STATICS

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- Vector Mechanics for Engineers: Statics, 9th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Statics, 5th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Statics"

Rigid Bodies:
Equivalent Systems of Forces

By: Kaveh Karami

Associate Prof. of Structural Engineering

https://prof.uok.ac.ir/Ka.Karami

Rigid Bodies: Equivalent Systems of Forces

❑ Introduction

•Treatment of a body as a single particle is not always possible. In general, the size of the body and the specific points of application of the forces must be considered.

- Most bodies in elementary mechanics are assumed to be rigid, i.e., the actual deformations are small and do not affect the conditions of equilibrium or motion of the body.

•Current chapter describes the effect of forces exerted on a rigid body and how to replace a given system of forces with a simpler equivalent system.

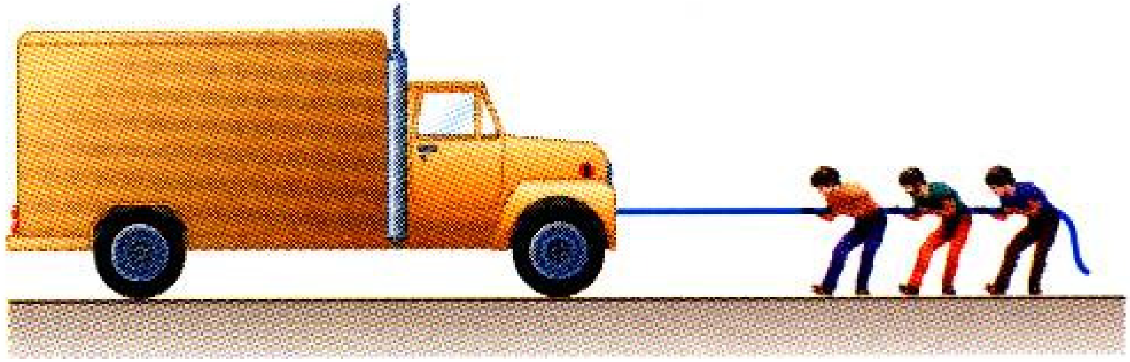
- moment of a force about a point
- moment of a force about an axis
- moment due to a couple

•Any system of forces acting on a rigid body can be replaced by an equivalent system consisting of one force acting at a given point and one couple.

Rigid Bodies: Equivalent Systems of Forces

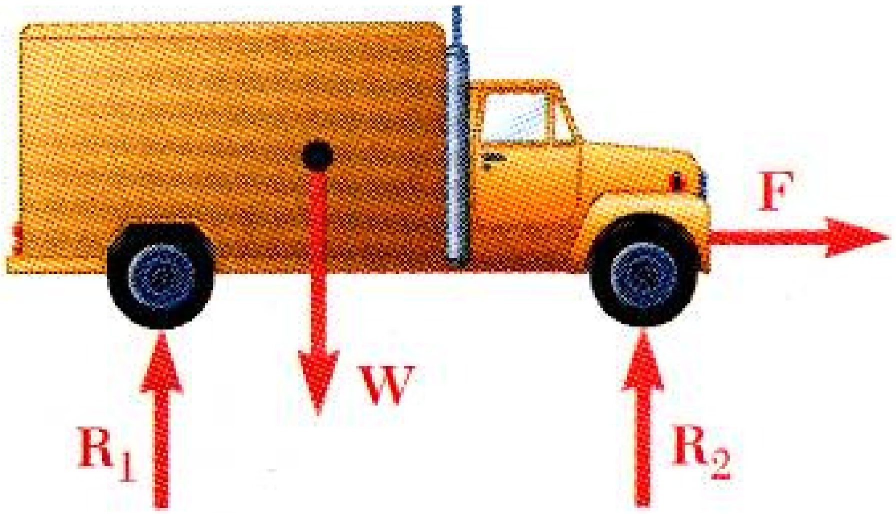
❑ External and Internal Forces

- Forces acting on rigid bodies are divided into two groups:
  - ***External forces***
  - ***Internal forces***



- External forces are shown in a free-body diagram.

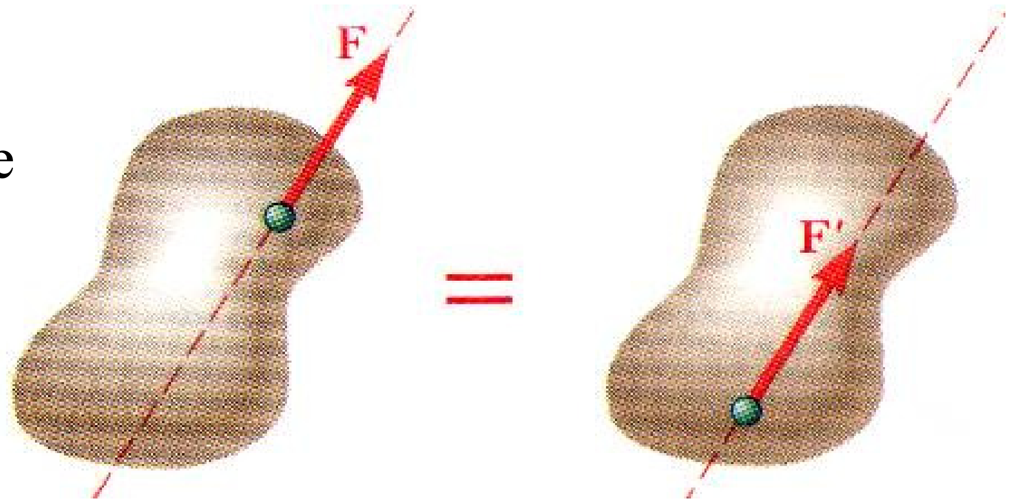
- If unopposed, ***each external force can impart a motion of translation or rotation, or both***.



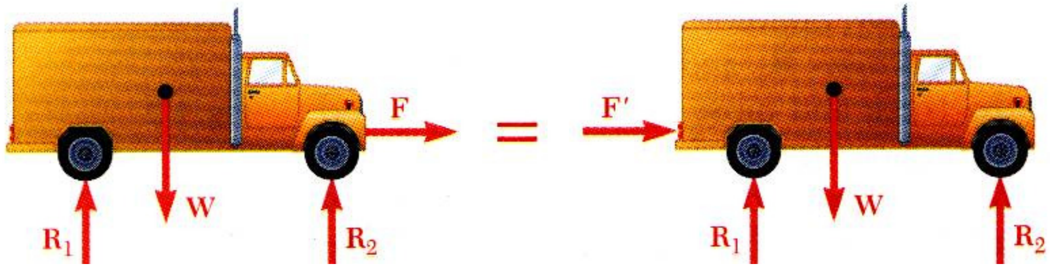
Rigid Bodies: Equivalent Systems of Forces

❑ Principle of Transmissibility: Equivalent Forces

- ***Principle of Transmissibility*** - Conditions of equilibrium or motion are not affected by *transmitting* a force along its line of action.
  NOTE: **F** and **F**' are equivalent forces.



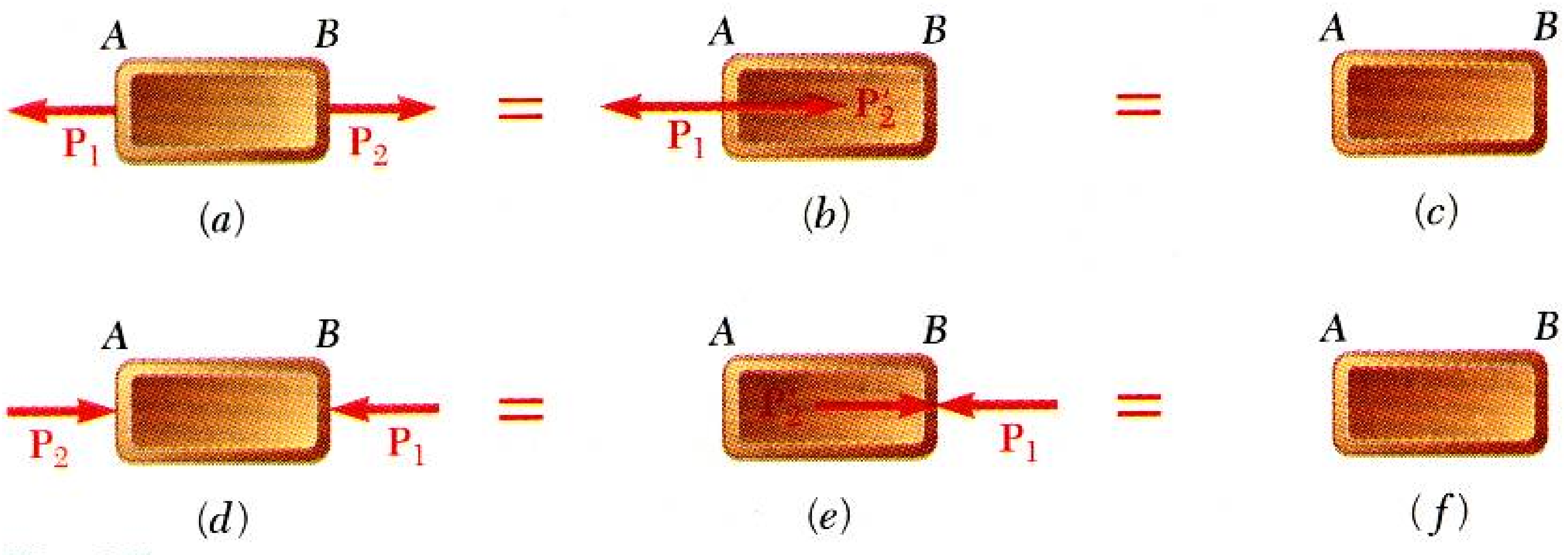
- Moving the point of application of the force **F** to the rear bumper does not affect the motion or the other forces acting on the truck.



Rigid Bodies: Equivalent Systems of Forces

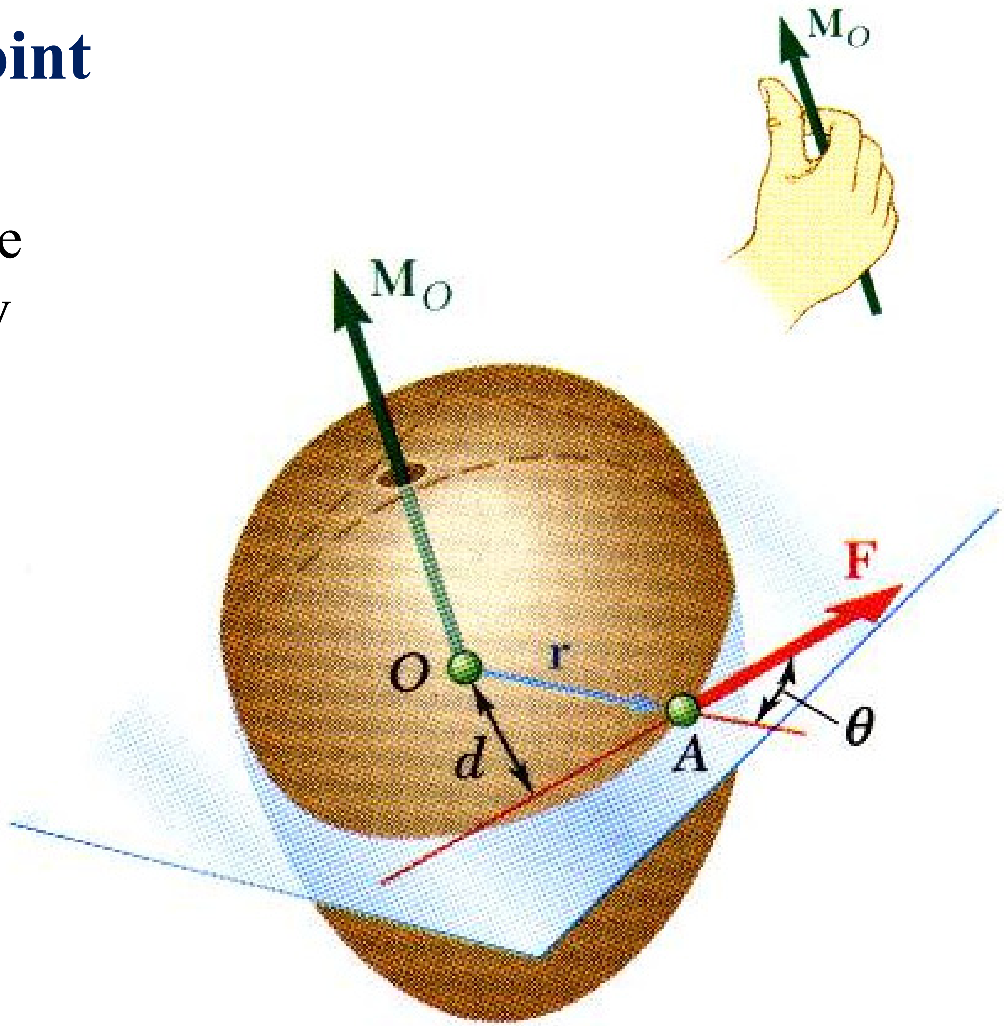
❑ Principle of Transmissibility: Equivalent Forces

- Principle of transmissibility may not always apply in determining internal forces and deformations.



Rigid Bodies: Equivalent Systems of Forces

❑ Moment of a Force About a Point



- A force vector is defined by its magnitude and direction. Its effect on the rigid body also depends on it point of application.

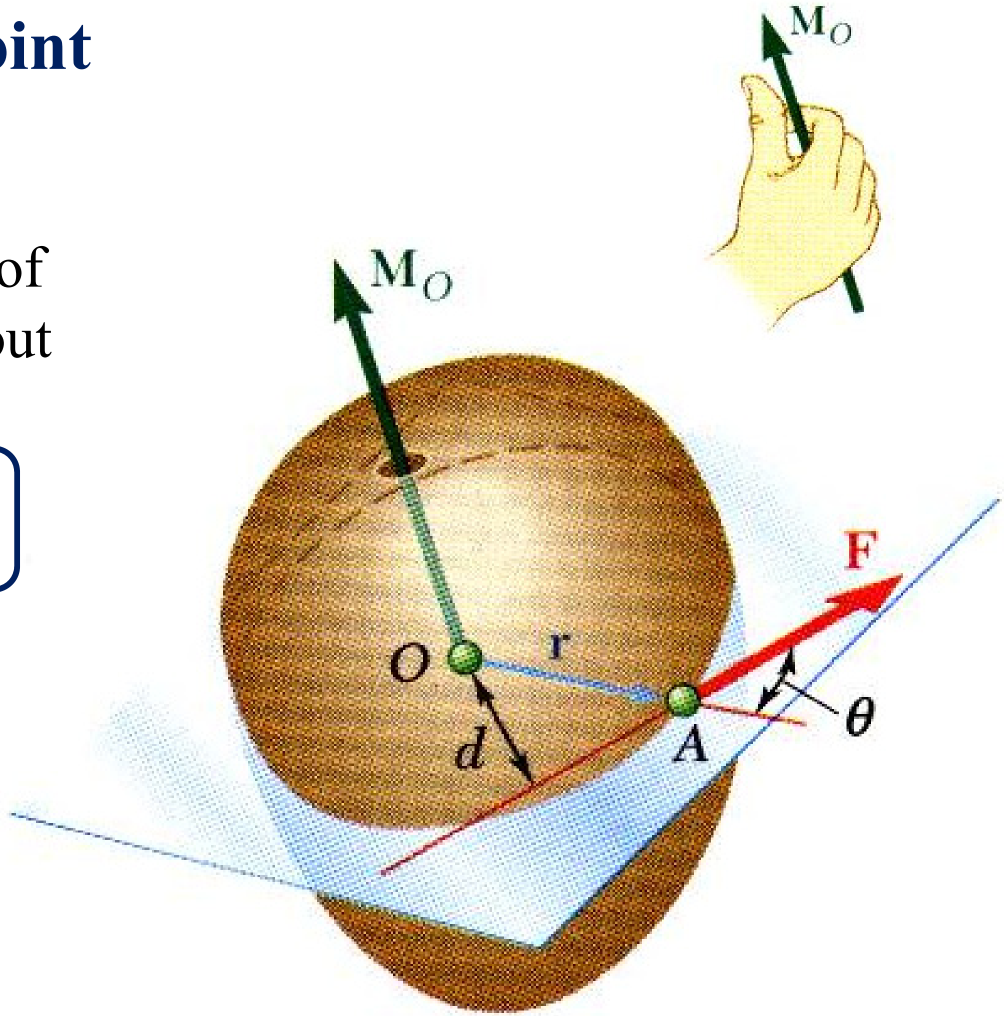
- The *moment* of ***F*** about *O* is defined as

$$\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$$

- The moment vector $\boldsymbol{M_O}$ is perpendicular to the plane containing *O* and the force ***F***.

Rigid Bodies: Equivalent Systems of Forces

❑ Moment of a Force About a Point



- Magnitude of $\boldsymbol{M_O}$ measures the tendency of the force to cause rotation of the body about an axis along $\boldsymbol{M_O}$.

$$M_O = rF\sin\theta = Fd$$

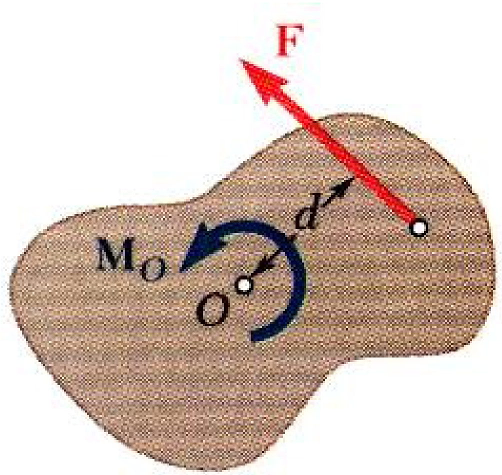
  The sense of the moment may be determined by the right-hand rule.

- Any force ***F'*** that has the same magnitude and direction as ***F***, is ***equivalent*** if it also has the same line of action and therefore, produces the same moment.

Rigid Bodies: Equivalent Systems of Forces

❑ Moment of a Force About a Point

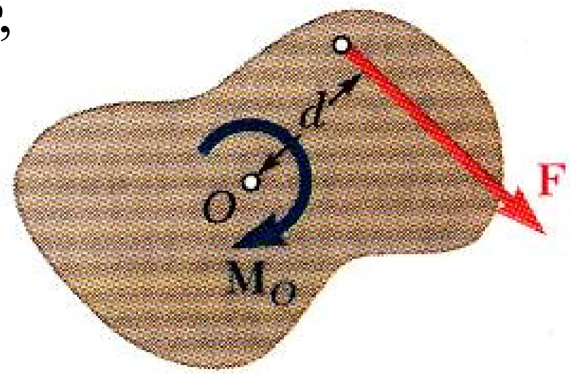
- ***Two-dimensional structures*** have length and breadth but negligible depth and are subjected to forces contained in the plane of the structure.



