# Nonlinear Analysis of Structures

## Geometrically Nonlinear Analysis of Plane Trusses

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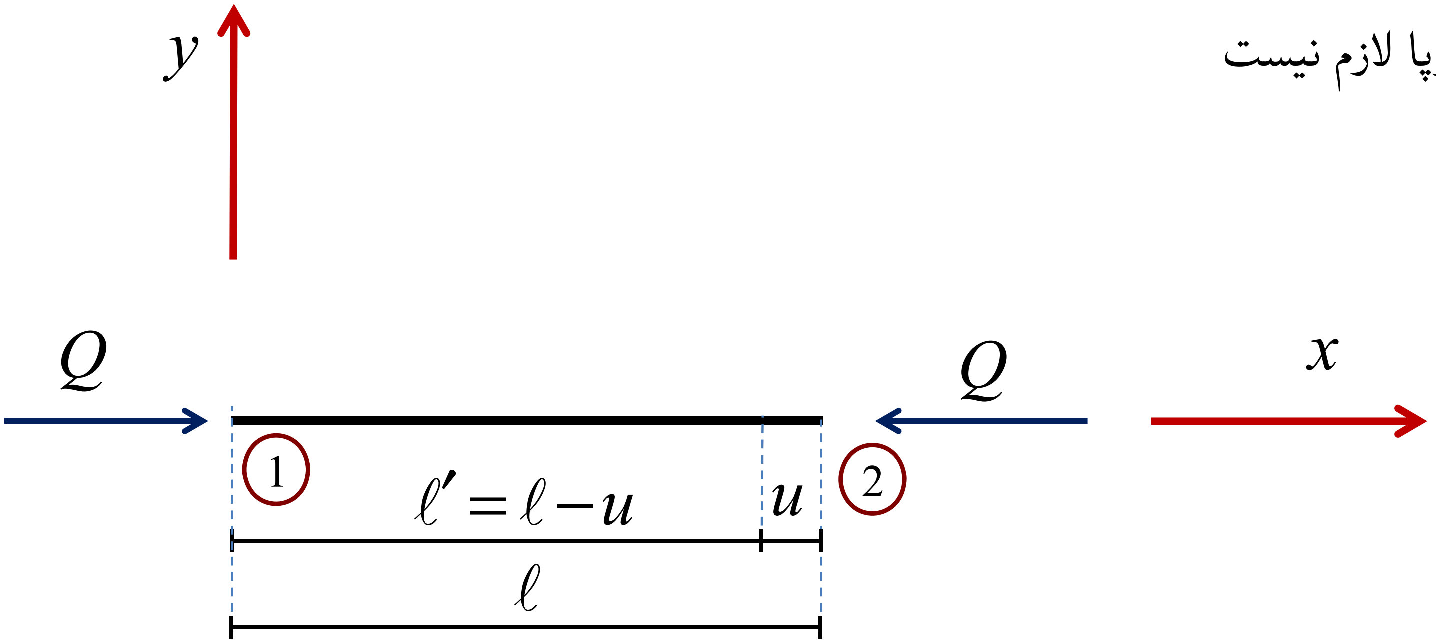
# Geometrically Nonlinear Analysis of Plane Trusses

- **Member, Force–Deformation Relationship in local Coordinate**

الف – روابط نیرو تغییر شکل در دستگاه مختصات محلی



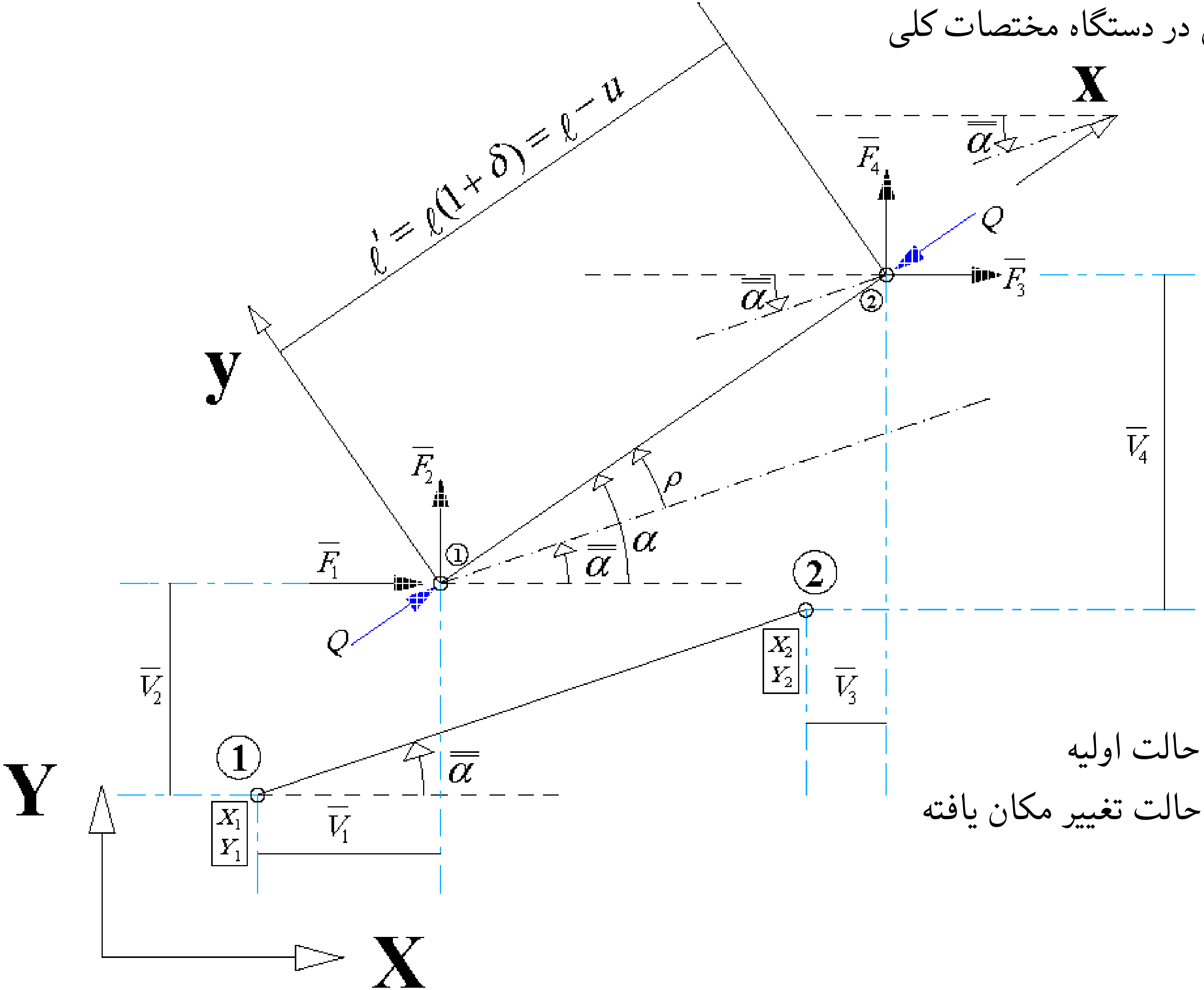

فرض انحنای کوچک برای خرپا لازم نیست

$$Q = \frac{AE}{\ell} u \quad (1)$$

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Member, Force–Deformation Relationship in Global Coordinate

