

Mechanics of Materials



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Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:

J.Wat Oler “Lectures notes on Mechanics od Materials”

Ibrahim A.Assakkaf “Lectures notes on Mechanics od Materials”

Torsion

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Torsion

□ Torsion of Noncircular Members

Introduction

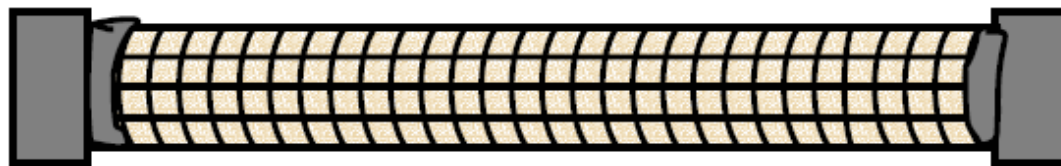
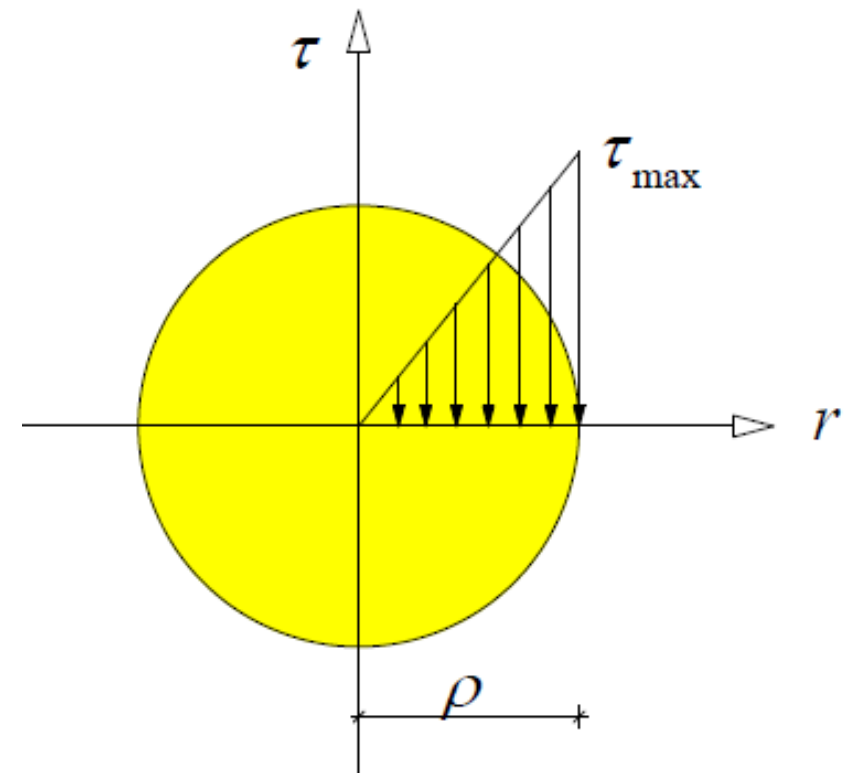
- The analysis of a noncircular torsion structural member is far *more complicated* than a circular shaft.
- The major difficulty basically lies in *determining the shear-strain distribution*.
- In these noncircular members, the discussion presented previously for circular shafts is not applicable.

Torsion

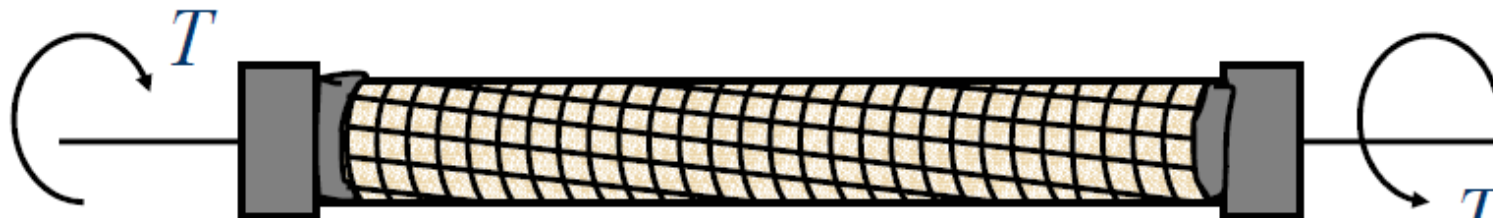
□ Torsion of Noncircular Members

Deformation of Circular Shaft Subjected to Torque T

plane transverse sections remain plane and the shear strain varies linearly from the geometric center.



