

STATICS



دانشگاه کردستان
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- Vector Mechanics for Engineers: Statics, 9th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Statics, 5th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self “Lectures notes on Statics”

Statics of Particles

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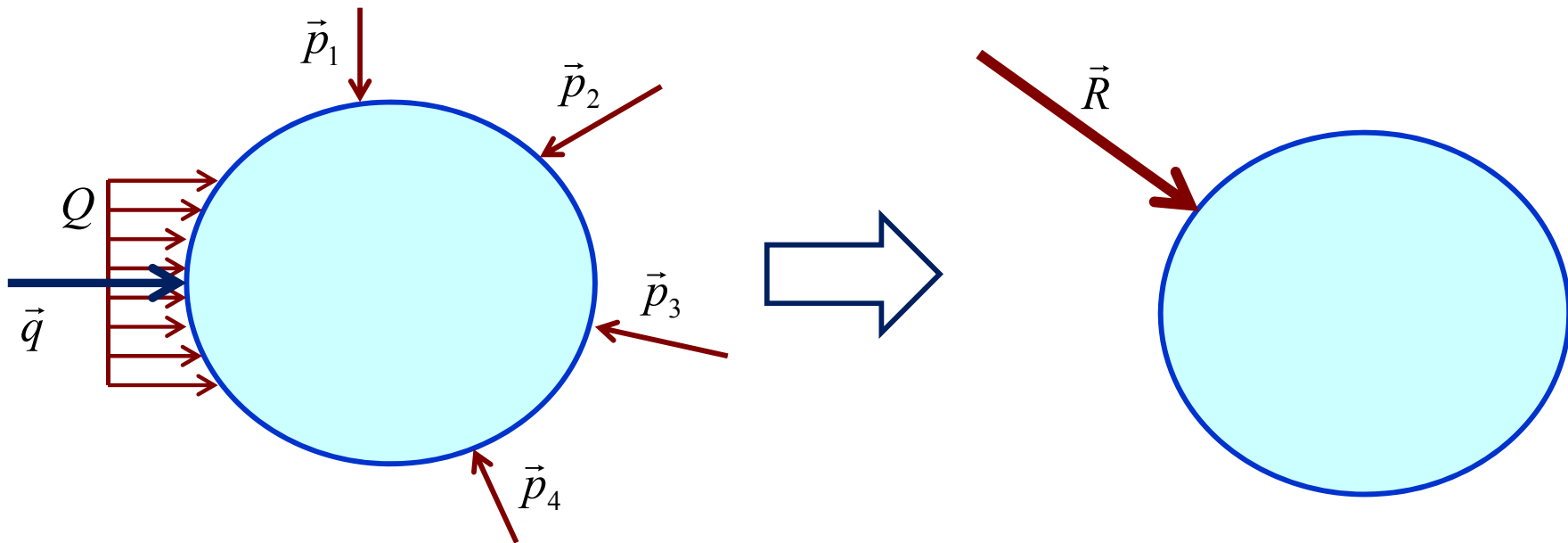
<https://prof.uok.ac.ir/Ka.Karami>

Statics of Particles

□ Introduction

The objective for the current chapter is to investigate the *effects of forces on particles*:

- a) Replacing multiple forces acting on a particle with a single equivalent or *resultant force*.



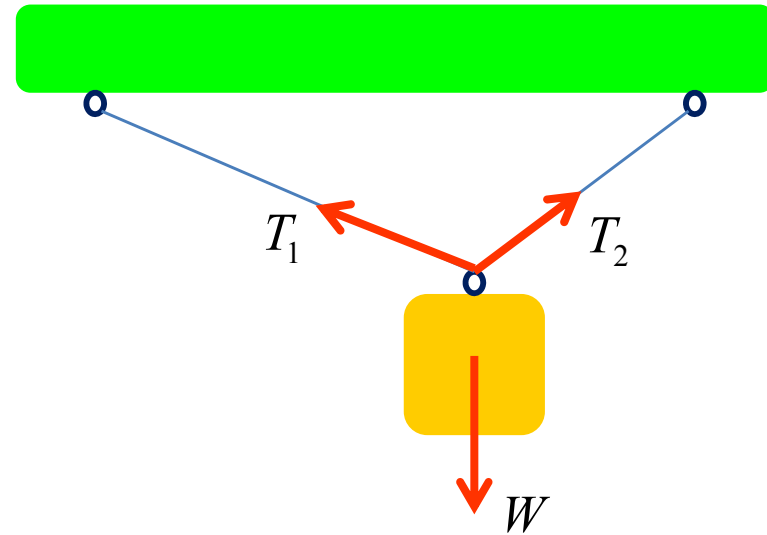
Statics of Particles

□ Introduction

b) relations between forces acting on a particle that is in a state of *equilibrium*.

$$T_1 = f(W)$$

$$T_2 = g(W)$$

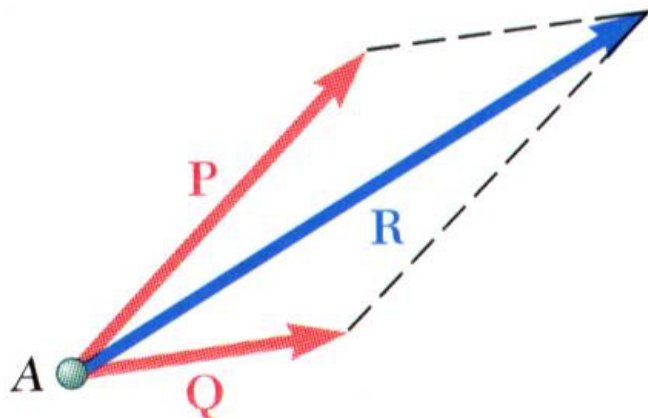
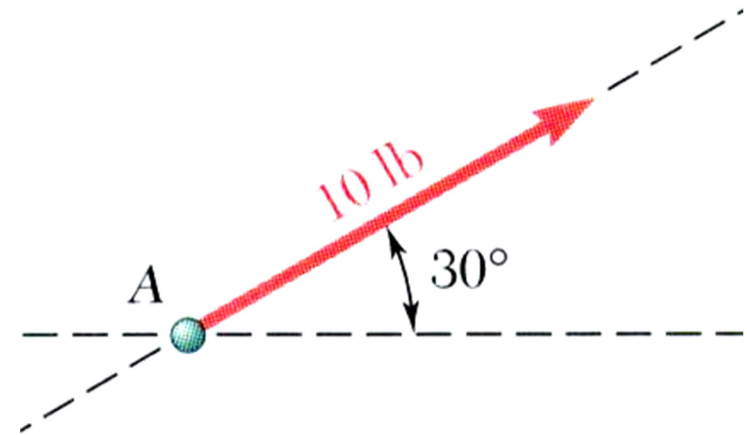


- The focus on *particles* does not imply a restriction to miniscule bodies. Rather, the study is restricted to analyses in which ***the size and shape of the bodies is not significant so that all forces may be assumed to be applied at a single point.***

Statics of Particles

□ Resultant of Two Forces

- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.
- Force is a *vector quantity*.



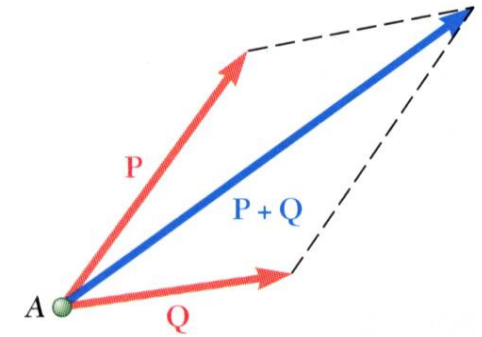
- Experimental evidence shows that the combined effect of two forces may be represented by a *single resultant force*.

- The *resultant* is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.

Statics of Particles

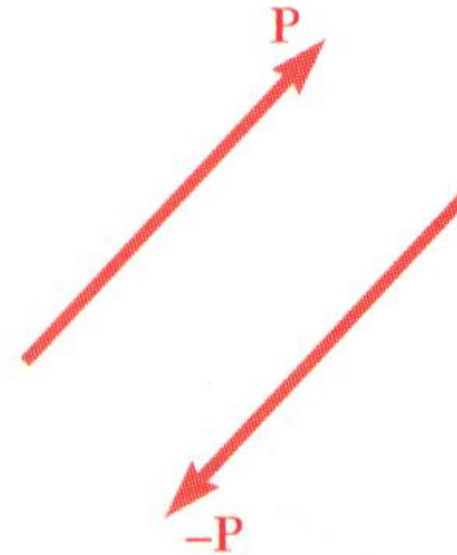
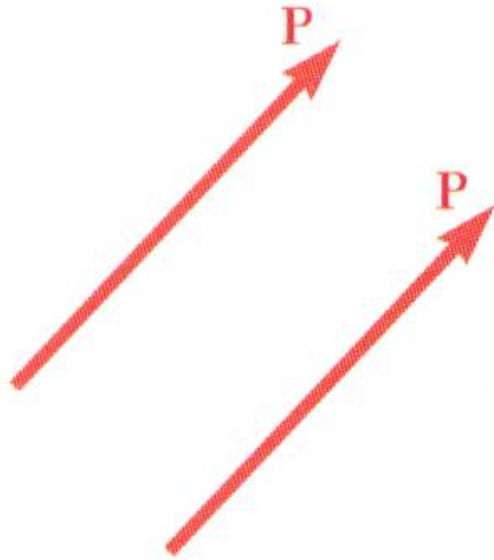
□ Vectors

- **Vector**: defined as mathematical expressions possessing **magnitude** and **direction** which add according to the parallelogram law. Examples: *displacements, velocities, accelerations*.
- **Scalar**: parameters possessing magnitude but not direction. Examples: *mass, volume, temperature*
- **Vector classifications:**
 - **Fixed or bound vectors** have well defined points of application that cannot be changed without affecting an analysis. Examples: Reaction Support
 - **Free vectors** may be freely moved in space without changing their effect on an analysis. Examples: Couples
 - **Sliding vectors** may be applied anywhere along their line of action without affecting an analysis.



Statics of Particles

□ Vectors



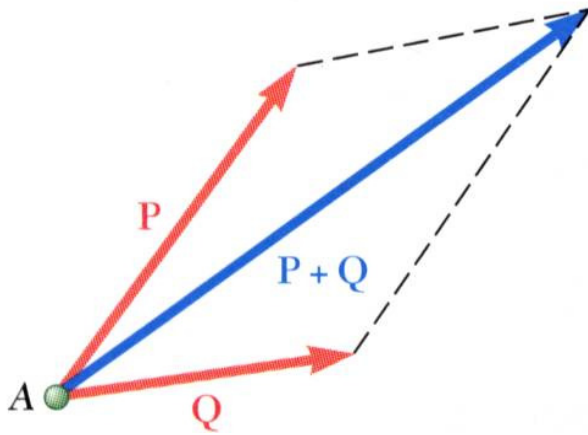
- *Equal vectors* have the same magnitude and direction.

- *Negative vector* of a given vector has the same magnitude and the opposite direction.

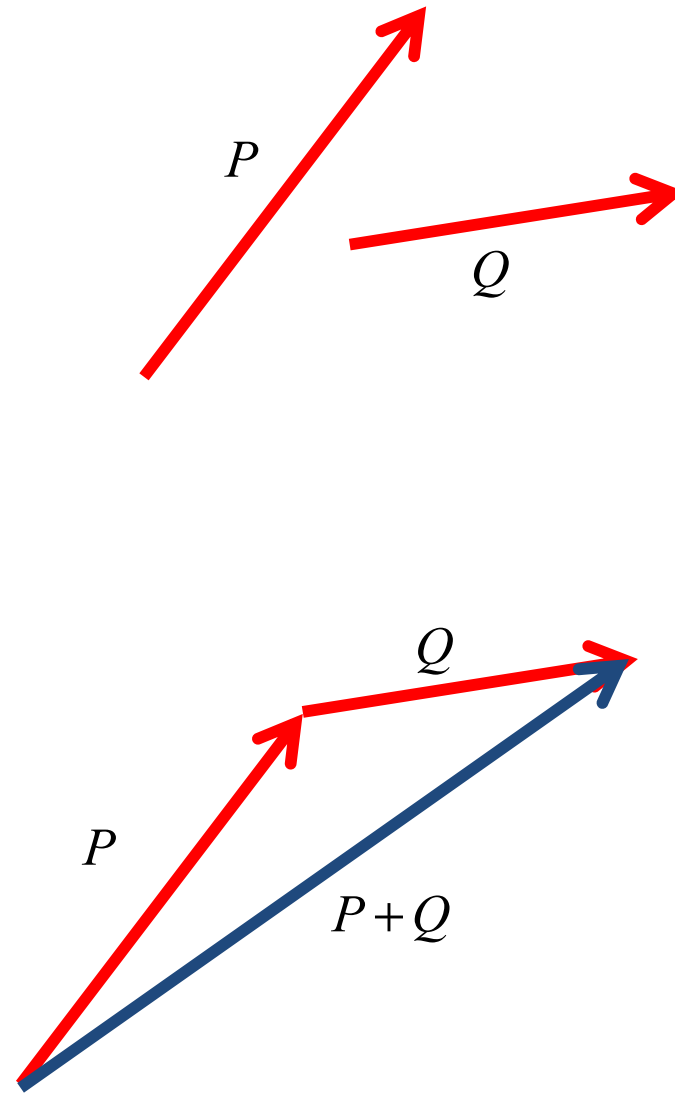
Statics of Particles

□ Addition of Vectors

- *Trapezoid* rule for vector addition



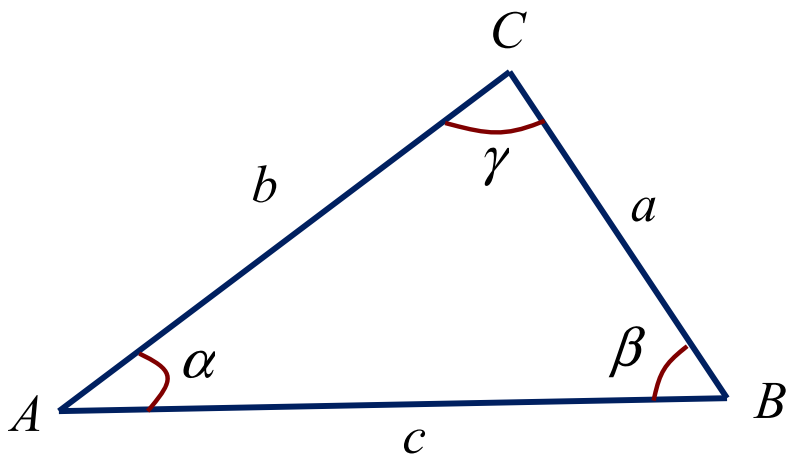
- *Triangle* rule for vector addition



Statics of Particles

□ Addition of Vectors

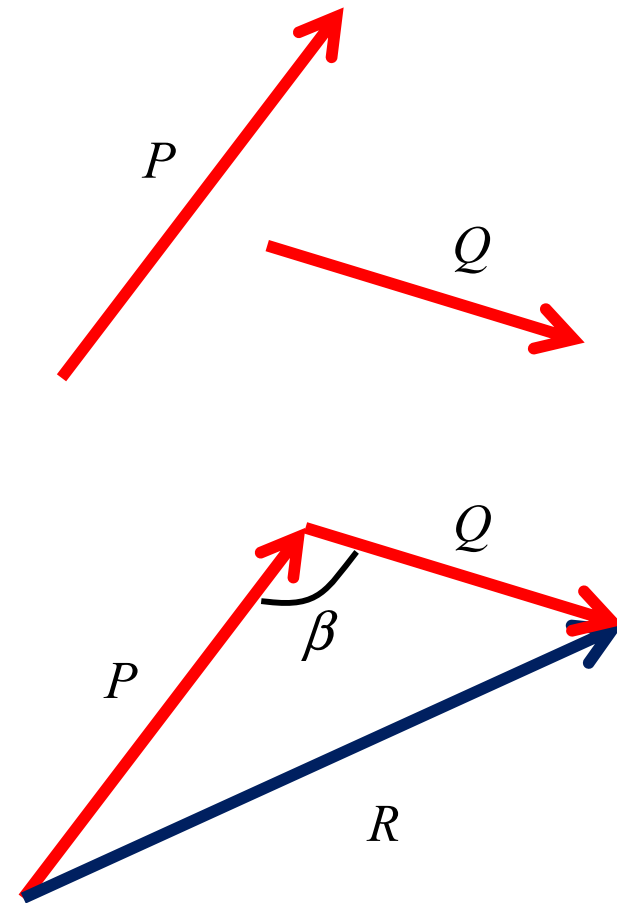
- *Law of cosines,*



$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$b^2 = c^2 + a^2 - 2ac \cos \beta$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$



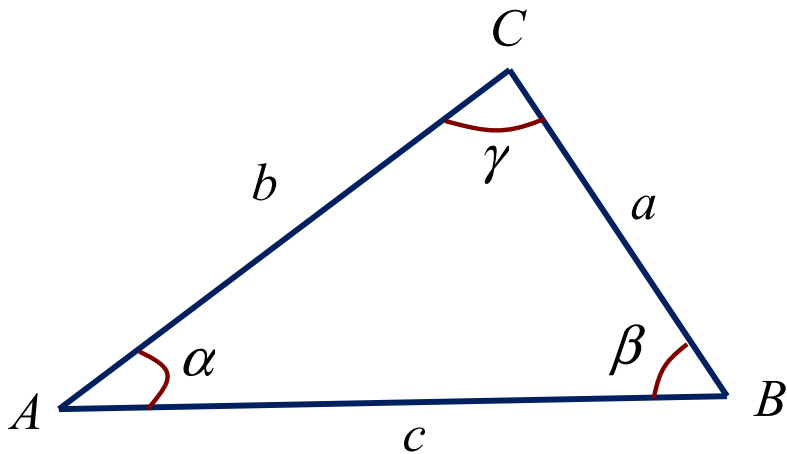
$$R^2 = P^2 + Q^2 - 2PQ \cos B$$

$$\vec{R} = \vec{P} + \vec{Q}$$

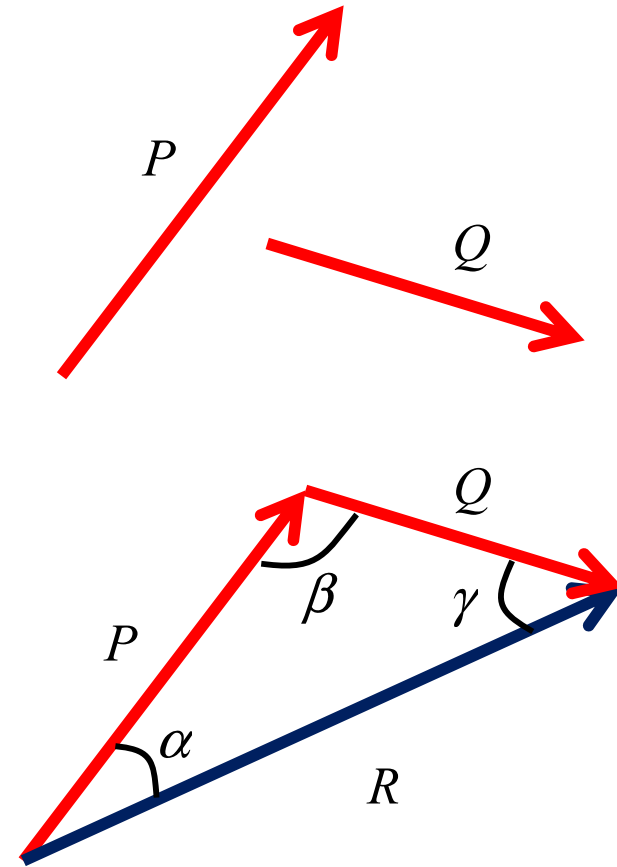
Statics of Particles

□ Addition of Vectors

- *Law of sines,*



$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$



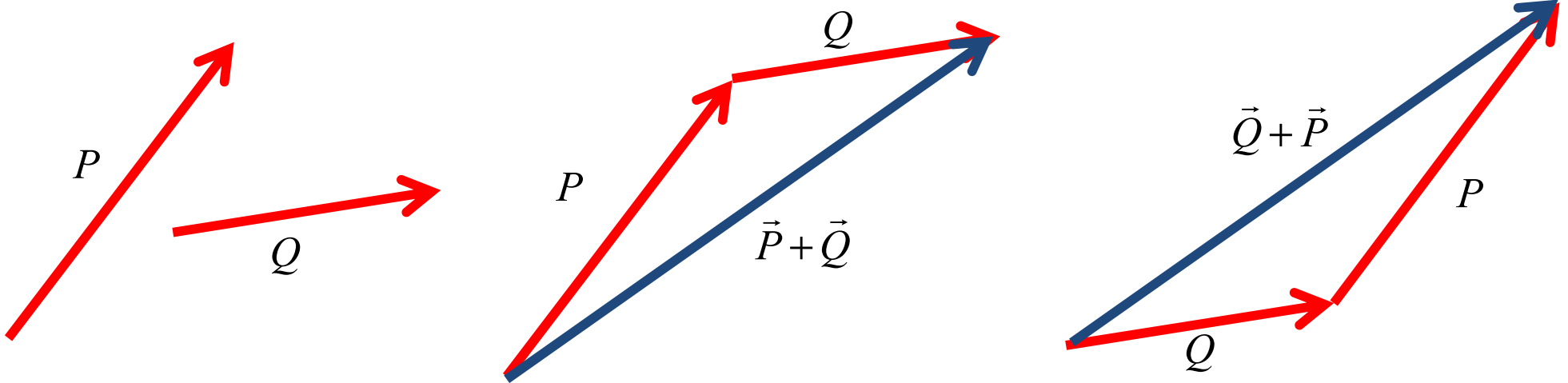
$$\frac{Q}{\sin \alpha} = \frac{R}{\sin \beta} = \frac{P}{\sin \gamma}$$

Statics of Particles

□ Addition of Vectors

- Vector addition is *commutative*,

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$



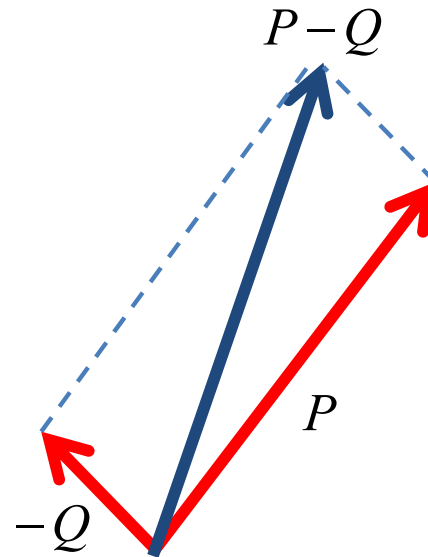
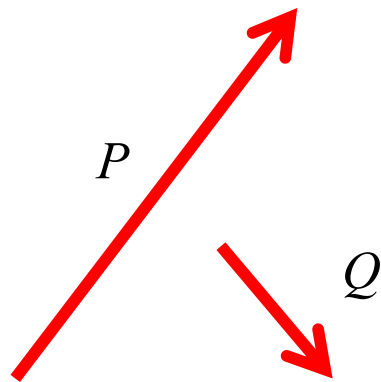
Statics of Particles