ب – روابط نیرو تغییر شکل در دستگاه مختصات کلی




$\bar{\bar{\alpha}}$ موقعیت محور عضو در حالت اولیه

$\alpha$ موقعیت محور عضو در حالت تغییر مکان یافته

$\rho$ دوران محور عضو

$\ell'$ طول کوتاه شدگی

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Member, Force–Deformation Relationship in Global Coordinate

نیروها در دستگاه مختصات کلی از رابطه زیر به دست می‌آید.

$$\{\overline{F}\}_{4\times1} = [\overline{B}]_{4\times1} Q \qquad (2)$$

که در آن

$\{\overline{F}\}$ نیروهای داخلی در دستگاه کلی

$$\{\overline{F}\} = \begin{Bmatrix} \overline{F}_1 \\ \overline{F}_2 \\ \overline{F}_3 \\ \overline{F}_4 \end{Bmatrix}$$

$[\overline{B}]$ ماتریس انتقال (Transformation Matrix)

$$[\overline{B}] = \begin{bmatrix} m \\ n \\ -m \\ -n \end{bmatrix} \quad , \quad \begin{matrix} m = \cos\alpha \\ n = \sin\alpha \end{matrix} \qquad (3)$$

با توجه به شکل

$$u = \ell - \ell'$$

$$\ell' = \left[ \left( (X_2 + \overline{V}_3) - (X_1 + \overline{V}_1) \right)^2 + \left( (Y_2 + \overline{V}_4) - (Y_1 + \overline{V}_2) \right)^2 \right]^{\frac{1}{2}} \qquad (4)$$

$$m = \frac{(X_2 + \overline{V}_3) - (X_1 + \overline{V}_1)}{\ell'} \quad , \quad n = \frac{(Y_2 + \overline{V}_4) - (Y_1 + \overline{V}_2)}{\ell'}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## Member, Force–Deformation Relationship in Global Coordinate

$$\{\Delta\overline{F}\}=[T]\{\Delta\overline{V}\} \qquad (5)$$

$$[T]=\left[\frac{\partial \overline{F}_i^{(i)}}{\partial \overline{V}_j}\right]$$ M.T.S.M in Global Coordinate

که در آن

$\dfrac{\partial \overline{F}_i^{(i)}}{\partial \overline{V}_j}$ مشتق $\overline{F}_i$ نسبت به هر یک از درجات آزادی

$\{\overline{V}\}$ تغییر مکان گره‌ای در دستگاه کلی

$$\{\overline{V}\}_{4\times1}=\begin{Bmatrix}\overline{V}_1\\ \overline{V}_2\\ \overline{V}_3\\ \overline{V}_4\end{Bmatrix}$$

شبیه $[t]$ در قاب‌ها است

$$[T]=\frac{AE}{\ell}[\overline{B}][\overline{B}]^T+Q\,[g] \qquad (6)$$ M.T.S.M in Global Coordinate

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Member, Force–Deformation Relationship in Global Coordinate

$$[g] = \frac{1}{\ell'} \begin{bmatrix} -n^2 & mn & n^2 & -mn \\ mn & -m^2 & -mn & m^2 \\ n^2 & -mn & -n^2 & mn \\ -mn & m^2 & mn & -m^2 \end{bmatrix} \qquad (7)$$

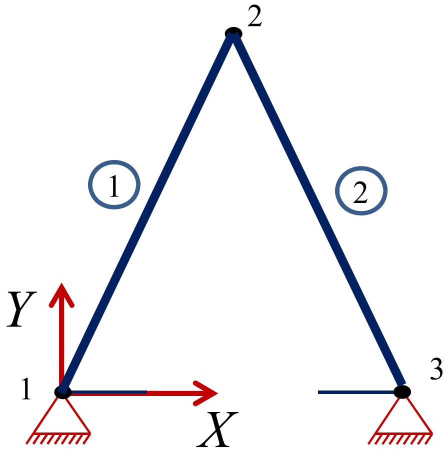
$$[\overline{B}][\overline{B}]^T = \begin{bmatrix} m^2 & mn & -m^2 & -mn \\ mn & n^2 & -mn & -n^2 \\ -m^2 & -mn & m^2 & mn \\ -mn & -n^2 & mn & n^2 \end{bmatrix} \qquad (8)$$

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Member, Force–Deformation Relationship in Global Coordinate

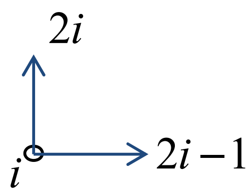
مراحل

$$\{\Delta P\}=[\tau]\{\Delta x\}$$




1) مختصات کلی تعیین می‌گردد $(X,Y)$ Establish General Coordinate

2) شماره‌گذاری اعضا




Node $i$

3) T برای هر عضو  به کمک رابطه (6) محاسبه می‌شود.

4) $[\tau]$ به کمک Assemble کردن تمام T ها به دست می‌آید.

$$[T^{(1)}]=\begin{bmatrix} 1 & 2 & 3 & 4 & \\ & & & & 1 \\ & & & & 2 \\ & & & & 3 \\ & & & & 4 \end{bmatrix} \qquad [T^{(2)}]=\begin{bmatrix} 3 & 4 & 5 & 6 & \\ & & & & 3 \\ & & & & 4 \\ & & & & 5 \\ & & & & 6 \end{bmatrix}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Member, Force–Deformation Relationship in Global Coordinate

$$
[\tau] = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & \\
T_{11}^{(1)} & T_{12}^{(1)} & T_{13}^{(1)} & T_{14}^{(1)} & 0 & 0 & 1 \\
T_{21}^{(1)} & T_{22}^{(1)} & T_{23}^{(1)} & T_{24}^{(1)} & 0 & 0 & 2 \\
T_{31}^{(1)} & T_{32}^{(1)} & T_{33}^{(1)} + T_{33}^{(2)} & T_{34}^{(1)} + T_{34}^{(2)} & T_{35}^{(2)} & T_{36}^{(2)} & 3 \\
T_{41}^{(1)} & T_{42}^{(1)} & T_{43}^{(1)} + T_{43}^{(2)} & T_{44}^{(1)} + T_{44}^{(2)} & T_{45}^{(2)} & T_{45}^{(2)} & 4 \\
0 & 0 & T_{53}^{(2)} & T_{54}^{(2)} & T_{55}^{(2)} & T_{55}^{(2)} & 5 \\
0 & 0 & T_{63}^{(2)} & T_{64}^{(2)} & T_{65}^{(2)} & T_{65}^{(2)} & 6
\end{bmatrix}
$$