(a)



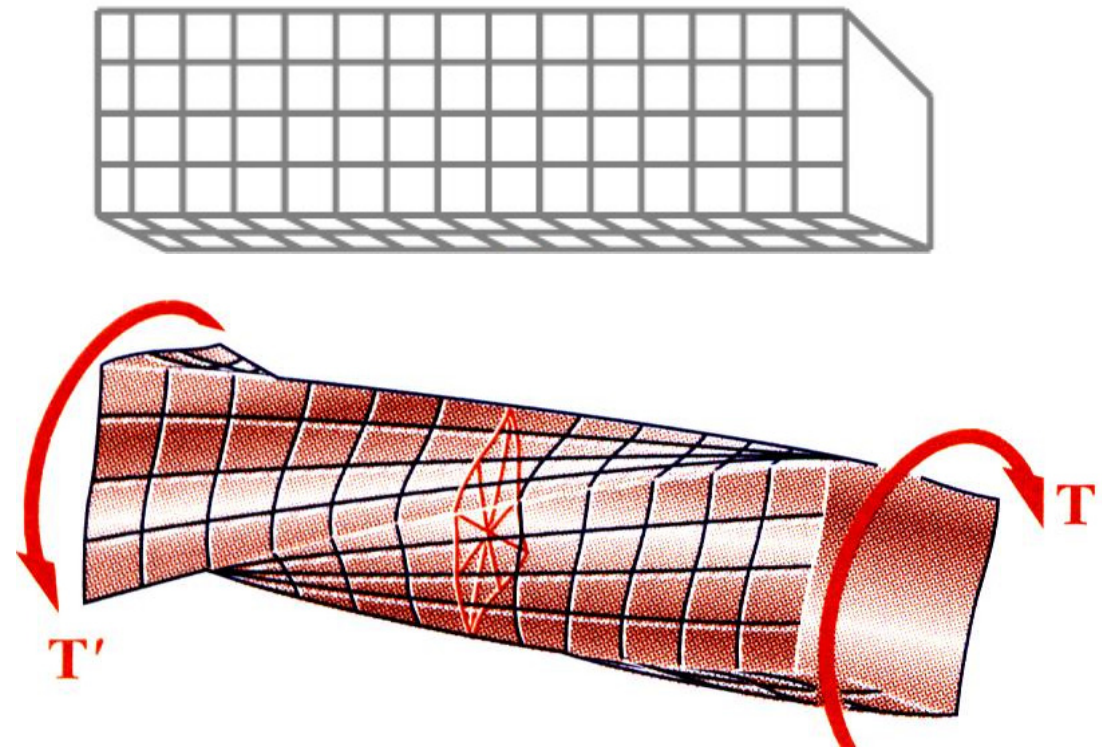
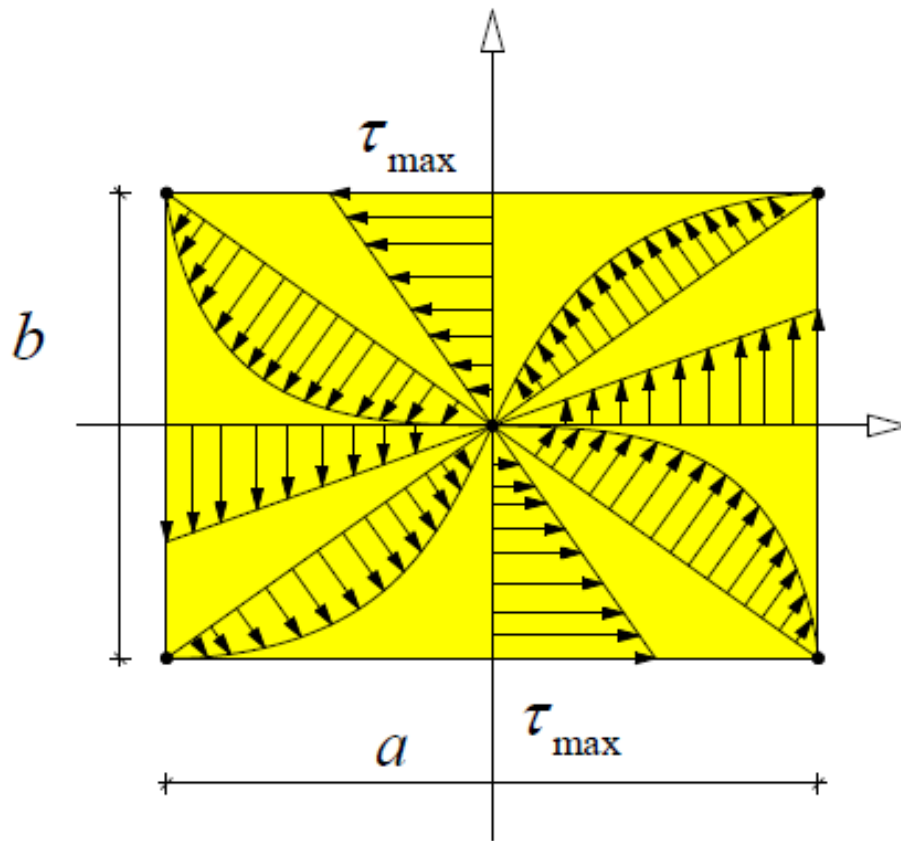
(b)