(a) $\mathbf{M}_O = +Fd$

- The plane of the structure contains the point O and the force $\boldsymbol{F}$. $\boldsymbol{M}_O$, the moment of the force about O is ***perpendicular to the plane***.

- If the force tends to rotate the structure ***counterclockwise***, the sense of the moment vector is ***out of the plane*** of the structure and the magnitude of the ***moment is positive***.



(b) $\mathbf{M}_O = -Fd$

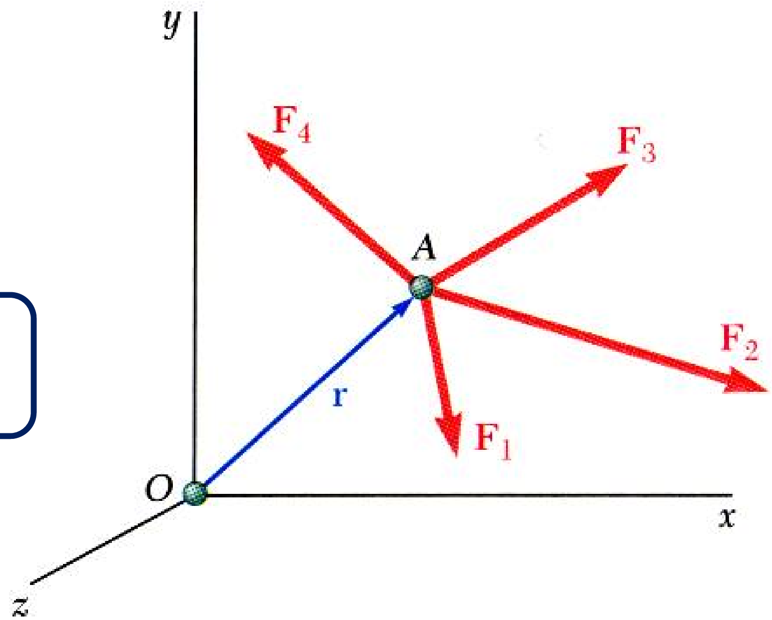
- If the force tends to rotate the structure ***clockwise***, the sense of the moment vector is ***into the plane*** of the structure and the magnitude of the ***moment is negative***.

Rigid Bodies: Equivalent Systems of Forces

❑ Varignon's Theorem

- The moment about a give point *O* of the resultant of several concurrent forces is equal to the sum of the moments of the various moments about the same point *O*.

$$\vec{r} \times (\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \cdots + \vec{r} \times \vec{F}_n$$

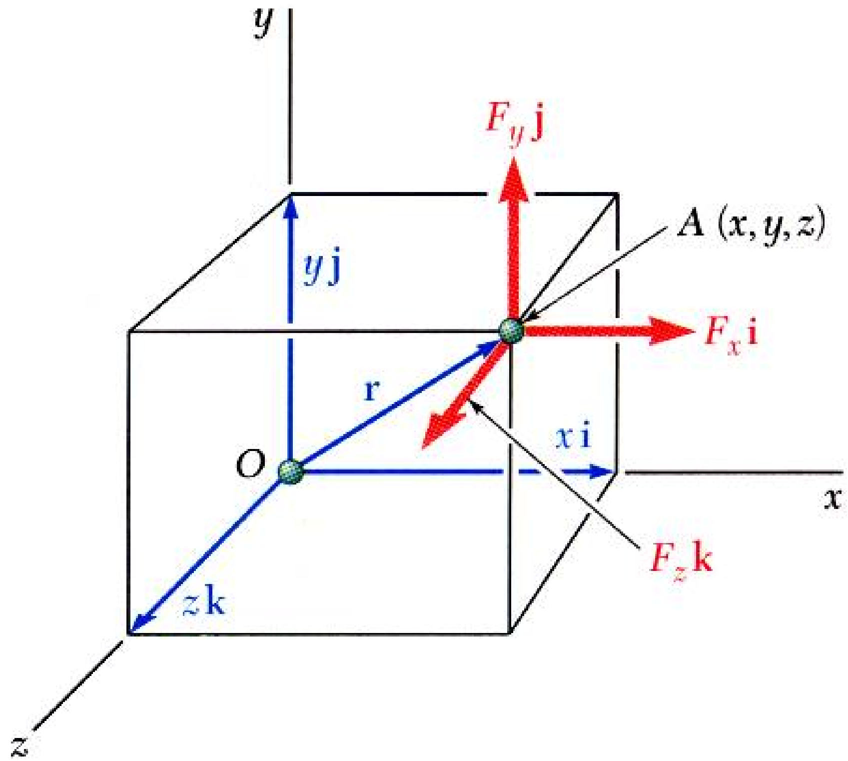


- Varigon's Theorem makes it possible to replace the direct determination of the moment of a force ***F*** by the moments of two or more component forces of ***F***.

Rigid Bodies: Equivalent Systems of Forces

Rectangular Components of the Moment of a Force

The moment of ***F*** about *O*,



$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$
$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} \quad \Rightarrow \quad \boxed{\vec{M}_O = \vec{r} \times \vec{F}}$$

$$\vec{M}_O = M_x\vec{i} + M_y\vec{j} + M_z\vec{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = (yF_z - zF_y)\vec{i} + (zF_x - xF_z)\vec{j} + (xF_y - yF_x)\vec{k}$$

Rigid Bodies: Equivalent Systems of Forces

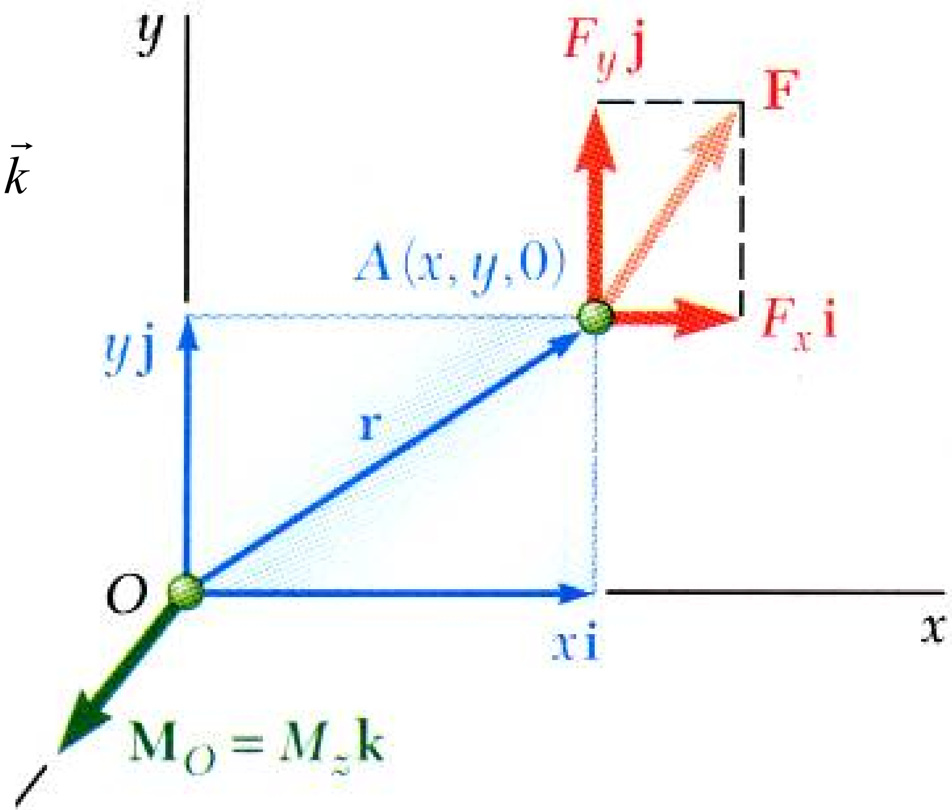
❑ Rectangular Components of the Moment of a Force

For two-dimensional structures,

$$\vec{M}_O = (yF_z - zF_y)\vec{i} + (zF_x - xF_z)\vec{j} + (xF_y - yF_x)\vec{k}$$

$$\Rightarrow \quad \boxed{\vec{M}_O = (xF_y - yF_x)\vec{k}}$$

$$\boxed{M_O = M_Z = xF_y - yF_x}$$



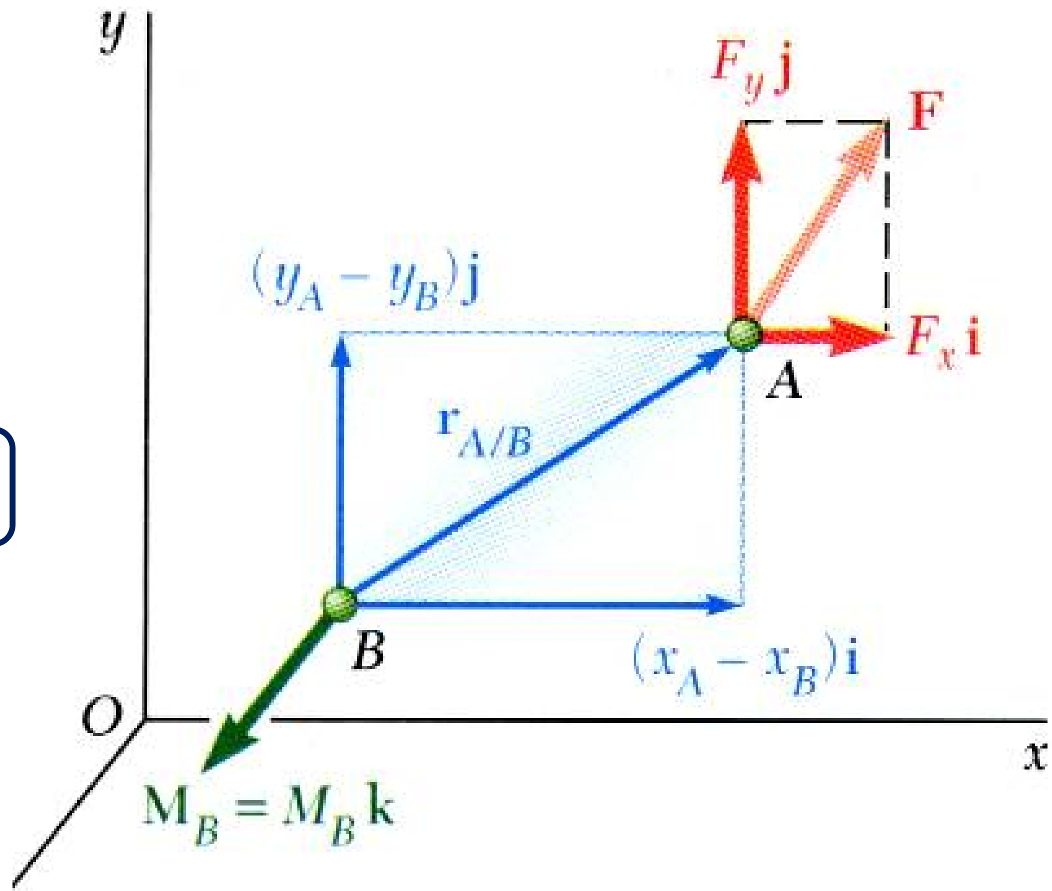
Rigid Bodies: Equivalent Systems of Forces

Rectangular Components of the Moment of a Force

For two-dimensional structures,

$$\vec{M}_O = [(x_A - x_B)F_y - (y_A - y_B)F_x]\vec{k}$$

$$M_O = M_Z = (x_A - x_B)F_y - (y_A - y_B)F_x$$



Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 01

A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at *O*.

Determine:

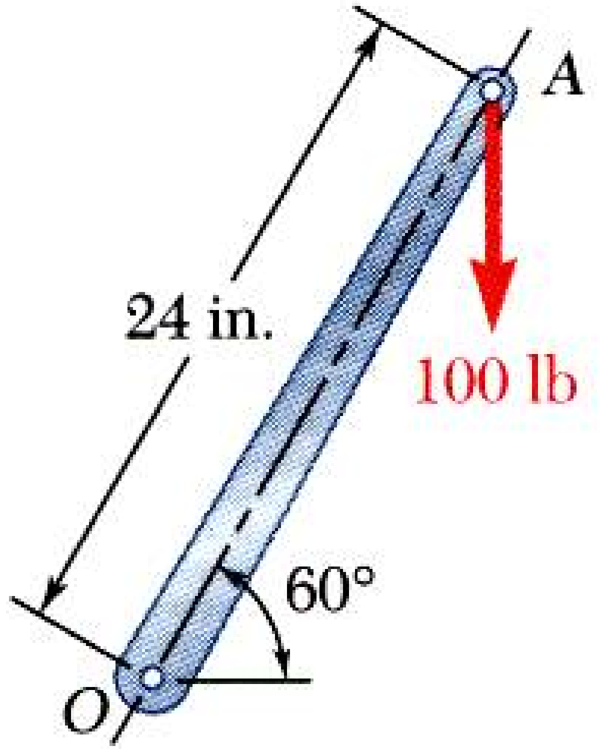
a) moment about *O*,

b) horizontal force at *A* which creates the same moment,

c) smallest force at A which produces the same moment,

d) location for a 240-lb vertical force to produce the same moment,

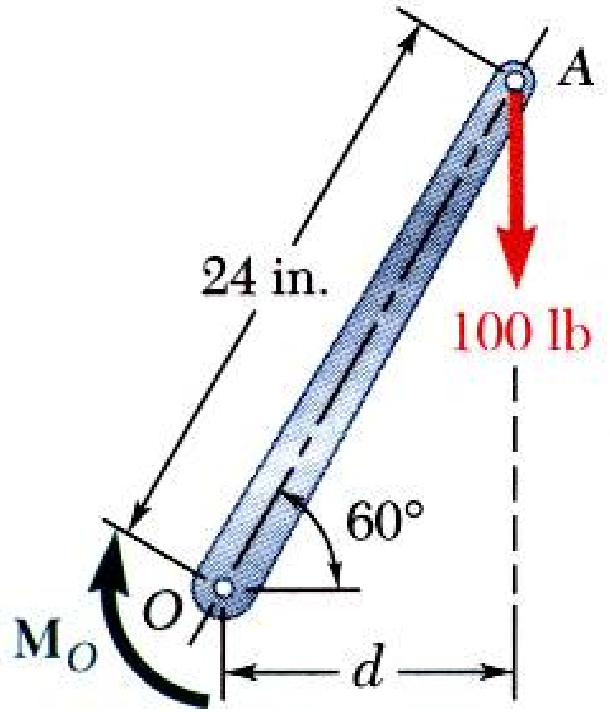
e) whether any of the forces from b, c, and d is equivalent to the original force?



❑ Sample Problem 01

SOLUTION:

a) Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.



$$d = 12\ (in)$$

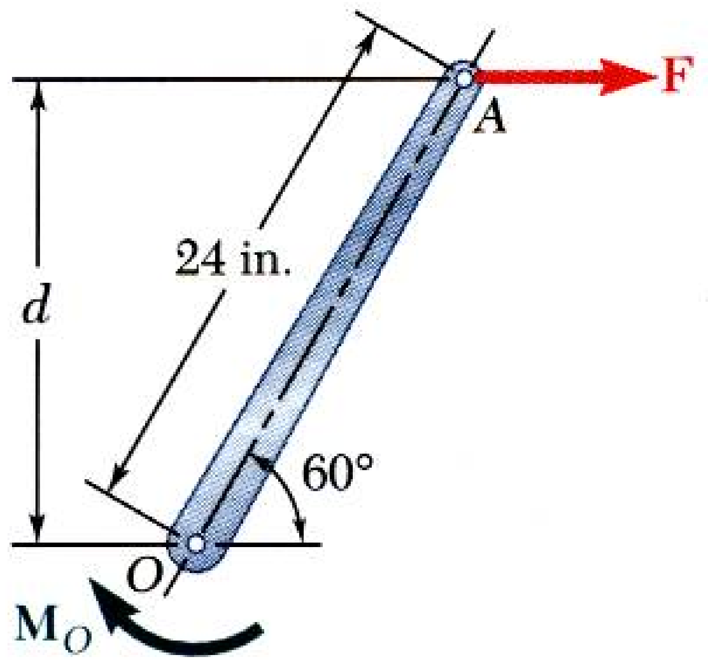
$$M_O = 1200\ (lb.in)$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 01

SOLUTION:

b) Horizontal force at A that produces the same moment,



$$M_O = 1200\ (lb.in)$$

$$d = 20.8\ (\text{in})$$

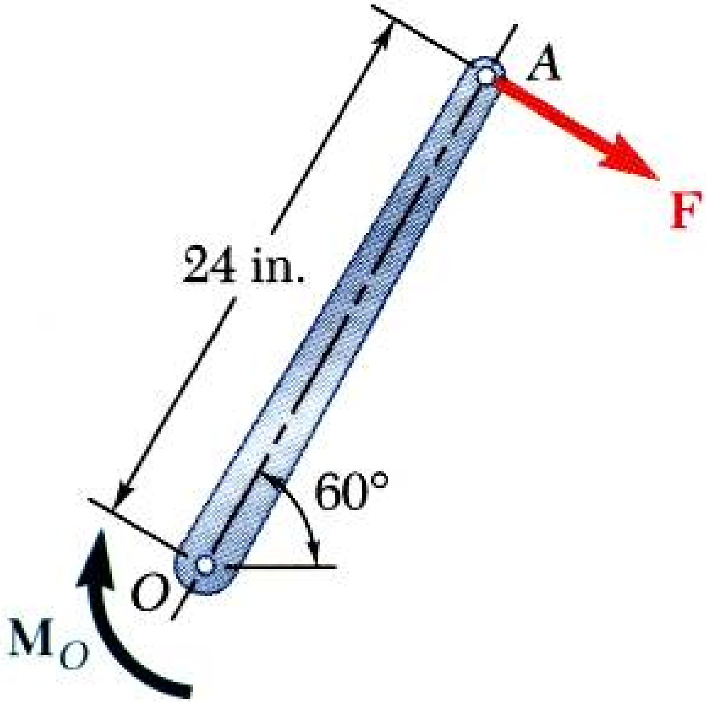
$$F = 57.7\ (lb)$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 01

SOLUTION:

c) The smallest force A to produce the same moment occurs when ***the perpendicular distance is a maximum or when F is perpendicular to OA***.