$$
\{\Delta P\}_{6\times1} = [\tau]_{6\times6}\{\Delta x\}_{6\times1} \quad \Rightarrow \quad
\begin{Bmatrix} \{\Delta P\}_{2\times1} \Big|_{3,4} \\ \{\Delta R\}_{4\times1} \Big|_{1,2,5,6} \end{Bmatrix}_{6\times1}
= \begin{bmatrix} \overset{3\;\;4}{[\tau_{PP}]_{2\times2}} & \overset{1\;\;2\;\;5\;\;6}{[\tau_{PR}]_{2\times4}} \\ [\tau_{RP}]_{4\times2} & [\tau_{RR}]_{4\times4} \end{bmatrix}_{6\times6}
\begin{Bmatrix} \{\Delta x\}_{2\times1} \Big|_{3,4} \\ \{\Delta x_{Support}\}_{4\times1} \Big|_{1,2,5,6} \end{Bmatrix}_{6\times1}
$$

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Member, Force–Deformation Relationship in Global Coordinate

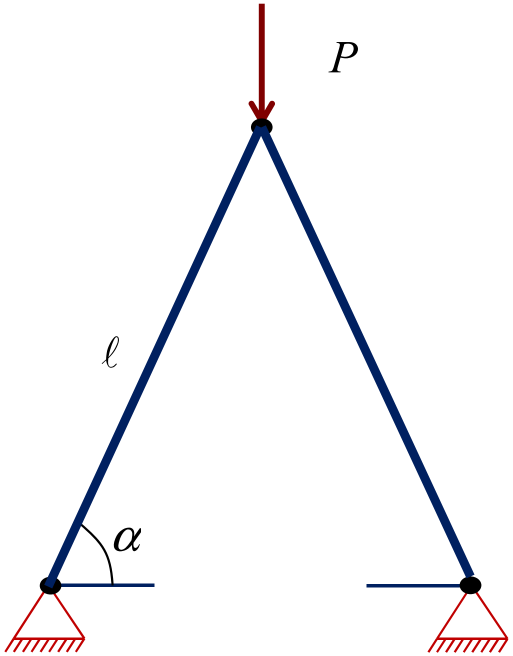
$$\begin{Bmatrix} \{\Delta P\}_{2\times1} \begin{vmatrix} 3 \\ 4 \end{vmatrix} \\ \{\Delta R\}_{4\times1} \begin{vmatrix} 1 \\ 2 \\ 5 \\ 6 \end{vmatrix} \end{Bmatrix}_{6\times1} = \begin{bmatrix} \overset{3 \quad 4}{[\tau_{PP}]_{2\times2}} & \overset{1 \quad 2 \quad 5 \quad 6}{[\tau_{PR}]_{2\times4}} \\ [\tau_{RP}]_{4\times2} & [\tau_{RR}]_{4\times4} \end{bmatrix}_{6\times6} \begin{Bmatrix} \{\Delta x\}_{2\times1} \begin{vmatrix} 3 \\ 4 \end{vmatrix} \\ \{\Delta x_{Support}\}_{4\times1} \begin{vmatrix} 1 \\ 2 \\ 5 \\ 6 \end{vmatrix} \end{Bmatrix}_{6\times1}$$

$$\Rightarrow \quad \{\Delta P\}_{2\times1} = [\tau_{PP}]_{2\times2}\{\Delta x\}_{2\times1} + [\tau_{PR}]_{2\times4}\{\Delta x_{Support}\}_{4\times1} \quad \Rightarrow \quad \{\Delta x\}_{2\times1} \ \checkmark$$

$$\Rightarrow \quad \{\Delta R\}_{4\times1} = [\tau_{RP}]_{4\times2}\{\Delta x\}_{2\times1} + [\tau_{RR}]_{4\times4}\{\Delta x_{Support}\}_{4\times1} \quad \Rightarrow \quad \{\Delta R\}_{4\times1} \ \checkmark$$

## ❑ Example

مثال: در خرپای نشان داده شده در شکل زیر مطلوب است تعیین الف ) بار بحرانی
و ب) جابجایی نقطه اثر بار  P=1500 kips .