Torsion

□ Torsion of Noncircular Members

Deformation of a Bar of Square Cross Section Subjected to Torque T

Planar cross-sections of noncircular shafts do not remain planar and stress and strain distribution do not vary linearly

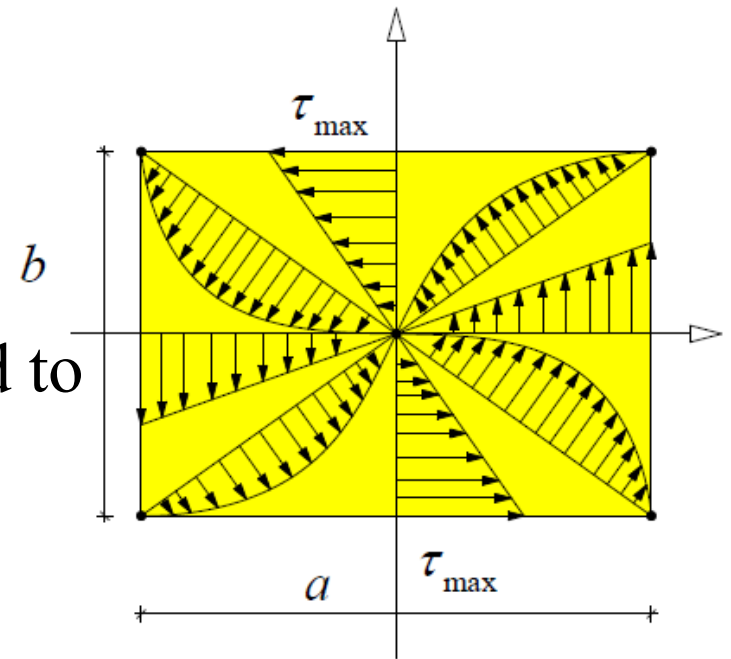


Torsion

□ Torsion of Noncircular Members

Determination of Shearing Stress

- Several rigorous methods have been derived to determine the shear strain distribution in noncircular torsion members.
- The solutions of many problems for solid noncircular torsion members can be found in several advanced books such as
 - Seely, F. B. and Smith, J. O., 1952. “*Advanced Mechanics of Materials*,” 2nd edition, Wiley.
 - Timoshenko, S. P. and Goodier, J. N., 1970. “*Theory of Elasticity*,” 3rd edition, McGraw-Hill, New York, section. 109.