□ Addition of Vectors

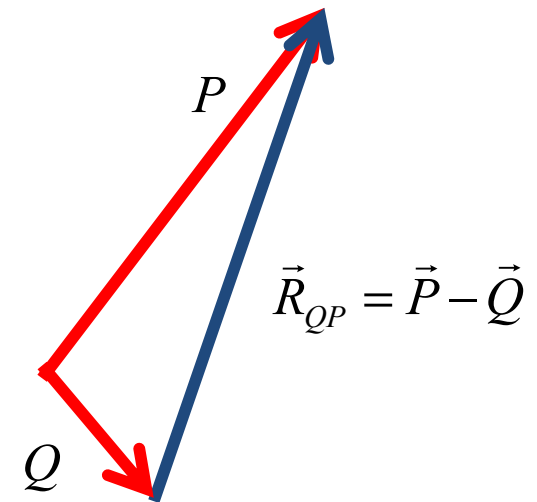
Vector *subtraction*

The subtraction of a vector is defined as the addition of the corresponding negative vector.

$$\vec{P} + (-\vec{Q}) = \vec{P} - \vec{Q}$$



Trapezoid

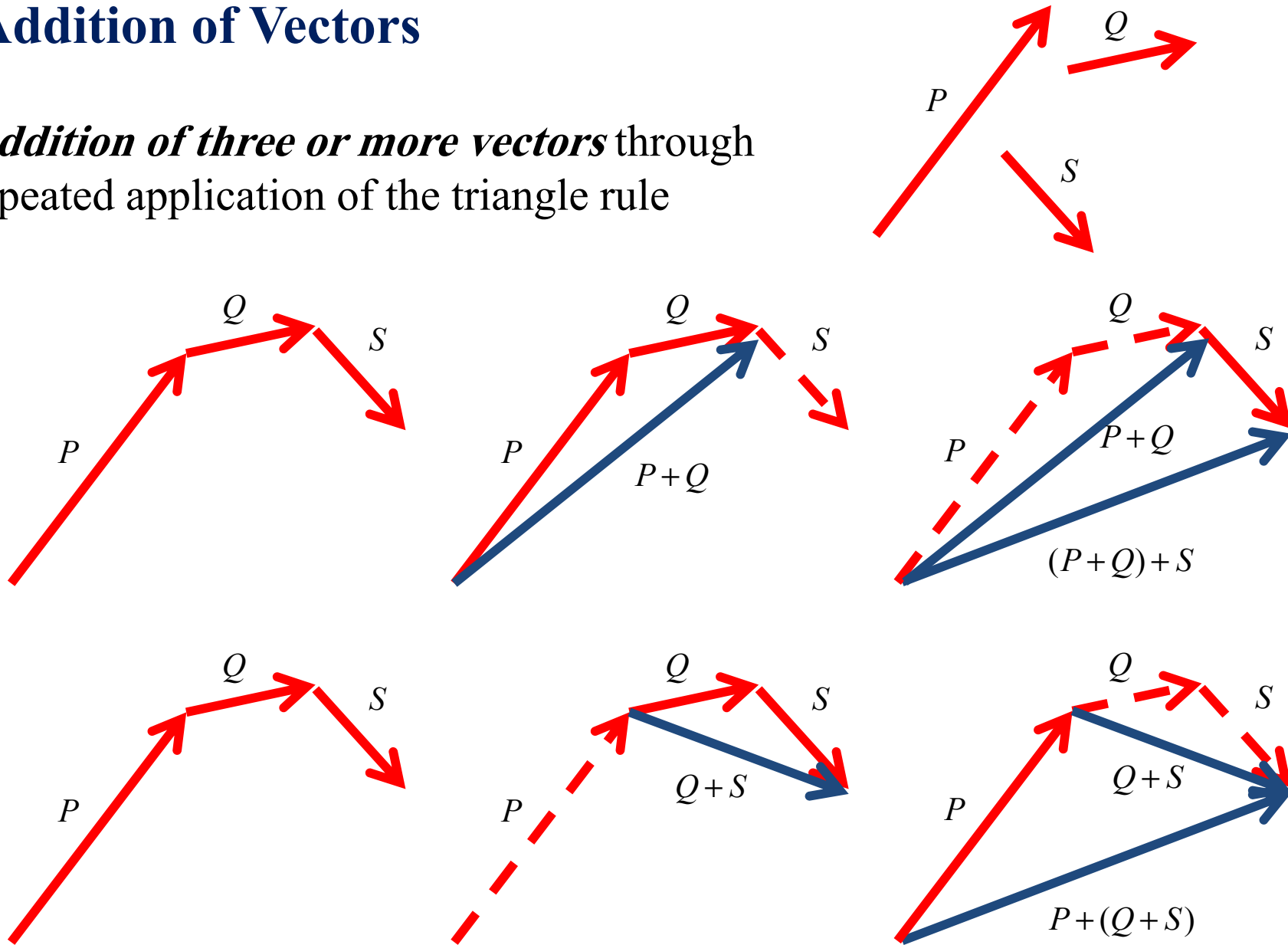


Triangle

Statics of Particles

□ Addition of Vectors

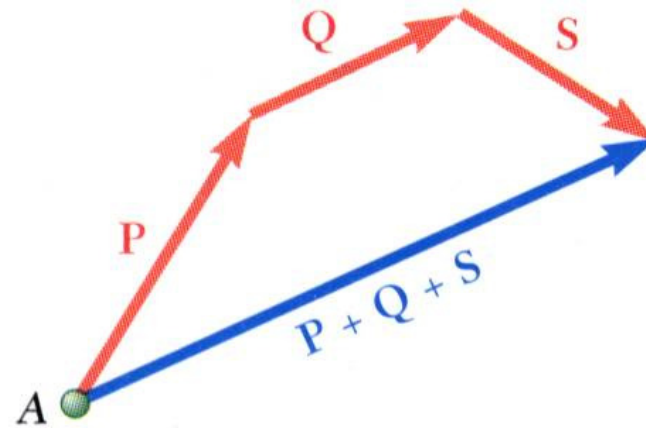
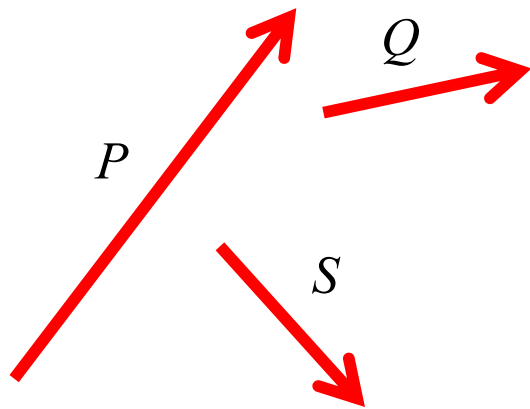
- *Addition of three or more vectors* through repeated application of the triangle rule



Statics of Particles

□ Addition of Vectors

- *The polygon rule for the addition* of three or more vectors.



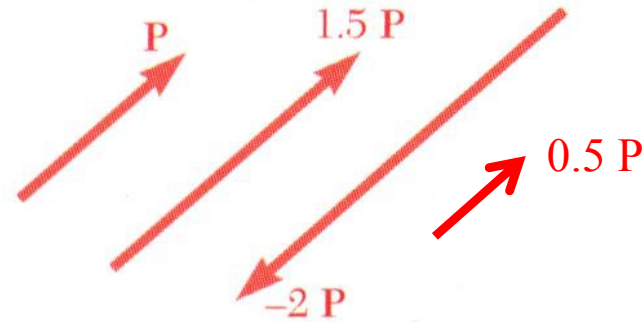
- Vector addition is *associative*,

$$\vec{P} + \vec{Q} + \vec{S} = (\vec{P} + \vec{Q}) + \vec{S} = \vec{P} + (\vec{Q} + \vec{S})$$

Statics of Particles

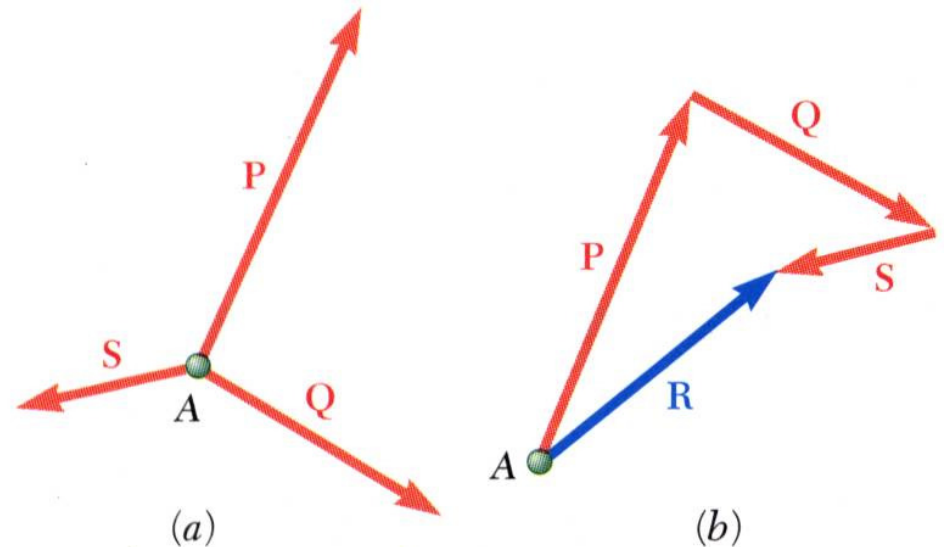
□ Resultant of Several Concurrent Forces

- *Multiplication of a vector by a scalar*



- *Concurrent forces*: set of forces which all pass through the same point.

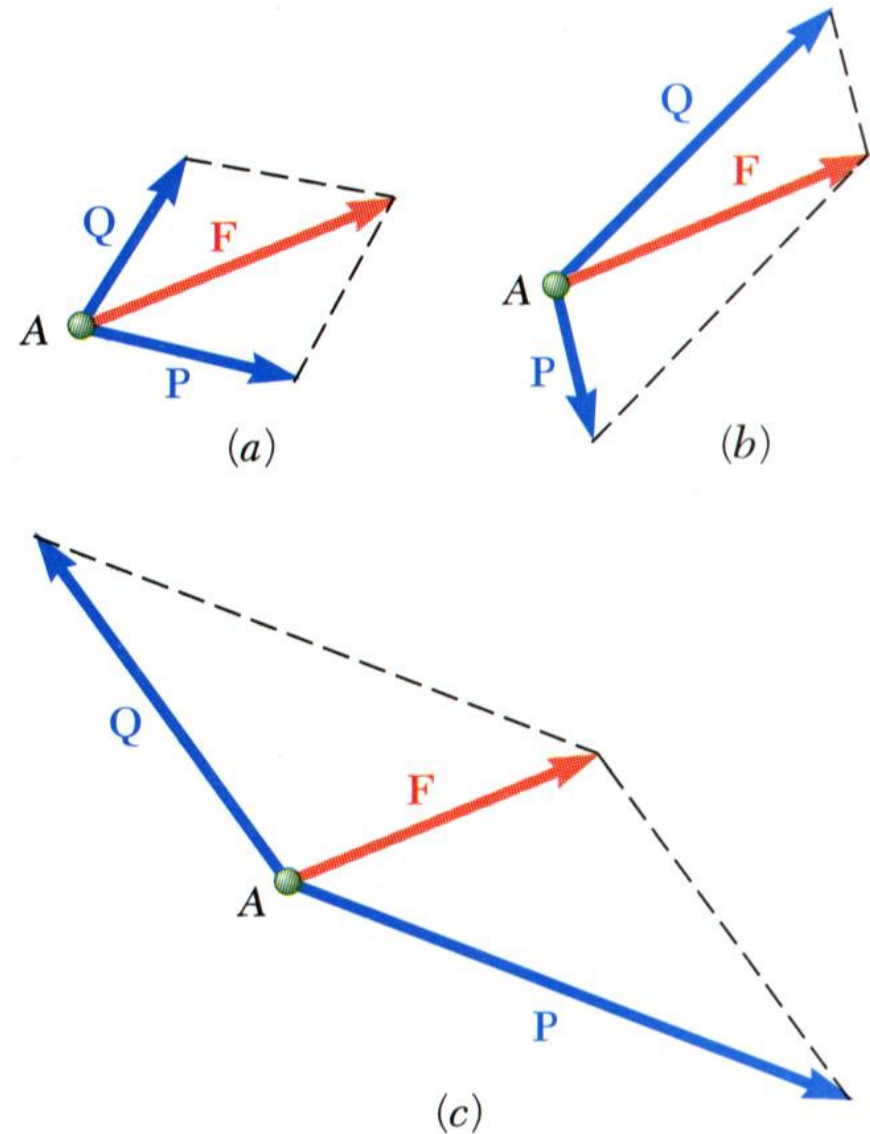
A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.



Statics of Particles

□ Resolution of a Force into Components

- **Vector force components:** a single force F acting on a particle may be replaced by two or more forces which, together, have the same effect on the particle. These forces are called the **components** of the Original force F , and the process of substituting them for F is called **resolving** the force F into components.

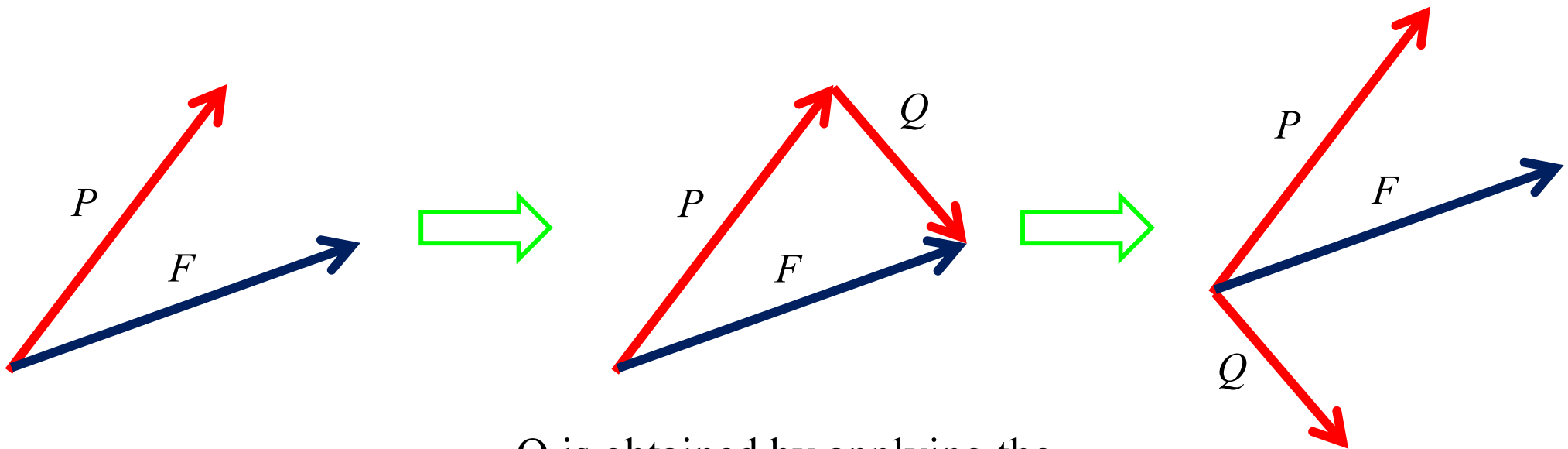


Statics of Particles

□ Resolution of a Force into Components

The number of ways in which a given force F may be resolved into two components is **unlimited**. Two cases are of particular interest:

I. One of the Two Components, P , Is Known

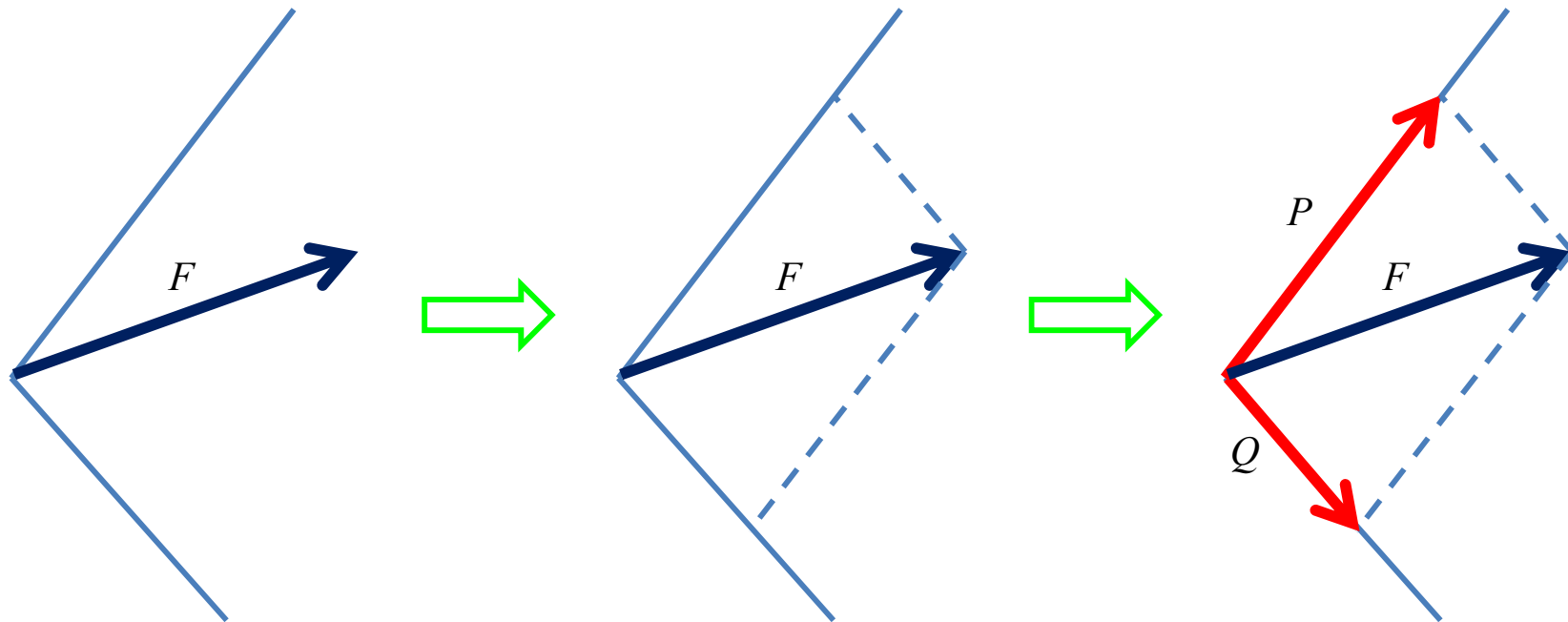


Q is obtained by applying the *triangle rule* and joining the tip of P to the tip of F

Statics of Particles

□ Resolution of a Force into Components

II. The Line of Action of Each Component Is Known

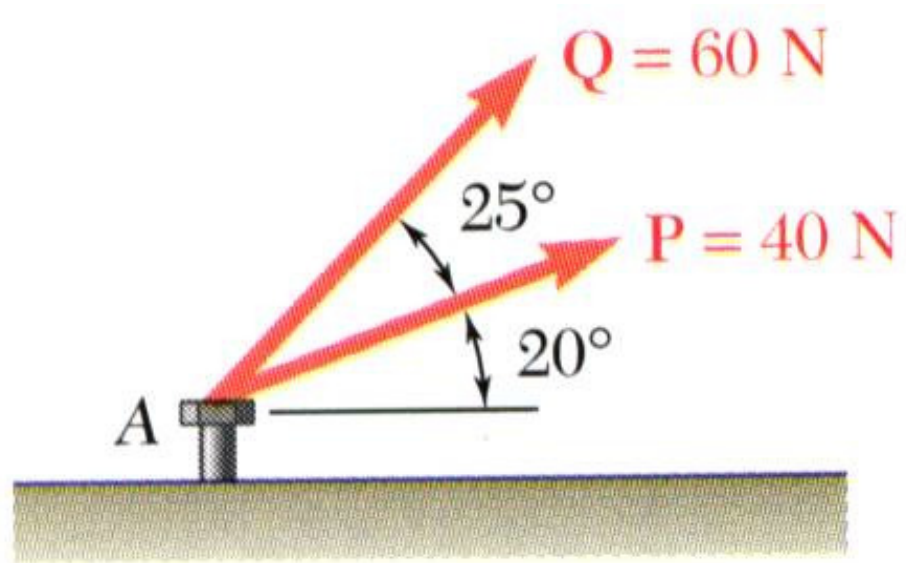


The magnitude and sense of the components are obtained by applying the ***parallelogram law*** and drawing lines, through the tip of F , parallel to the given lines of action

Statics of Particles

□ Sample Problem 01

The two forces act on a bolt at A . Determine their resultant.