$\alpha = 30^0$

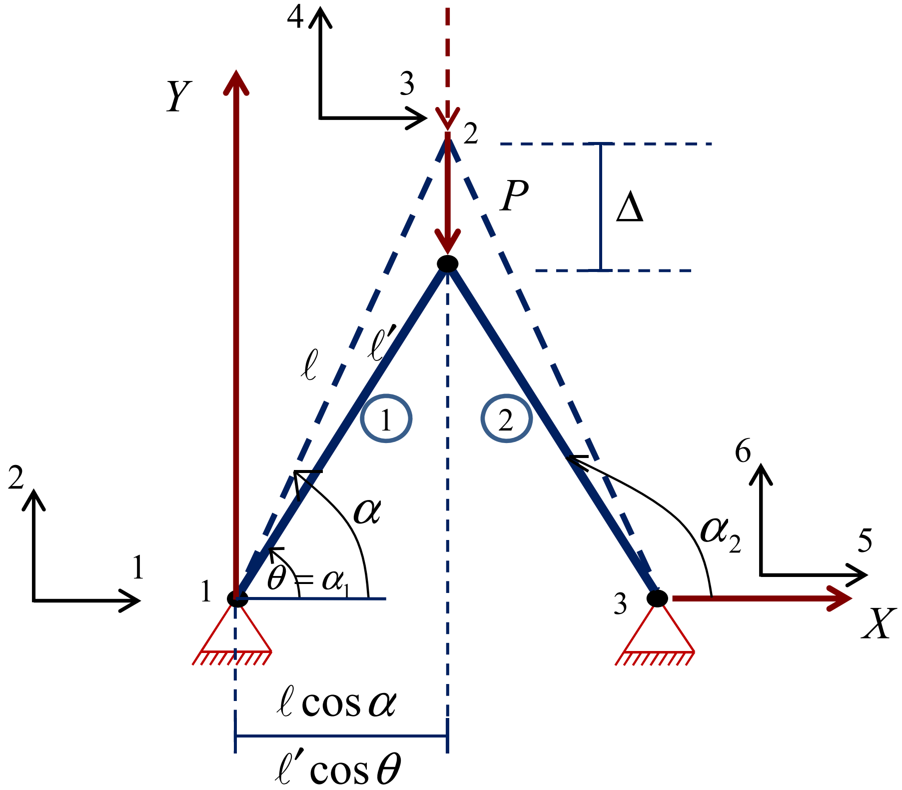
$\ell = 100\ (in.)$

$E = 30000\ (ksi)$

$A = 1\ (in.^2)$

# Geometrically Nonlinear Analysis of Plane Trusses

- **Example**



# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example

$$
[T_1]=\begin{bmatrix}
1 & 2 & 3 & 4 & \\
\left(\frac{AE}{\ell}\right)_1 m_1{}^2-\frac{Q_1}{\ell'_1}n_1{}^2 & \left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1 & -\left(\frac{AE}{\ell}\right)_1 m_1{}^2+\frac{Q_1}{\ell'_1}n_1{}^2 & -\left(\frac{AE}{\ell}\right)_1 m_1n_1-\frac{Q_1}{\ell'_1}m_1n_1 & 1\\
\left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1 & \left(\frac{AE}{\ell}\right)_1 n_1{}^2-\frac{Q_1}{\ell'_1}m_1{}^2 & -\left(\frac{AE}{\ell}\right)_1 m_1n_1-\frac{Q_1}{\ell'_1}m_1n_1 & -\left(\frac{AE}{\ell}\right)_1 n_1{}^2+\frac{Q_1}{\ell'_1}m_1{}^2 & 2\\
-\left(\frac{AE}{\ell}\right)_1 m_1{}^2+\frac{Q_1}{\ell'_1}n_1{}^2 & -\left(\frac{AE}{\ell}\right)_1 m_1n_1-\frac{Q_1}{\ell'_1}m_1n_1 & \left(\frac{AE}{\ell}\right)_1 m_1{}^2-\frac{Q_1}{\ell'_1}n_1{}^2 & \left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1 & 3\\
-\left(\frac{AE}{\ell}\right)_1 m_1n_1-\frac{Q_1}{\ell'_1}m_1n_1 & -\left(\frac{AE}{\ell}\right)_1 n_1{}^2+\frac{Q_1}{\ell'_1}m_1{}^2 & \left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1 & \left(\frac{AE}{\ell}\right)_1 n_1{}^2-\frac{Q_1}{\ell'_1}m_1{}^2 & 4
\end{bmatrix}
$$

$$
[T_2]=\left(\frac{AE}{\ell}\right)_2[\overline{B}_2][\overline{B}_2]^T+Q_2\,[g_2]=\left(\frac{AE}{\ell}\right)_2\begin{bmatrix} m_2\\ n_2\\ -m_2\\ -n_2\end{bmatrix}\begin{bmatrix} m_2\\ n_2\\ -m_2\\ -n_2\end{bmatrix}^T+\frac{Q_2}{\ell'_2}\begin{bmatrix} -n_2{}^2 & m_2n_2 & n_2{}^2 & -m_2n_2\\ m_2n_2 & -m_2{}^2 & -m_2n_2 & m_2{}^2\\ n_2{}^2 & -m_2n_2 & -n_2{}^2 & m_2n_2\\ -m_2n_2 & m_2{}^2 & m_2n_2 & -m_2{}^2\end{bmatrix}
$$

## ❑ Example

$$
[T_2]=\left[\begin{array}{ccccc}
3 & 4 & 5 & 6 & \\
\left(\frac{AE}{\ell}\right)_2 m_2{}^2-\frac{Q_2}{\ell'_2}n_2{}^2 & \left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & -\left(\frac{AE}{\ell}\right)_2 m_2{}^2+\frac{Q_2}{\ell'_2}n_2{}^2 & -\left(\frac{AE}{\ell}\right)_2 m_2n_2-\frac{Q_2}{\ell'_2}m_2n_2 & 3 \\
\left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & \left(\frac{AE}{\ell}\right)_2 n_2{}^2-\frac{Q_2}{\ell'_2}m_2{}^2 & -\left(\frac{AE}{\ell}\right)_2 m_2n_2-\frac{Q_2}{\ell'_2}m_2n_2 & -\left(\frac{AE}{\ell}\right)_2 n_2{}^2+\frac{Q_2}{\ell'_2}m_2{}^2 & 4 \\
-\left(\frac{AE}{\ell}\right)_2 m_2{}^2+\frac{Q_2}{\ell'_2}n_2{}^2 & -\left(\frac{AE}{\ell}\right)_2 m_2n_2-\frac{Q_2}{\ell'_2}m_2n_2 & \left(\frac{AE}{\ell}\right)_2 m_2{}^2-\frac{Q_2}{\ell'_2}n_2{}^2 & \left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & 5 \\
-\left(\frac{AE}{\ell}\right)_2 m_2n_2-\frac{Q_2}{\ell'_2}m_2n_2 & -\left(\frac{AE}{\ell}\right)_2 n_2{}^2+\frac{Q_2}{\ell'_2}m_2{}^2 & \left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & \left(\frac{AE}{\ell}\right)_2 n_2{}^2-\frac{Q_2}{\ell'_2}m_1{}^2 & 6
\end{array}\right]
$$

$$
[\tau]=\left[\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & \\
T_{11}{}^{(1)} & T_{12}{}^{(1)} & T_{13}{}^{(1)} & T_{14}{}^{(1)} & 0 & 0 & 1 \\
T_{21}{}^{(1)} & T_{22}{}^{(1)} & T_{23}{}^{(1)} & T_{24}{}^{(1)} & 0 & 0 & 2 \\
T_{31}{}^{(1)} & T_{32}{}^{(1)} & T_{33}{}^{(1)}+T_{33}{}^{(2)} & T_{34}{}^{(1)}+T_{34}{}^{(2)} & T_{35}{}^{(2)} & T_{36}{}^{(2)} & 3 \\
T_{41}{}^{(1)} & T_{42}{}^{(1)} & T_{43}{}^{(1)}+T_{43}{}^{(2)} & T_{44}{}^{(1)}+T_{44}{}^{(2)} & T_{45}{}^{(2)} & T_{45}{}^{(2)} & 4 \\
0 & 0 & T_{53}{}^{(2)} & T_{54}{}^{(2)} & T_{55}{}^{(2)} & T_{55}{}^{(2)} & 5 \\
0 & 0 & T_{63}{}^{(2)} & T_{64}{}^{(2)} & T_{65}{}^{(2)} & T_{65}{}^{(2)} & 6
\end{array}\right]
$$