Torsion

□ Torsion of Noncircular Members

For uniform rectangular cross-sections,

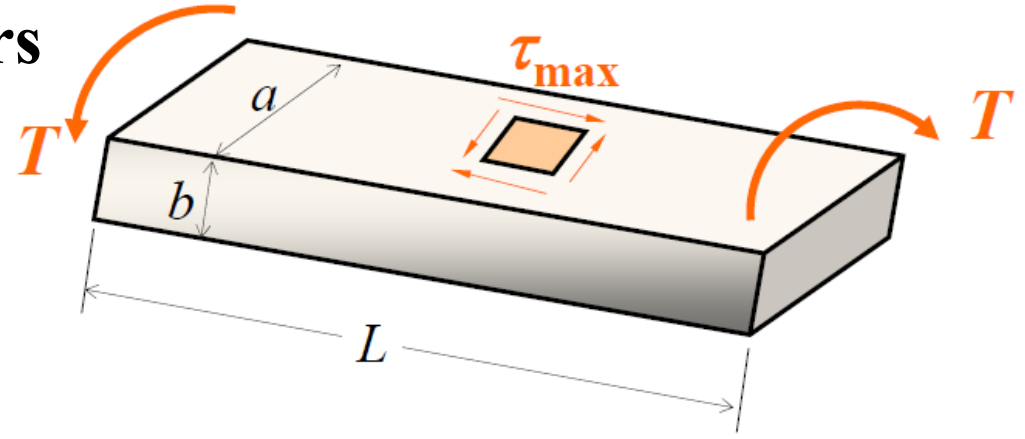


TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

The maximum shearing stress occurs along the center line of the wider face of the bar and is equal to