Statics of Particles

□ Sample Problem 01

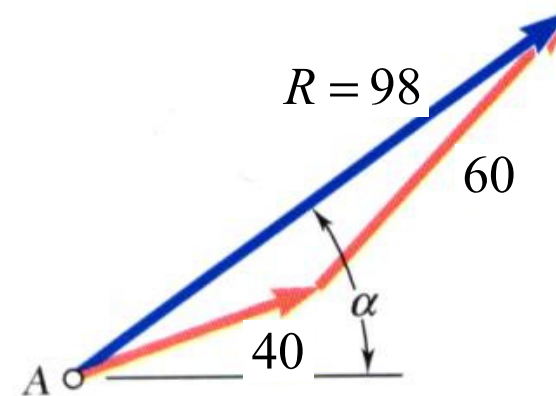
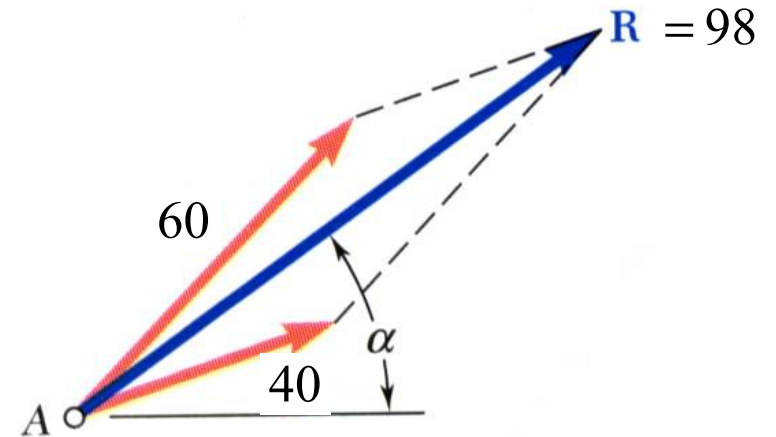
SOLUTION: Graphical solution

- A *parallelogram* with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

$$\mathbf{R} = 98 \text{ N} \quad \alpha = 35^\circ$$

- A *triangle* is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\mathbf{R} = 98 \text{ N} \quad \alpha = 35^\circ$$



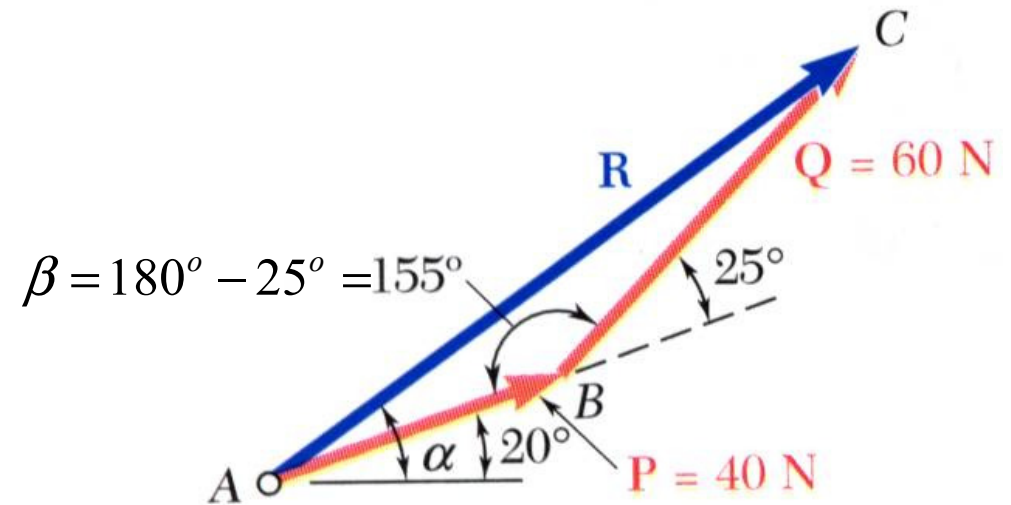
Statics of Particles

□ Sample Problem 01

SOLUTION: Trigonometric solution

- *The Law of Cosines,*

$$\Rightarrow R = 97.73 \text{ (N)}$$



-
- *The Law of Sines,*

$$\Rightarrow A = 15.04^\circ$$

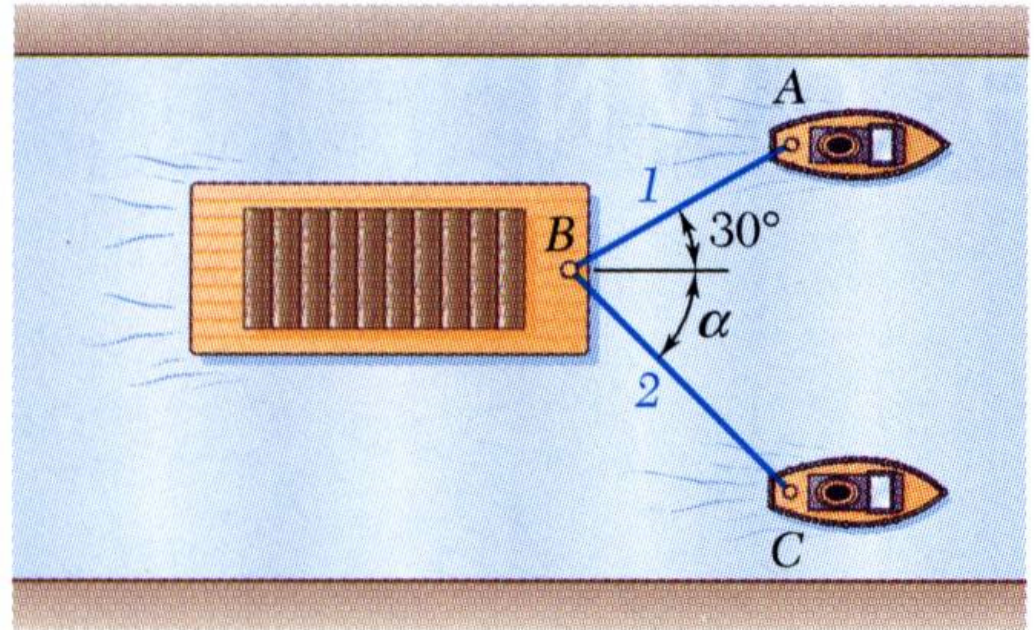
$$\alpha = 35.04^\circ$$

Statics of Particles

□ Sample Problem 02

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

- the tension in each of the ropes for $\alpha = 45^\circ$,
- the value of α for which the tension in rope 2 is a minimum.



Statics of Particles

□ Sample Problem 02

SOLUTION:

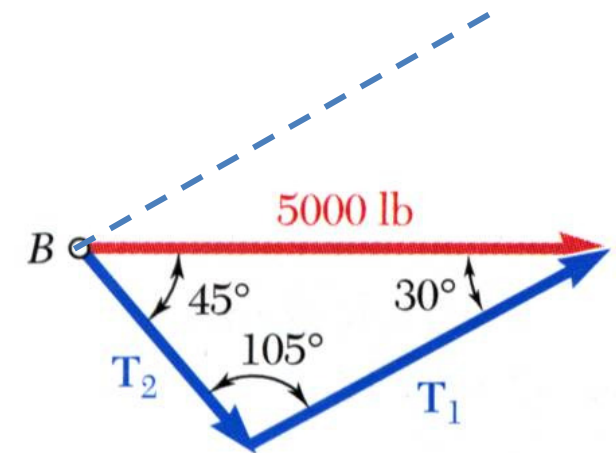
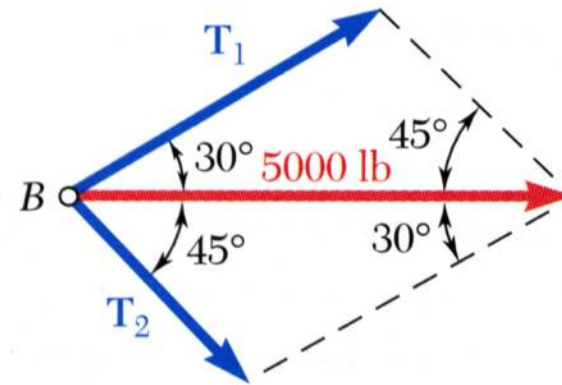
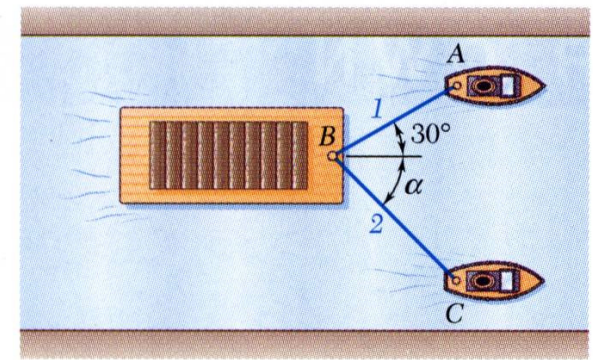
- **Graphical solution** - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.

$$T_1 = 3700 \text{ (lbf)} \quad T_2 = 2600 \text{ (lbf)}$$

- **Trigonometric solution** - Triangle Rule with Law of Sines

$$T_1 = 3660 \text{ (lbf)}$$

$$T_2 = 2590 \text{ (lbf)}$$



Statics of Particles

□ Sample Problem 02

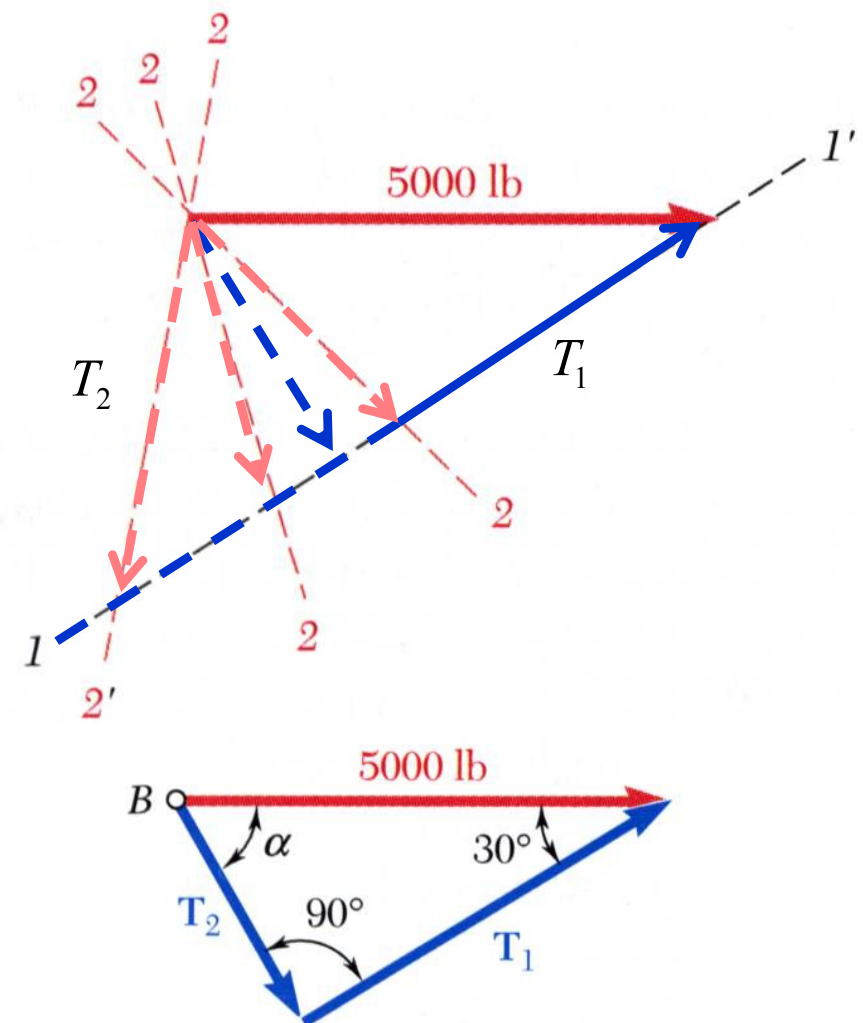
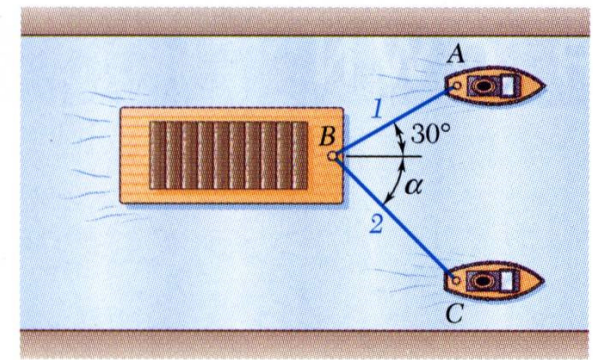
SOLUTION:

- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α .
- The minimum tension in rope 2 occurs when T_1 and T_2 are *perpendicular*.

$$T_2 = 2500 \text{ (lbf)}$$

$$T_1 = 4330 \text{ (lbf)}$$

$$\alpha = 60^\circ$$



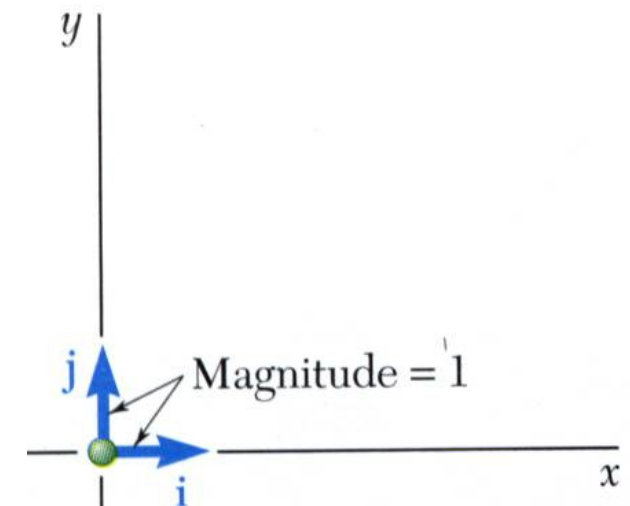
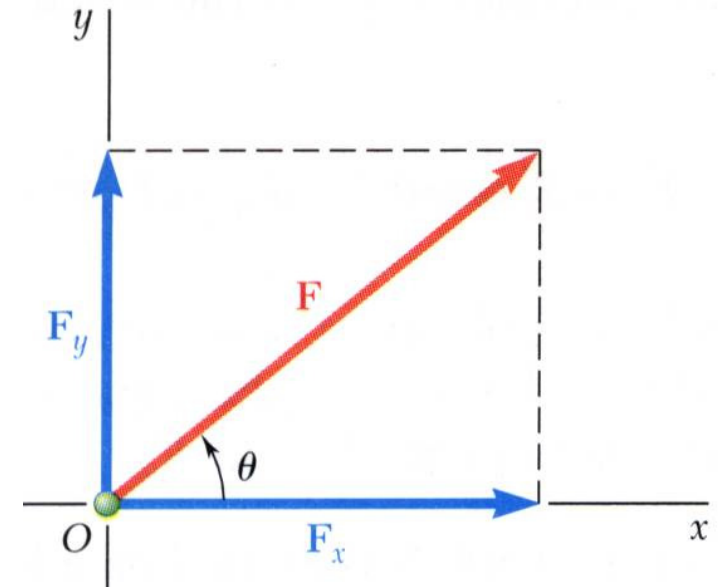
Statics of Particles

□ Rectangular Components of a Force: Unit Vectors

- May resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. \vec{F}_x and \vec{F}_y are referred to as **rectangular vector components** and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

- Define perpendicular **unit vectors** \vec{i} and \vec{j} which are parallel to the x and y axes.



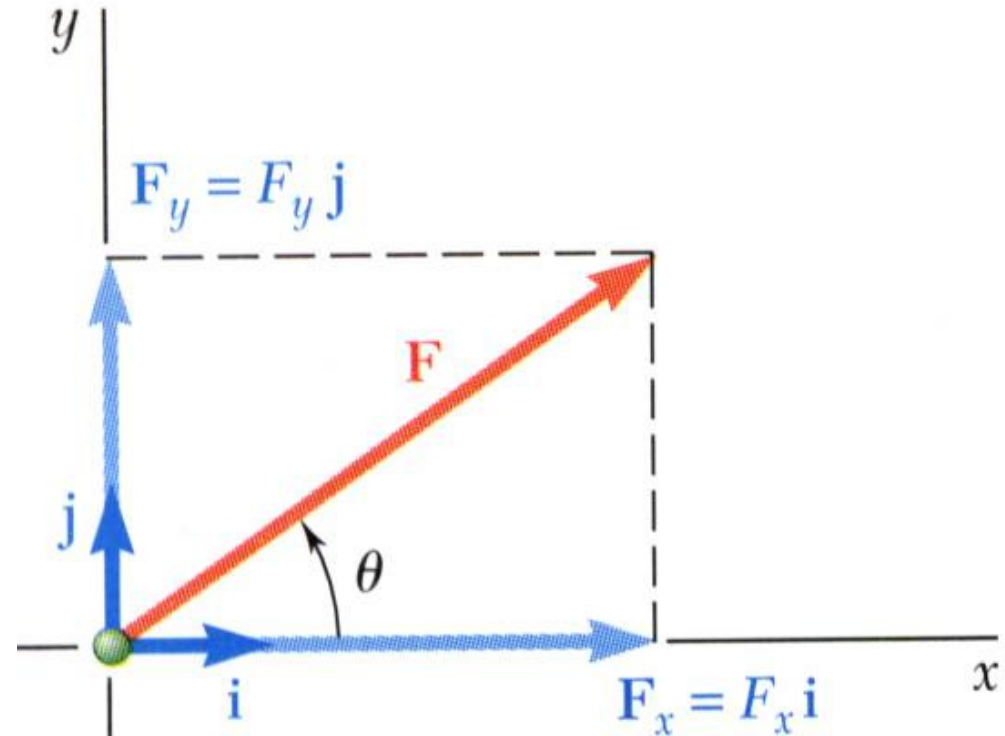
Statics of Particles

□ Rectangular Components of a Force: Unit Vectors

- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = F_x \vec{i} + F_y \vec{j}$$

F_x and F_y are referred to as the *scalar components* of \vec{F}



Statics of Particles

□ Addition of Forces by Summing Components

- Wish to find the resultant of 3 or more concurrent forces,

$$\vec{P} = P_x \vec{i} + P_y \vec{j} \quad \vec{Q} = Q_x \vec{i} + Q_y \vec{j} \quad \vec{S} = S_x \vec{i} + S_y \vec{j}$$

$$\vec{R} = \vec{P} + \vec{Q} + \vec{S} = R_x \vec{i} + R_y \vec{j}$$

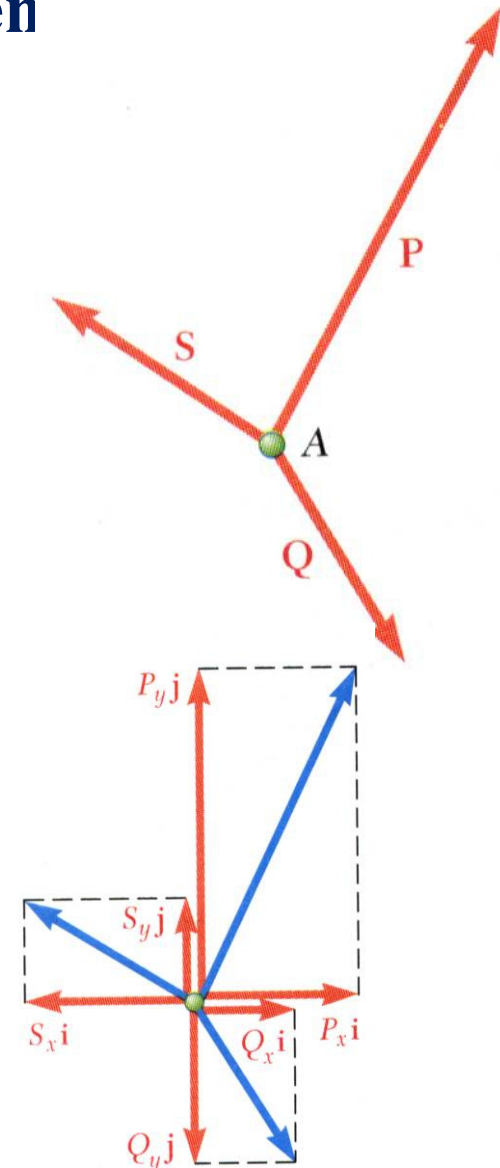
- Resolve each force into rectangular components

$$\vec{R} = R_x \vec{i} + R_y \vec{j} = P_x \vec{i} + P_y \vec{j} + Q_x \vec{i} + Q_y \vec{j} + S_x \vec{i} + S_y \vec{j}$$

$$\Rightarrow \vec{R} = (P_x + Q_x + S_x) \vec{i} + (P_y + Q_y + S_y) \vec{j}$$

$$R_x = P_x + Q_x + S_x = \sum F_x$$

$$R_y = P_y + Q_y + S_y = \sum F_y$$



Statics of Particles

□ Addition of Forces by Summing Components

- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

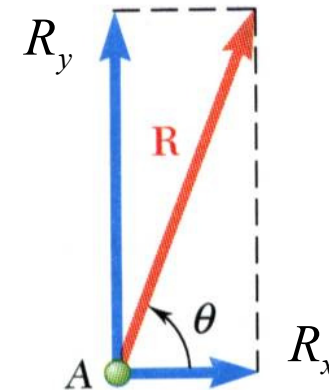
$$\vec{R} = R_x \vec{i} + R_y \vec{j}$$

$$R_x = \sum_{i=1}^n Fx_i$$

$$R_y = \sum_{i=1}^n Fy_i$$

- To find the resultant magnitude and direction,

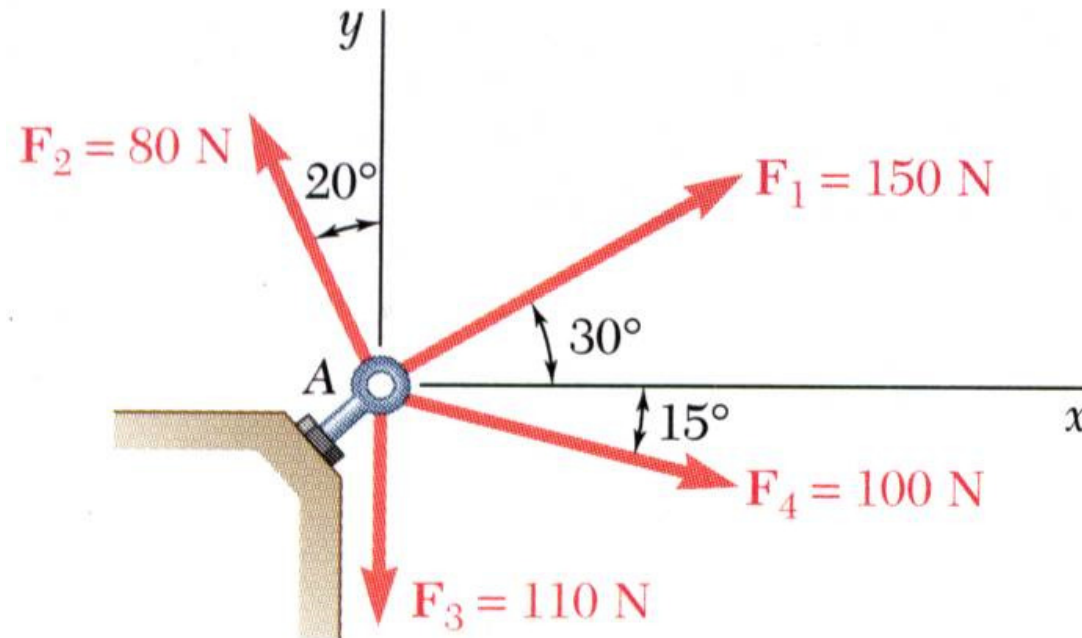
$$R = \sqrt{R_x^2 + R_y^2} \quad \& \quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Statics of Particles

□ Sample Problem 03

Four forces act on bolt A as shown. Determine the resultant of the force on the bolt.