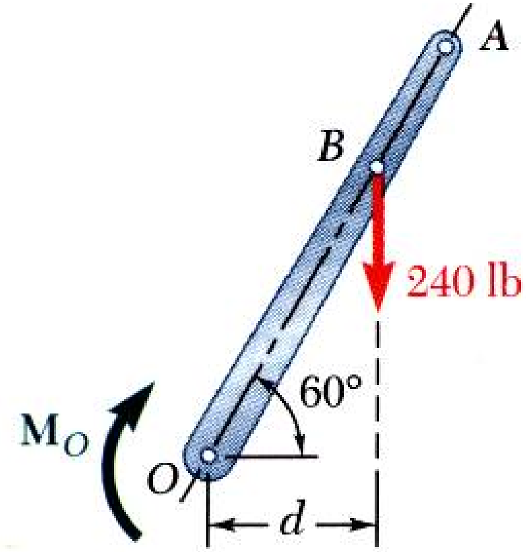
$$F = 50\,(lb)$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 01

SOLUTION:

d) To determine the point of application of a 240 lb force to produce the same moment,



$$M_O = 1200\ (lb.in)$$

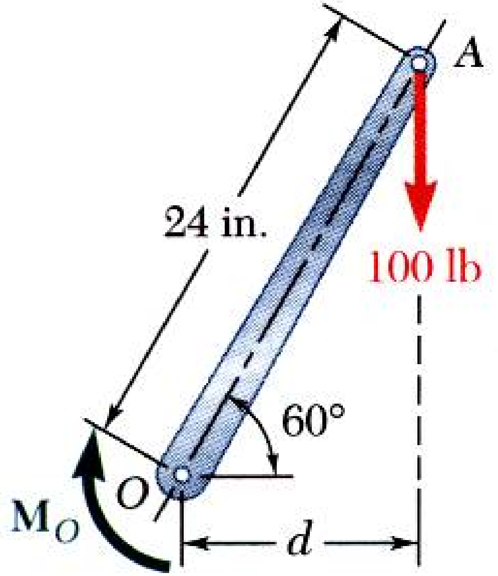
$$OB = 10\ \text{(in)}$$

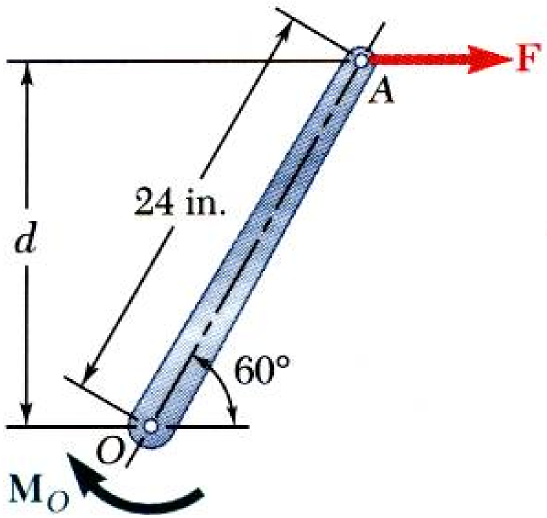
Rigid Bodies: Equivalent Systems of Forces

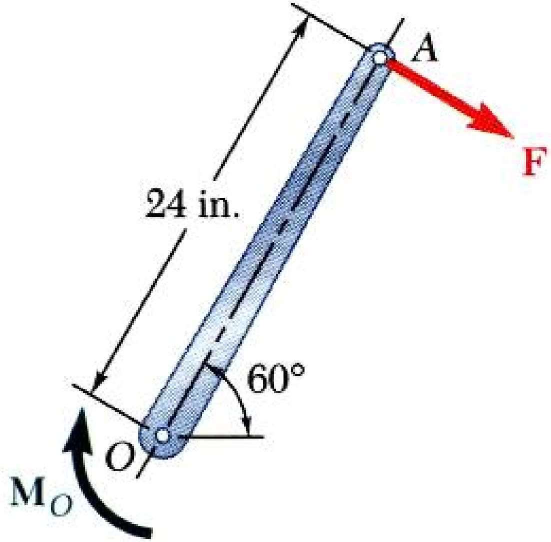
❑ Sample Problem 01

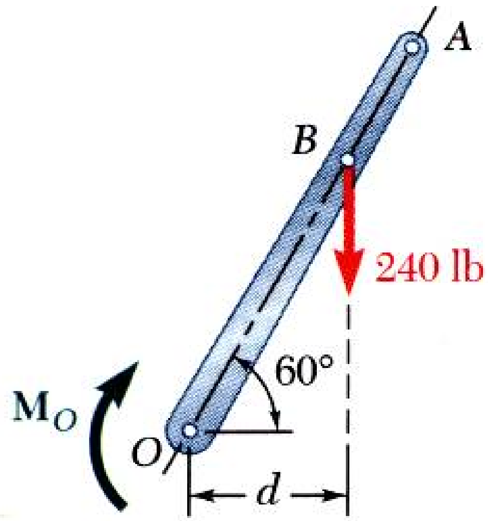
SOLUTION:

e) Although each of the forces in parts b), c), and d) produces the same moment as the 100 lb force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 100 lb force.



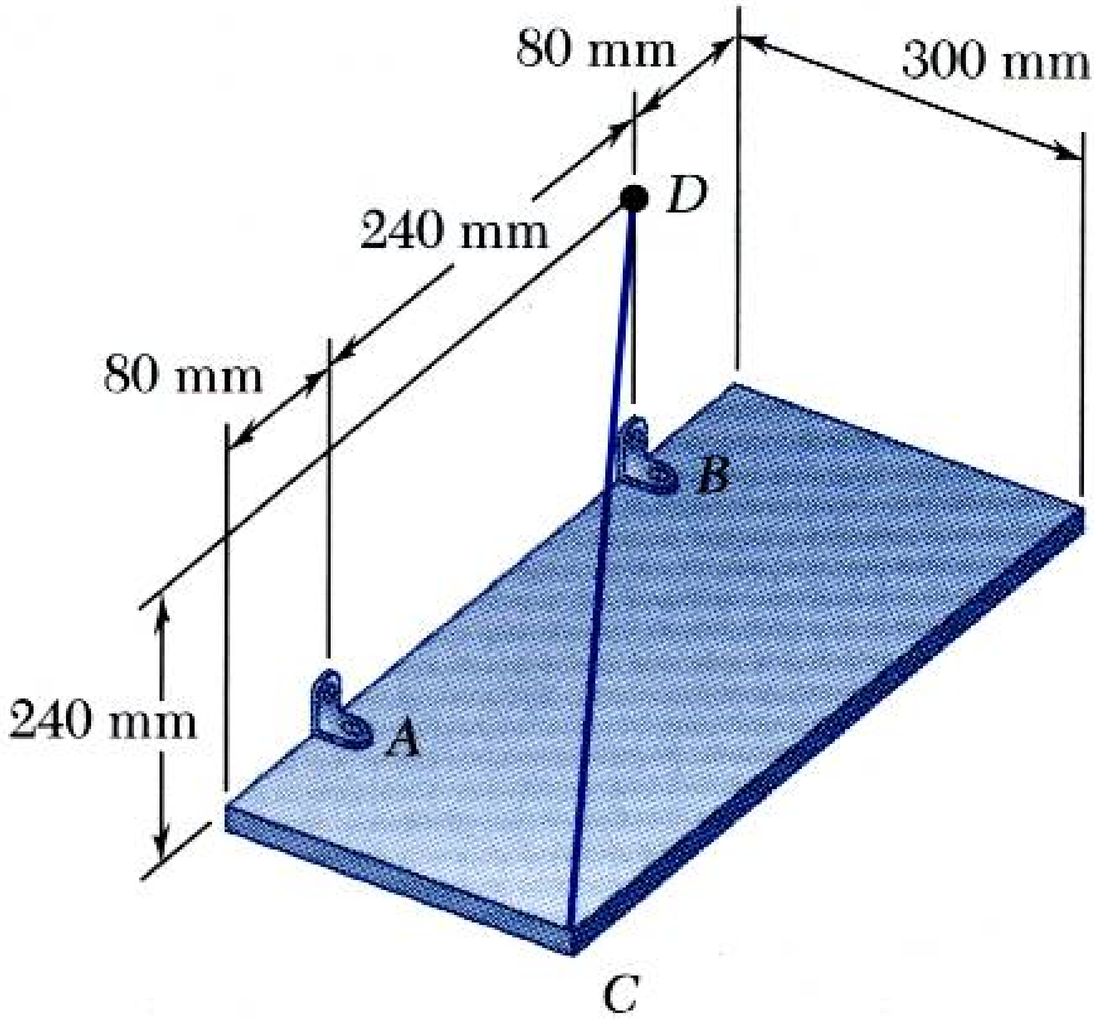






❑ Sample Problem 02

The rectangular plate is supported by the brackets at *A* and *B* and by a wire *CD*. Knowing that the tension in the wire is 200 N, determine the moment about *A* of the force exerted by the wire at *C*.

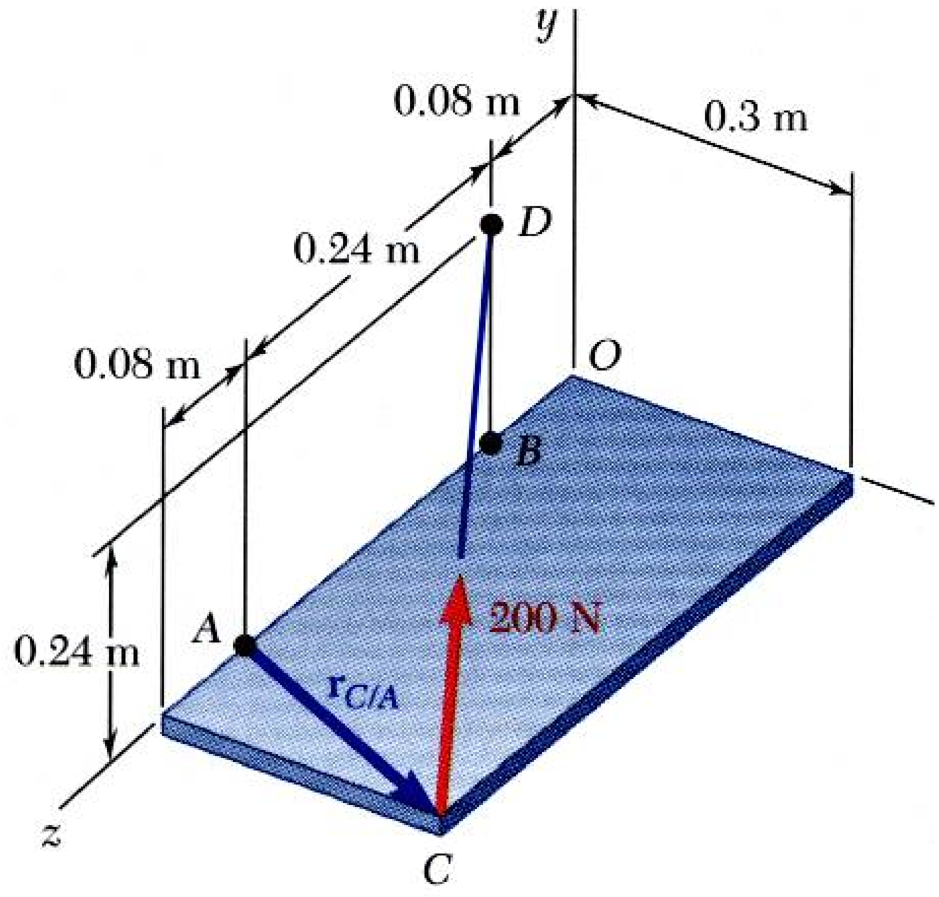


Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 02

SOLUTION:

$$\Rightarrow \vec{r}_{C/A} = 0.3\vec{i} + 0.08\vec{k}$$



$$\vec{F} = -(120\text{ N})\vec{i} + (96\text{ N})\vec{j} - (128\text{ N})\vec{k}$$

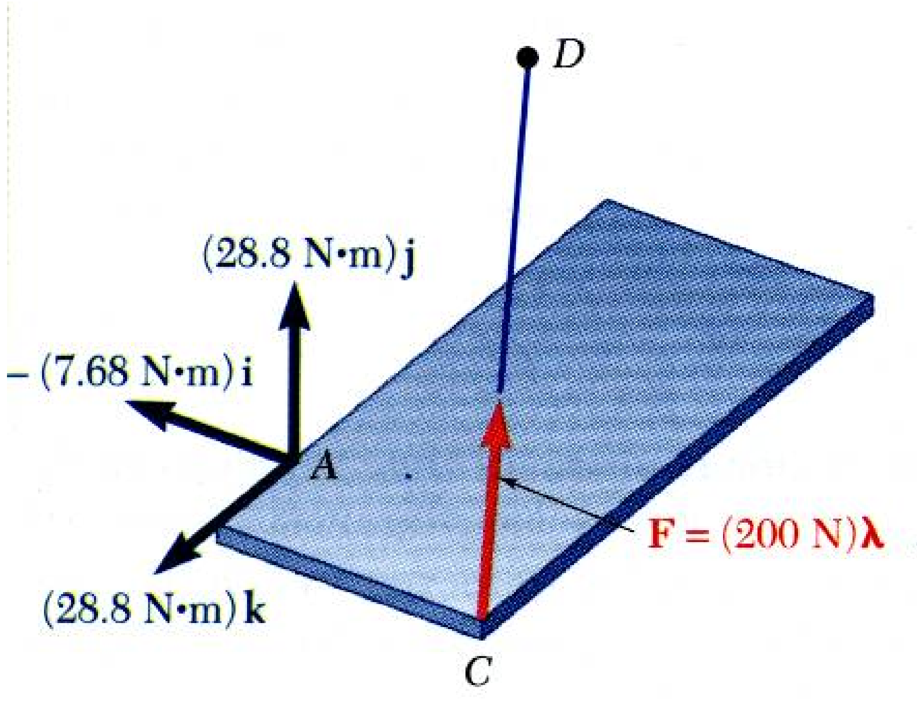
Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 02

SOLUTION:

$$\vec{r}_{C/A} = 0.3\vec{i} + 0.08\vec{k}$$

$$\vec{F} = -(120\ \text{N})\vec{i} + (96\ \text{N})\vec{j} - (128\ \text{N})\vec{k}$$



$$\Rightarrow \vec{M}_A = -(7.68\ \text{N}\cdot\text{m})\vec{i} + (28.8\ \text{N}\cdot\text{m})\vec{j} + (28.8\ \text{N}\cdot\text{m})\vec{k}$$

Rigid Bodies: Equivalent Systems of Forces

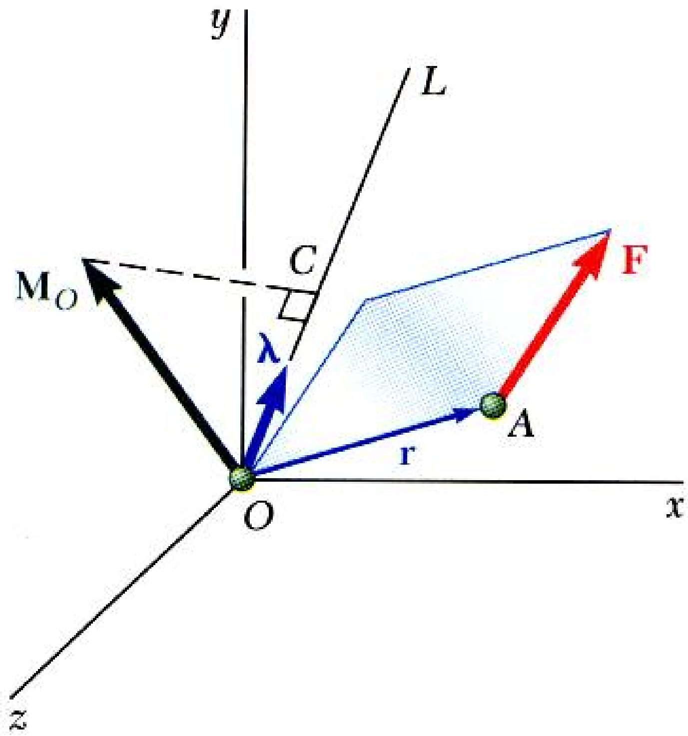
❑ Moment of a Force About a Given Axis

- Moment $\boldsymbol{M_O}$ of a force $\boldsymbol{F}$ applied at the point $\boldsymbol{A}$ about a point $\boldsymbol{O}$,

$$\vec{M}_O = \vec{r} \times \vec{F}$$

- Scalar moment M_{OL} about an axis $\boldsymbol{OL}$ is the projection of the moment vector $\boldsymbol{M_O}$ onto the axis,

$$M_{OL} = \vec{\lambda} \bullet \vec{M}_O = \vec{\lambda} \bullet (\vec{r} \times \vec{F})$$



- Moments of $\boldsymbol{F}$ about the coordinate axes,

$$\vec{M}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = (yF_z - zF_y)\vec{i} + (zF_x - xF_z)\vec{j} + (xF_y - yF_x)\vec{k} \quad \Rightarrow \quad \begin{aligned} M_x &= yF_z - zF_y \\ M_y &= zF_x - xF_z \\ M_z &= xF_y - yF_x \end{aligned}$$

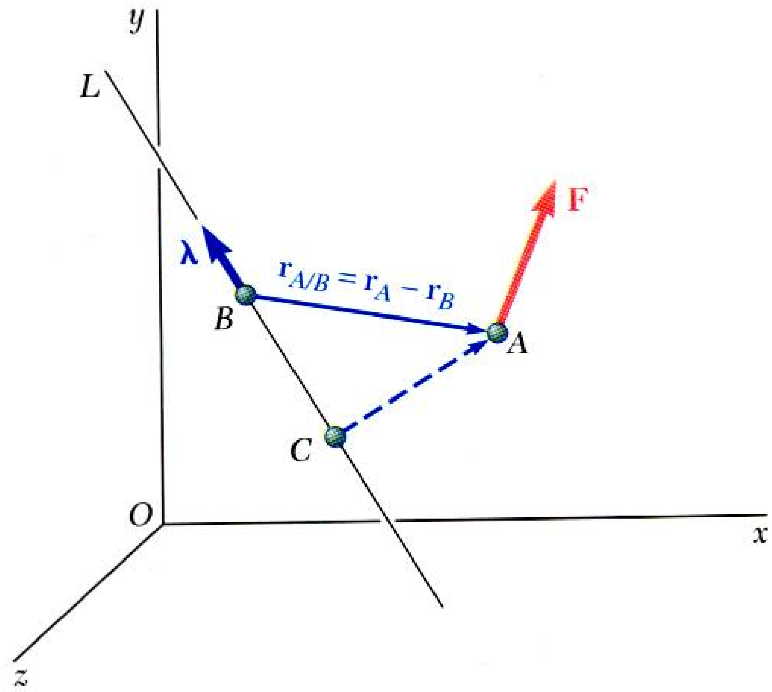
Rigid Bodies: Equivalent Systems of Forces

❑ Moment of a Force About a Given Axis

- Moment of a force about an arbitrary axis,

$$M_{BL} = \vec{\lambda} \bullet \vec{M}_B \quad \Rightarrow \quad \boxed{M_{BL} = \vec{\lambda} \bullet (\vec{r}_{A/B} \times \vec{F})}$$

$$\boxed{\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B}$$



- ***The result is independent of the point B along the given axis.***

It should be noted that the result obtained is independent of the choice of the point B on the given axis. Indeed, denoting by MCL the result obtained with a different point C, we have

