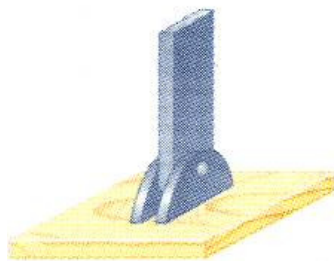


Equilibrium of Rigid Bodies

□ Reactions at Supports and Connections for a Three-Dimensional Structure



Hinge and bearing supporting radial load only



Pin and bracket

Hinge and bearing supporting axial thrust and radial load

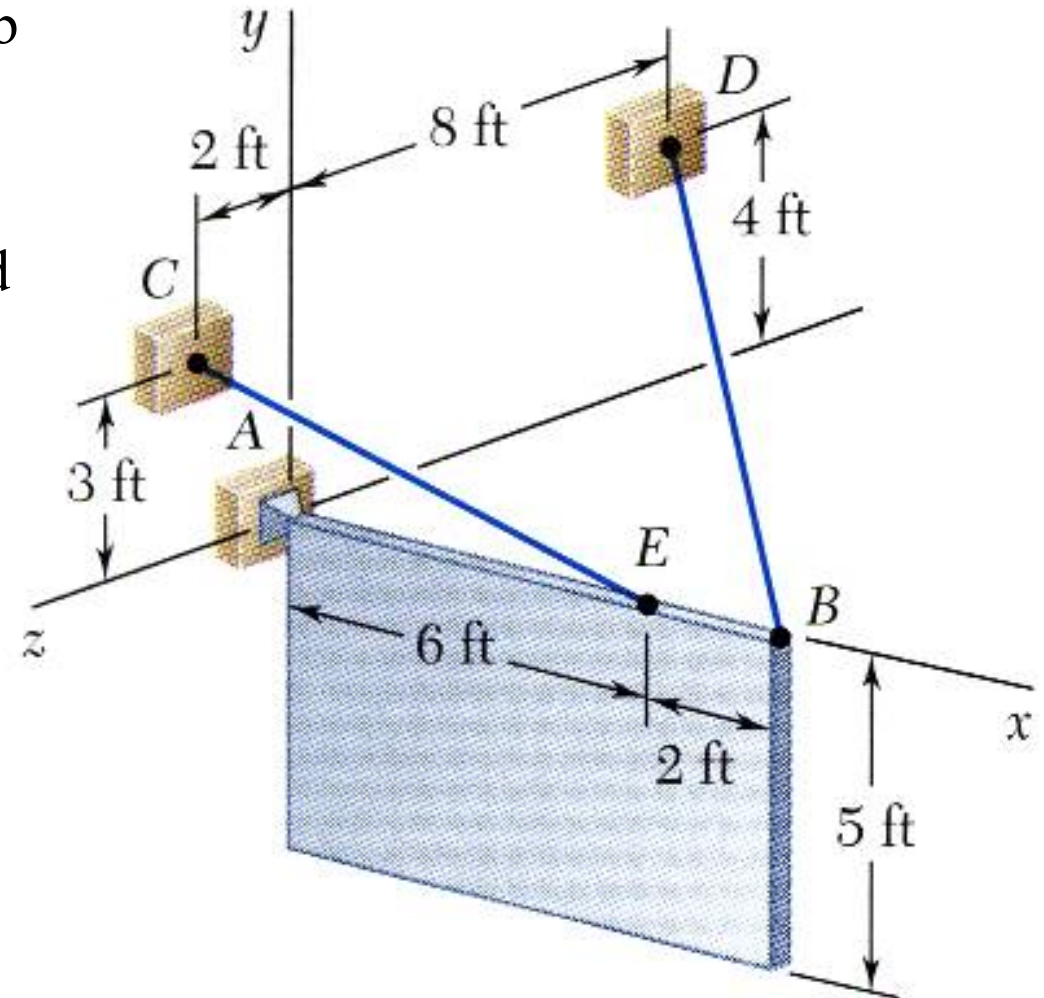
Three force components (and two couples)

Equilibrium of Rigid Bodies

□ Sample Problem 05

A sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at A and by two cables.