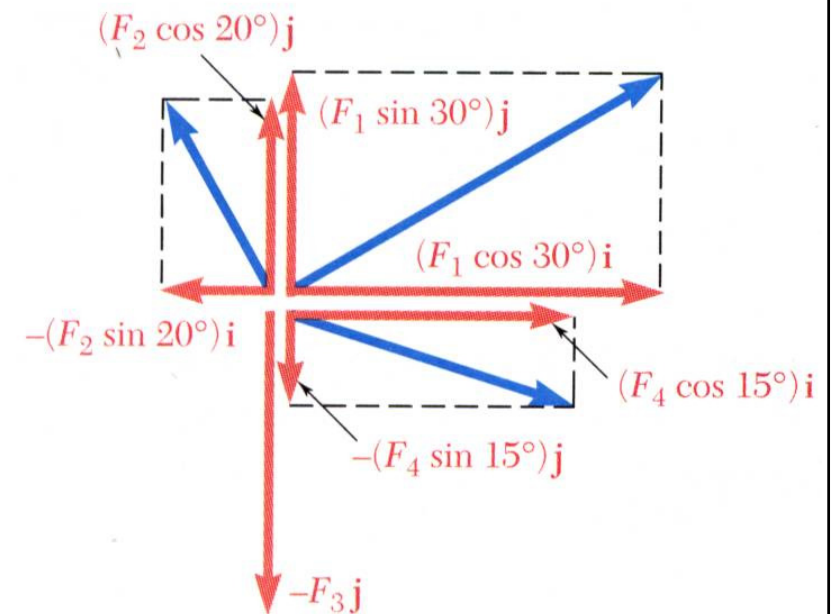
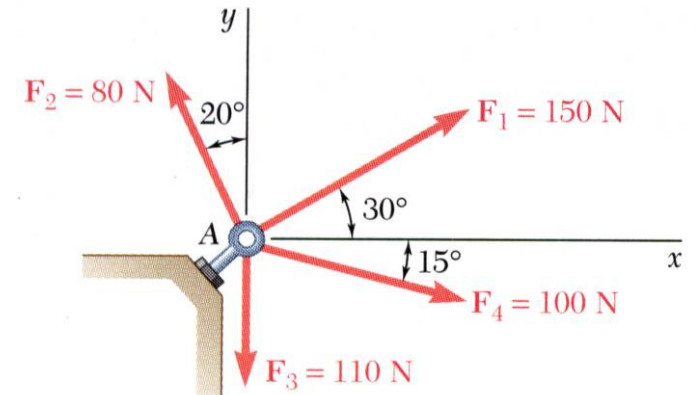
Statics of Particles

□ Sample Problem 03

SOLUTION:

- Resolve each force into rectangular components.

<i>force</i>	<i>mag</i>	<i>x - comp</i>	<i>y - comp</i>
\vec{F}_1			
\vec{F}_2			
\vec{F}_3			
\vec{F}_4			

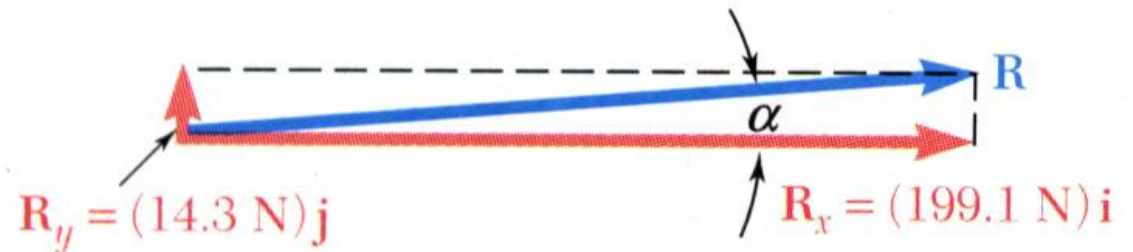
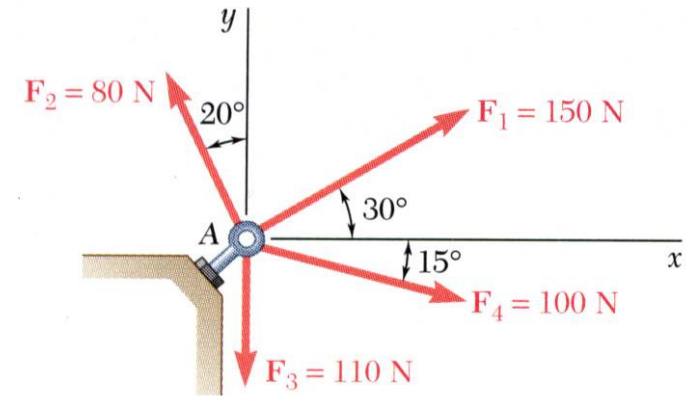


Statics of Particles

□ Sample Problem 03

SOLUTION:

- Calculate the magnitude and direction.



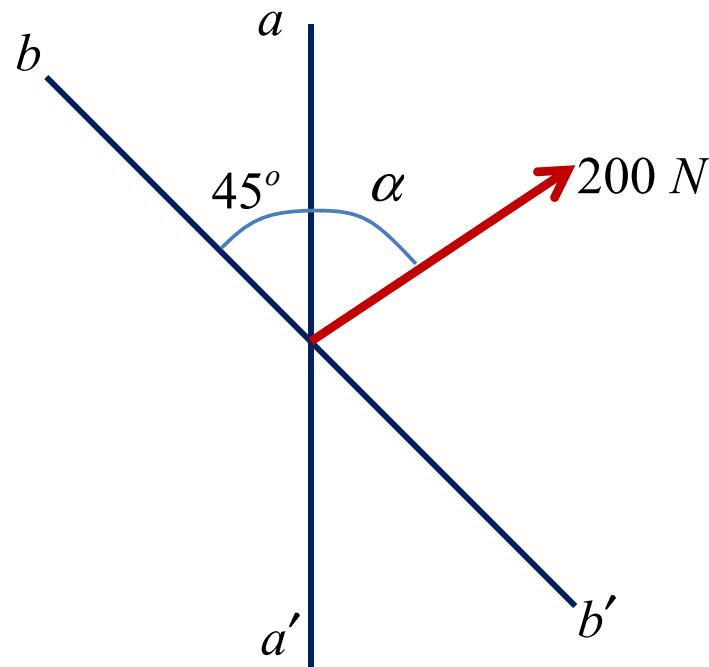
$$R = 199.6\text{ (N)}$$

$$\alpha = 4.1^\circ$$

Statics of Particles

□ Sample Problem 04

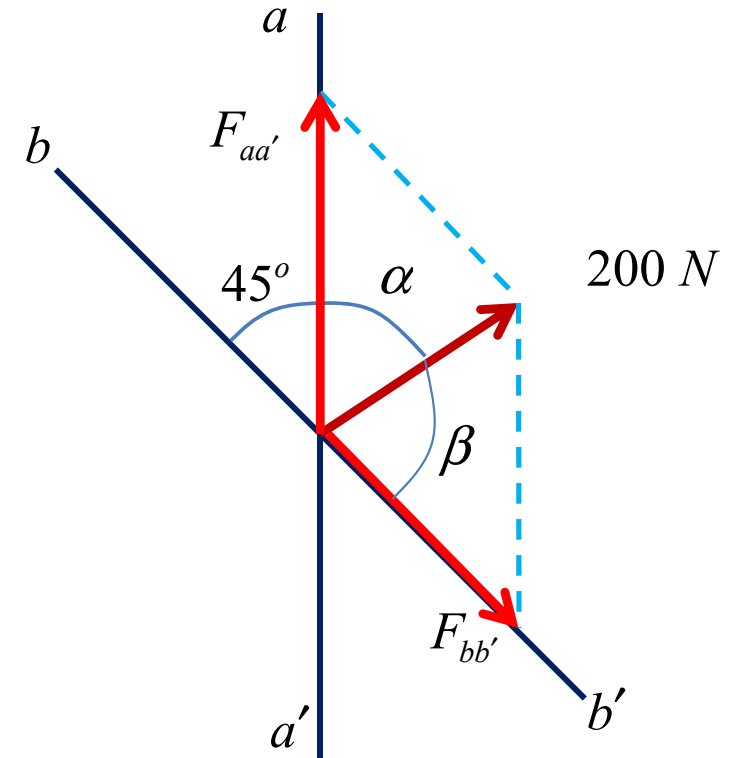
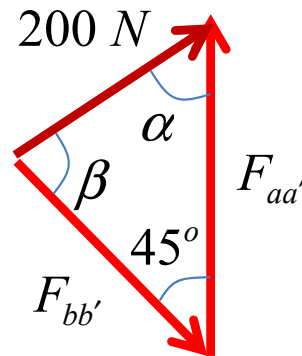
The 200-N force is to be resolved into components along lines a-a' and b-b'. (a) Determine the angle α using trigonometry knowing that the component along a-a' is to be 150 N. (b) What is the corresponding value of the component along b-b'?



Statics of Particles

□ Sample Problem 04

SOLUTION:



$$\beta = 32.03^\circ$$

$$\alpha = 102.97^\circ \approx 103^\circ$$

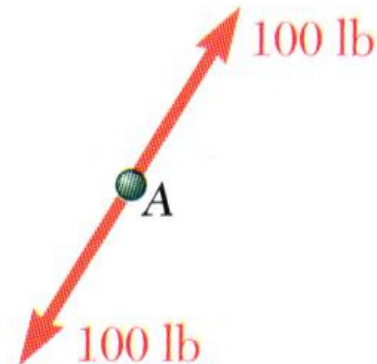
$$F_{bb'} = 275.63 \text{ N}$$

Statics of Particles

□ Equilibrium of a Particle

When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.

- ***Newton's First Law***: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.
- ***Particle acted upon by two forces***:
 - equal magnitude
 - same line of action
 - opposite sense



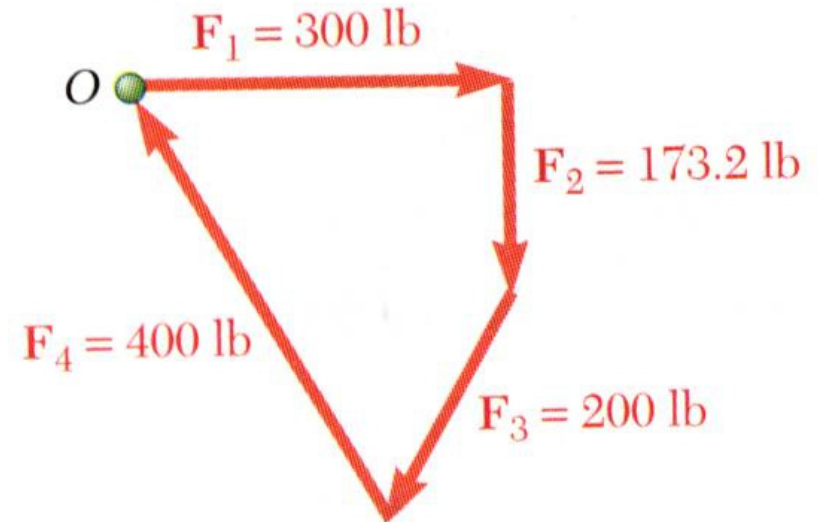
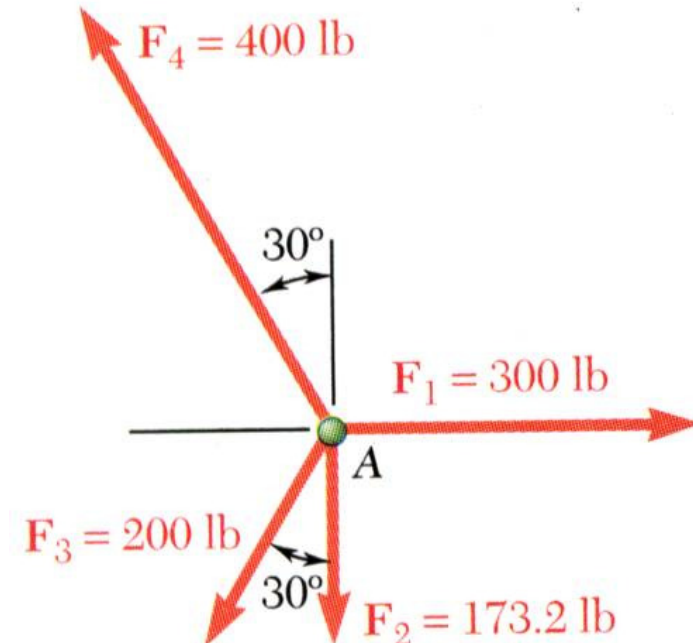
Statics of Particles

□ Equilibrium of a Particle

- *Particle acted upon by three or more forces:*
 - graphical solution yields a closed polygon
 - algebraic solution

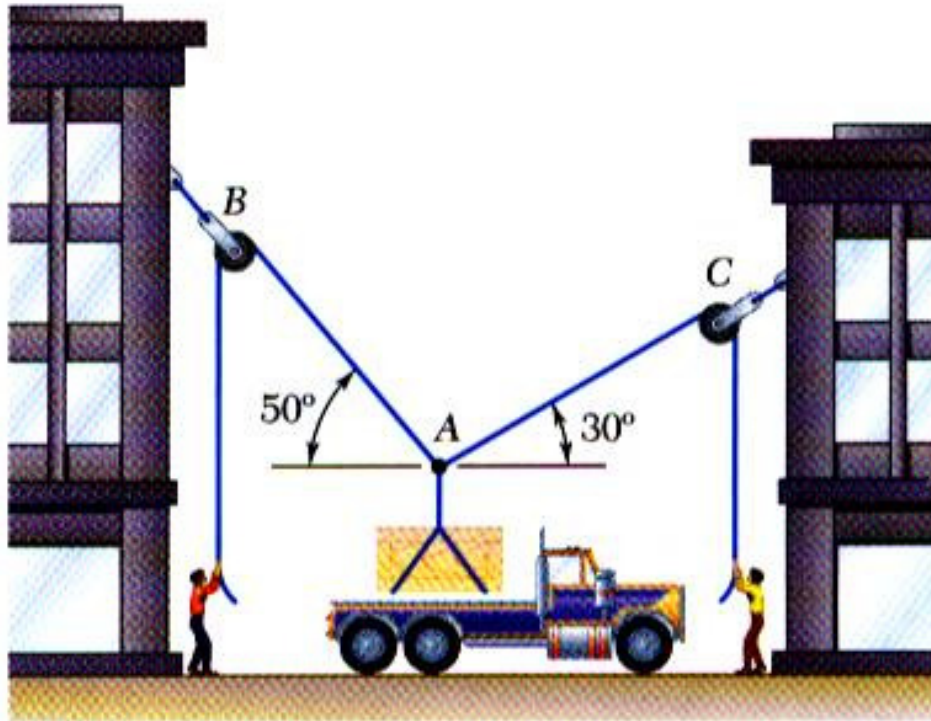
$$\vec{R} = \sum \vec{F} = 0 \Rightarrow$$

$$\sum F_x = 0 \quad \& \quad \sum F_y = 0$$

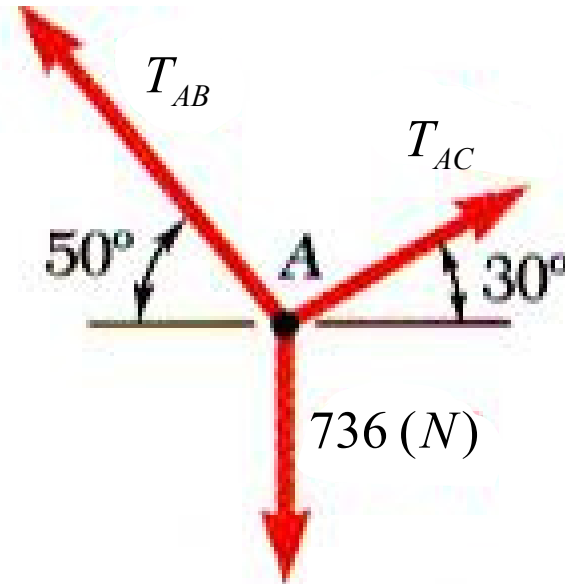


Statics of Particles

□ Free-Body Diagrams



Space Diagram: A sketch showing the physical conditions of the problem.

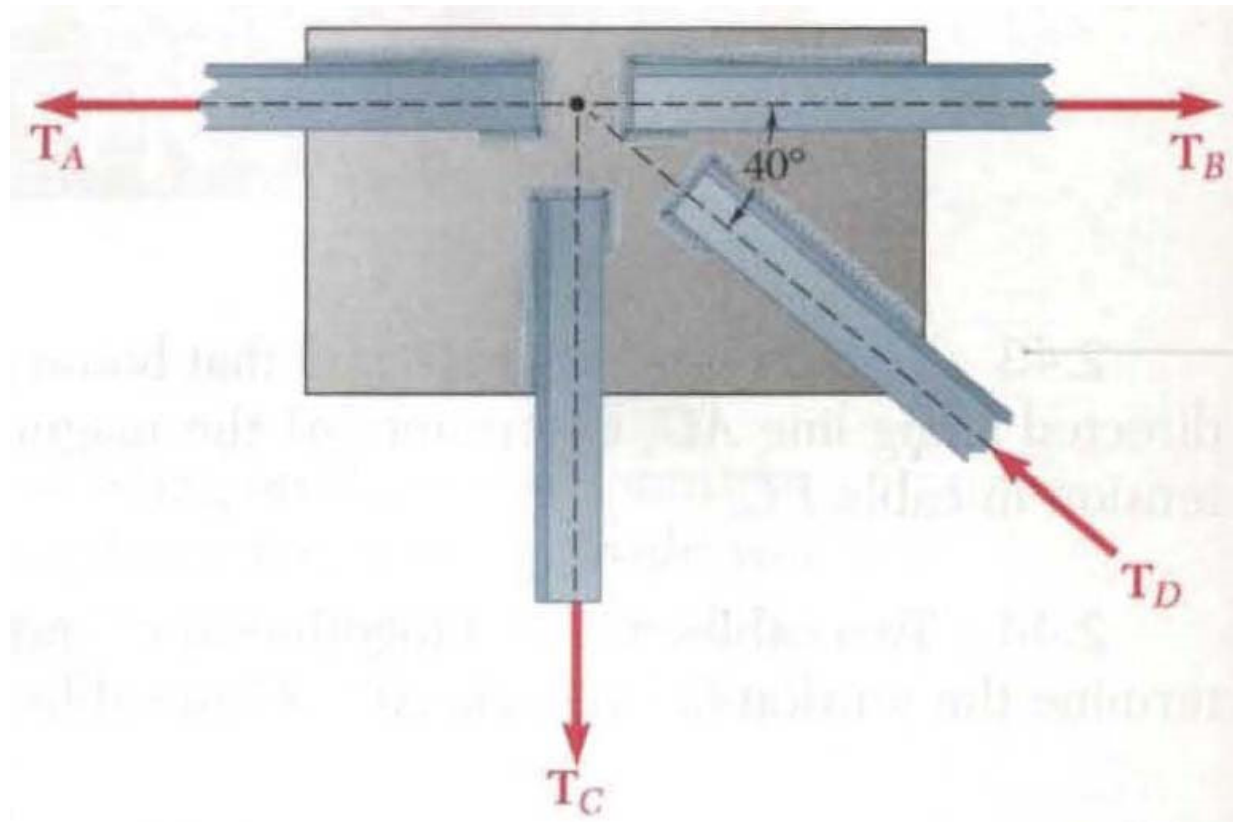


Free-Body Diagram: A sketch showing only the forces on the selected particle.

Statics of Particles

□ Sample Problem 05

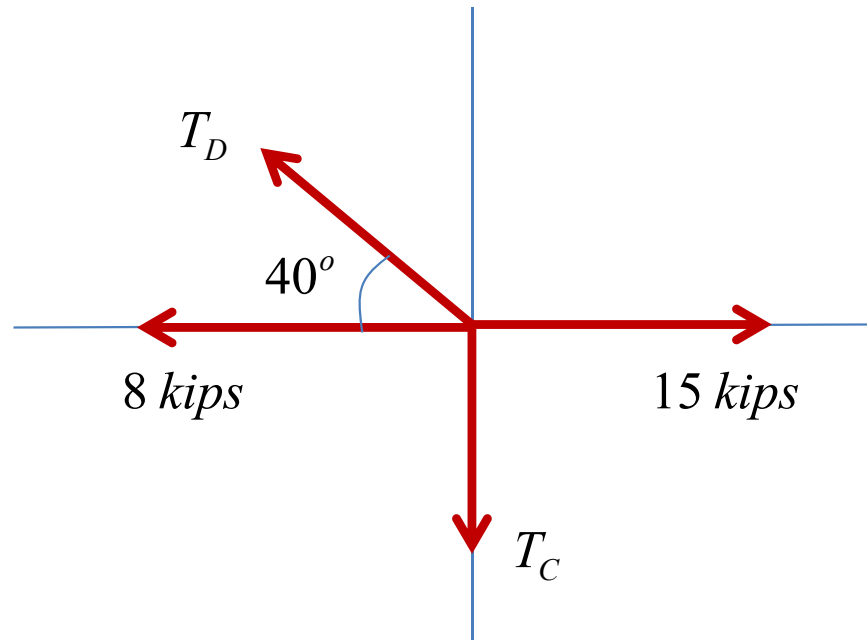
Two forces of magnitude $T_A = 8 \text{ kips}$ and $T_B = 15 \text{ kips}$ are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces T_C and T_D .