## ❑ Example

$$[\tau_{PP}]_{2\times 2} = \begin{bmatrix} & 3 & 4 & \\ & \left(\frac{AE}{\ell}\right)_1 m_1^{\ 2} - \frac{Q_1}{\ell'_1} n_1^{\ 2} + \left(\frac{AE}{\ell}\right)_2 m_2^{\ 2} - \frac{Q_2}{\ell'_2} n_2^{\ 2} & \left(\frac{AE}{\ell}\right)_1 m_1 n_1 + \frac{Q_1}{\ell'_1} m_1 n_1 + \left(\frac{AE}{\ell}\right)_2 m_2 n_2 + \frac{Q_2}{\ell'_2} m_2 n_2 & 3 \\ & \left(\frac{AE}{\ell}\right)_1 m_1 n_1 + \frac{Q_1}{\ell'_1} m_1 n_1 + \left(\frac{AE}{\ell}\right)_2 m_2 n_2 + \frac{Q_2}{\ell'_2} m_2 n_2 & \left(\frac{AE}{\ell}\right)_1 n_1^{\ 2} - \frac{Q_1}{\ell'_1} m_1^{\ 2} + \left(\frac{AE}{\ell}\right)_2 n_2^{\ 2} - \frac{Q_2}{\ell'_2} m_2^{\ 2} & 4 \end{bmatrix}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## Example

$$\left(\frac{AE}{\ell}\right)_1 = \left(\frac{AE}{\ell}\right)_2 = \frac{AE}{\ell} \quad \& \quad \frac{Q_1}{\ell'_1} = \frac{Q_2}{\ell'_2} = \frac{Q}{\ell'} \Rightarrow$$

$$[\tau_{PP}]_{2\times2} = \frac{AE}{\ell}\begin{bmatrix} 3 & 4 & \\ m_1{}^2 + m_2{}^2 & m_1 n_1 + m_2 n_2 & 3 \\ m_1 n_1 + m_2 n_2 & n_1{}^2 + n_2{}^2 & 4 \end{bmatrix} + \frac{Q}{\ell'}\begin{bmatrix} 3 & 4 & \\ -n_1{}^2 - n_2{}^2 & m_1 n_1 + m_2 n_2 & 3 \\ m_1 n_1 + m_2 n_2 & -m_1{}^2 - m_2{}^2 & 4 \end{bmatrix}$$

$$m = m_1 = -m_2 = \cos\theta$$
$$n = n_1 = n_2 = \sin\theta$$

$$\Rightarrow \quad [\tau_{PP}] = \frac{AE}{\ell}\begin{bmatrix} 2m^2 & 0 \\ 0 & 2n^2 \end{bmatrix} + \frac{Q}{\ell'}\begin{bmatrix} -2n^2 & 0 \\ 0 & -2m^2 \end{bmatrix}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example

$$\Rightarrow \left[\tau_{PP}\right] = \frac{2AE}{\ell \cos\alpha} \begin{bmatrix} \cos\alpha - \sin^2\theta\cos\theta & 0 \\ 0 & \cos\alpha - \cos^3\theta \end{bmatrix}$$

# Geometrically Nonlinear Analysis of Plane Trusses

- **Example**

خرپا چه زمانی ناپایدار می‌شود؟

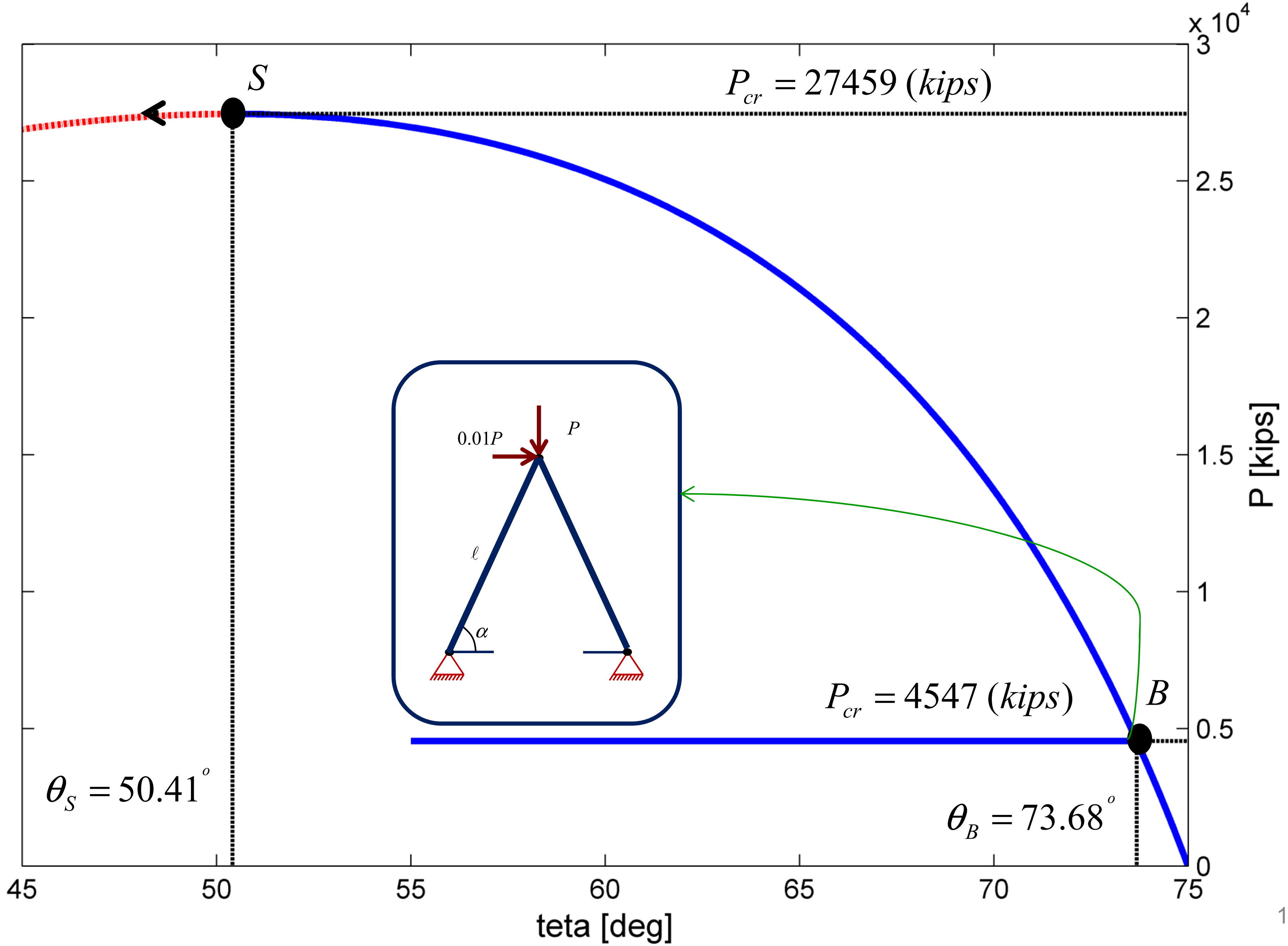
# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example

در چه حالتی مودهای ناپایداری Bifurcation و Snap through همزمان اتفاق می‌افتد.