Determine the tension in each cable and the reaction at A .



Equilibrium of Rigid Bodies

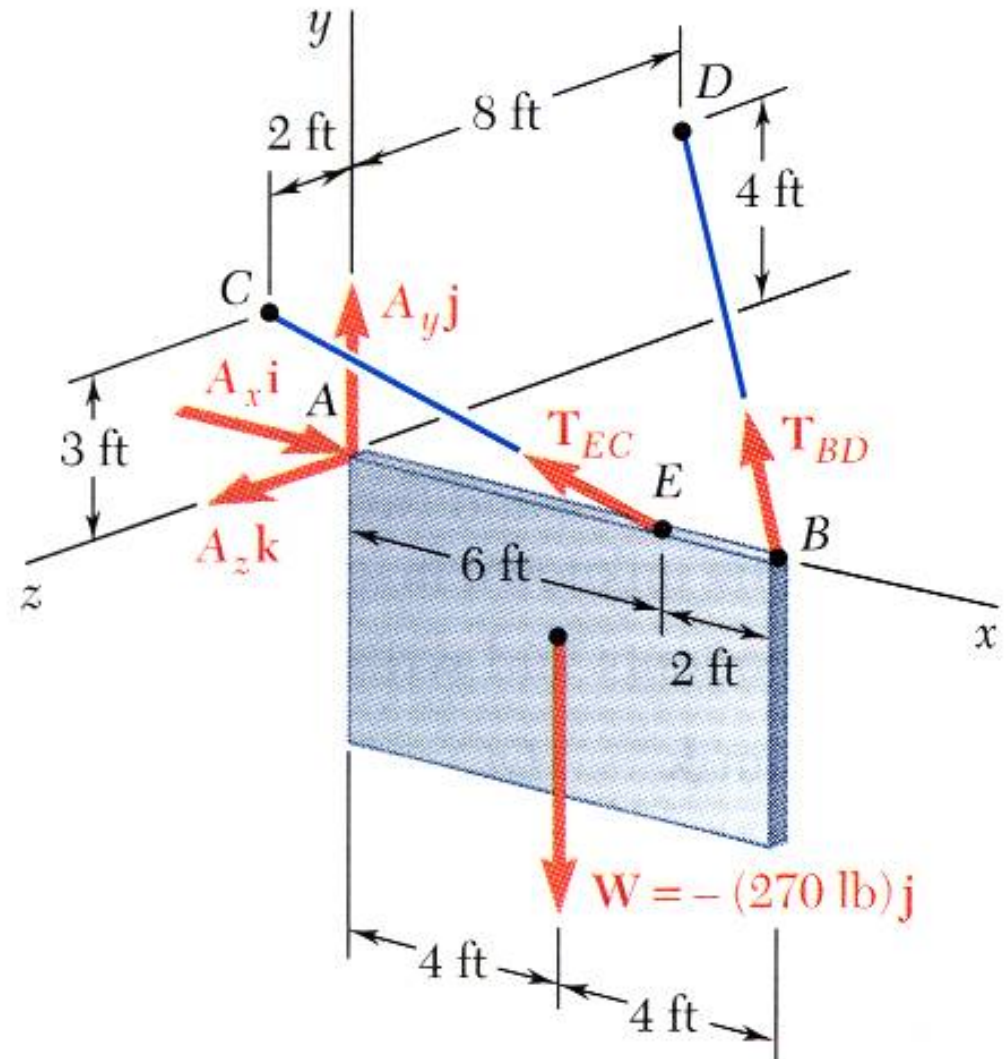
□ Sample Problem 05

SOLUTION:

- Create a free-body diagram for the sign.

$$\vec{T}_{BD} = T_{BD} \left(-\frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k} \right)$$

$$\vec{T}_{EC} = T_{EC} \left(-\frac{6}{7}\vec{i} + \frac{3}{7}\vec{j} + \frac{2}{7}\vec{k} \right)$$



Equilibrium of Rigid Bodies

□ Sample Problem 05

SOLUTION:

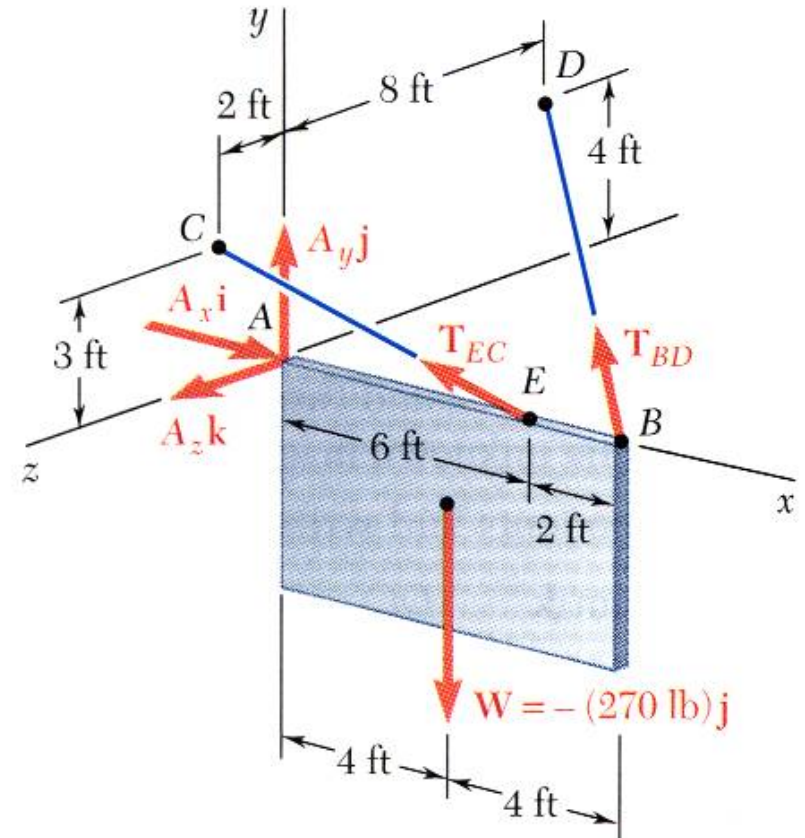
- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

$$\sum \vec{F} = \vec{A} + \vec{T}_{BD} + \vec{T}_{EC} + \vec{W} = 0$$

$$\vec{i} : \sum F_x = 0 \Rightarrow A_x - \frac{2}{3}T_{BD} - \frac{6}{7}T_{EC} = 0$$

$$\vec{j} : \sum F_y = 0 \Rightarrow A_y + \frac{1}{3}T_{BD} + \frac{3}{7}T_{EC} - 270 \text{ lb} = 0 \quad (I)$$

$$\vec{k} : \sum F_z = 0 \Rightarrow A_z - \frac{2}{3}T_{BD} + \frac{2}{7}T_{EC} = 0$$