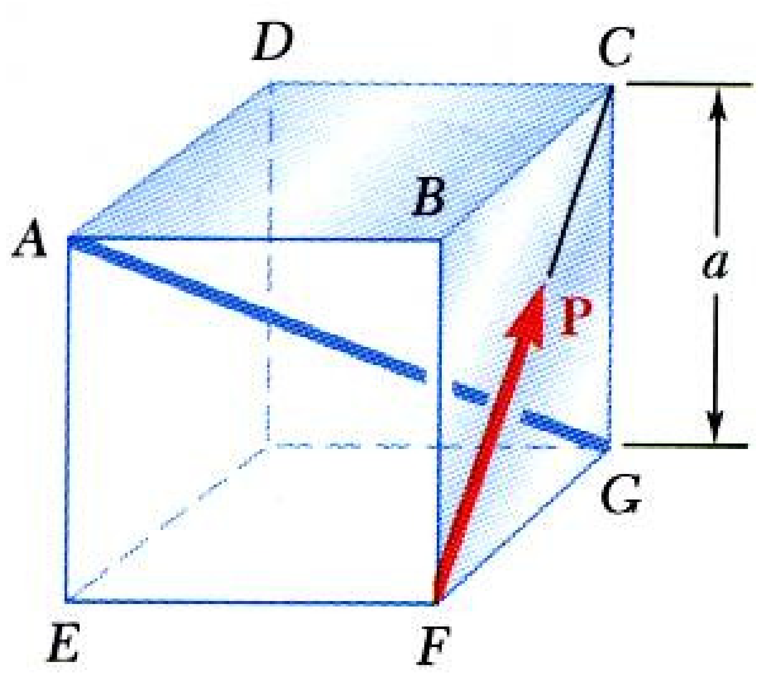
$$M_{CL} = \vec{\lambda} \bullet (\vec{r}_{A/C} \times \vec{F}) = \vec{\lambda} \bullet [(\vec{r}_A - \vec{r}_C) \times \vec{F}] =$$

$$\vec{\lambda} \bullet [(\vec{r}_A - \vec{r}_B) \times \vec{F}] + \vec{\lambda} \bullet [(\vec{r}_B - \vec{r}_C) \times \vec{F}]$$

$$\Rightarrow \quad M_{CL} = M_{BL}$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 03



A cube is acted on by a force ***P*** as shown. Determine the moment of ***P***

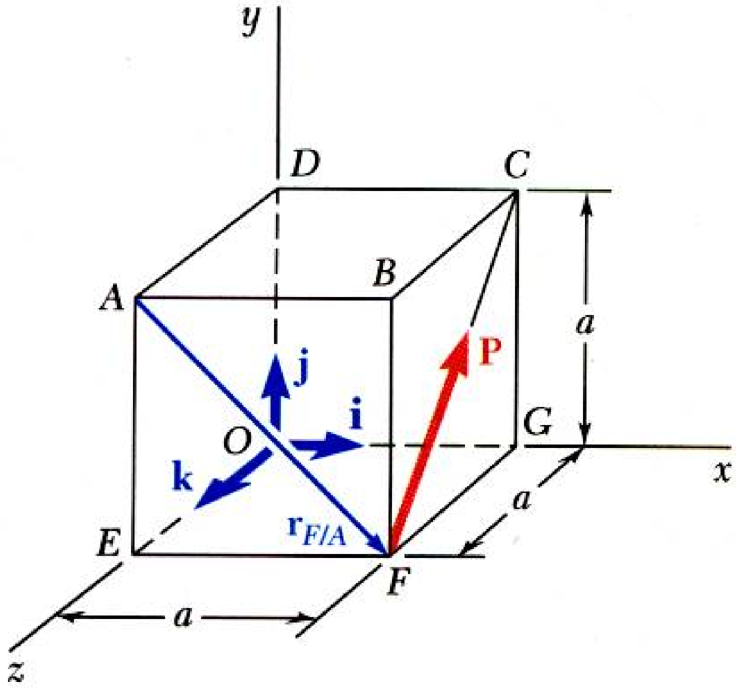
a) about *A*
b) about the edge *AB* and
c) about the diagonal *AG* of the cube.
d) Determine the perpendicular distance between *AG* and *FC*.

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 03

SOLUTION:

- Moment of ***P*** about *A*,



$$\vec{r}_{F/A} = a(\vec{i} - \vec{j})$$

$$\vec{P} = \frac{P}{\sqrt{2}}(\vec{j} - \vec{k})$$

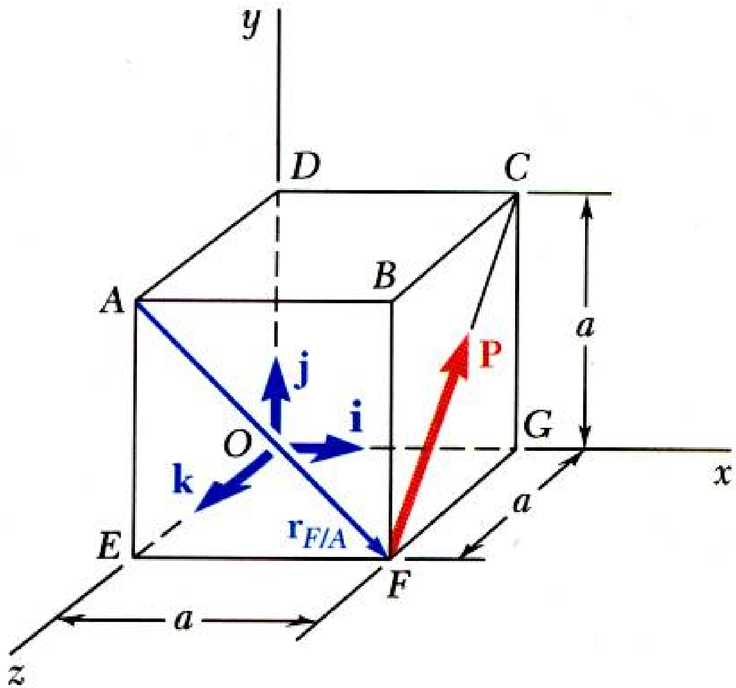
$$\Rightarrow \vec{M}_A = (\frac{P}{\sqrt{2}}a)(\vec{i} + \vec{j} + \vec{k})$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 03

SOLUTION:

- Moment of ***P*** about *AB,*



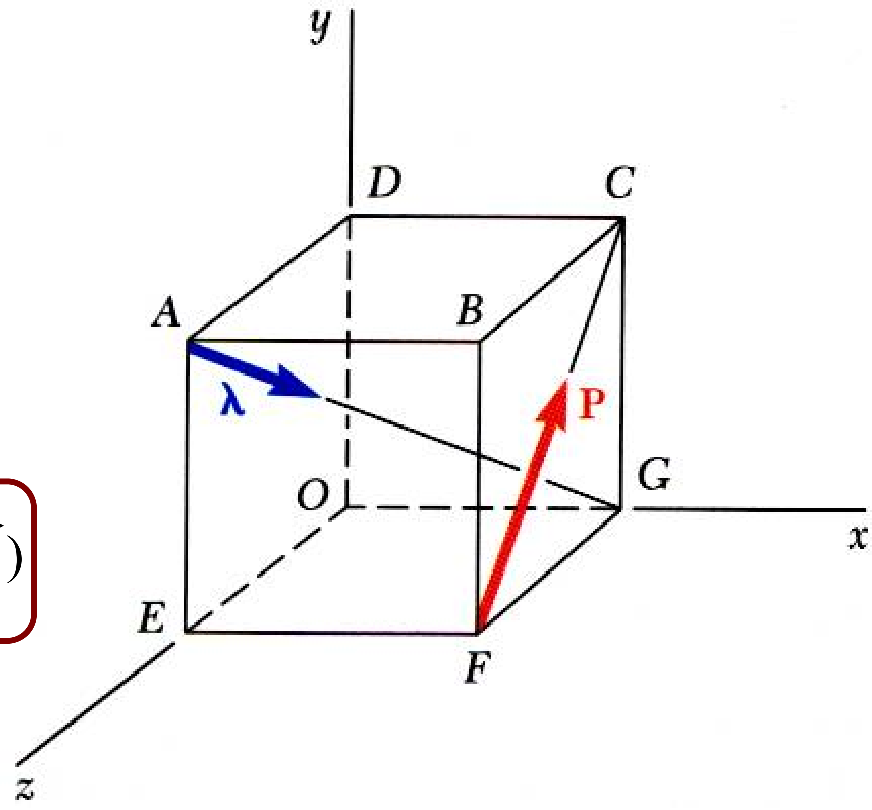
$$\vec{\lambda}_{AB} = \vec{i}$$

$$M_{AB} = \frac{P}{\sqrt{2}} a$$

❑ Sample Problem 03

SOLUTION:

- Moment of ***P*** about the diagonal *AG*,



$$\vec{\lambda} = \frac{1}{\sqrt{3}}(\vec{i} - \vec{j} - \vec{k})$$

$$\vec{M}_A = \frac{P}{\sqrt{2}} a(\vec{i} + \vec{j} + \vec{k})$$

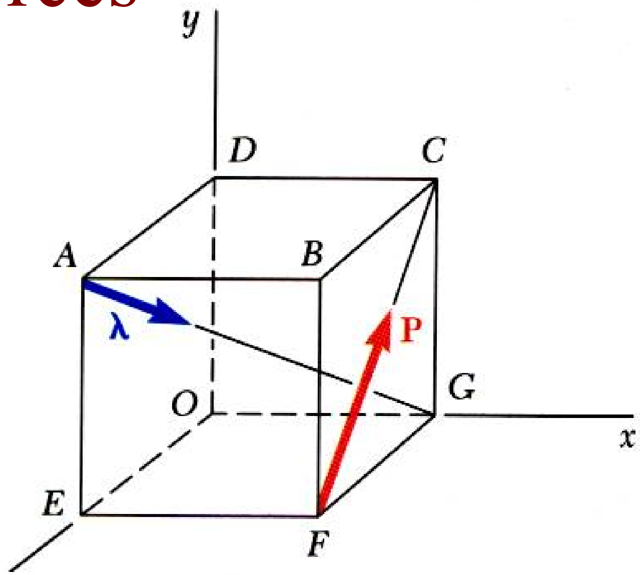
$$M_{AG} = -\frac{Pa}{\sqrt{6}}$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 03

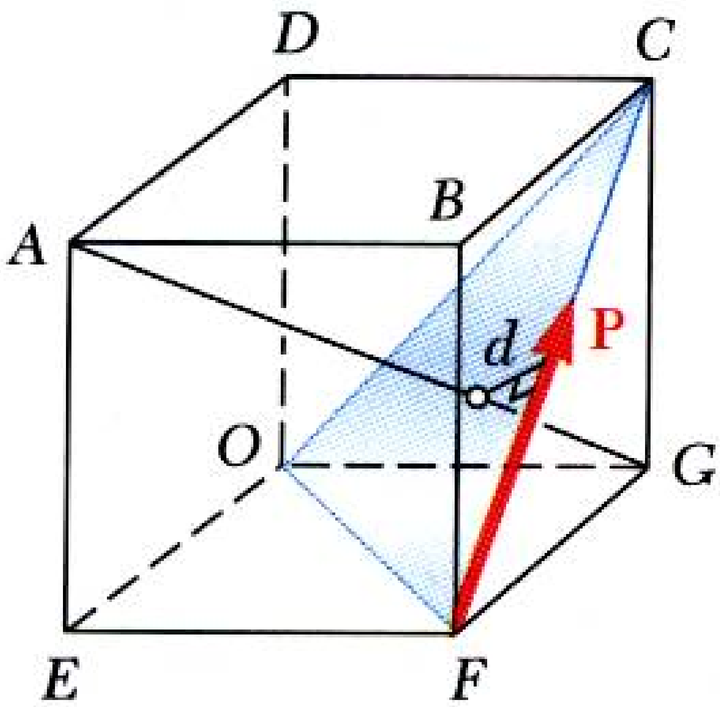
SOLUTION:

- Perpendicular distance between *AG* and *FC*,



⇒ ***Therefore, P is perpendicular to AG.***

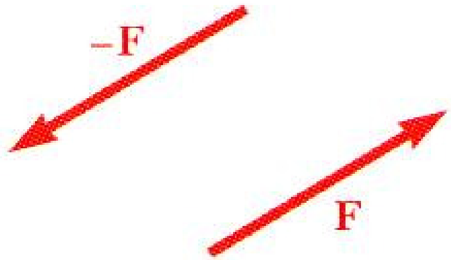
$$d = \frac{a}{\sqrt{6}}$$



Rigid Bodies: Equivalent Systems of Forces

❑ Moment of a Couple

- Two forces ***F*** and -***F*** having the same magnitude, parallel lines of action, and opposite sense are said to form a ***couple***.

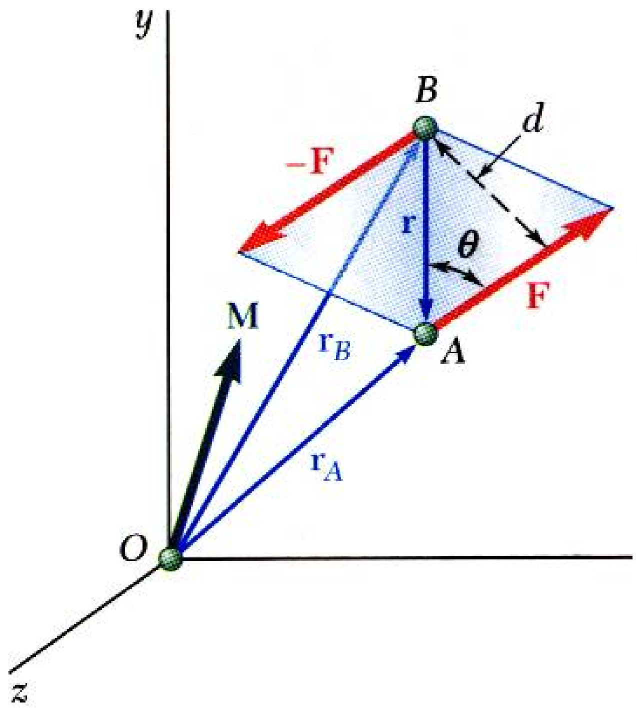


- Moment of the couple,

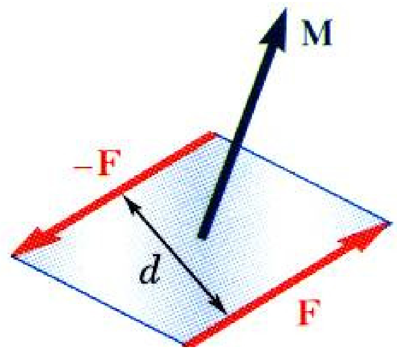
$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F}) = (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

$$\Rightarrow \quad \vec{M} = \vec{r} \times \vec{F}$$

$$\Rightarrow \quad M = rF\sin\theta \quad \Rightarrow \quad \boxed{M = Fd}$$



- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.



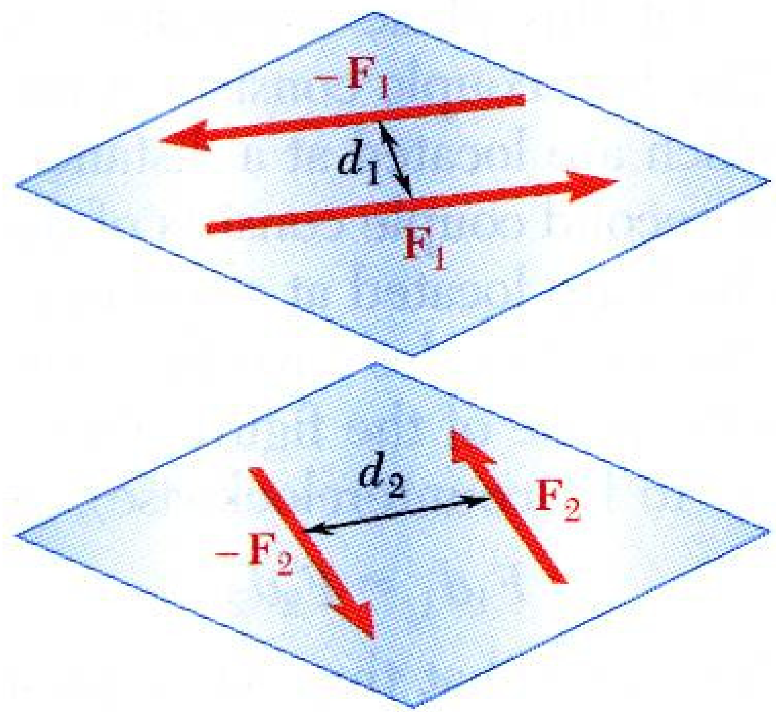
❑ Moment of a Couple

Two couples will have equal moments if

$$F_1 d_1 = F_2 d_2$$

⇒

- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.