$$\tau_{\max} = \frac{T}{c_1 ab^2}$$

The angle of twist

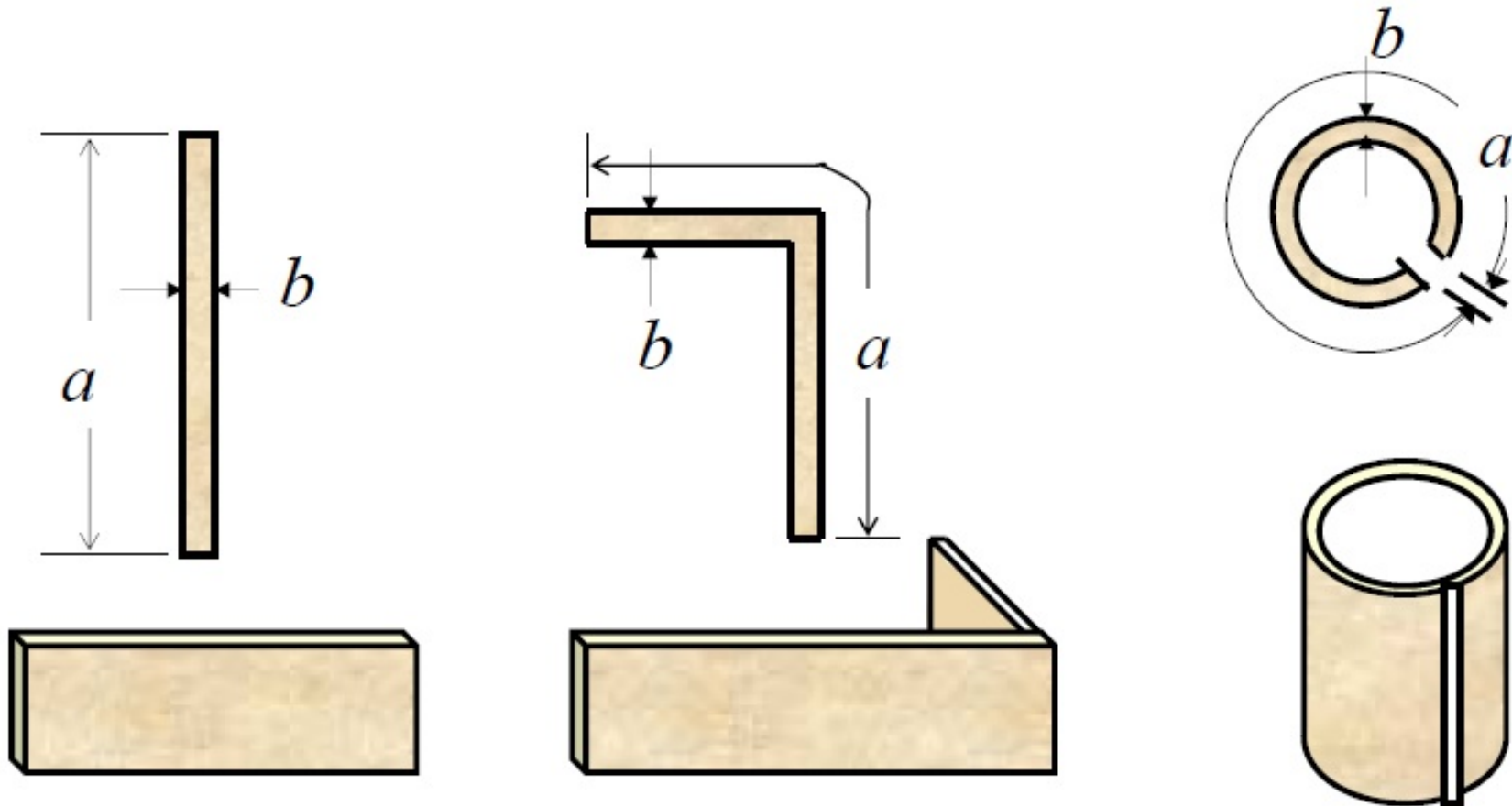
$$\phi = \frac{TL}{c_2 ab^3 G}$$

$$\text{If } \frac{a}{b} \geq 5 \Rightarrow C_1 = C_2 = \frac{1}{3} \left(1 - \frac{0.630}{(a/b)} \right)$$

Torsion

□ Torsion of Noncircular Members

Thin-Walled Member of Uniform Thickness and Arbitrary Shape



Torsion

□ Torsion of Noncircular Members

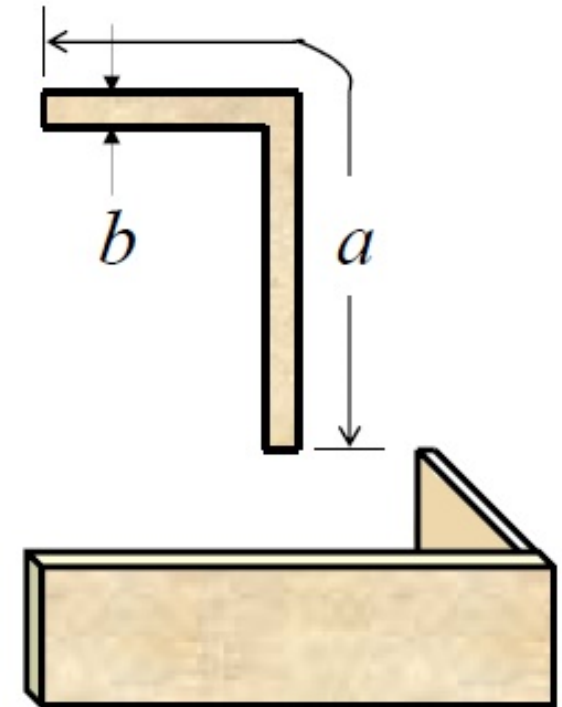
Thin-Walled Member of Uniform Thickness and Arbitrary Shape

Based on the membrane analogy (Timoshenko and Goodier 1970), for a thin walled member of uniform thickness and arbitrary shape, ***the maximum shearing stress is the same as for a rectangular bar with a very large value of a/b***