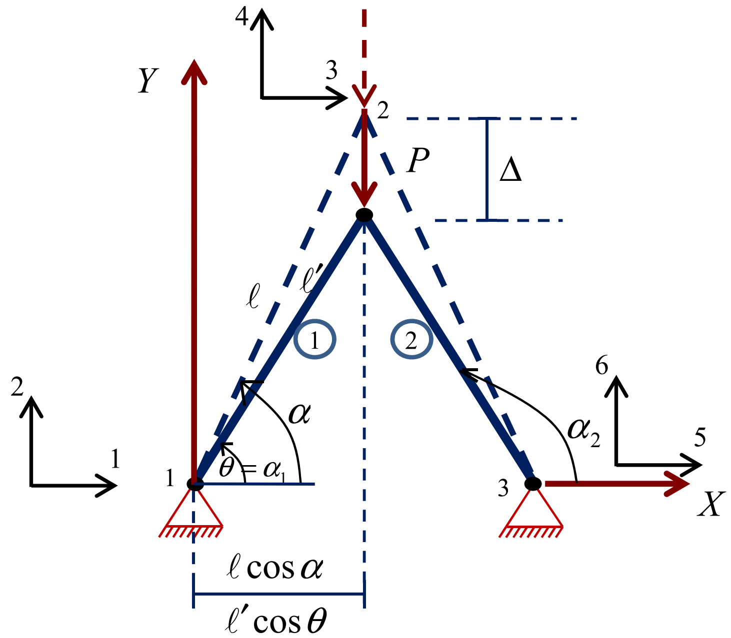
# Geometrically Nonlinear Analysis of Plane Trusses

- **Example**



# Geometrically Nonlinear Analysis of Plane Trusses

- **Example**




$$\Rightarrow \boxed{\ell'_1 = \ell\left[1 - 2\frac{\Delta}{\ell}\sin\alpha + \left(\frac{\Delta}{\ell}\right)^2\right]^{\frac{1}{2}}}$$

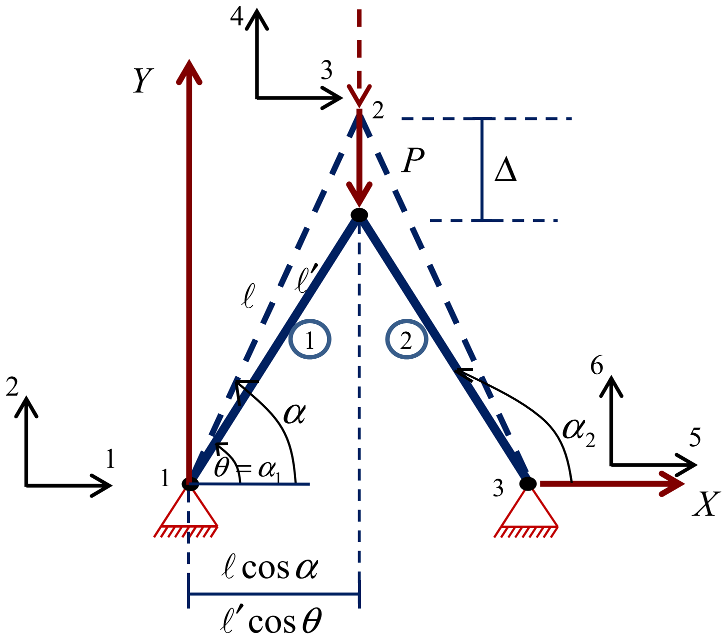
$$\Rightarrow \boxed{\ell'_2 = \ell'_1 = \ell' = \ell\left[1 - 2\frac{\Delta}{\ell}\sin\alpha + \left(\frac{\Delta}{\ell}\right)^2\right]^{\frac{1}{2}}}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example

$$m_1 = \frac{\ell \cos\alpha}{\ell'}$$

$$n_1 = \frac{\ell \sin\alpha - \Delta}{\ell'}$$




$$m_2 = -\frac{\ell \cos\alpha}{\ell'}$$

$$n_2 = \frac{\ell \sin\alpha - \Delta}{\ell'}$$

$$\Rightarrow \quad [\tau_{PP}] = \frac{AE}{\ell}\begin{bmatrix} 2m^2 & 0 \\ 0 & 2n^2 \end{bmatrix} + \frac{Q}{\ell'}\begin{bmatrix} -2n^2 & 0 \\ 0 & -2m^2 \end{bmatrix}$$

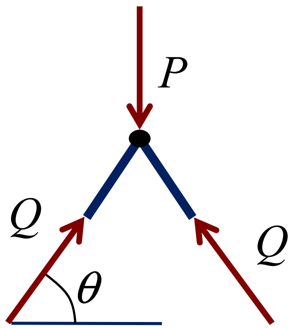
# Geometrically Nonlinear Analysis of Plane Trusses

- **Example**

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example

در هر مرحله باید تعادل گره‌ای بررسی شود برای این منظور باید تعادل را در دو راستای 3 و 4 کنترل کرد. اما چون در راستای 3 نیرویی وارد نمی‌شود نیازی به کنترل تعادل در این راستا نیست.



# Geometrically Nonlinear Analysis of Plane Trusses

## Example

یادآوری

$$[\tau_{PP}]=\frac{AE}{\ell}\begin{bmatrix}2m^2 & 0\\ 0 & 2n^2\end{bmatrix}+\frac{Q}{\ell'}\begin{bmatrix}-2n^2 & 0\\ 0 & -2m^2\end{bmatrix}$$

$$\Rightarrow\quad [\tau_{PP}]=\frac{AE}{\ell}\begin{bmatrix}2\left(\frac{\ell\cos\alpha}{\ell\beta}\right)^2 & 0\\ 0 & 2\left(\frac{\ell\sin\alpha-\Delta}{\ell\beta}\right)^2\end{bmatrix}+\frac{AE}{\ell}\left(\frac{1-\beta}{\beta}\right)\begin{bmatrix}-2\left(\frac{\ell\sin\alpha-\Delta}{\ell\beta}\right)^2 & 0\\ 0 & -2\left(\frac{\ell\cos\alpha}{\ell\beta}\right)^2\end{bmatrix}$$