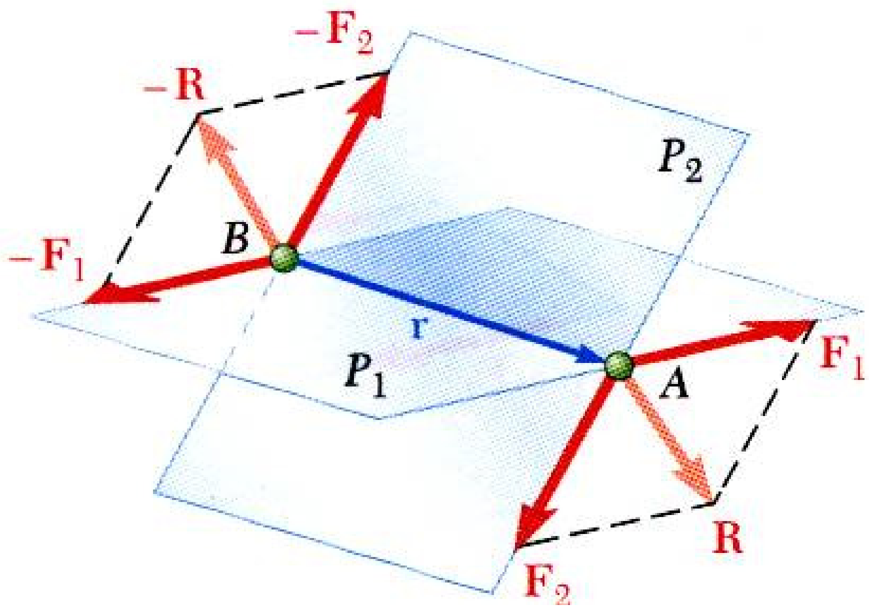


Rigid Bodies: Equivalent Systems of Forces

Addition of Couples

- Consider two intersecting planes P_1 and P_2 with each containing a couple

$$\vec{M}_1 = \vec{r} \times \vec{F}_1 \text{ in plane } P_1$$
$$\vec{M}_2 = \vec{r} \times \vec{F}_2 \text{ in plane } P_2$$

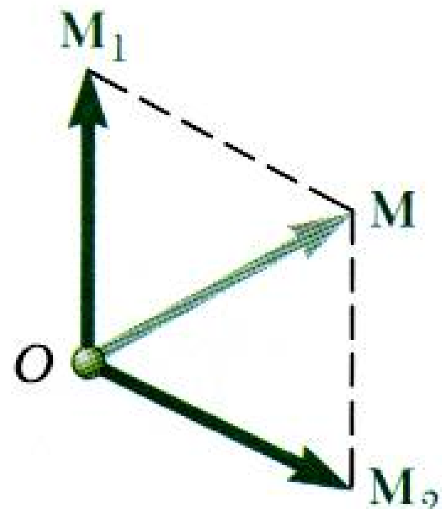


- Resultants of the vectors also form a couple

$$\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times (\vec{F}_1 + \vec{F}_2)$$

- By Varigon's theorem

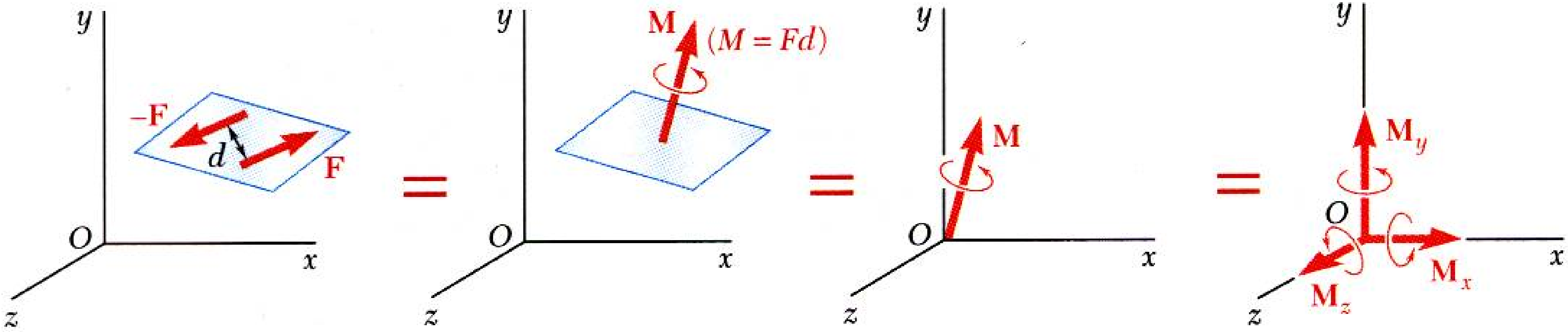
$$\vec{M} = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 \quad \Rightarrow \quad \vec{M} = \vec{M}_1 + \vec{M}_2$$



- Sum of two couples is also a couple that is equal to the vector sum of the two couples

Rigid Bodies: Equivalent Systems of Forces

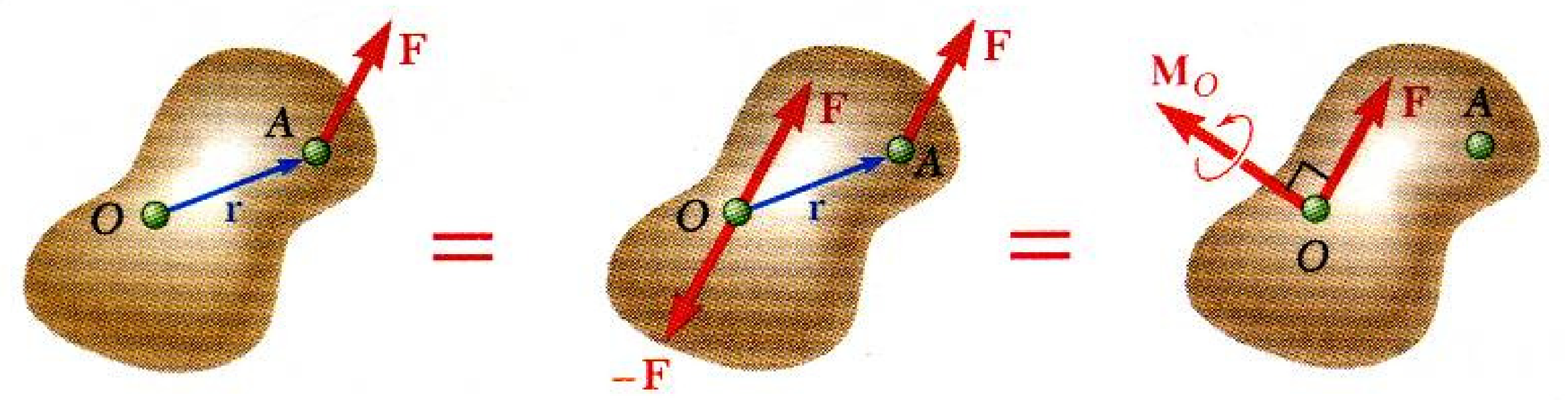
Couples Can Be Represented by Vectors



- A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.
- *Couple vectors* obey the law of addition of vectors.
- Couple vectors are free vectors, i.e., the point of application is not significant.
- Couple vectors may be resolved into component vectors.

Rigid Bodies: Equivalent Systems of Forces

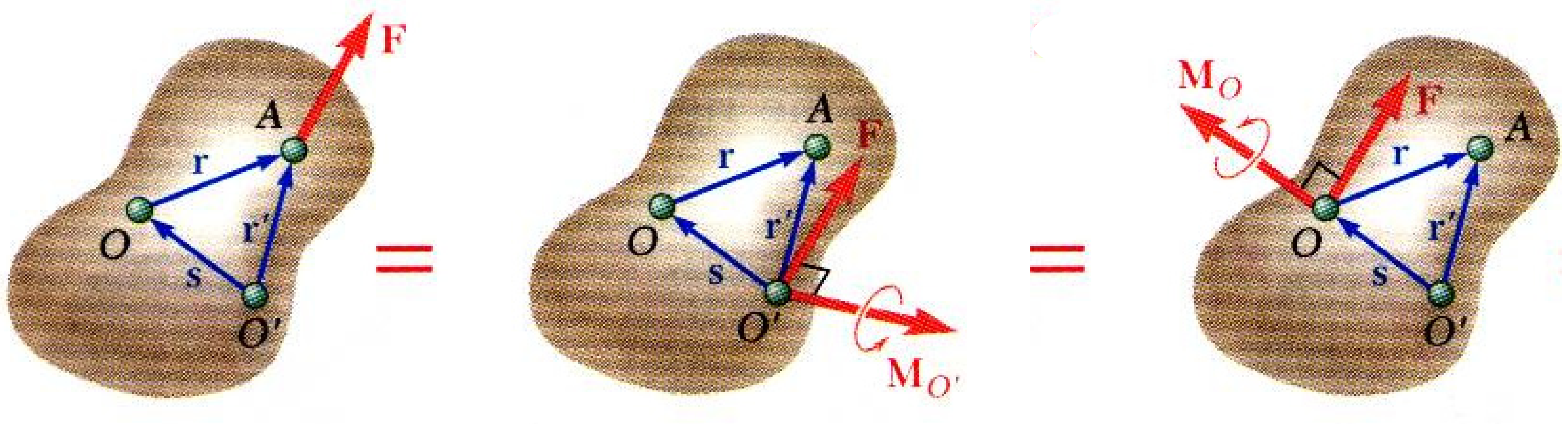
❑ Resolution of a Force Into a Force at O and a Couple



- Force vector ***F*** can not be simply moved to *O* without modifying its action on the body.
- Attaching equal and opposite force vectors at *O* produces no net effect on the body.
- The three forces may be replaced by an equivalent force vector and couple vector, i.e, ***a force-couple system***.

Rigid Bodies: Equivalent Systems of Forces

❑ Resolution of a Force Into a Force at O and a Couple



- Moving $\boldsymbol{F}$ from A to a different point O' requires the addition of a different couple vector $\boldsymbol{M}_{O'}$ $\boxed{\vec{M}_{O'} = \vec{r}\,' \times \vec{F}}$

- The moments of $\boldsymbol{F}$ about O and O' are related,

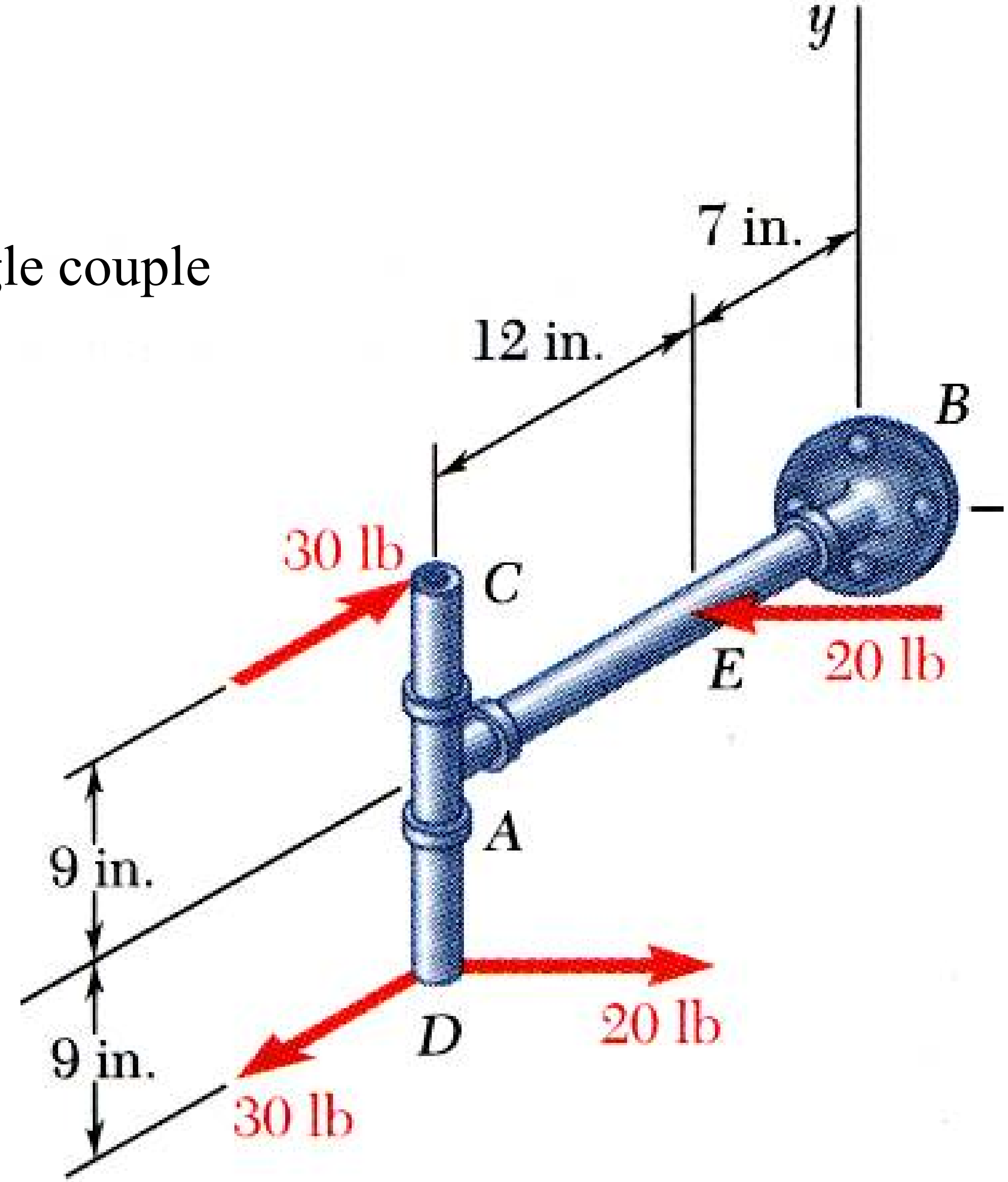
$$\vec{M}_{O'} = \vec{r}\,' \times \vec{F} = (\vec{s} + \vec{r}) \times \vec{F} = \vec{s} \times \vec{F} + \vec{r} \times \vec{F} \quad \Rightarrow \quad \boxed{\vec{M}_{O'} = \vec{s} \times \vec{F} + \vec{M}_O}$$

- Moving the force-couple system from O to O' requires the addition of the moment of the force at O about O'.

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 04

Determine the components of the single couple equivalent to the couples shown.



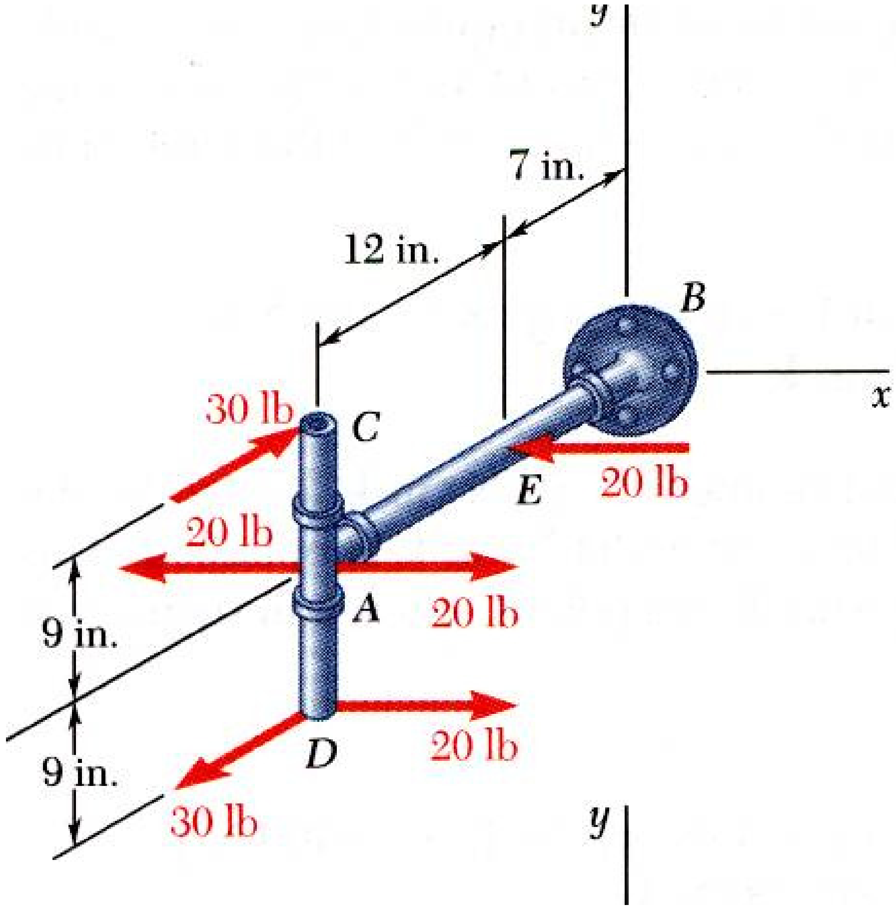
Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 04

SOLUTION:

- Attach equal and opposite 20 lb forces in the $\pm x$ direction at A

- The three couples may be represented by three couple vectors,

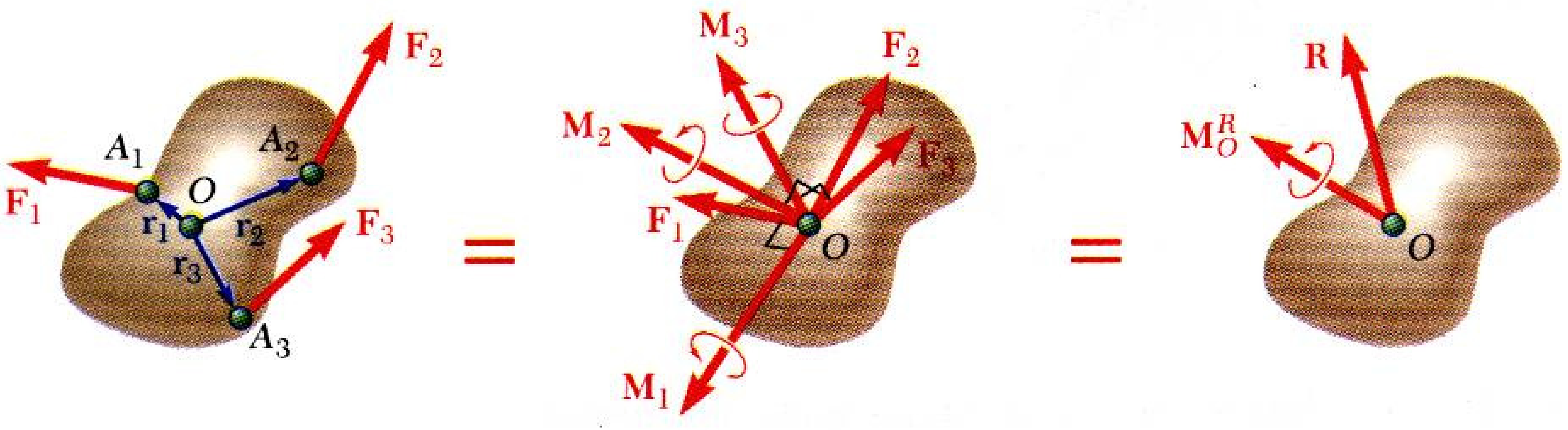


$$M_x = -540\ \text{lb}\cdot\text{in.}$$
$$M_y = +240\text{lb}\cdot\text{in.}$$
$$M_z = +180\ \text{lb}\cdot\text{in.}$$

$$\Rightarrow \vec{M} = -(540\ \text{lb}\cdot\text{in.})\vec{i} + (240\text{lb}\cdot\text{in.})\vec{j} + (180\ \text{lb}\cdot\text{in.})\vec{k}$$