Statics of Particles

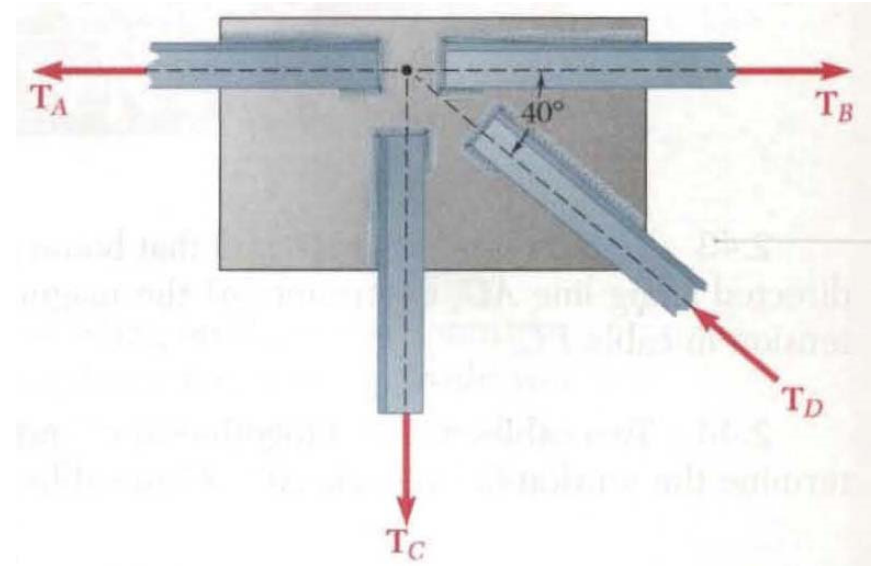
□ Sample Problem 05

SOLUTION:



$$T_D = 9.14 \text{ kips}$$

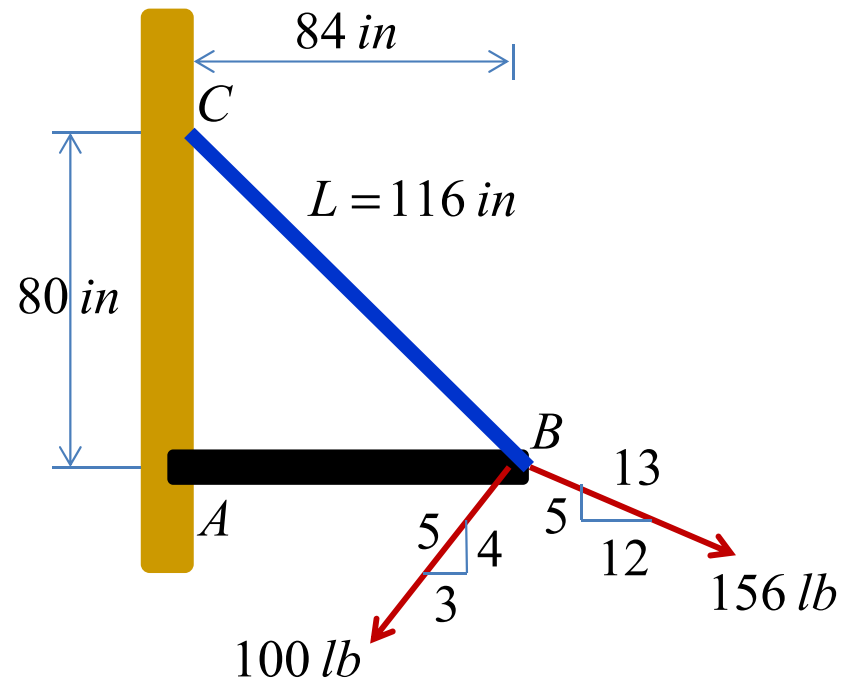
$$T_C = 5.87 \text{ kips}$$



Statics of Particles

□ Sample Problem 06

Knowing that the tension in cable BC is 145 lb, determine the resultant of the three forces exerted at point B of beam AB.



Statics of Particles

□ Sample Problem 06

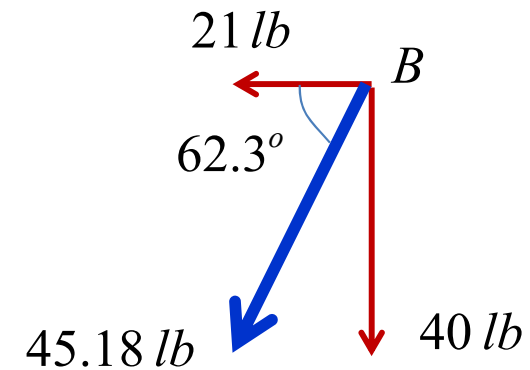
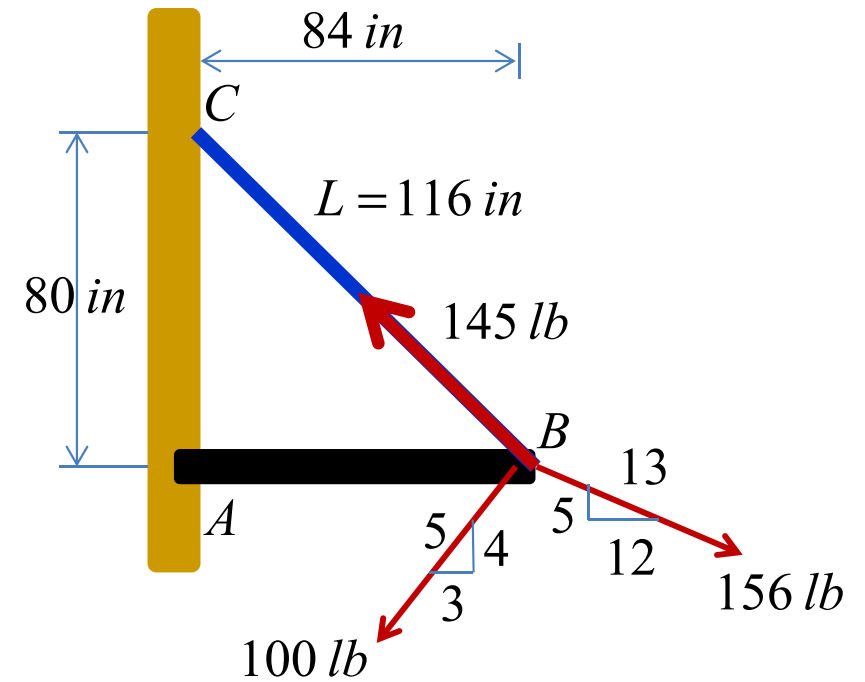
SOLUTION:

$$\Rightarrow F_x = -21 \text{ lb}$$

$$\Rightarrow F_y = -40 \text{ lb}$$

$$F = 45.18 \text{ lb}$$

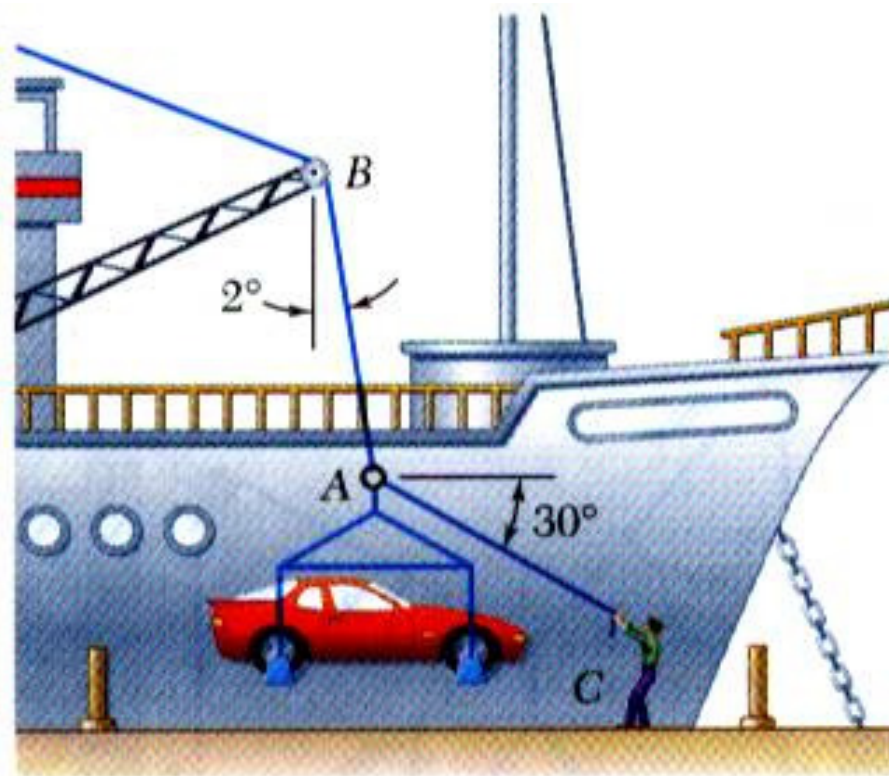
$$\theta = 62.3^\circ$$



Statics of Particles

□ Sample Problem 07

In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

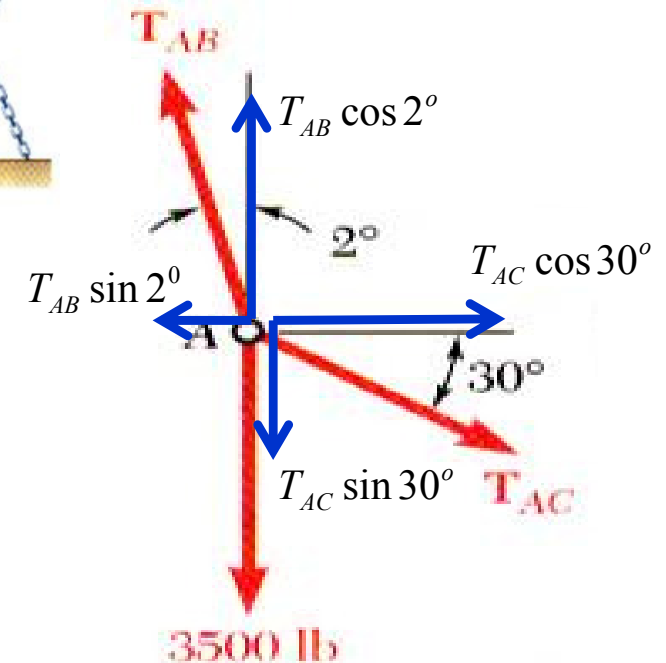
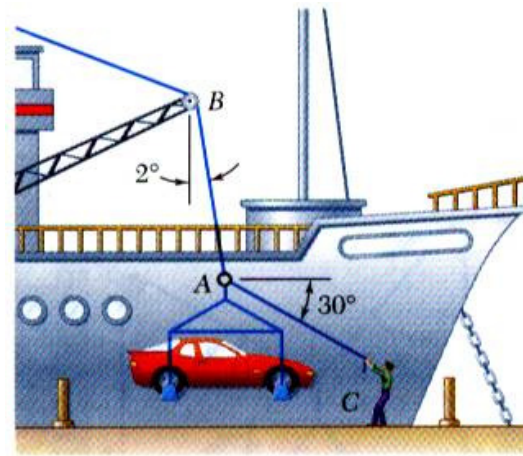


Statics of Particles

□ Sample Problem 07

SOLUTION:

- Construct a free-body diagram for the particle at A .
- Apply the conditions for equilibrium.



$$T_{AC} = T_{AB} \frac{\sin 2^\circ}{\cos 30^\circ} \quad (I)$$

$$T_{AB} = 3574.2 \text{ (lb)}$$

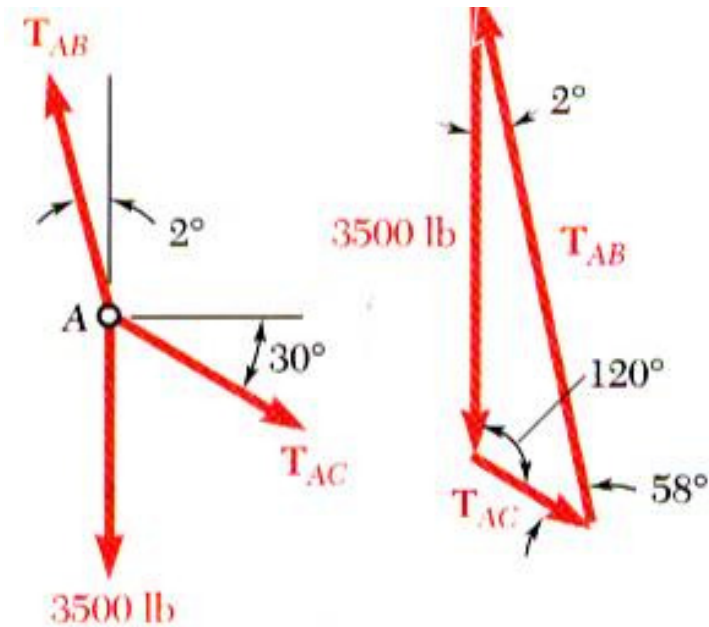
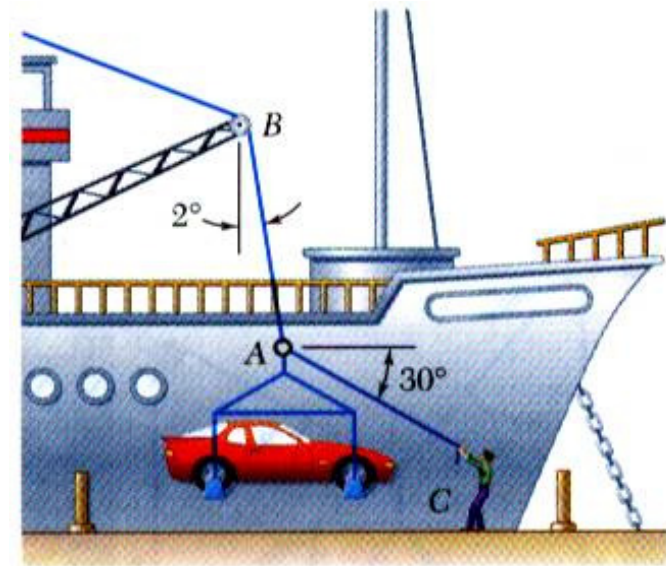
$$T_{AC} = 144 \text{ (lb)}$$

Statics of Particles

□ Sample Problem 07

SOLUTION:

- Alternative solution : *Law of sines*



$$T_{AB} = 3574.2 \text{ (lb)}$$

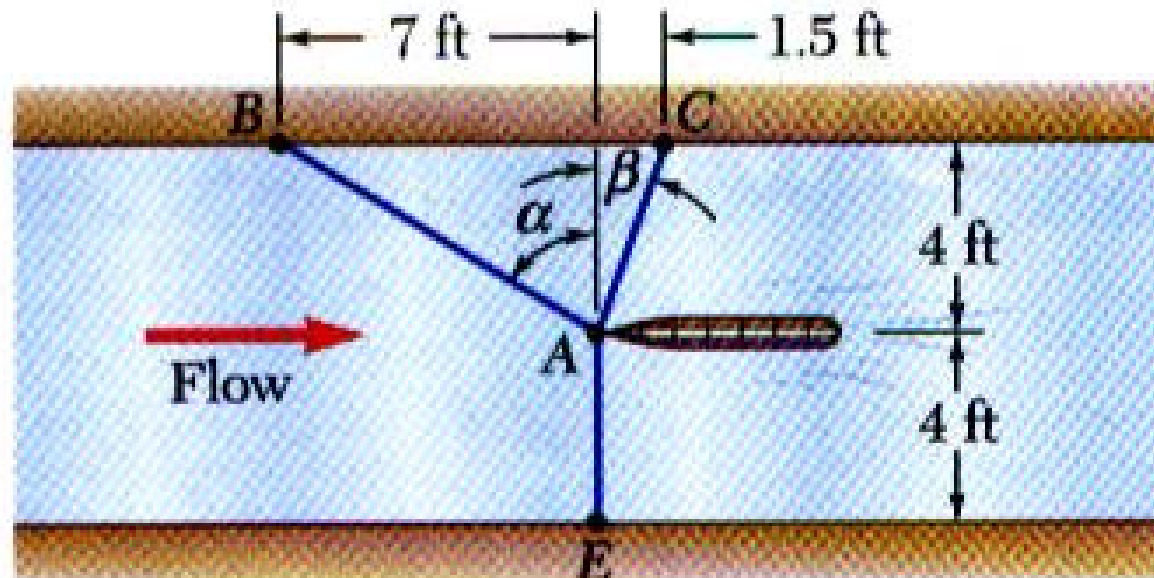
$$T_{AC} = 144 \text{ (lb)}$$

Statics of Particles

□ Sample Problem 08

It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 40 lb in cable AB and 60 lb in cable AE .

Determine the drag force exerted on the hull and the tension in cable AC .

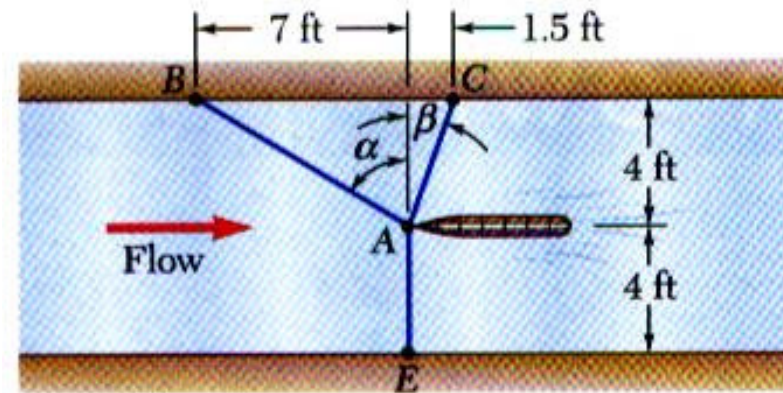


Statics of Particles

□ Sample Problem 08

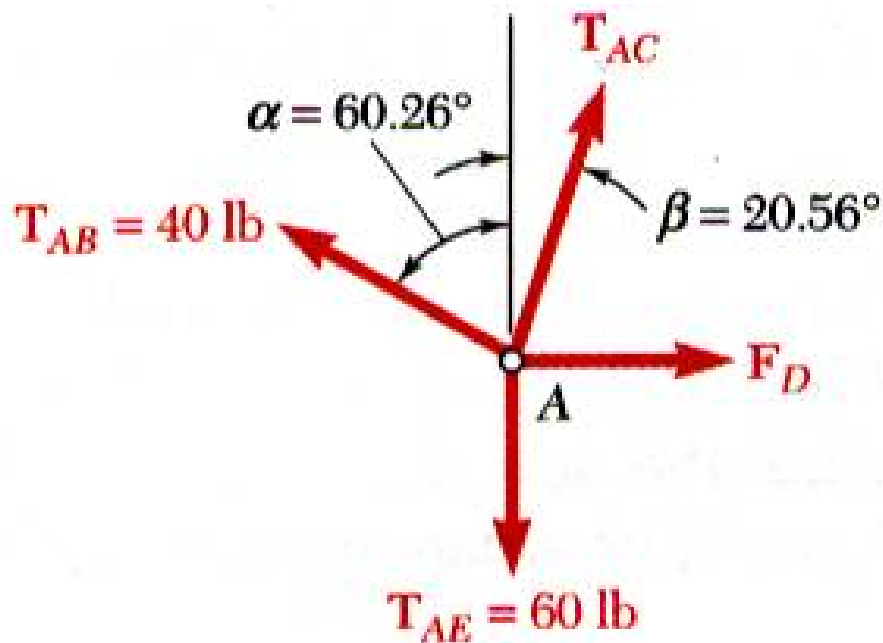
SOLUTION:

- Choosing the hull as the free body, draw a free-body diagram.



$$\alpha = 60.26^\circ$$

$$\beta = 20.56^\circ$$



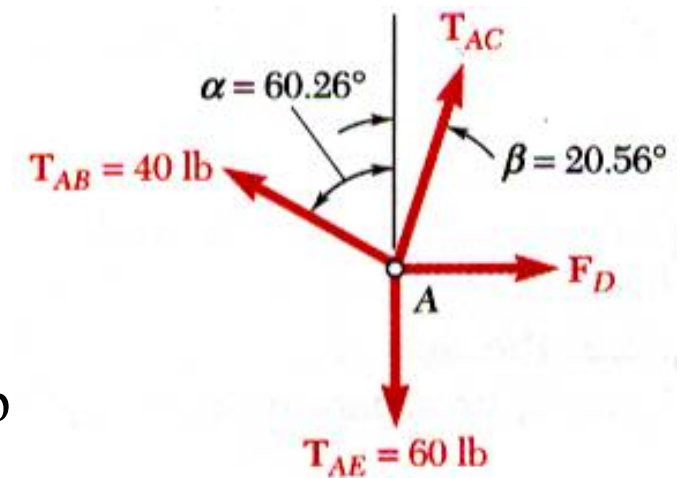
- Express the condition for *equilibrium* for the hull by writing that the sum of all forces must be zero.