$$\frac{a}{b} \gg 5 \Rightarrow C_1 = C_2 = \frac{1}{3}$$

$$\Rightarrow \tau_{\max} = \frac{3Tb_{\max}}{\sum_{i=1}^n a_i b_i^3}$$

$$\phi = \frac{3TL}{G \sum_{i=1}^n a_i b_i^3}$$



Torsion

□ Torsion of Noncircular Members

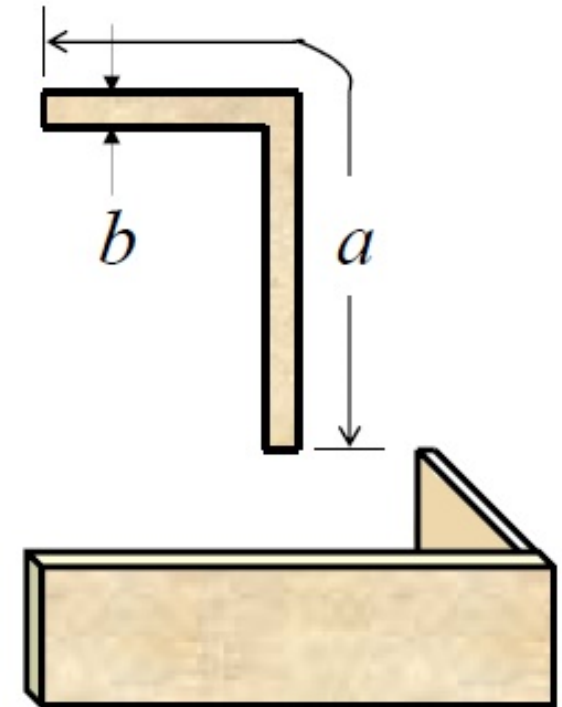
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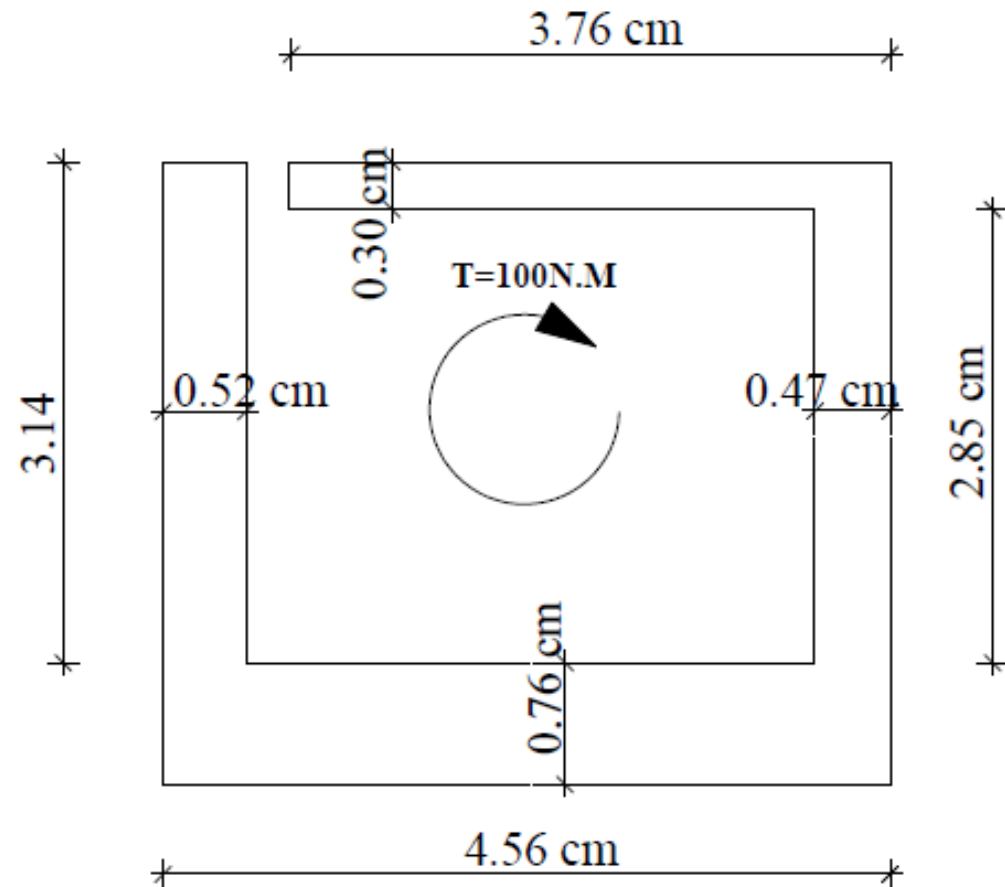