Rigid Bodies: Equivalent Systems of Forces

❑ System of Forces: Reduction to a Force and Couple

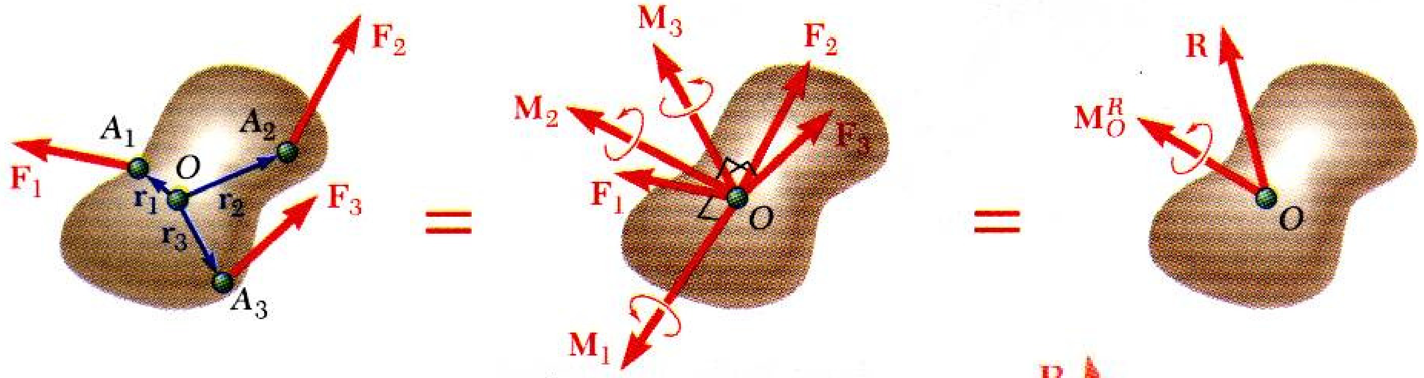


- A system of forces may be replaced by a collection of force-couple systems acting a given point O
- The force and couple vectors may be combined into a resultant force vector and a resultant couple vector,

$$\vec{R} = \sum \vec{F} \qquad \vec{M}_O^R = \sum (\vec{r} \times \vec{F})$$

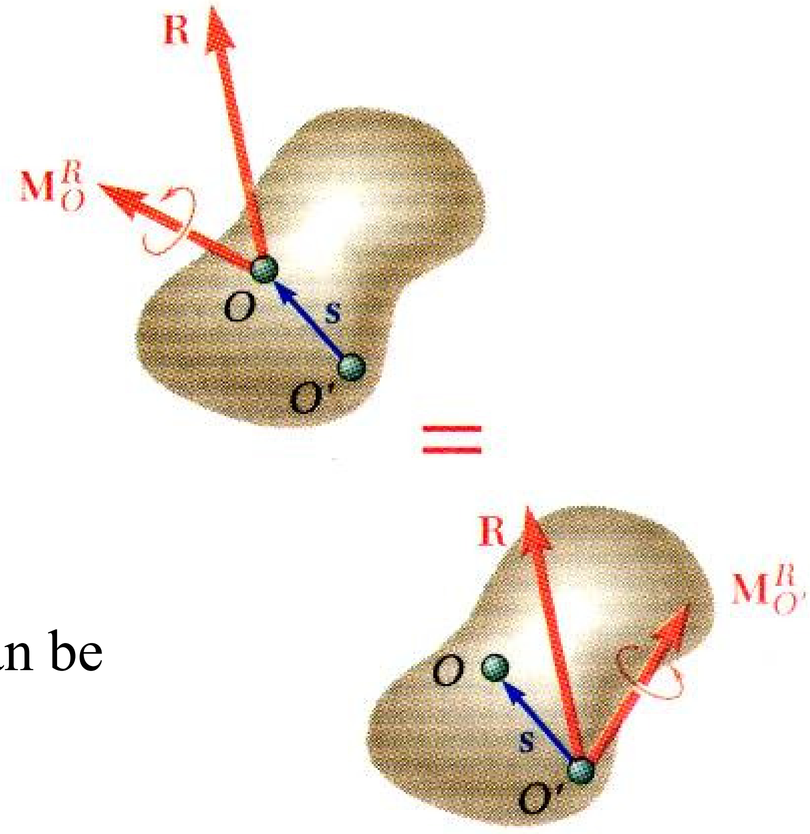
Rigid Bodies: Equivalent Systems of Forces

❑ System of Forces: Reduction to a Force and Couple



- The force-couple system at O may be moved to O' with the addition of the moment of $\boldsymbol{R}$ about O',

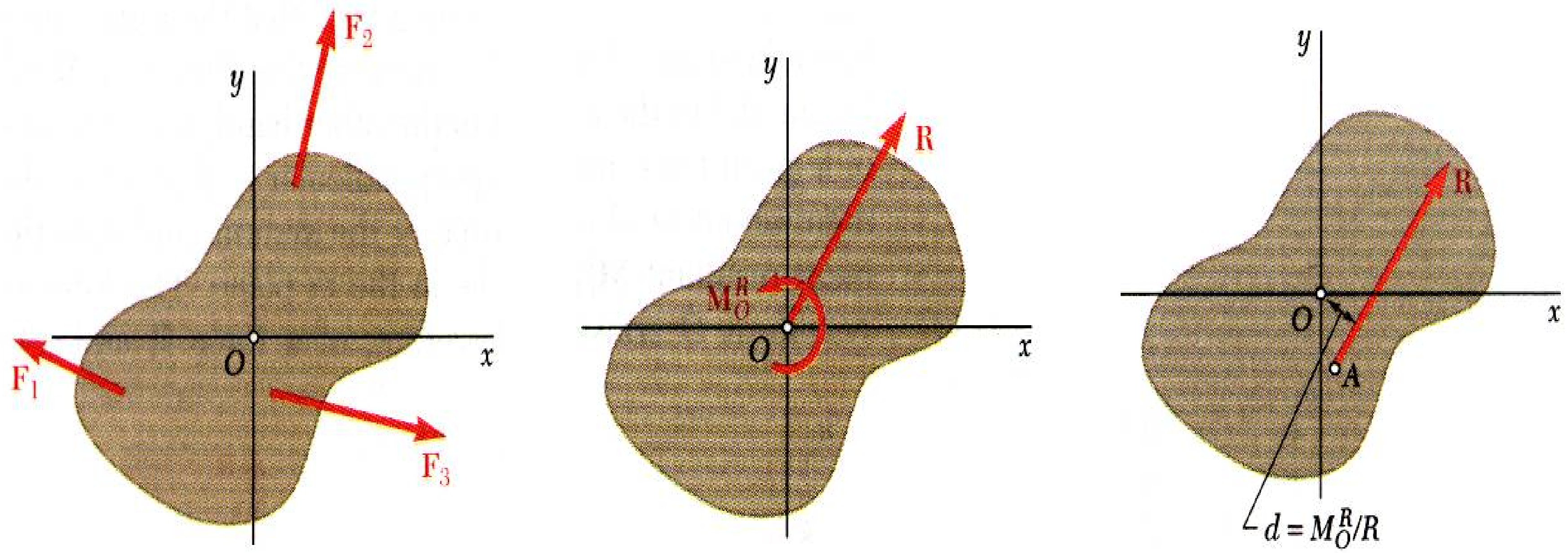
$$\vec{M}_{O'}^{R} = \vec{M}_{O}^{R} + \vec{s} \times \vec{R}$$



- Two systems of forces are equivalent if they can be reduced to the same force-couple system.

Rigid Bodies: Equivalent Systems of Forces

❑ Further Reduction of a System of Forces

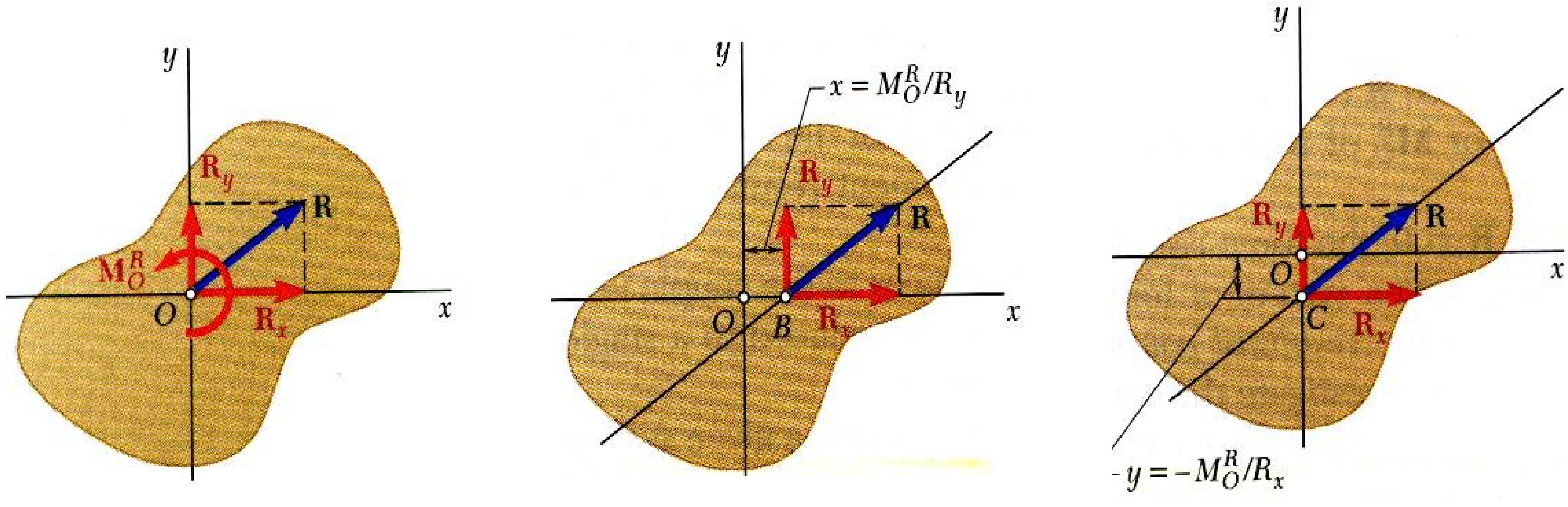


- System of coplanar forces is reduced to a force-couple system $\vec{R}$ and $\vec{M}_O^R$ that is mutually perpendicular.

- System can be reduced to a single force by moving the line of action of $\vec{R}$ until its moment about O becomes $\vec{M}_O^R$

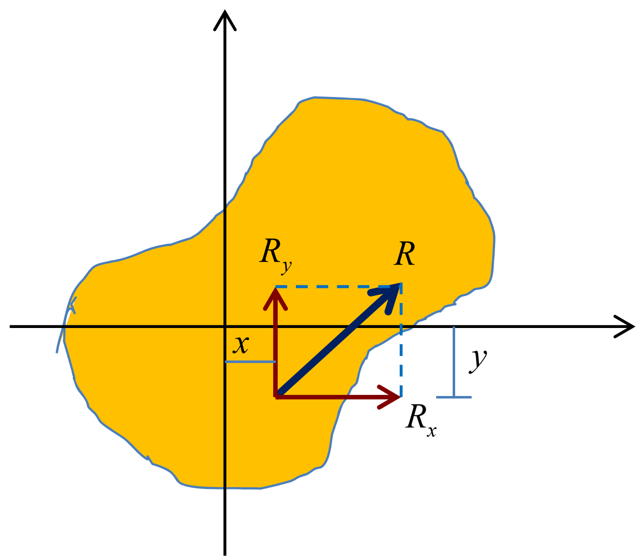
Rigid Bodies: Equivalent Systems of Forces

❑ Further Reduction of a System of Forces



- In terms of rectangular coordinates,

$$xR_y + yR_x = M_O^R$$

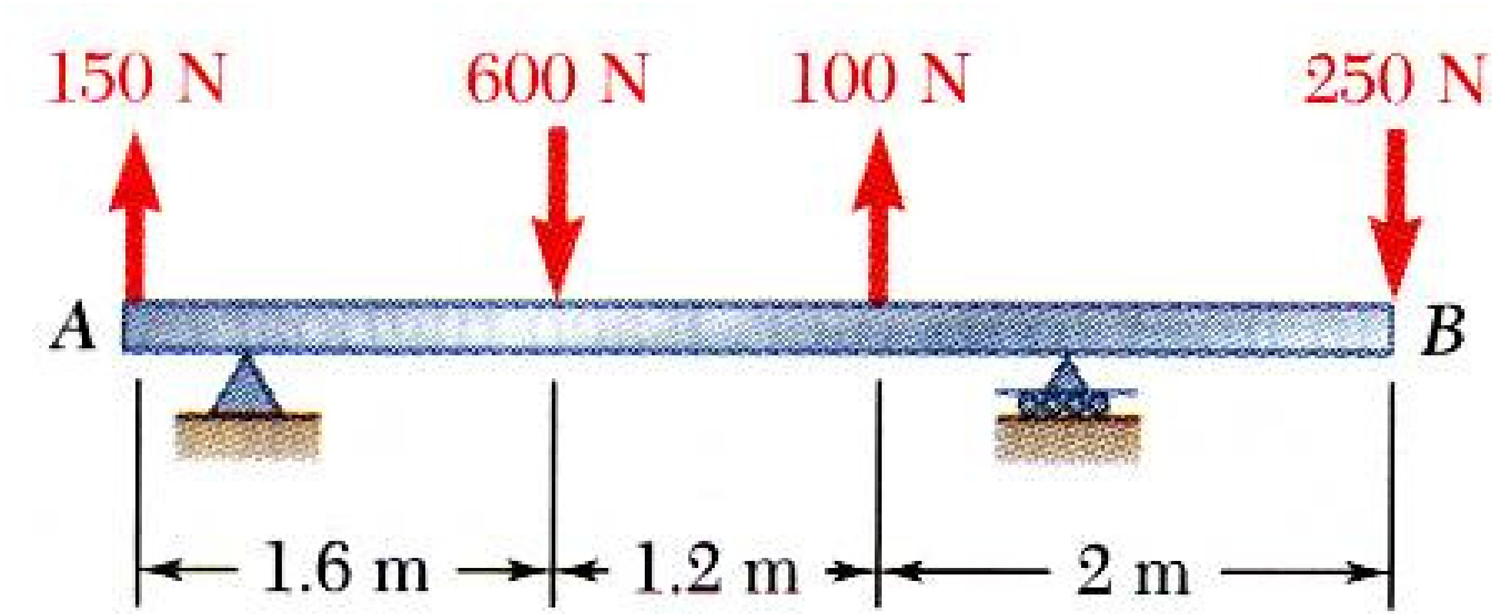


Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 05

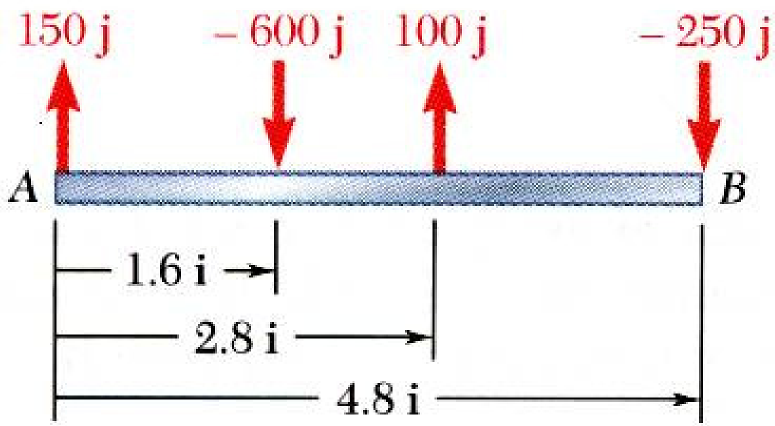
For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at *A*, (b) an equivalent force couple system at *B*, and (c) a single force or resultant.

Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.



Rigid Bodies: Equivalent Systems of Forces

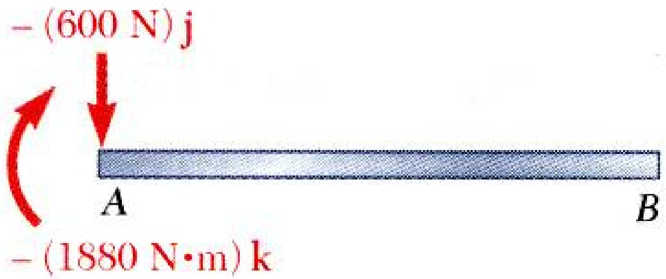
Sample Problem 05



SOLUTION:

a) Compute the resultant force and the resultant couple at *A*.

$$\Rightarrow \vec{R} = -(600\ \text{N})\vec{j}$$



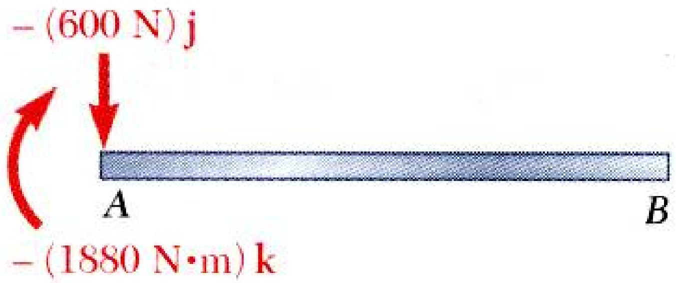
$$\Rightarrow \vec{M}_A^R = -(1880\ \text{N}\cdot\text{m})\vec{k}$$

Rigid Bodies: Equivalent Systems of Forces

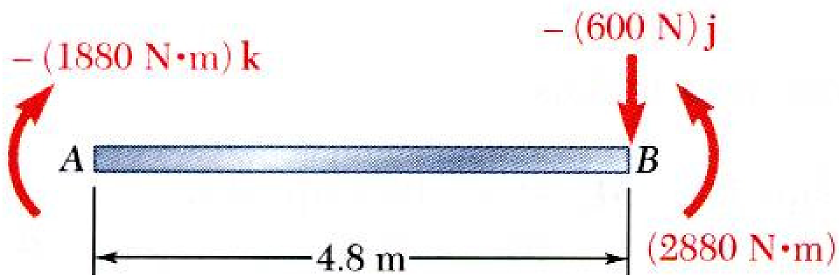
❑ Sample Problem 05

SOLUTION:

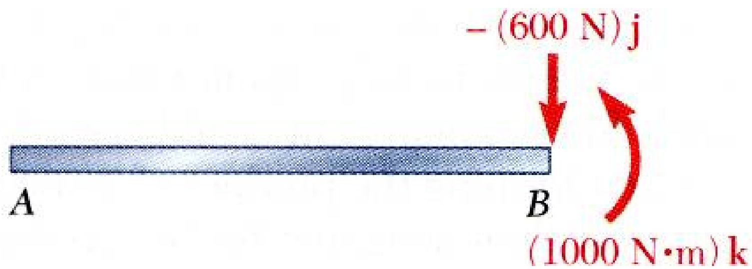
b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.



The force is unchanged by the movement of the force-couple system from *A* to *B*.