Statics of Particles

□ Sample Problem 08

SOLUTION:

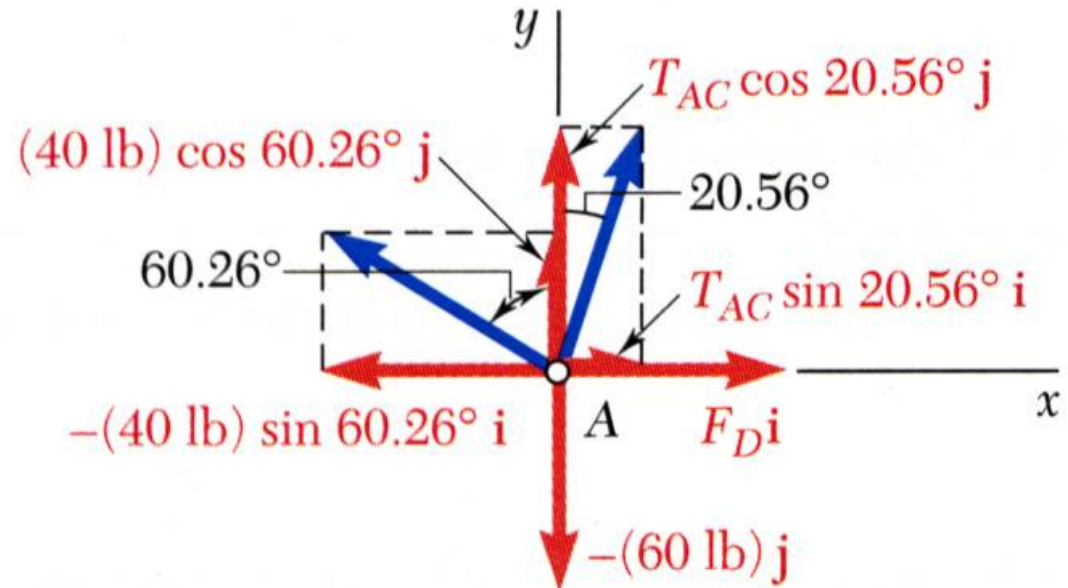
- Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.



$$\vec{T}_{AB} = -(34.73)\vec{i} + (19.84)\vec{j}$$

$$\vec{T}_{AC} = (0.3512T_{AC})\vec{i} + (0.9363T_{AC})\vec{j}$$

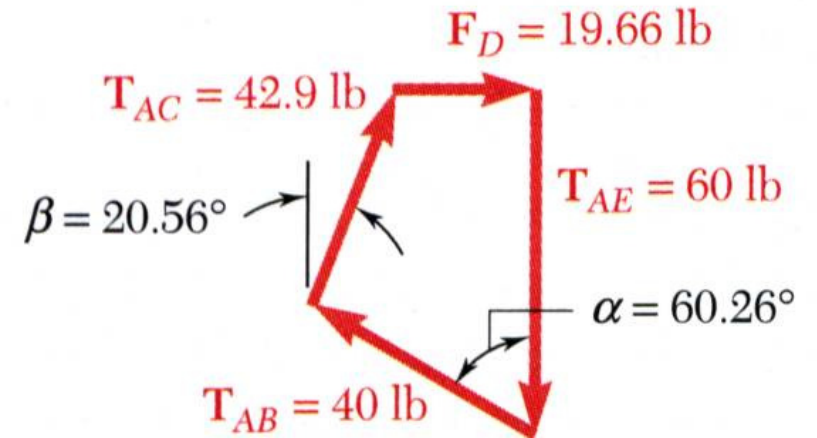
$$\vec{T}_{AE} = -(60)\vec{j} \quad \& \quad \vec{F}_D = F_D\vec{i}$$



Statics of Particles

□ Sample Problem 08

SOLUTION:



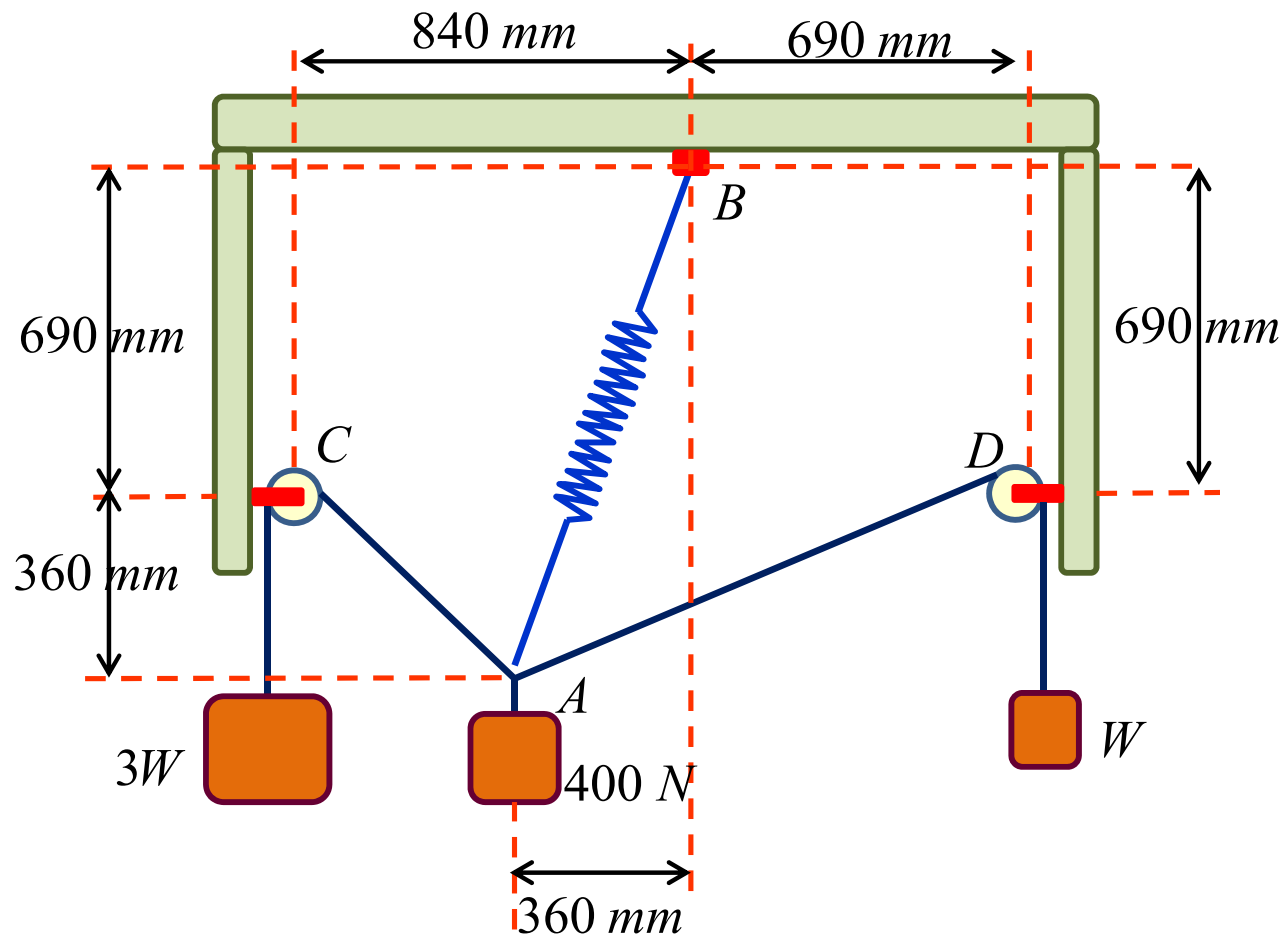
This equation is satisfied only if each component of the resultant is equal to zero

$$(I) \ \& \ (II) \ \Rightarrow \ \boxed{\begin{cases} T_{AC} = 42.9 \text{ (lb)} \\ F_D = 19.66 \text{ (lb)} \end{cases}}$$

Statics of Particles

□ Sample Problem 09

A load of weight 400 N is suspended from a spring and two cords that are attached to blocks of weights $3W$ and W as shown. Knowing that the constant of the spring is 800 N/m , determine (a) the value of W , (b) the unstretched length of the spring.

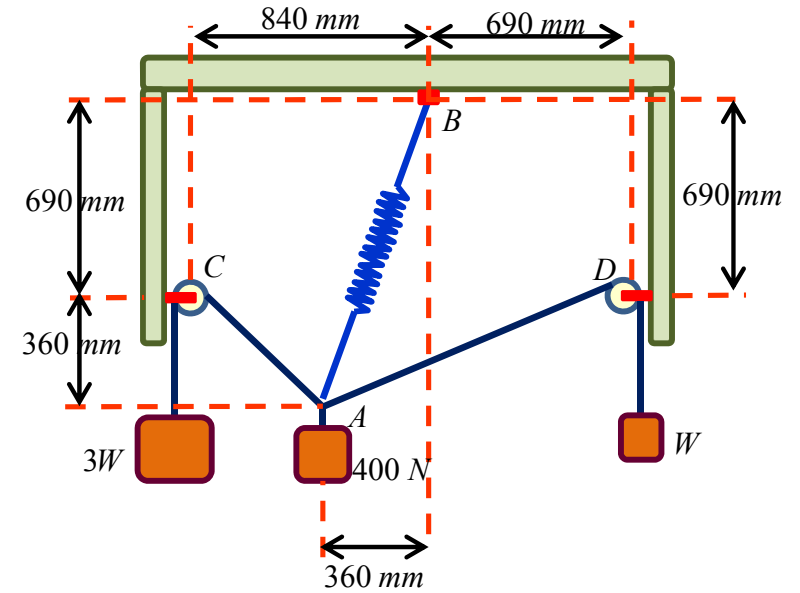


Statics of Particles

□ Sample Problem 09

SOLUTION:

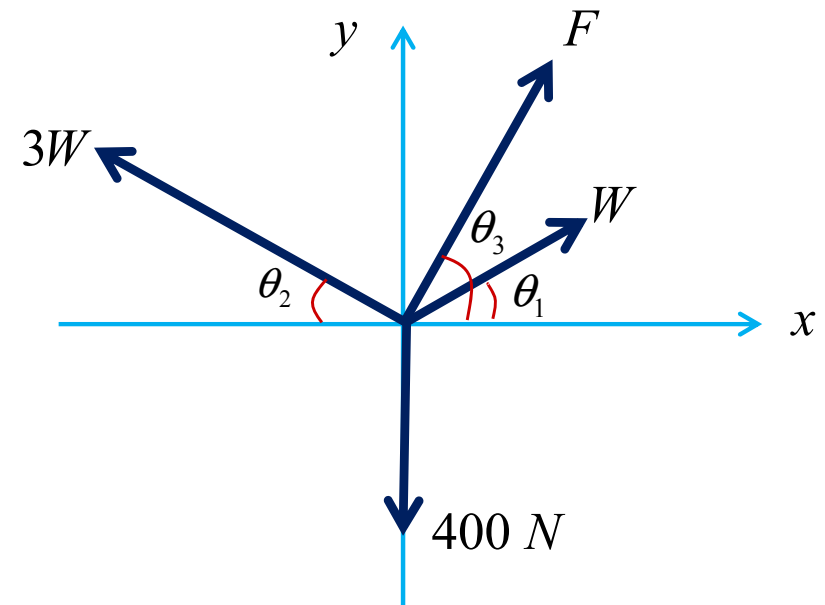
draw a free-body diagram.



$$\theta_1 = 18.92^\circ$$

$$\theta_2 = 36.87^\circ$$

$$\theta_3 = 71.08^\circ$$

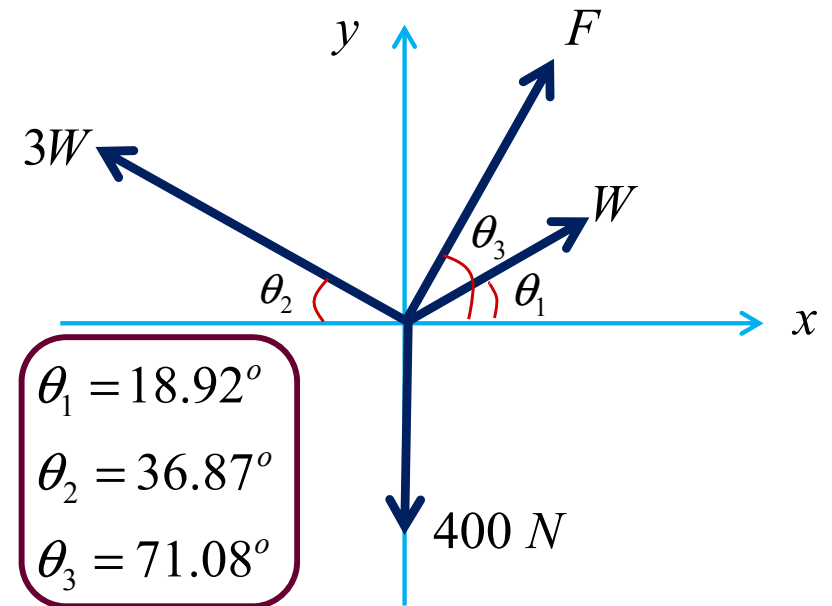


Statics of Particles

□ Sample Problem 09

SOLUTION:

Apply the conditions for equilibrium.



$$\Rightarrow 0.32F - 1.45W = 0 \quad (I)$$

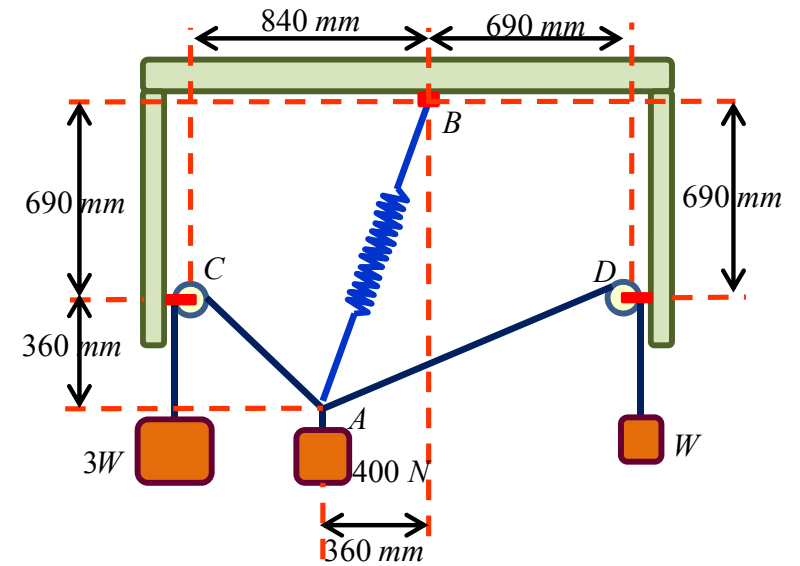
$$\Rightarrow 0.95F + 2.12W = 400 \quad (II)$$

$$(I) \ \& \ (II) \Rightarrow \begin{cases} W = 62.31 \text{ (N)} \\ F = 282.24 \text{ (N)} \end{cases}$$

Statics of Particles

□ Sample Problem 09

SOLUTION:



$$\Delta x = 352.8 \text{ (mm)}$$

$$L_1 = 757.20 \text{ (mm)}$$

Statics of Particles

□ Unit Vector

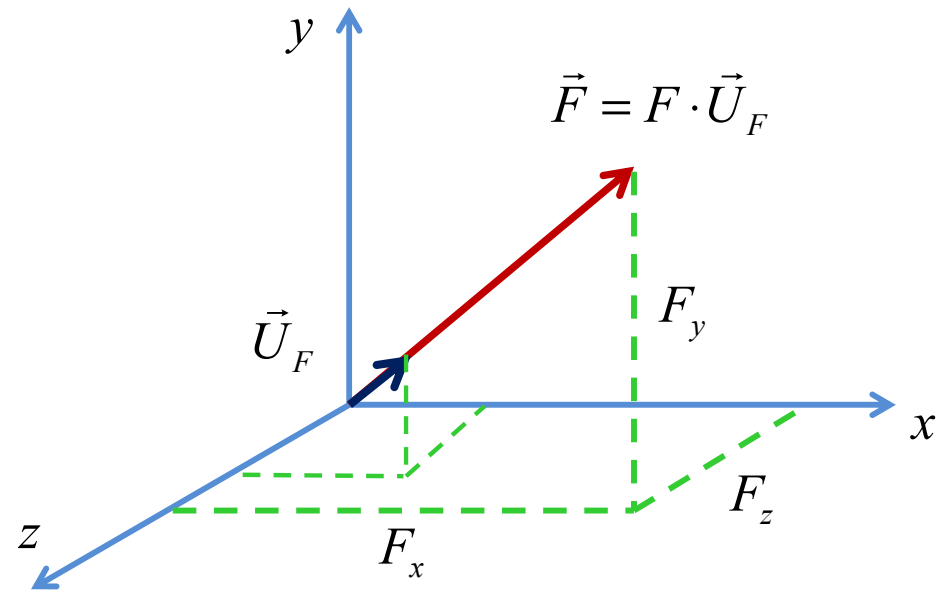
A **unit vector** is a vector who has the unit length.

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

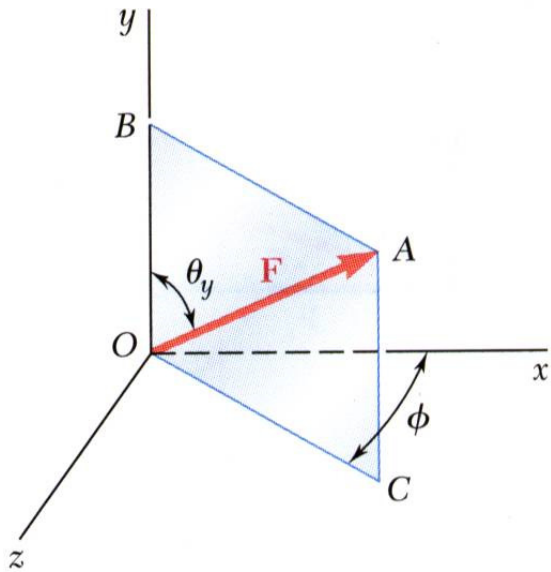
$$\vec{U}_F = \frac{\vec{F}}{F} \Rightarrow \vec{U}_F = \frac{F_x}{F} \vec{i} + \frac{F_y}{F} \vec{j} + \frac{F_z}{F} \vec{k}$$

$$U_F = \sqrt{\left(\frac{F_x}{F}\right)^2 + \left(\frac{F_y}{F}\right)^2 + \left(\frac{F_z}{F}\right)^2} = 1$$

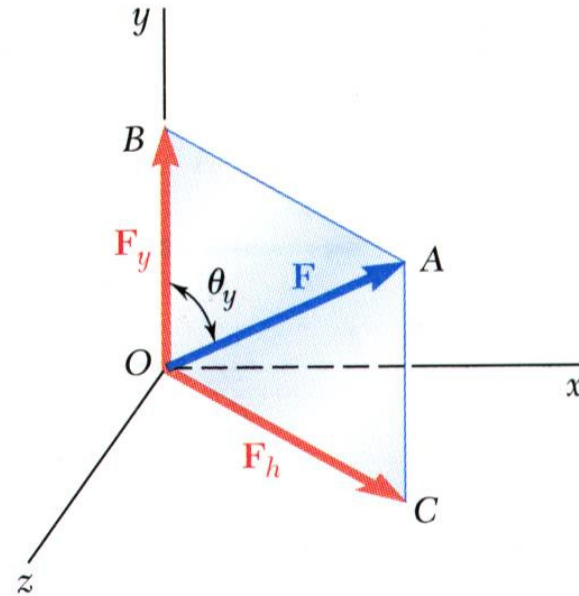


Statics of Particles

□ Rectangular Components in Space



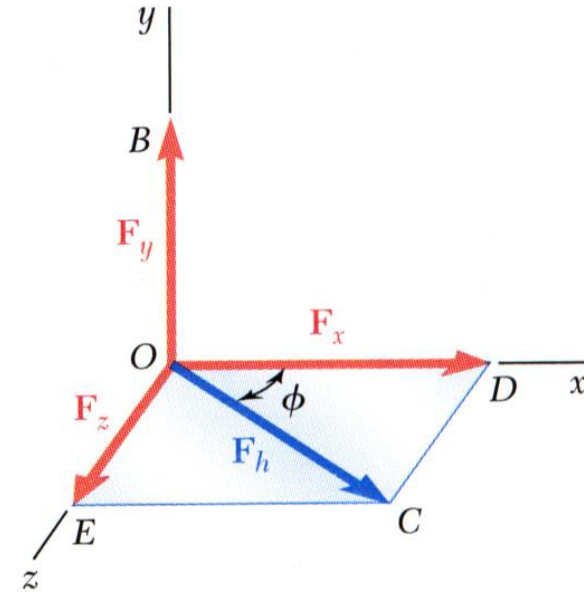
- The vector \vec{F} is contained in the plane $OBAC$.



- Resolve \vec{F} into horizontal and vertical components.

$$F_y = F \cos \theta_y$$

$$F_h = F \sin \theta_y$$



- Resolve F_h into rectangular components

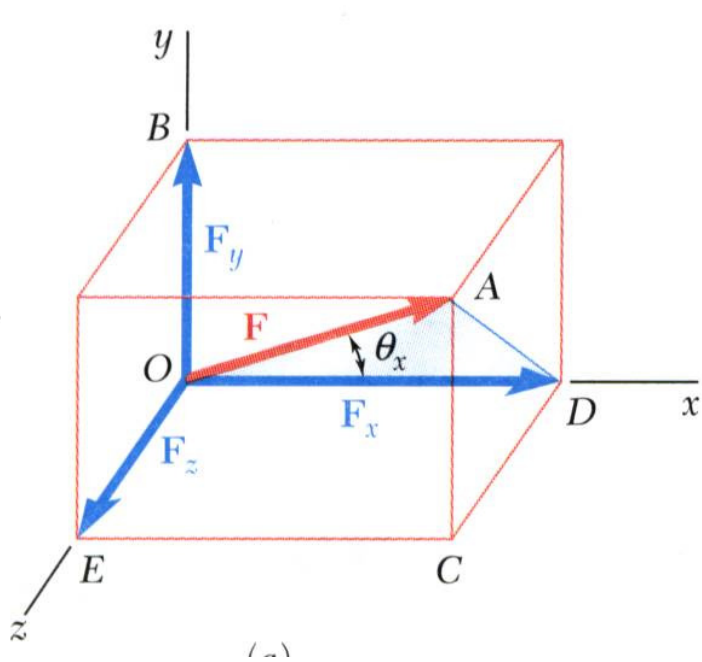
$$F_x = F_h \cos \phi \Rightarrow F_x = F \sin \theta_y \cos \phi$$

$$F_z = F_h \sin \phi \Rightarrow F_z = F \sin \theta_y \sin \phi$$

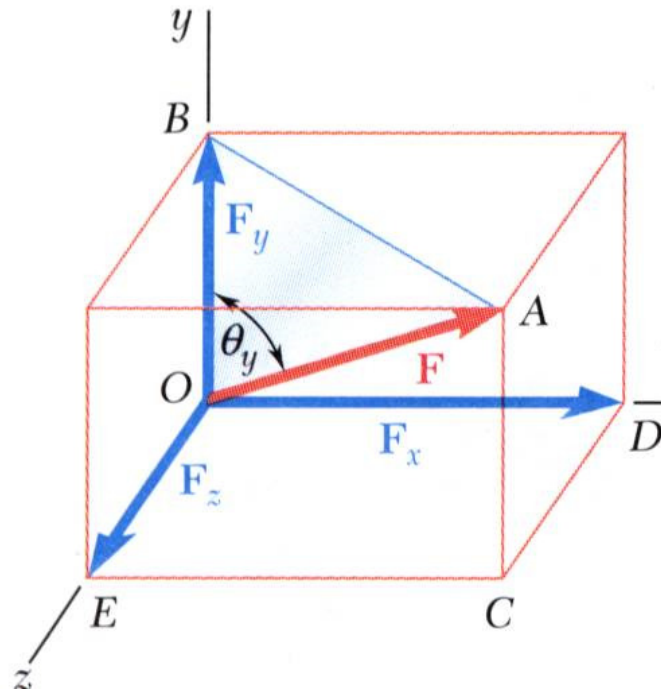
Statics of Particles

□ Rectangular Components in Space

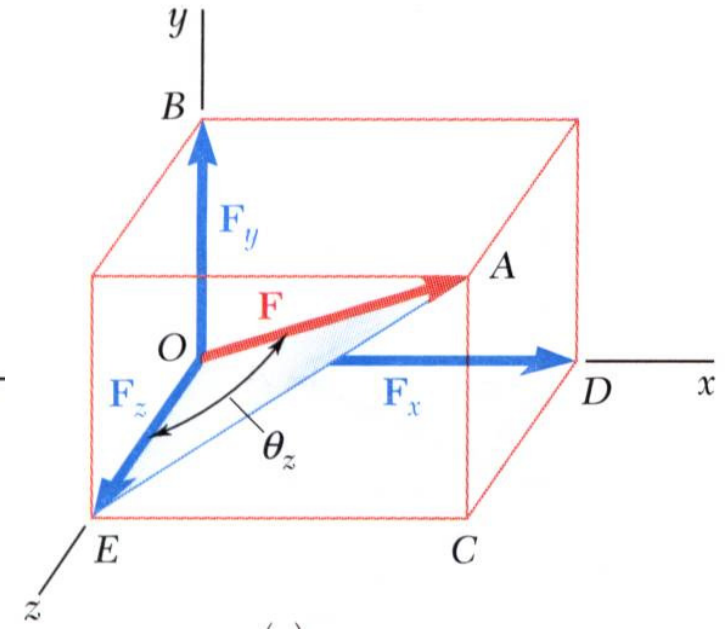
- With the angles between \vec{F} and the axes,



$$F_x = F \cos \theta_x$$



$$F_y = F \cos \theta_y$$



$$F_z = F \cos \theta_z$$

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k} \Rightarrow \vec{F} = F(\cos \theta_x \vec{i} + \cos \theta_y \vec{j} + \cos \theta_z \vec{k})$$