$$\Rightarrow\quad [\tau_{PP}]=\frac{2AE}{\ell}\begin{bmatrix}\left(\frac{\ell\cos\alpha}{\ell\beta}\right)^2-\left(\frac{1-\beta}{\beta}\right)\left(\frac{\sin\alpha-\frac{\Delta}{\ell}}{\beta}\right)^2 & 0\\ 0 & \left(\frac{\sin\alpha-\frac{\Delta}{\ell}}{\beta}\right)^2-\left(\frac{1-\beta}{\beta}\right)\left(\frac{\ell\cos\alpha}{\ell\beta}\right)^2\end{bmatrix}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## Example

$$\Rightarrow \quad [\tau_{PP}] = \frac{2AE}{\ell\beta^2}\begin{bmatrix} \cos^2\alpha - \left(\frac{1}{\beta}-1\right)\left(\sin\alpha - \frac{\Delta}{\ell}\right)^2 & 0 \\ 0 & \left(\sin\alpha - \frac{\Delta}{\ell}\right)^2 - \left(\frac{1}{\beta}-1\right)\cos^2\alpha \end{bmatrix}$$

$$\Rightarrow \quad [\tau_{PP}] = \frac{2AE}{\ell\beta^2}\begin{bmatrix} \cos^2\alpha - \frac{\left(\sin\alpha - \frac{\Delta}{\ell}\right)^2}{\beta} + \left(\sin\alpha - \frac{\Delta}{\ell}\right)^2 & 0 \\ 0 & \left(\sin\alpha - \frac{\Delta}{\ell}\right)^2 - \frac{\cos^2\alpha}{\beta} + \cos^2\alpha \end{bmatrix}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example

$$\Rightarrow \quad [\tau_{PP}] = \frac{2AE}{\ell\beta^2}\begin{bmatrix} \cos^2\alpha - \dfrac{\beta^2 - \cos^2\alpha}{\beta} + \beta^2 - \cos^2\alpha & 0 \\ 0 & \beta^2 - \cos^2\alpha - \dfrac{\cos^2\alpha}{\beta} + \cos^2\alpha \end{bmatrix}$$

$$\Rightarrow \quad [\tau_{PP}] = \frac{2AE}{\ell}\begin{bmatrix} 1 - \dfrac{\beta^2 - \cos^2\alpha}{\beta^3} & 0 \\ 0 & 1 - \dfrac{\cos^2\alpha}{\beta^3} \end{bmatrix} \quad (II)$$

$$\beta = \left[1 - 2\frac{\Delta}{\ell}\sin\alpha + \left(\frac{\Delta}{\ell}\right)^2\right]^{\frac{1}{2}}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example

خرپا چه زمانی ناپایدار می‌شود؟

با استفاده از روش Linear Incremental Method with Iteration

Updated Newton Raphson Iteration (Updated N.R.It)

- **Example**

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example

②

$$\{\mathbf{x}^{(2)}\} = \{\mathbf{x}^{(1)}\} + \{\Delta\mathbf{x}^{(1)}\} = \begin{Bmatrix} 0 \\ 10 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 4.18 \end{Bmatrix} \Rightarrow \boxed{\{\mathbf{x}^{(2)}\} = \begin{Bmatrix} 0 \\ 14.18 \end{Bmatrix} (in)} \overset{(I)}{\Rightarrow} \boxed{\{\mathbf{P}_{\mathbf{x}^{(2)}}^{(2)}\} = \begin{Bmatrix} 0 \\ 1440.6 \end{Bmatrix} (kips)}$$

$$\{\Delta\mathbf{Q}^{(2)}\} = \begin{Bmatrix} 0 \\ 1500 \end{Bmatrix} - \{\mathbf{P}_{\mathbf{x}^{(2)}}^{(2)}\} = \begin{Bmatrix} 0 \\ 1500 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 1440.6 \end{Bmatrix} \Rightarrow \boxed{\{\Delta\mathbf{Q}^{(2)}\} = \begin{Bmatrix} 0 \\ 59.384 \end{Bmatrix} (kips)}$$

$$(II) \Rightarrow \boxed{[\boldsymbol{\tau}_{\mathbf{x}^{(2)}}^{(2)}] = \begin{bmatrix} 506.47 & 0 \\ 0 & 53.308 \end{bmatrix} (kips/in)}$$

$$\overset{(Equation\ \ 2-22)}{\Rightarrow} \{\Delta\mathbf{x}^{(2)}\} = [\boldsymbol{\tau}^{(2)}]^{-1}\{\Delta\mathbf{Q}^{(2)}\} \Rightarrow \boxed{\{\Delta\mathbf{x}^{(2)}\} = \begin{Bmatrix} 0 \\ 1.114 \end{Bmatrix} (in.)}$$

# Geometrically Nonlinear Analysis of Plane Trusses

- **Example**

③

$$\{\mathbf{x}^{(3)}\} = \{\mathbf{x}^{(2)}\} + \{\Delta\mathbf{x}^{(2)}\} = \begin{Bmatrix} 0 \\ 14.18 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1.114 \end{Bmatrix} \Rightarrow \boxed{\{\mathbf{x}^{(3)}\} = \begin{Bmatrix} 0 \\ 15.294 \end{Bmatrix} (in)} \overset{(I)}{\Rightarrow} \boxed{\{\mathbf{P}_{\mathbf{x}^{(3)}}^{(3)}\} = \begin{Bmatrix} 0 \\ 1495.9 \end{Bmatrix} (kips)}$$

$$\{\Delta\mathbf{Q}^{(3)}\} = \begin{Bmatrix} 0 \\ 1500 \end{Bmatrix} - \{P_{\{\mathbf{x}^{(3)}\}}^{(3)}\} = \begin{Bmatrix} 0 \\ 1500 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 1495.9 \end{Bmatrix} \Rightarrow \boxed{\{\Delta\mathbf{Q}^{(3)}\} = \begin{Bmatrix} 0 \\ 4.1372 \end{Bmatrix} (kips)}$$

$$(II) \Rightarrow \boxed{[\boldsymbol{\tau}_{\mathbf{x}^{(3)}}^{(3)}] = \begin{bmatrix} 511.01 & 0 \\ 0 & 45.89 \end{bmatrix} (kips/in)}$$

$$\overset{(Equation\ 2-22)}{\Rightarrow} \{\Delta\mathbf{x}^{(3)}\} = [\boldsymbol{\tau}^{(3)}]^{-1}\{\Delta\mathbf{Q}^{(3)}\} \Rightarrow \boxed{\{\Delta\mathbf{x}^{(3)}\} = \begin{Bmatrix} 0 \\ 0.090156 \end{Bmatrix} (in.)}$$