Torsion

□ Torsion of Noncircular Members

Example 01

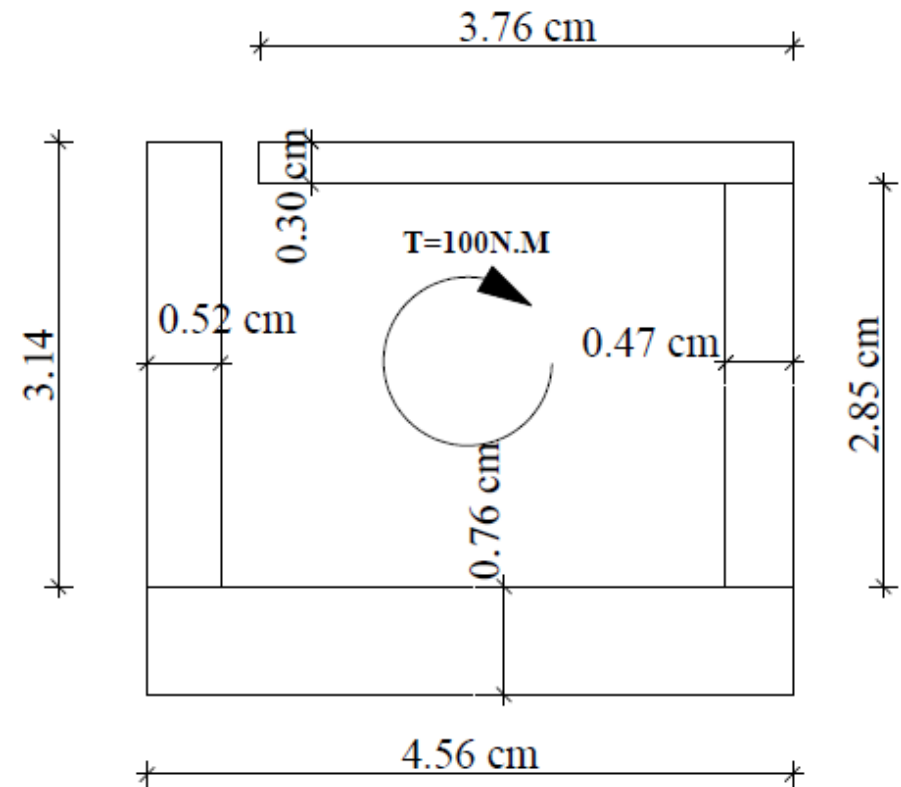
Determine the maximum shear stress due to exerted torque.