Statics of Particles

□ Rectangular Components in Space

- $\vec{\lambda}$ is a unit vector along the line of action of \vec{F} and $\cos \theta_x$, $\cos \theta_y$, and $\cos \theta_z$ are the direction cosines for \vec{F}

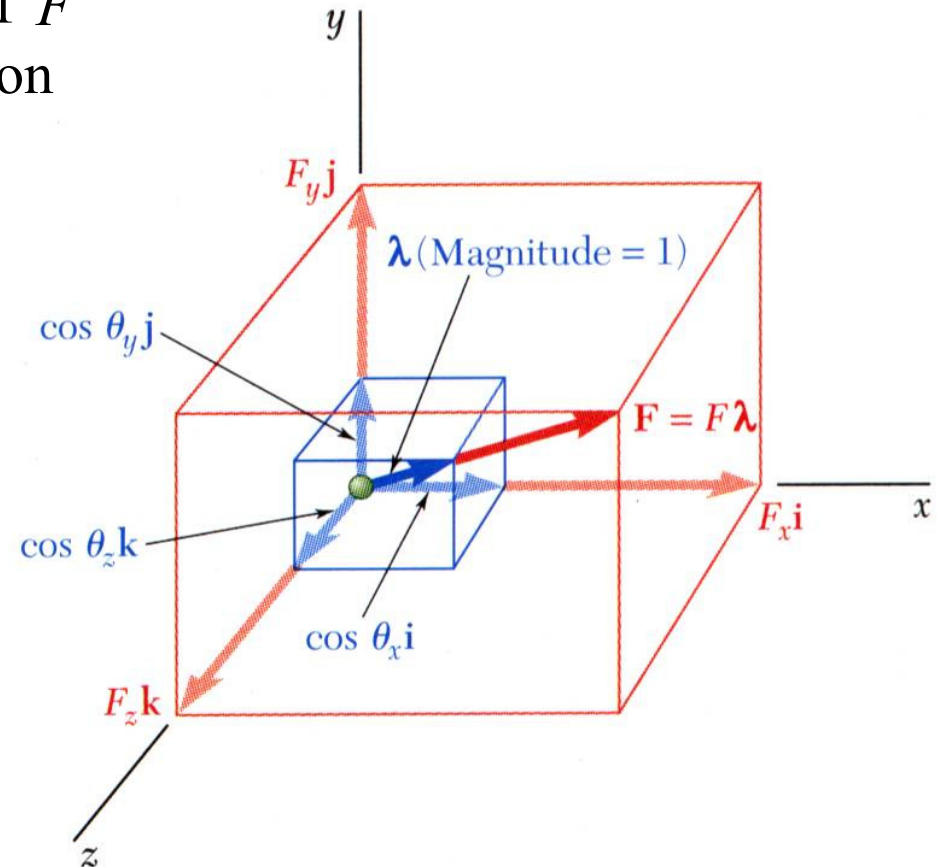
$$\vec{F} = F\vec{\lambda}$$

$$\vec{\lambda} = \cos \theta_x \vec{i} + \cos \theta_y \vec{j} + \cos \theta_z \vec{k}$$

$$\theta_x = \cos^{-1} \left(\frac{F_x}{F} \right)$$

$$\theta_y = \cos^{-1} \left(\frac{F_y}{F} \right)$$

$$\theta_z = \cos^{-1} \left(\frac{F_z}{F} \right)$$



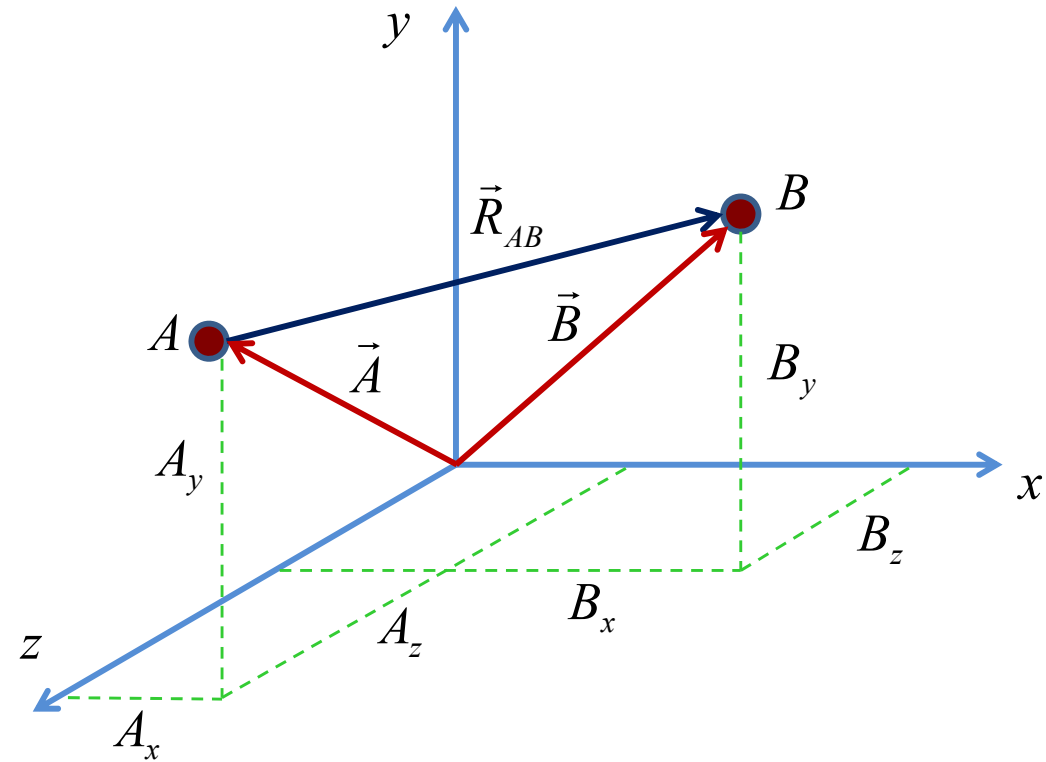
$$\lambda = 1 \Rightarrow \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

Statics of Particles

□ Rectangular Components in Space

Direction of the vector is defined by the location of two points,

$A(A_x, A_y, A_z)$ and $B(B_x, B_y, B_z)$



$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$

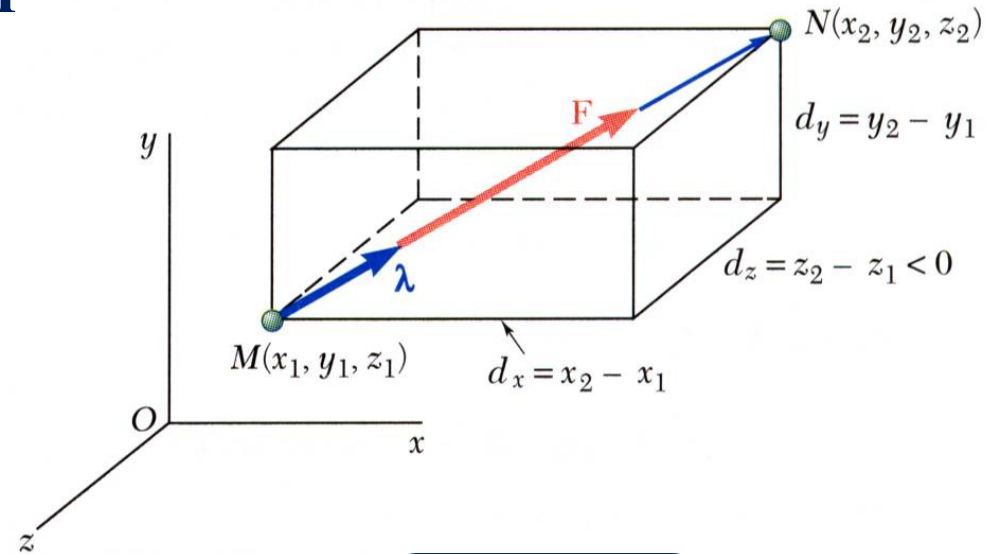
$$\vec{R}_{AB} = \vec{B} - \vec{A} \Rightarrow \vec{R}_{AB} = (B_x - A_x) \vec{i} + (B_y - A_y) \vec{j} + (B_z - A_z) \vec{k}$$

Statics of Particles

□ Rectangular Components in Space

Direction of the force is defined by
the location of two points,

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



vector joining M and N : $\vec{d} = d_x \vec{i} + d_y \vec{j} + d_z \vec{k}$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$d_x = (x_2 - x_1)$$

$$d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\vec{\lambda} = \frac{1}{d} (d_x \vec{i} + d_y \vec{j} + d_z \vec{k}) \quad \Rightarrow \quad \vec{F} = F \vec{\lambda}$$

$$F_x = \frac{F d_x}{d} \quad F_y = \frac{F d_y}{d} \quad F_z = \frac{F d_z}{d}$$

Statics of Particles

□ Addition of n Forces by Summing Components in Space

$$\vec{F}_1 = Fx_1 \vec{i} + Fy_1 \vec{j} + Fz_1 \vec{k}$$

$$\vec{F}_2 = Fx_2 \vec{i} + Fy_2 \vec{j} + Fz_2 \vec{k}$$

⋮

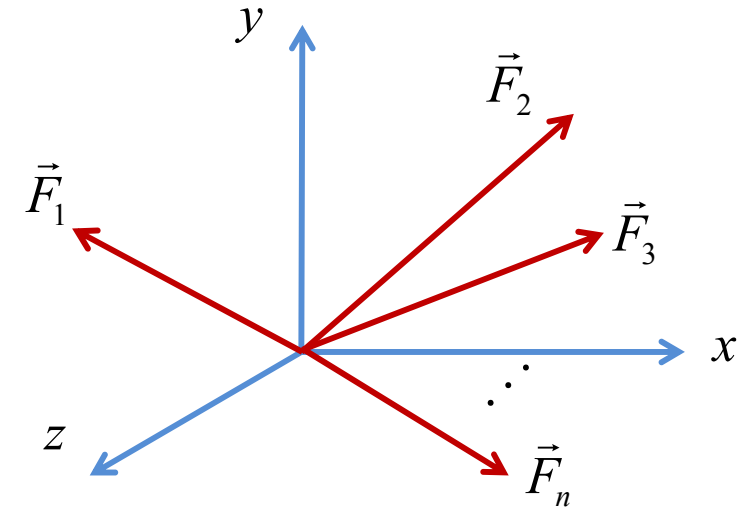
$$\vec{F}_n = Fx_n \vec{i} + Fy_n \vec{j} + Fz_n \vec{k}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

$$\Rightarrow \vec{F} = (Fx_1 + Fx_2 + \cdots + Fx_n) \vec{i} + (Fy_1 + Fy_2 + \cdots + Fy_n) \vec{j} + (Fz_1 + Fz_2 + \cdots + Fz_n) \vec{k}$$

$$\Rightarrow \vec{F} = \left(\sum_{r=1}^n Fx_r \right) \vec{i} + \left(\sum_{r=1}^n Fy_r \right) \vec{j} + \left(\sum_{r=1}^n Fz_r \right) \vec{k}$$

$$\Rightarrow F = \sqrt{\left(\sum_{r=1}^n Fx_r \right)^2 + \left(\sum_{r=1}^n Fy_r \right)^2 + \left(\sum_{r=1}^n Fz_r \right)^2}$$



$$\theta_x = \cos^{-1} \left(\frac{\sum_{r=1}^n Fx_r}{F} \right)$$

$$\theta_y = \cos^{-1} \left(\frac{\sum_{r=1}^n Fy_r}{F} \right)$$

$$\theta_z = \cos^{-1} \left(\frac{\sum_{r=1}^n Fz_r}{F} \right)$$

Statics of Particles

□ Note on Vectors

- *Dot or inner product*

The dot product is a method for multiplying two vectors. Because the product of the multiplication is a scalar, the dot product is sometimes referred to as the scalar product.

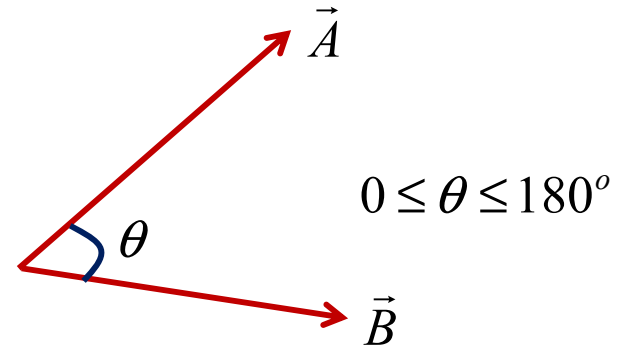
$$\vec{A} \cdot \vec{B} = A \cdot B \cos \theta \Rightarrow \theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{A \cdot B} \right)$$

$$\text{if } \vec{A} \cdot \vec{B} = 0 \Rightarrow \theta = 90^\circ \Rightarrow \vec{A} \perp \vec{B}$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$a(\vec{A} \cdot \vec{B}) = (a\vec{A}) \cdot \vec{B} = \vec{A} \cdot (a\vec{B}) = (\vec{A} \cdot \vec{B})a$$

$$\vec{A} \cdot (\vec{B} + \vec{D}) = (\vec{A} \cdot \vec{B}) + (\vec{A} \cdot \vec{D})$$



$$\vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{k} \cdot \vec{j} = (1)(1) \cos 90^\circ = 0$$

$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = (1)(1) \cos 0^\circ = 1$$

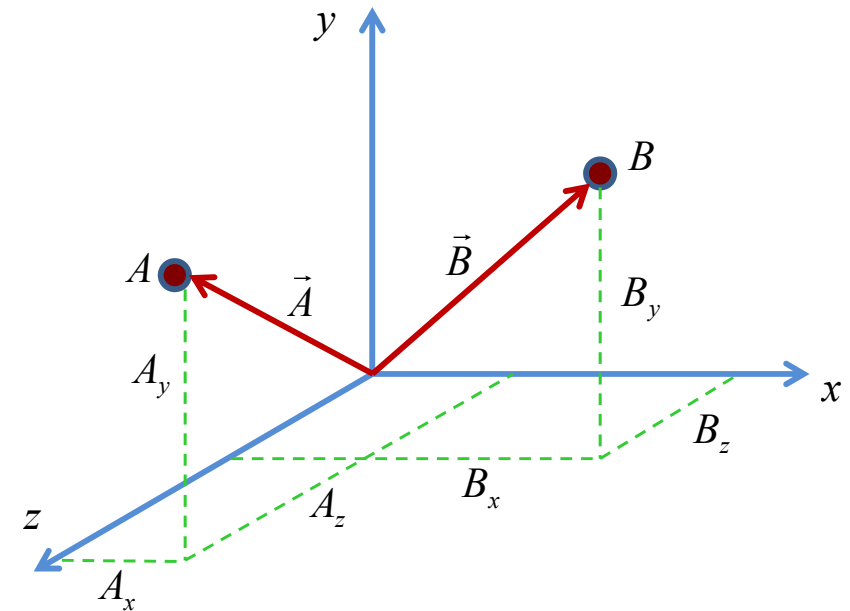
Statics of Particles

□ Note on Vectors

The dot product to vectors in space

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$



$$\vec{A} \cdot \vec{B} = (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \cdot (B_x \vec{i} + B_y \vec{j} + B_z \vec{k})$$

$$= A_x B_x (\vec{i} \cdot \vec{i}) + A_x B_y (\vec{i} \cdot \vec{j}) + A_x B_z (\vec{i} \cdot \vec{k})$$

$$+ A_y B_x (\vec{j} \cdot \vec{i}) + A_y B_y (\vec{j} \cdot \vec{j}) + A_y B_z (\vec{j} \cdot \vec{k})$$

$$+ A_z B_x (\vec{k} \cdot \vec{i}) + A_z B_y (\vec{k} \cdot \vec{j}) + A_z B_z (\vec{k} \cdot \vec{k})$$

$$\Rightarrow \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Statics of Particles

□ Note on Vectors

- *Projection of a vector on a given axis:*

$$P_{OL} = P \cos \theta = \text{projection of } P \text{ along } OL$$

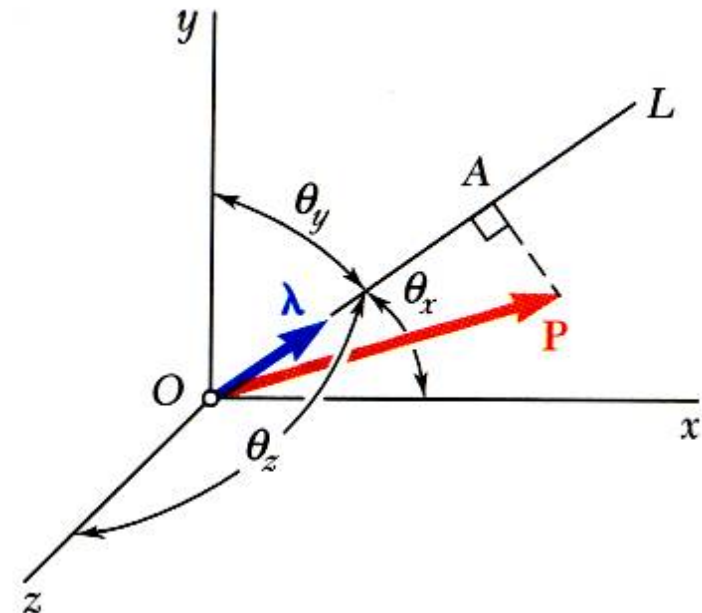
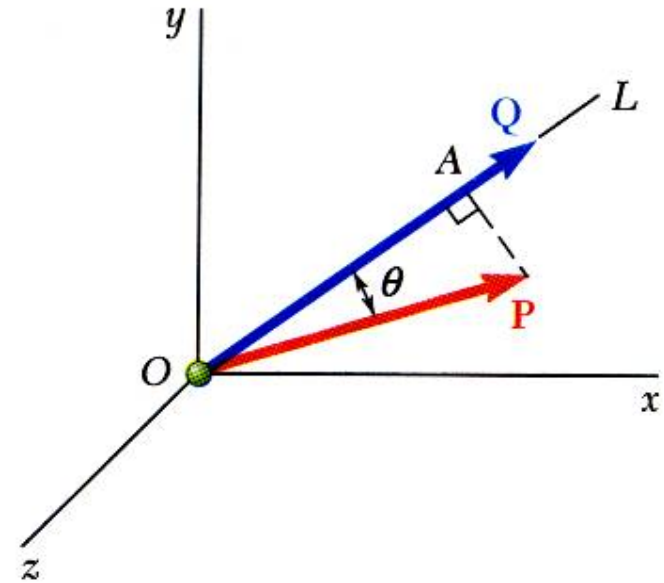
$$\vec{P} \cdot \vec{Q} = PQ \cos \theta$$

$$\frac{\vec{P} \cdot \vec{Q}}{Q} = P \cos \theta = P_{OL}$$

- *For an axis defined by a unit vector:*

$$P_{OL} = \vec{P} \cdot \vec{\lambda}$$

$$= P_x \cos \theta_x + P_y \cos \theta_y + P_z \cos \theta_z$$



Statics of Particles

□ Note on Vectors

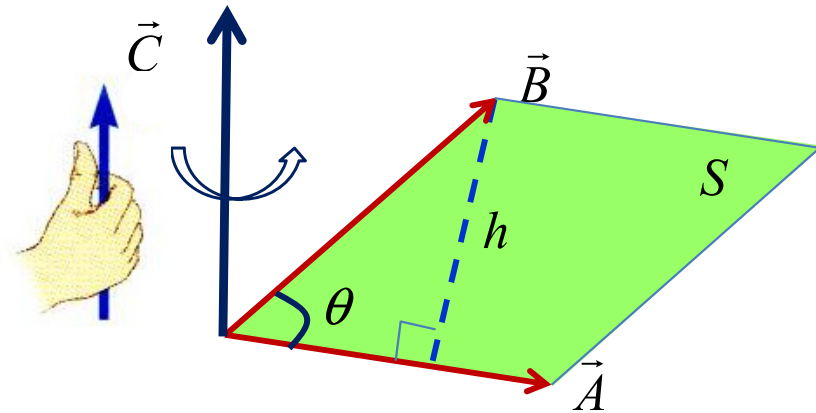
• *The Cross Product (Vector Product)*

The cross product $\vec{A} \times \vec{B}$ between two vectors \vec{A} and \vec{B} is a new vector perpendicular to the plane defined by the original two vectors.

$$\vec{C} = \vec{A} \times \vec{B} \quad , \quad \vec{C} \perp (\vec{A} \& \vec{B})$$

$$C = A \cdot B \sin \theta$$

$$S = h \cdot A = (B \sin \theta) \cdot A \quad \Rightarrow \quad C = S$$



$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

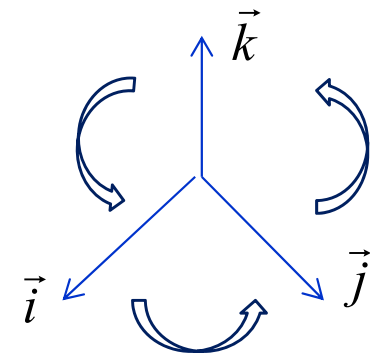
$$(a\vec{A}) \times \vec{B} = a(\vec{A} \times \vec{B})$$

$$(\vec{A} + \vec{B}) \times \vec{C} = (\vec{A} \times \vec{C}) + (\vec{B} \times \vec{C})$$

$$(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$$

$$\vec{i} \times \vec{j} = \vec{k} \quad \vec{j} \times \vec{k} = \vec{i} \quad \vec{k} \times \vec{i} = \vec{j}$$