# Geometrically Nonlinear Analysis of Plane Trusses

## Example

4

$$\{\mathbf{x}^{(4)}\} = \{\mathbf{x}^{(3)}\} + \{\Delta\mathbf{x}^{(3)}\} = \begin{Bmatrix} 0 \\ 15.294 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0.0902 \end{Bmatrix} \Rightarrow \boxed{\{\mathbf{x}^{(4)}\} = \begin{Bmatrix} 0 \\ 15.38416 \end{Bmatrix} (in)} \overset{(I)}{\Rightarrow} \boxed{\{\mathbf{P}_{\mathbf{x}^{(4)}}^{(4)}\} = \begin{Bmatrix} 0 \\ 1499.97 \end{Bmatrix} (kips)}$$

$$\{\Delta\mathbf{Q}^{(4)}\} = \begin{Bmatrix} 0 \\ 1500 \end{Bmatrix} - \{P_{\{\mathbf{x}^{(4)}\}}^{(4)}\} = \begin{Bmatrix} 0 \\ 1500 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 1499.97 \end{Bmatrix} \Rightarrow \boxed{\{\Delta\mathbf{Q}^{(4)}\} = \begin{Bmatrix} 0 \\ 0.0269 \end{Bmatrix} (kips)}$$

$$(II) \Rightarrow \boxed{\left[\boldsymbol{\tau}_{\mathbf{x}^{(4)}}^{(4)}\right] = \begin{bmatrix} 511.38 & 0 \\ 0 & 45.287 \end{bmatrix} (kips/in)}$$

$$\overset{(Equation\ 2-22)}{\Rightarrow} \{\Delta\mathbf{x}^{(4)}\} = \left[\boldsymbol{\tau}^{(4)}\right]^{-1} \{\Delta\mathbf{Q}^{(4)}\} \Rightarrow \boxed{\{\Delta\mathbf{x}^{(4)}\} = \begin{Bmatrix} 0 \\ 0.0006 \end{Bmatrix} (in.)}$$

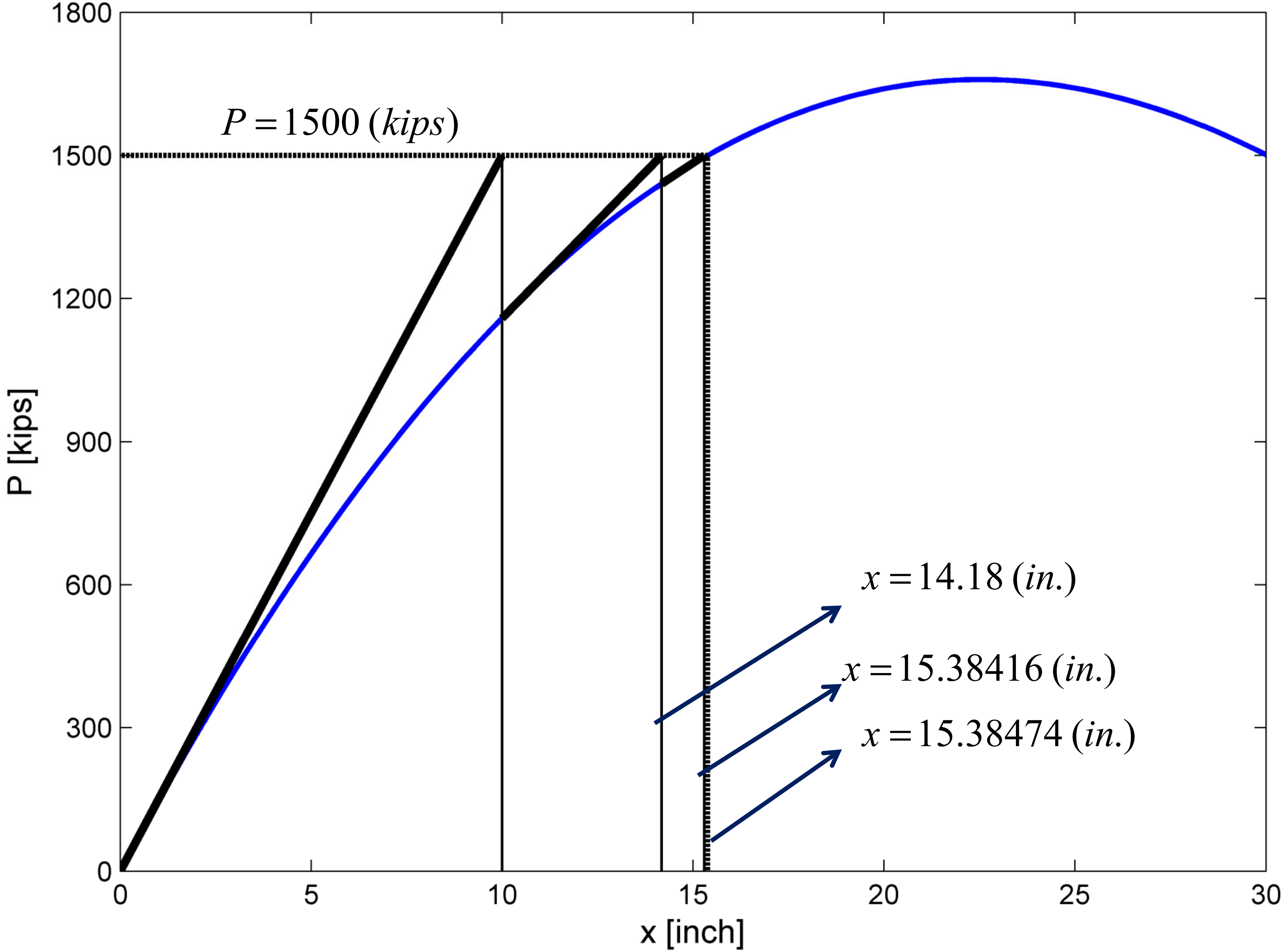
# Geometrically Nonlinear Analysis of Plane Trusses

## Example

| **Iteration** | $P^{(i)}\ (kips)$ | $x^{(i)}\ (in.)$ | $\tau^{(i)}\ (kips/in.)$ | $\Delta Q^{(i)}\ (kips)$ | $\Delta x^{(i)}\ (in.)$ | $x^{(i+1)}\ (in.)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 150.000 | 1500 | 10 | 10 |
| 1 | 1158.836 | 10 | 81.617 | 341.1639 | 4.1800 | 14.18004 |
| 2 | 1440.618 | 14.18004 | 53.308 | 59.38181 | 1.1139 | 15.29399 |
| 3 | 1495.862 | 15.29399 | 45.890 | 4.137854 | 0.0902 | 15.38416 |
| 4 | 1499.973 | 15.38416 | 45.292 | 0.026937 | 0.0006 | 15.38475 |
| 5 | **1500** | 15.38475 | 45.288 | 1.17E-06 | 0.0000 | **15.38475** |

# Geometrically Nonlinear Analysis of Plane Trusses

## ❑ Example




Updated Newton Raphson Iteration (Updated N.R.It)