The couple at *B* is equal to the moment about *B* of the force-couple system found at *A*.



$$\Rightarrow \vec{M}_B^R = +(1000\ \text{N}\cdot\text{m})\,\vec{k}$$

Rigid Bodies: Equivalent Systems of Forces

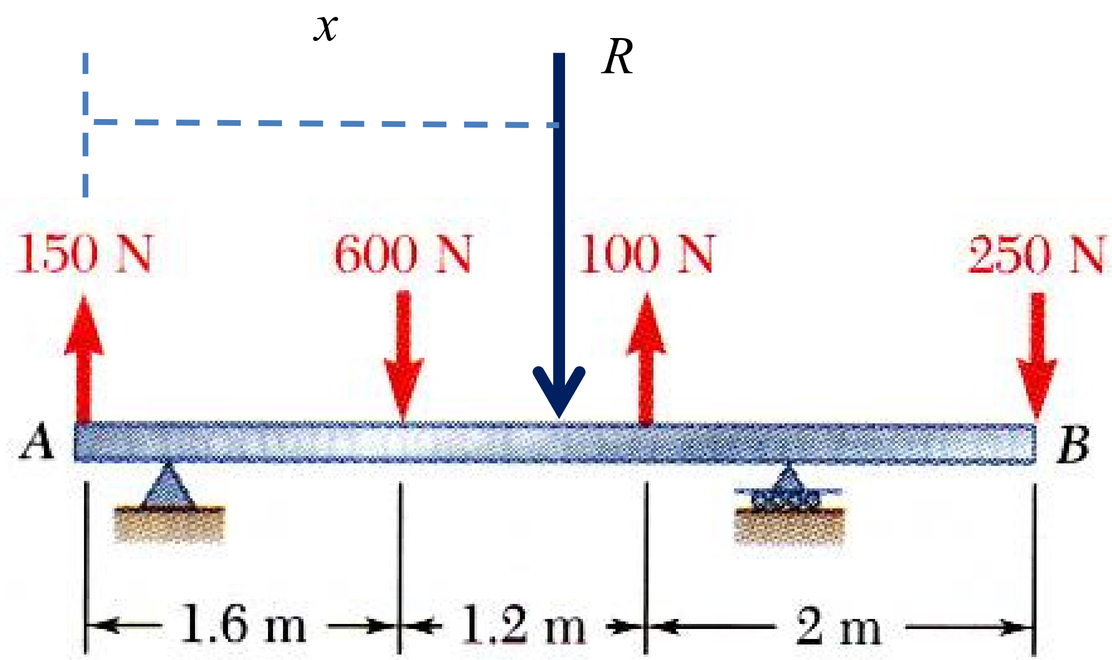
❑ Sample Problem 05

SOLUTION:

b) a single force or resultant.

$$R = -600 \text{ N}$$

$$M_A^R = -1880 \text{ N} \cdot \text{m}$$

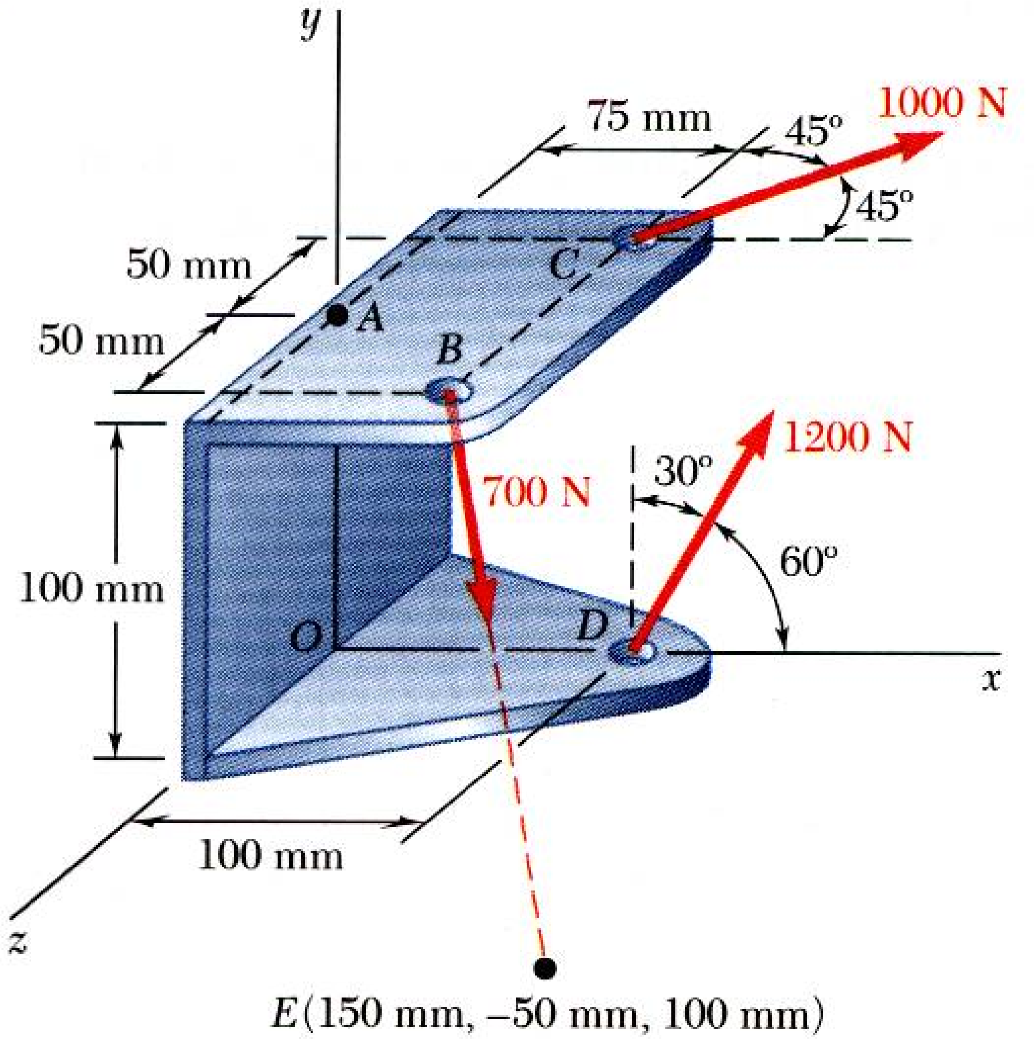


$$x = 3.13\ m$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 06

Three cables are attached to the bracket as shown. Replace the forces with an equivalent force-couple system at *A*.



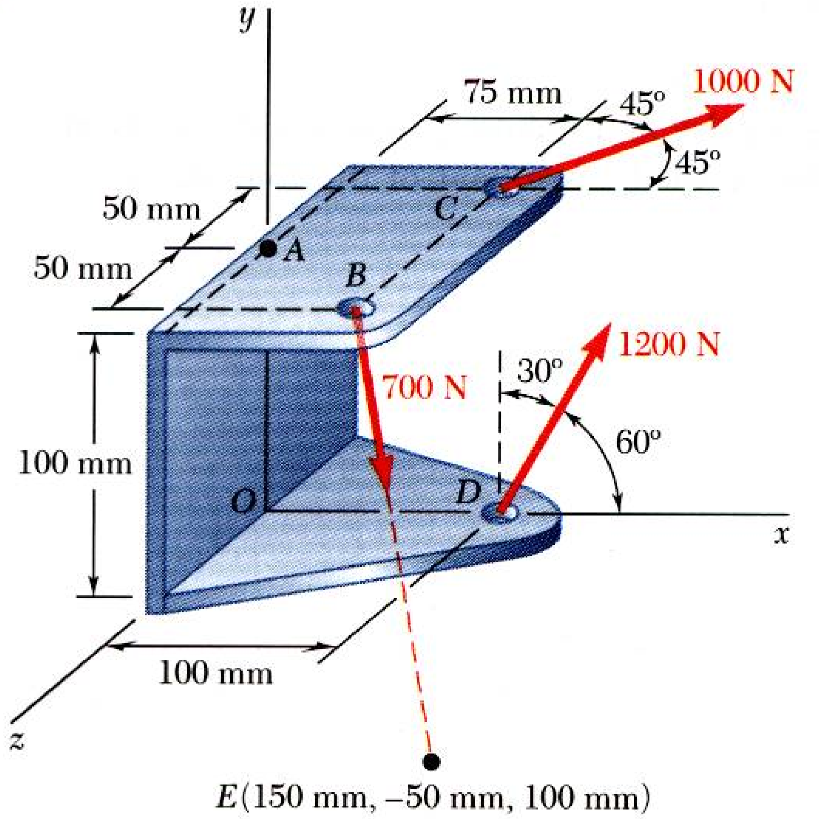
Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 06

SOLUTION:

- Determine the relative position vectors with respect to *A*.

$$\vec{r}_{C/A} = 0.075\vec{i} - 0.050\vec{k} \text{ (m)}$$
$$\vec{r}_{D/A} = 0.100\vec{i} - 0.100\vec{j} \text{ (m)}$$



- Resolve the forces into rectangular components.

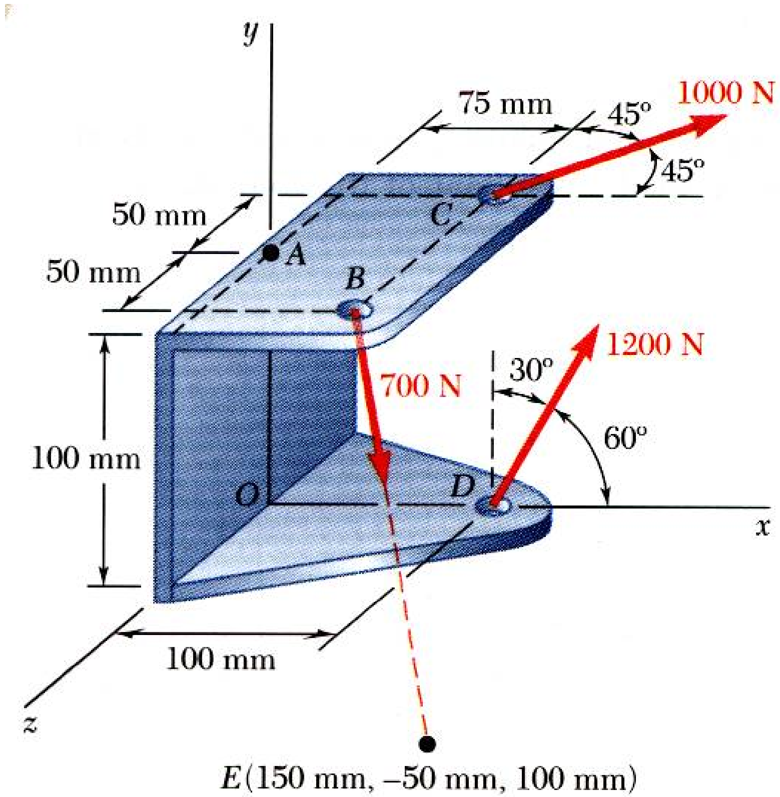
$$\vec{\lambda}_B = 0.429\vec{i} - 0.857\vec{j} + 0.289\vec{k}$$

$$\vec{F}_B = 300\vec{i} - 600\vec{j} + 200\vec{k} \text{ (N)}$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 06

SOLUTION:



- Resolve the forces into rectangular components.

$$\vec{F}_C = 707\vec{i} - 707\vec{k} \quad \text{(N)}$$

$$\vec{F}_D = 600\vec{i} + 1039\vec{j} \quad \text{(N)}$$

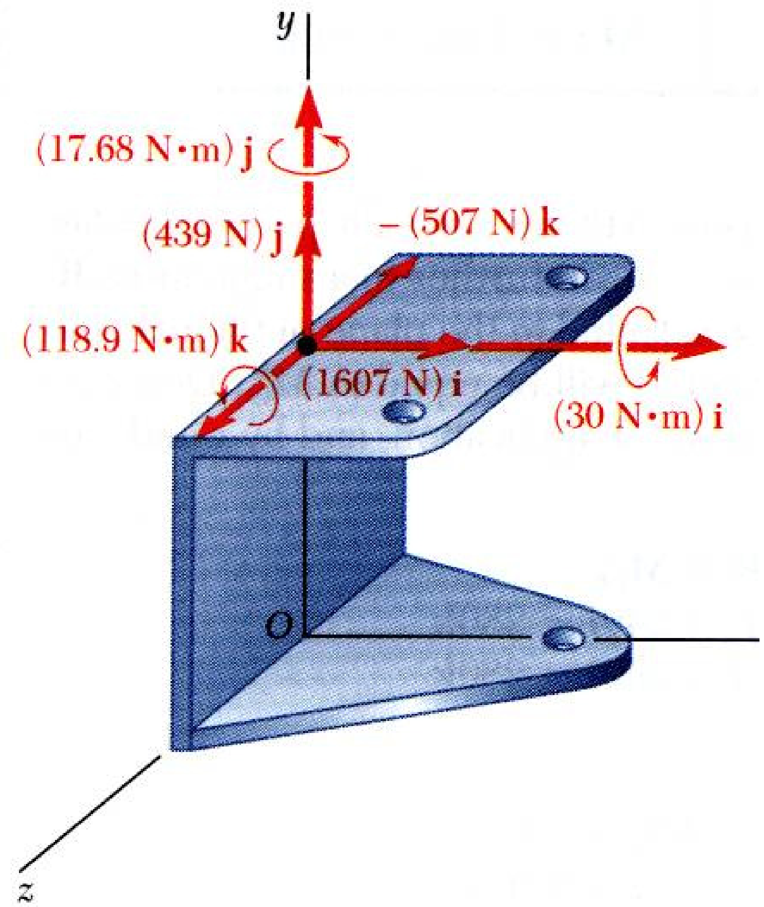
Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 06

SOLUTION:

- Compute the equivalent force,

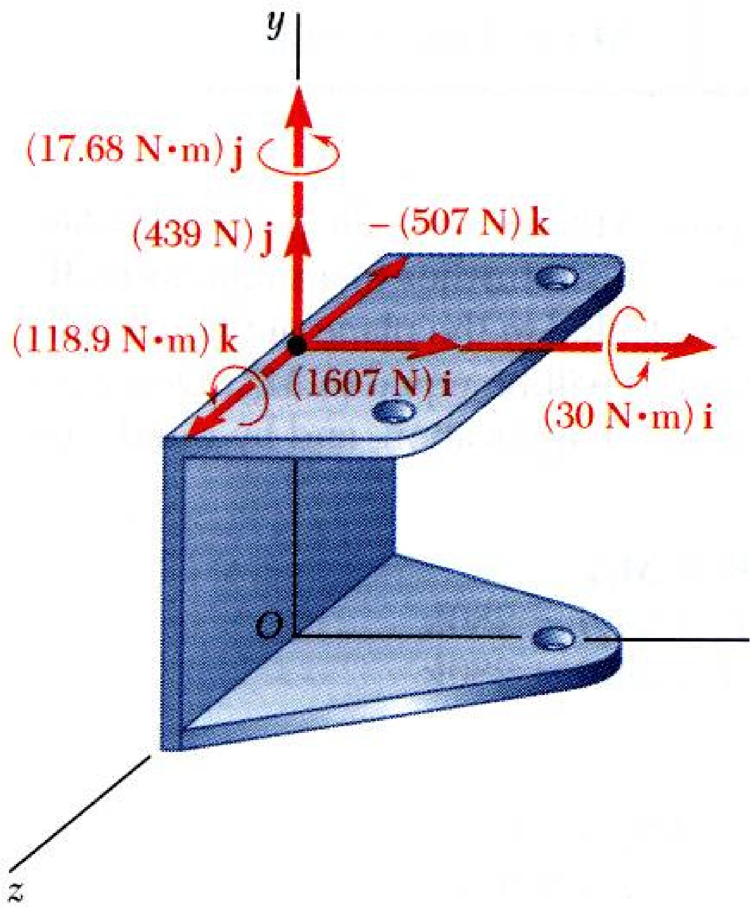
$$\Rightarrow \boxed{\vec{R} = 1607\vec{i} + 439\vec{j} - 507\vec{k} \ \ (\text{N})}$$



❑ Sample Problem 06

SOLUTION:

- Compute the equivalent couple,



$$\vec{r}_{C/A} \times \vec{F}_c = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.075 & 0 & -0.050 \\ 707 & 0 & -707 \end{vmatrix} = 17.68\vec{j}$$

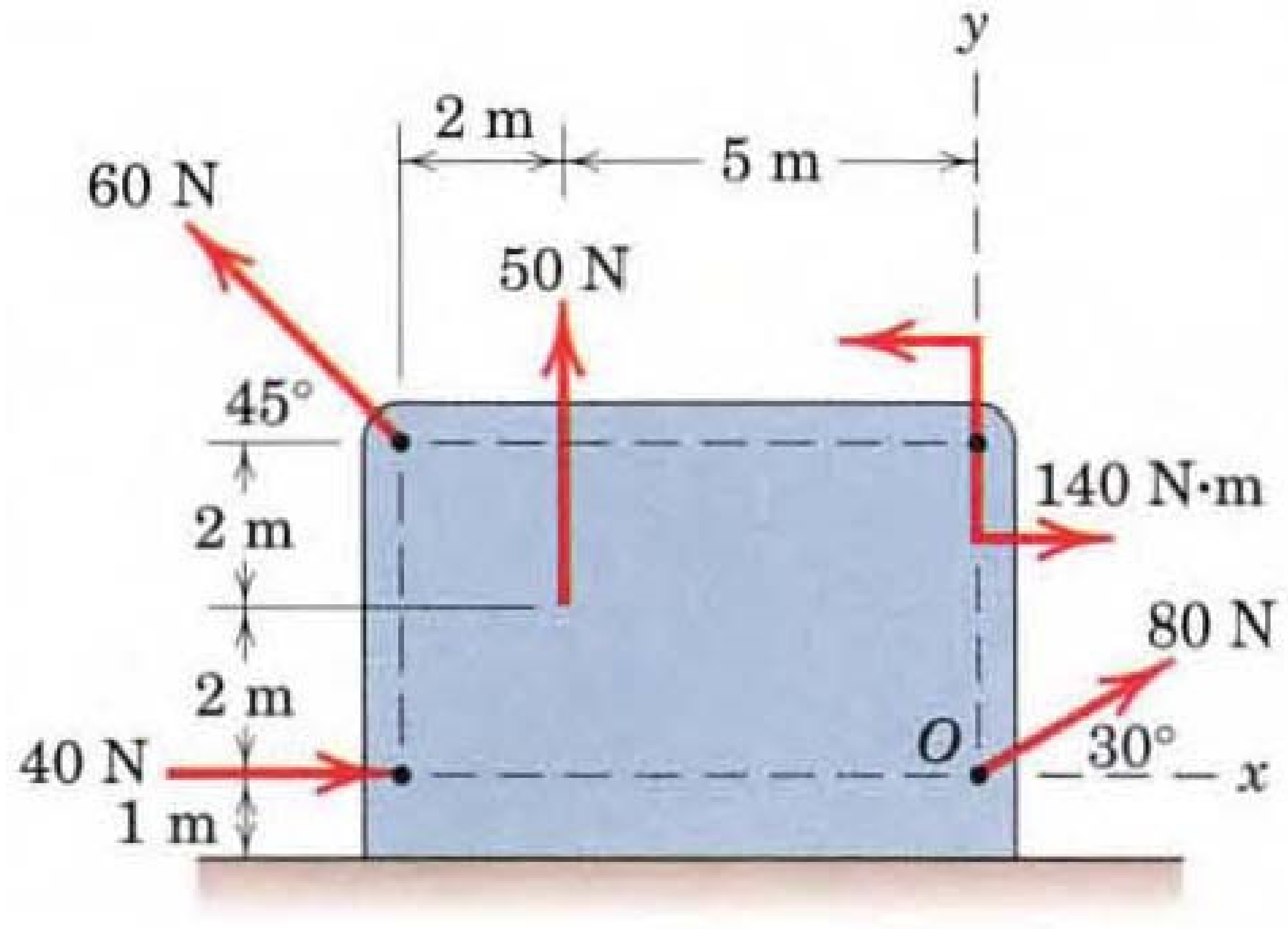
$$\vec{r}_{D/A} \times \vec{F}_D = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.100 & -0.100 & 0 \\ 600 & 1039 & 0 \end{vmatrix} = 163.9\vec{k}$$

$$\Rightarrow \boxed{\vec{M}_A^R = 30\vec{i} + 17.68\vec{j} + 118.9\vec{k}}$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 07

Determine a) the resultant of the four forces and one couple which act on the plate shown at point O. b) an equivalent force system



Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 07

SOLUTION:

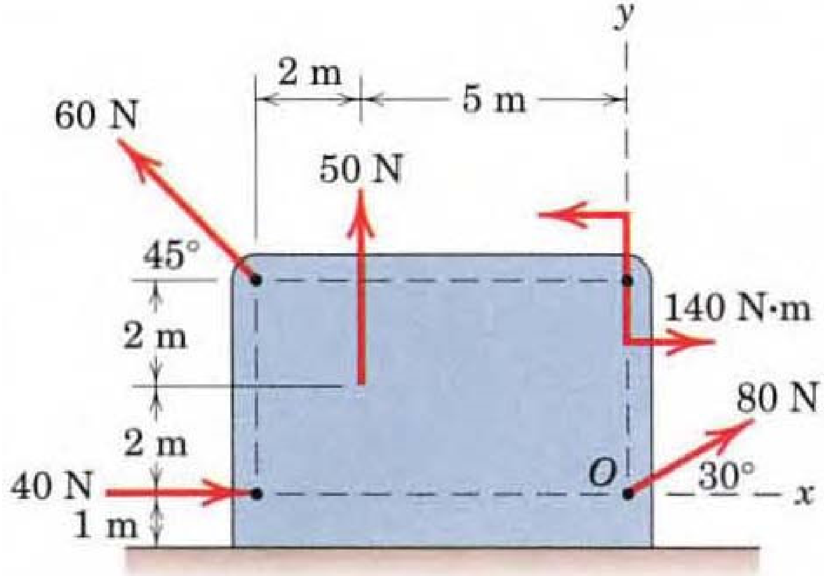
a) an equivalent force-couple system,

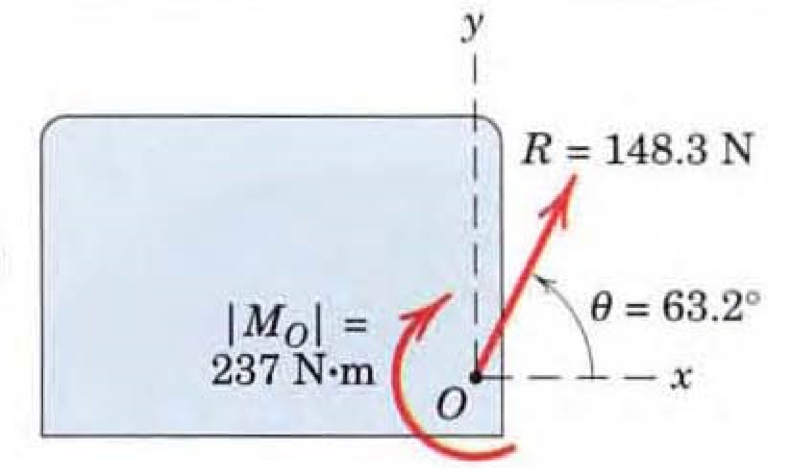
$\Rightarrow$ $R_{xo} = 66.9N$

$\Rightarrow$ $R_{yo} = 132.4N$

$\Rightarrow$ $R_o = 148.3N$

$\theta = 63.2^o$





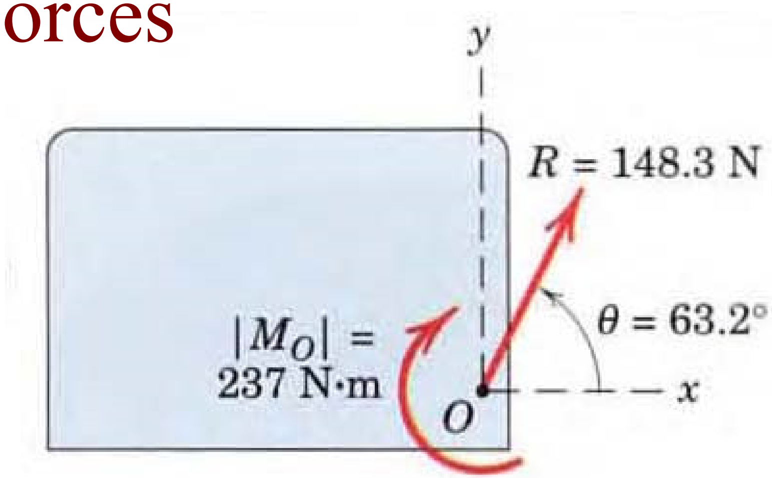
$M_o = -237\ (N.m)$

Rigid Bodies: Equivalent Systems of Forces

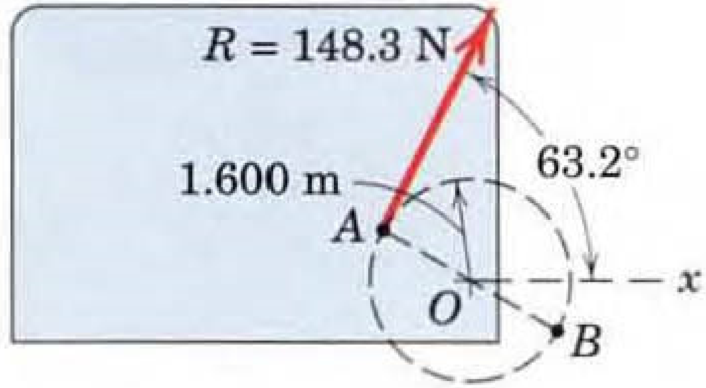
❑ Sample Problem 07

SOLUTION:

a) an equivalent force system,

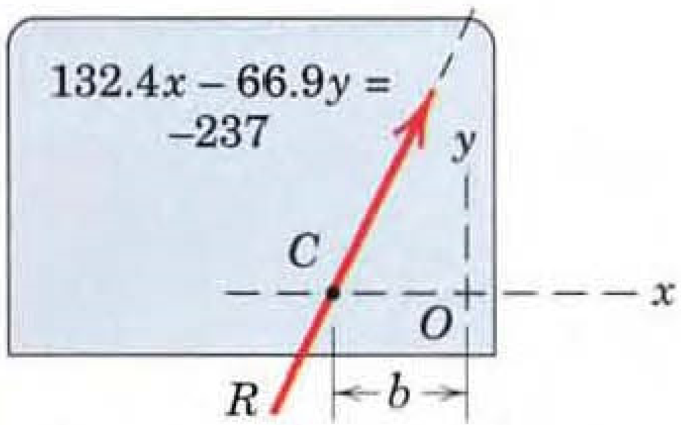


$$d = 1.6\ m$$



Suppose $\vec{r} = x\vec{i} + y\vec{j}$

$$132.4x - 66.9y = -237$$



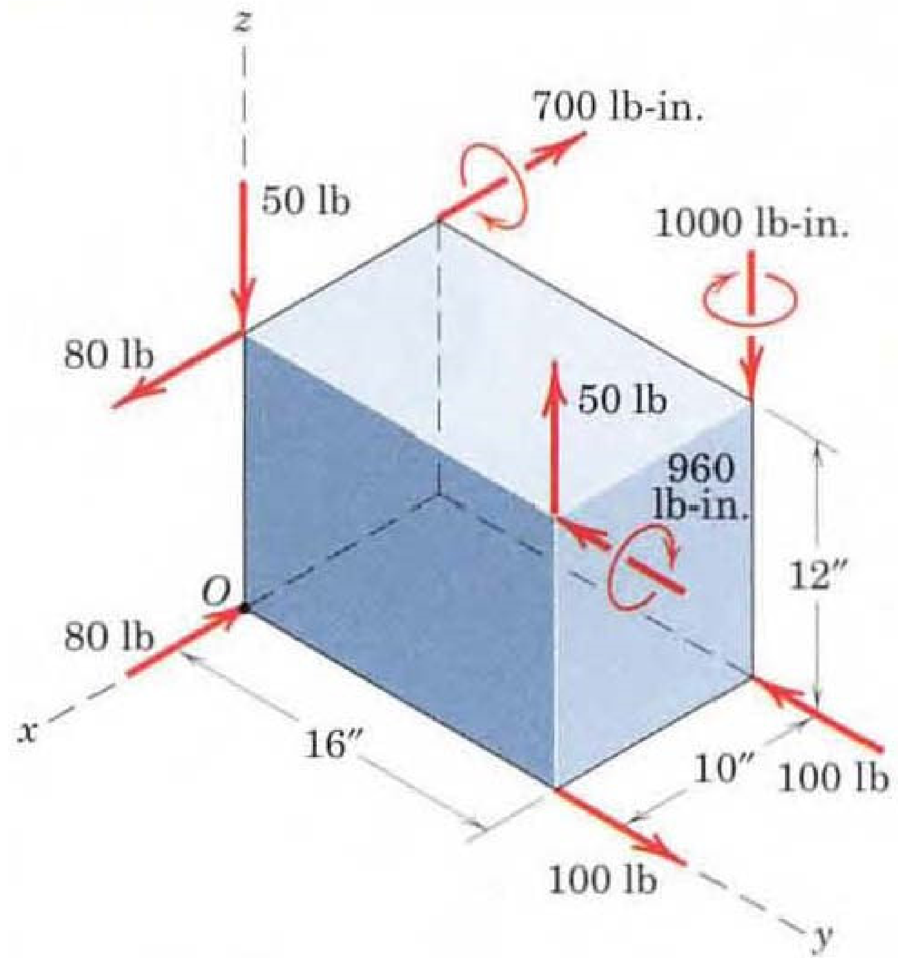
for example

$$b = -1.792\ m$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 08

Determine the resultant of the four forces and one couple which act on the plate shown at point O

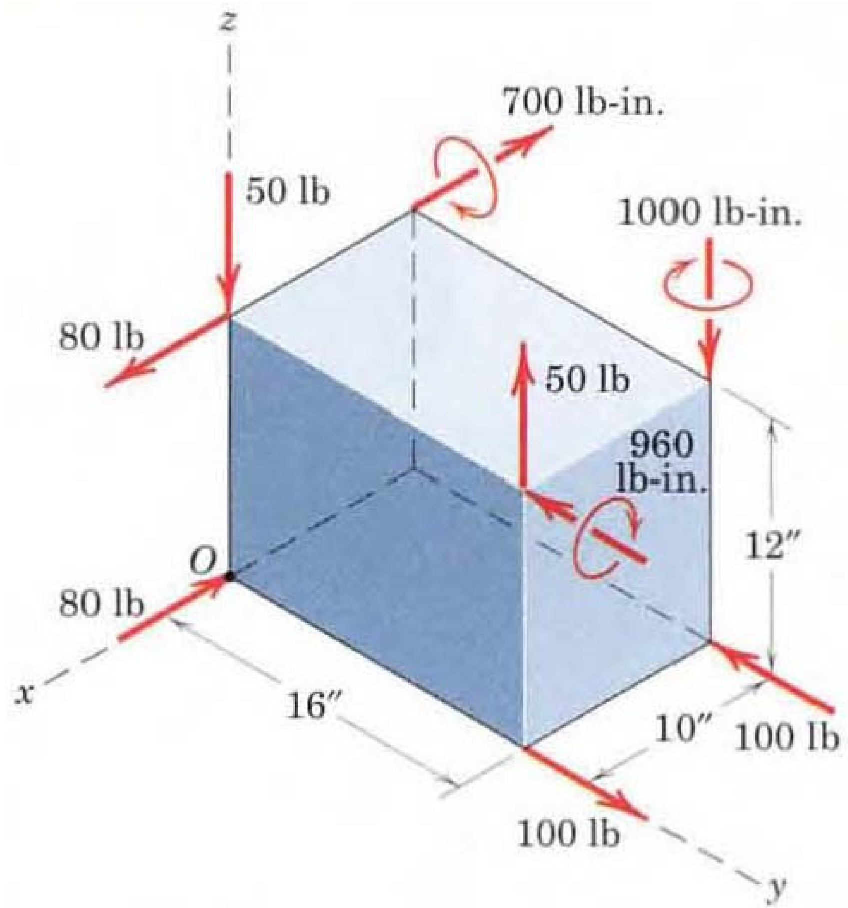


Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 08

SOLUTION:

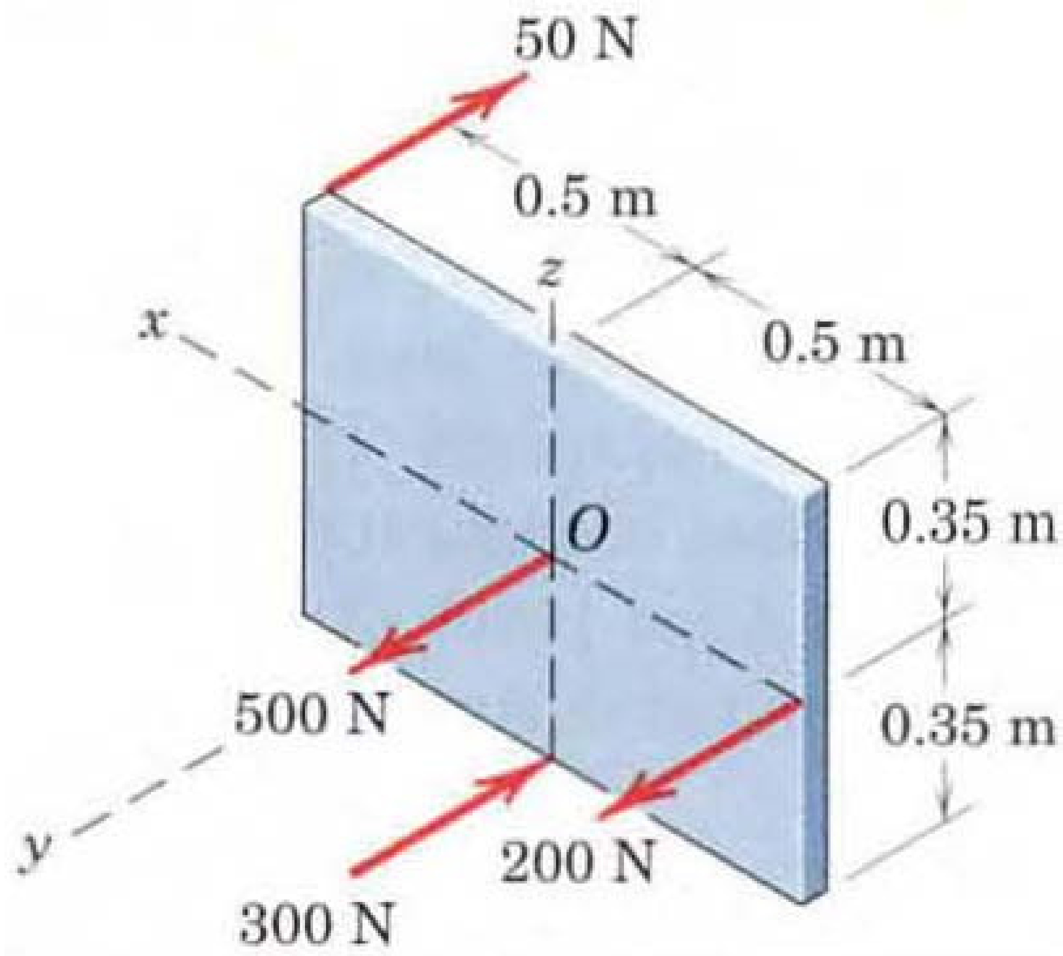
$$\Rightarrow \boxed{\vec{R}_o = 0}$$



$$\Rightarrow \boxed{\vec{M}_o = 100\,\vec{i}\ (lb.in)}$$

❑ Sample Problem 09

Determine the resultant of the system of parallel forces which act on the plate. Solve with a vector approach.

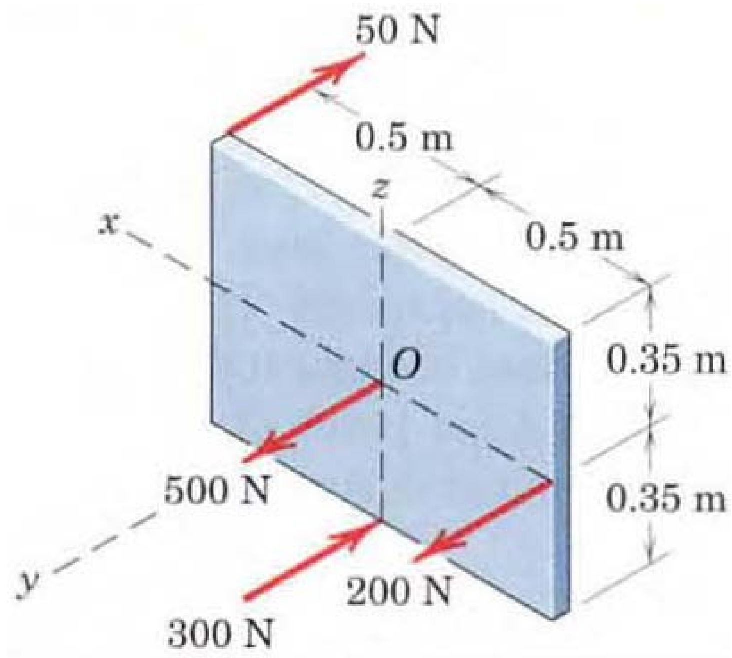


Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 09

SOLUTION:

Transfer of all forces to point 0 results in the force-couple system



$$\Rightarrow \vec{R}_o = 350\vec{j}\ (N)$$

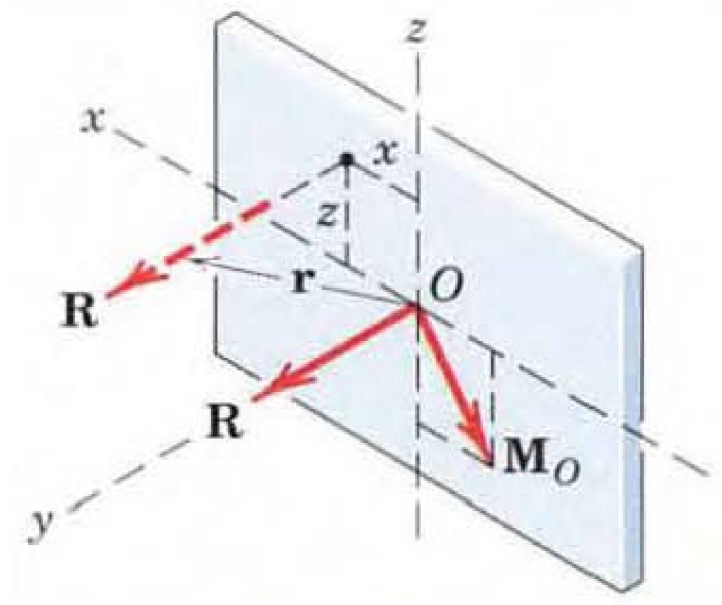
$$\Rightarrow \vec{M}_o = -87.5\,\vec{i} - 125\,\vec{k}$$

Rigid Bodies: Equivalent Systems of Forces

❑ Sample Problem 09

SOLUTION:

The placement of R so that it alone represents the above force-couple system is determined by the principle of moments in vector form



$$\vec{R}_o = 350\vec{j}\ (N)$$

$$\vec{M}_o = -87.5\,\vec{i} - 125\,\vec{k}$$

Suppose

$$\begin{cases} z = 0.250\ m \\ x = -0.375\ m \end{cases}$$