Equilibrium of Rigid Bodies

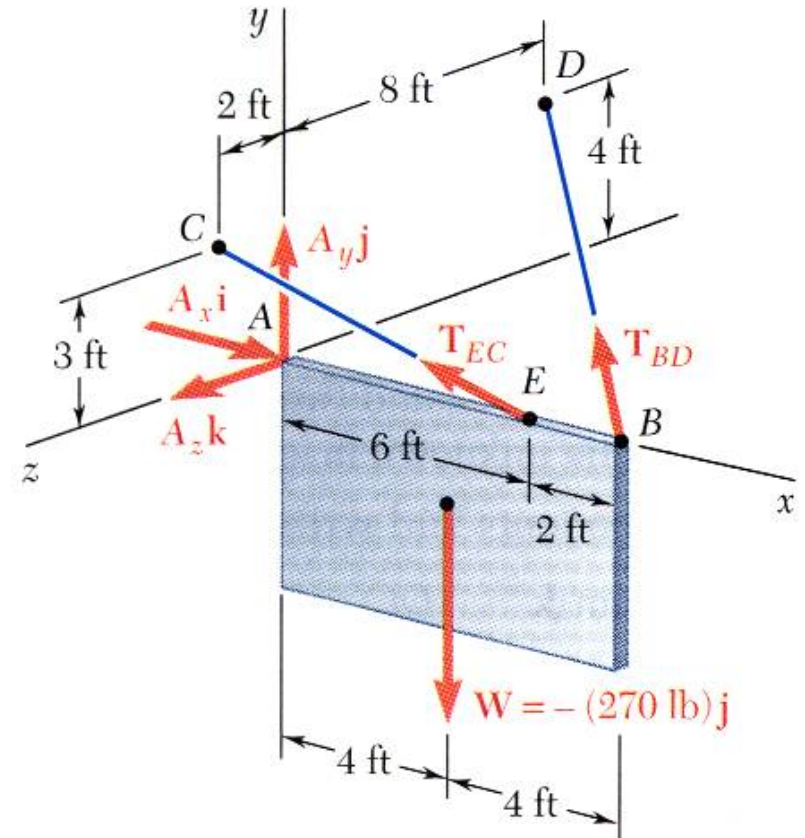
□ Sample Problem 05

SOLUTION:

$$\vec{M}_{\vec{T}_{BD}/A} = \left(\frac{16}{3} T_{BD} \right) \vec{j} + \left(\frac{8}{3} T_{BD} \right) \vec{k}$$

$$\vec{M}_{\vec{T}_{EC}/A} = \left(-\frac{12}{7} T_{EC} \right) \vec{j} + \left(\frac{18}{7} T_{EC} \right) \vec{k}$$

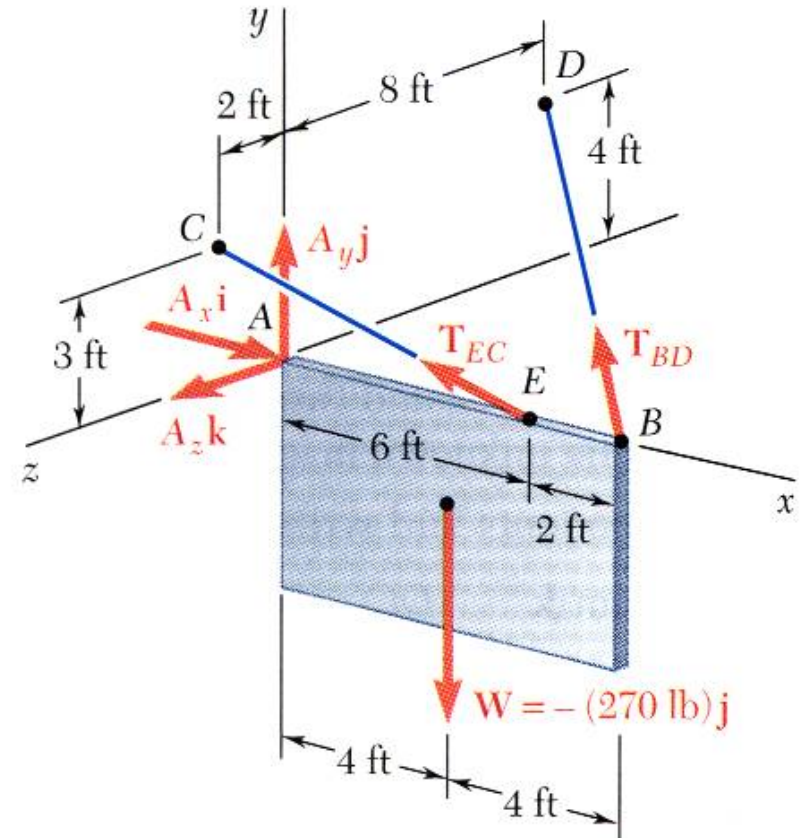
$$\vec{M}_{\vec{W}/A} = (-1080) \vec{k}$$



Equilibrium of Rigid Bodies

□ Sample Problem 05

SOLUTION:



(I) & (II) \Rightarrow Solve the 5 equations for the 5 unknowns,

$$T_{BD} = 101.3 \text{ (lb)}$$

$$T_{EC} = 315 \text{ (lb)}$$

$$\vec{A} = (338 \text{ lb})\vec{i} + (101.2 \text{ lb})\vec{j} - (22.5 \text{ lb})\vec{k}$$