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$



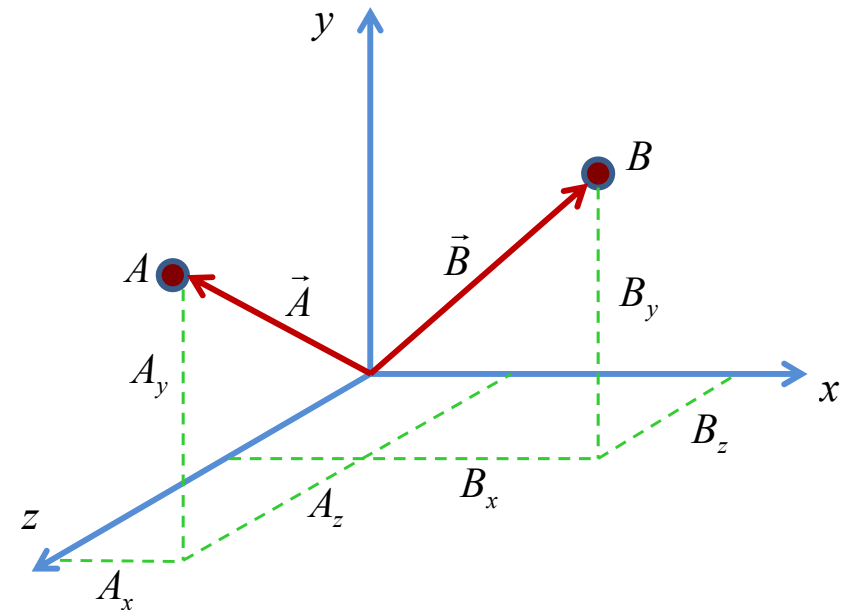
Statics of Particles

□ Note on Vectors

The Cross product to vectors in space

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$



$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \times (B_x \vec{i} + B_y \vec{j} + B_z \vec{k}) = A_x B_x (\vec{i} \times \vec{i}) + A_x B_y (\vec{i} \times \vec{j}) + A_x B_z (\vec{i} \times \vec{k}) \\ &+ A_y B_x (\vec{j} \times \vec{i}) + A_y B_y (\vec{j} \times \vec{j}) + A_y B_z (\vec{j} \times \vec{k}) + A_z B_x (\vec{k} \times \vec{i}) + A_z B_y (\vec{k} \times \vec{j}) + A_z B_z (\vec{k} \times \vec{k}) \end{aligned}$$

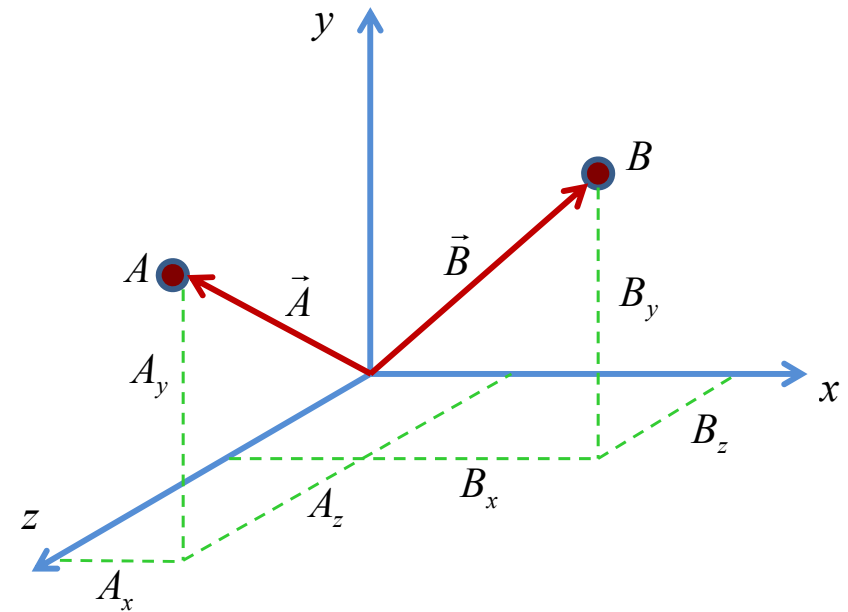
$$\Rightarrow \vec{A} \times \vec{B} = A_x B_y (\vec{k}) + A_x B_z (-\vec{j}) + A_y B_x (-\vec{k}) + A_y B_z (\vec{i}) + A_z B_x (\vec{j}) + A_z B_y (-\vec{i})$$

$$\Rightarrow \vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \vec{i} - (A_x B_z - A_z B_x) \vec{j} + (A_x B_y - A_y B_x) \vec{k}$$

Statics of Particles

□ Note on Vectors

The Cross product to vectors in space



$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$

$$\Rightarrow \vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y) \vec{i} - (A_x B_z - A_z B_x) \vec{j} + (A_x B_y - A_y B_x) \vec{k}$$

Statics of Particles

□ Note on Vectors

- Mixed triple product of three vectors,

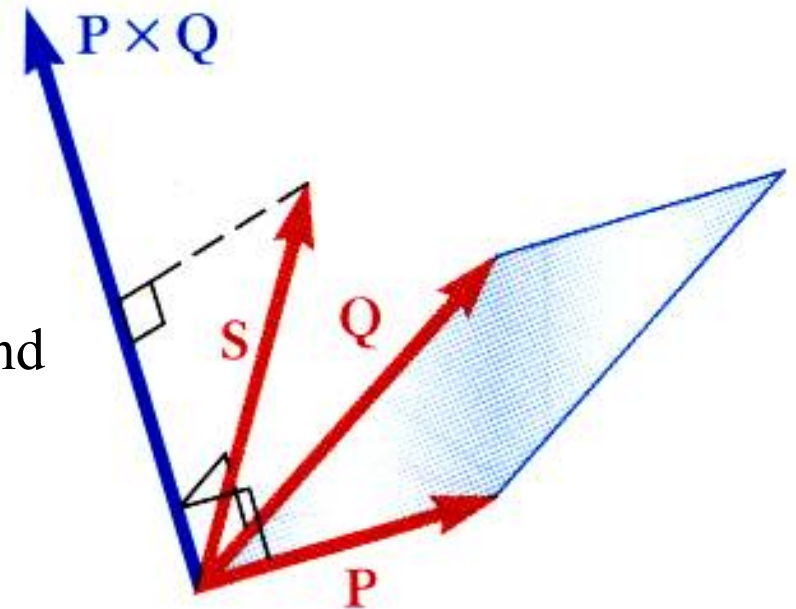
$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = \text{scalar result}$$

- The six mixed triple products formed from \mathbf{S} , \mathbf{P} , and \mathbf{Q} have equal magnitudes but not the same sign,

$$\begin{aligned}\vec{S} \cdot (\vec{P} \times \vec{Q}) &= \vec{P} \cdot (\vec{Q} \times \vec{S}) = \vec{Q} \cdot (\vec{S} \times \vec{P}) \\ &= -\vec{S} \cdot (\vec{Q} \times \vec{P}) = -\vec{P} \cdot (\vec{S} \times \vec{Q}) = -\vec{Q} \cdot (\vec{P} \times \vec{S})\end{aligned}$$

- Evaluating the mixed triple product,

$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = S_x(P_y Q_z - P_z Q_y) + S_y(P_z Q_x - P_x Q_z) + S_z(P_x Q_y - P_y Q_x) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

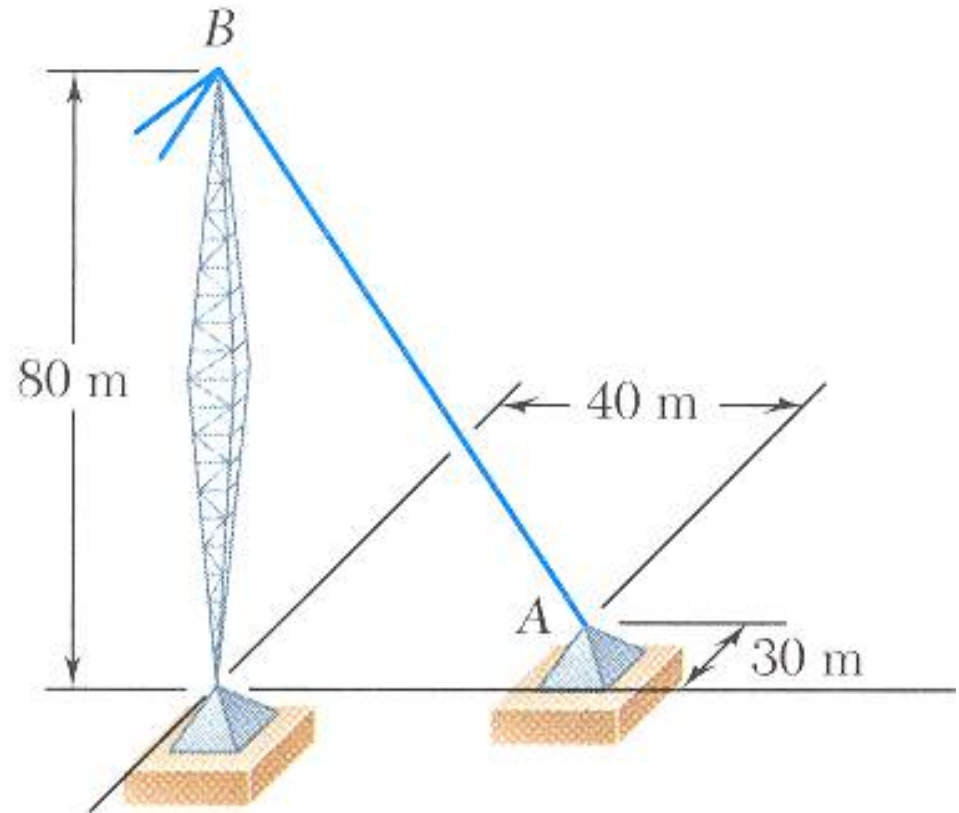


Statics of Particles

□ Sample Problem 10

The tension in the guy wire is 2500 N. Determine:

- components F_x , F_y , F_z of the force acting on the bolt at A ,
- the angles θ_x , θ_y , θ_z defining the direction of the force



Statics of Particles

□ Sample Problem 10

SOLUTION:

- Determine the unit vector pointing from A towards B .

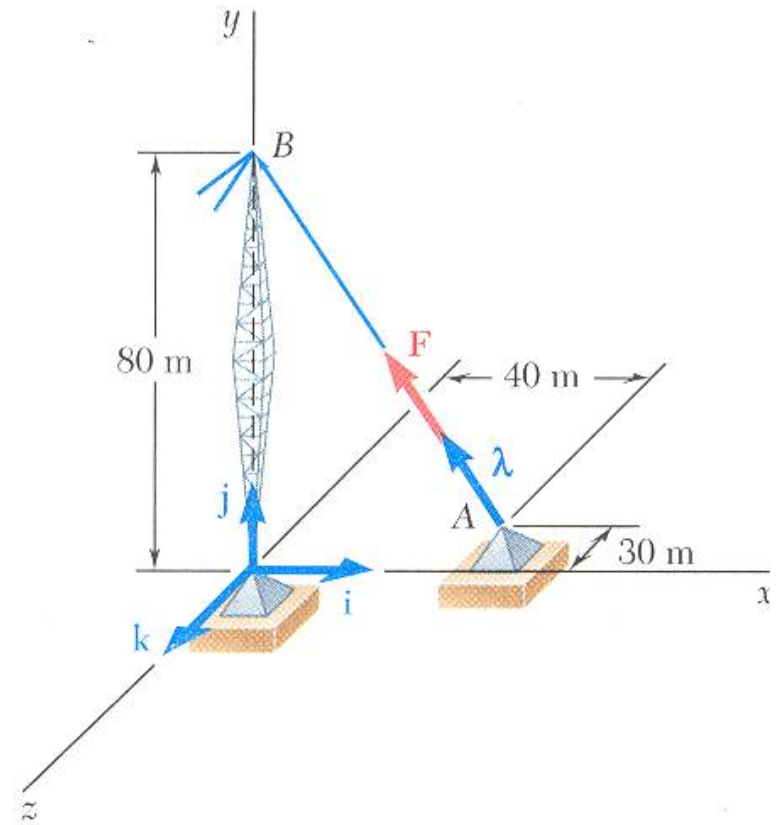
$$\vec{AB} = (-40)\vec{i} + (80)\vec{j} + (30)\vec{k}$$

$$AB = 94.3 \text{ m}$$

- Determine the components of the force.

$$\vec{\lambda} = -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\vec{F} = (-1060 \text{ N})\vec{i} + (2120 \text{ N})\vec{j} + (795 \text{ N})\vec{k}$$

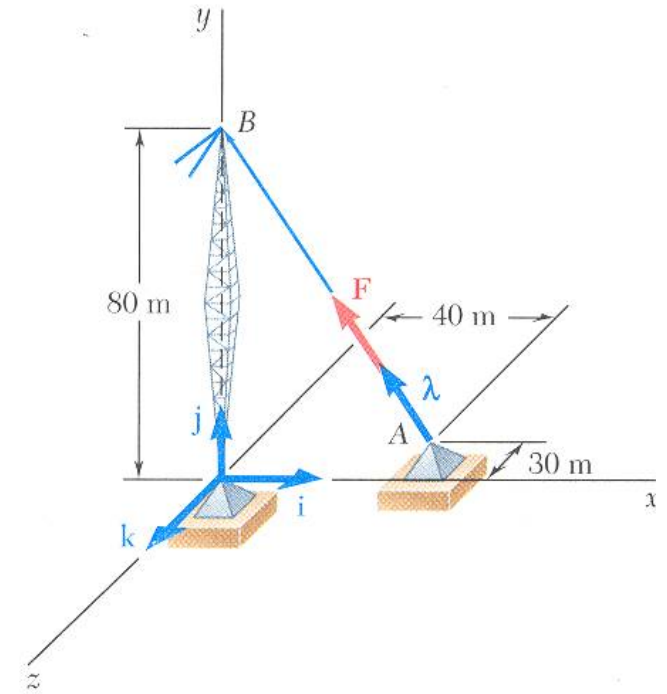


Statics of Particles

□ Sample Problem 10

SOLUTION:

- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.



\Rightarrow

$$\theta_x = 115.1^\circ$$

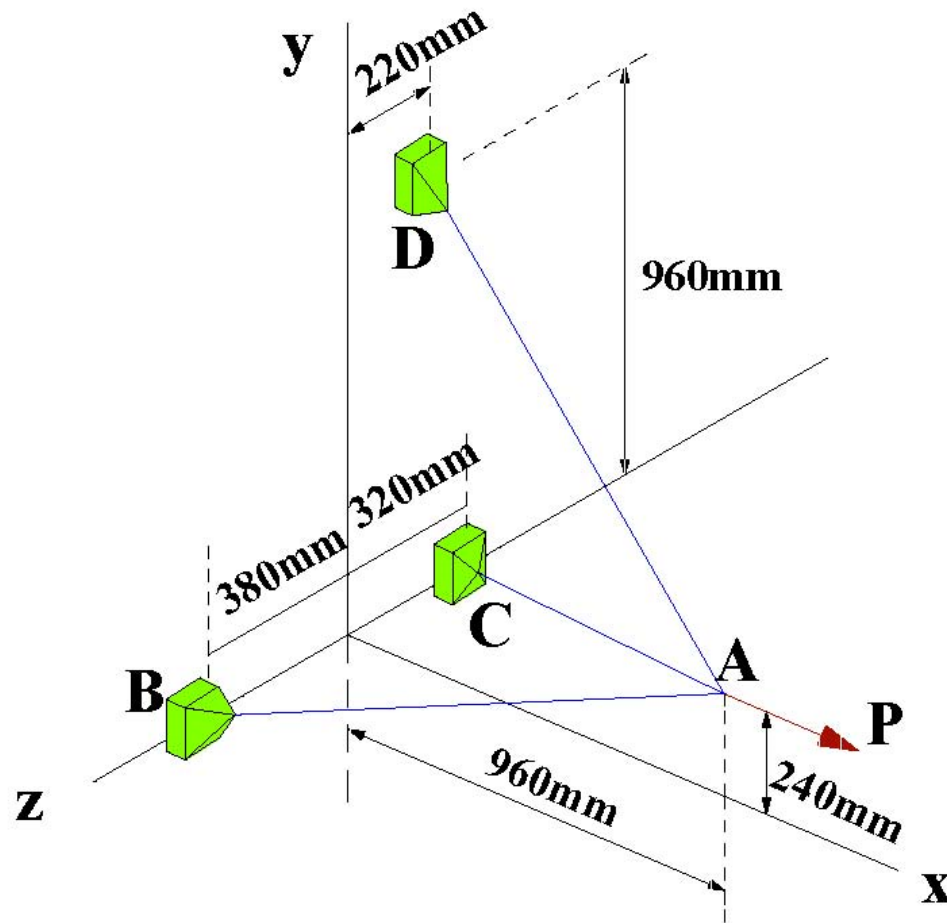
$$\theta_y = 32.0^\circ$$

$$\theta_z = 71.5^\circ$$

Statics of Particles

□ Sample Problem 11

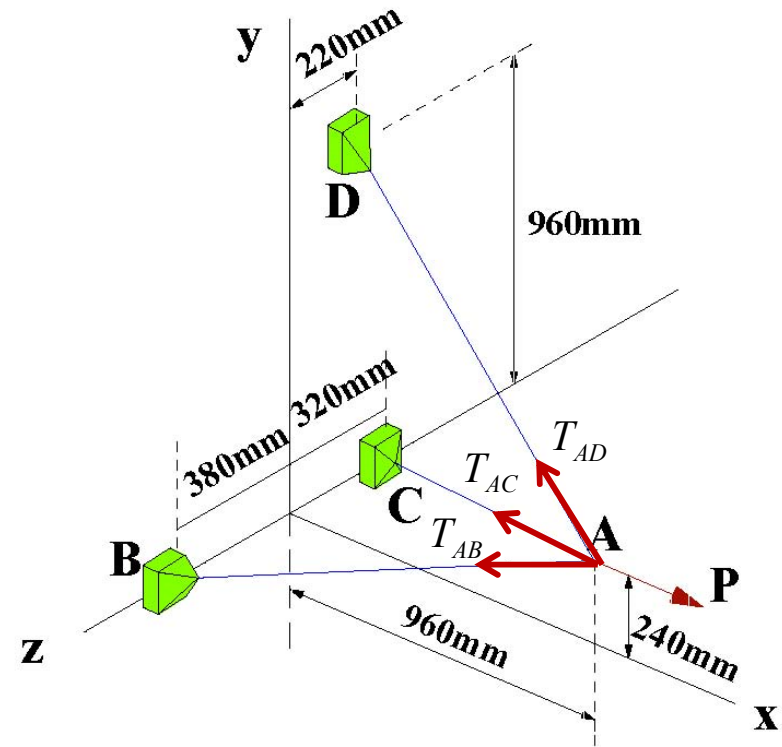
Determine the magnitude of P if there is 305 N tension force in Cable AD .



Statics of Particles

□ Sample Problem 11

SOLUTION:



$$AB = 1060 \text{ mm}$$

$$AC = 1040 \text{ mm}$$

$$AD = 1220 \text{ mm}$$

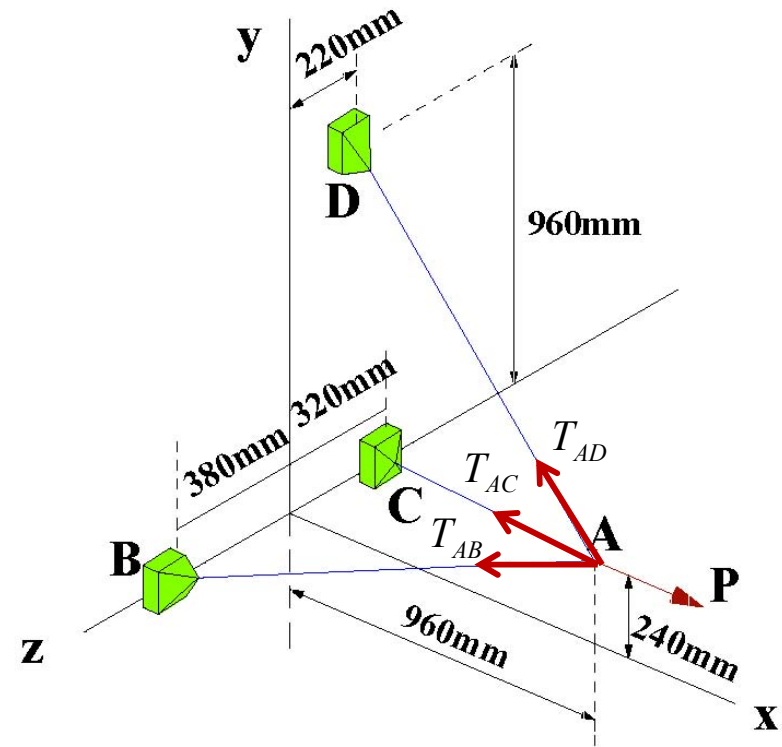
Statics of Particles

□ Sample Problem 11

SOLUTION:

$$\vec{\lambda}_{AC} = \frac{\vec{AC}}{AC} = -\left(\frac{12}{13}\right)\vec{i} - \left(\frac{3}{13}\right)\vec{j} - \left(\frac{4}{13}\right)\vec{k}$$

$$\vec{\lambda}_{AD} = \frac{\vec{AD}}{AD} = -\left(\frac{48}{61}\right)\vec{i} + \left(\frac{36}{61}\right)\vec{j} - \left(\frac{11}{61}\right)\vec{k}$$



$$\vec{T}_{AC} = \vec{\lambda}_{AC} \cdot T_{AC} = \left(-\frac{12}{13}\vec{i} - \frac{3}{13}\vec{j} - \frac{4}{13}\vec{k}\right) \cdot T_{AC}$$

$$\vec{T}_{AD} = \vec{\lambda}_{AD} \cdot (305) = -240\vec{i} + 180\vec{j} - 55\vec{k}$$

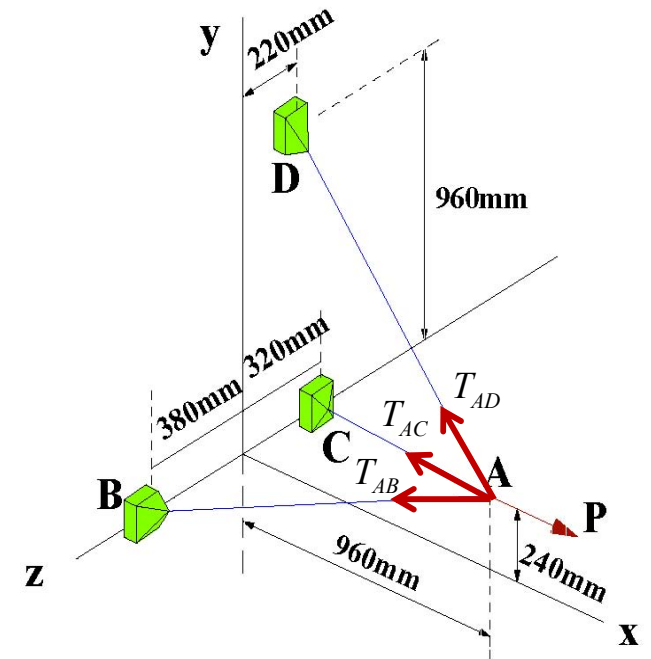
$$\vec{P} = P\vec{i}$$

Statics of Particles

□ Sample Problem 11

SOLUTION:

- Apply the conditions for equilibrium.



$$(I), (II), (III) \Rightarrow \begin{cases} T_{AB} = 446.71 (N) \\ T_{AC} = 341.71 (N) \\ P = 4960 (N) \end{cases}$$