Torsion

□ Torsion of Noncircular Members

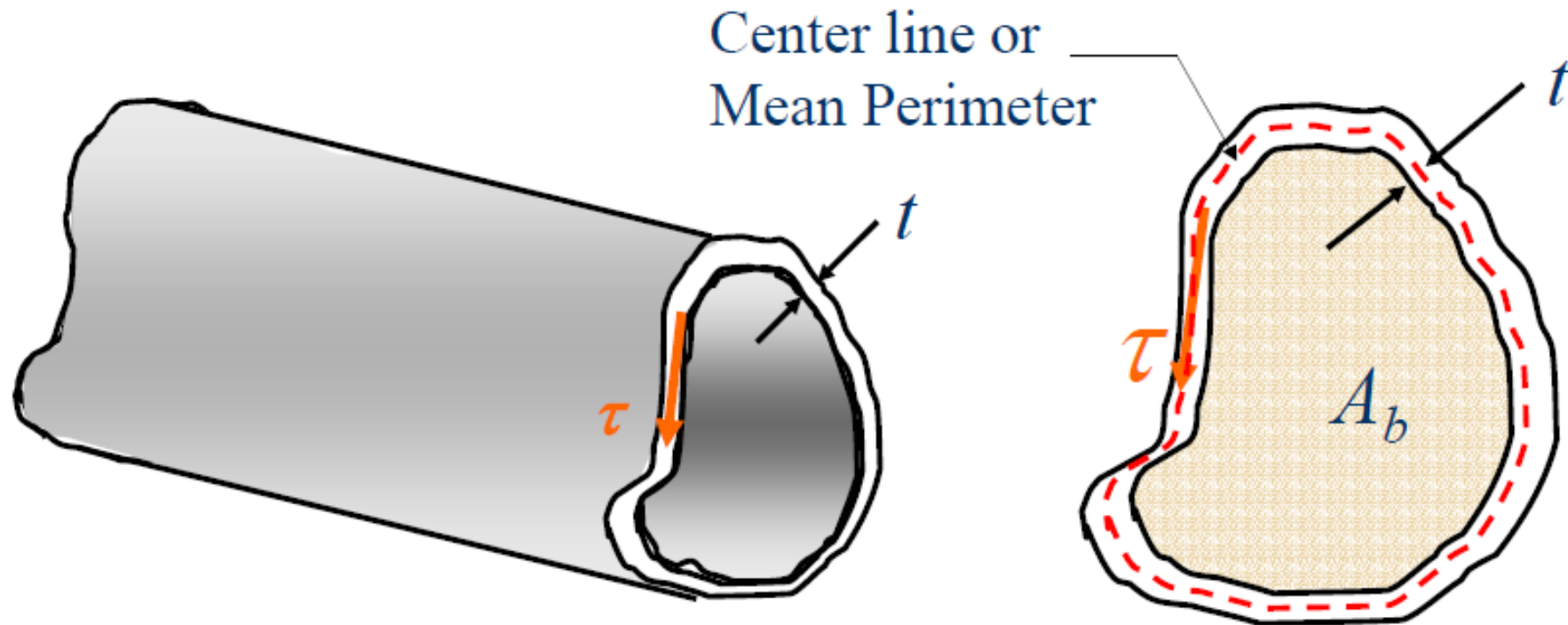
Example 01



Torsion

□ Torsion of Noncircular Members

Thin-Walled Hollow Shafts



Torsion

□ Torsion of Noncircular Members

Thin-Walled Hollow Shafts

- Summing forces in the x-direction on AB ,

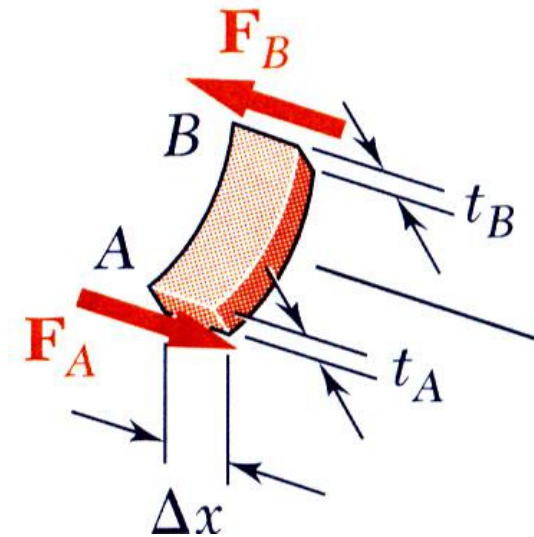
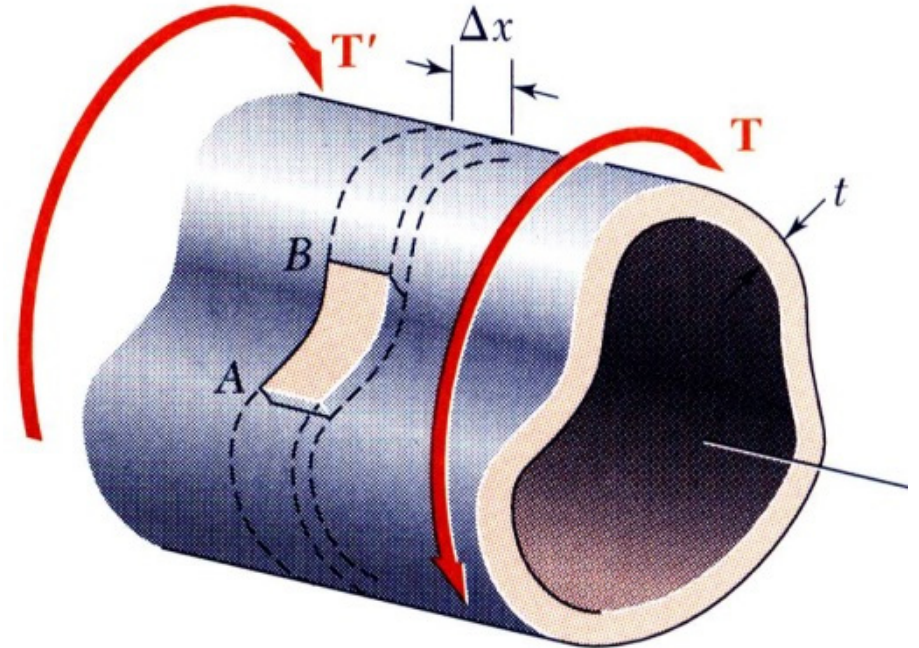
$$\sum F_x = 0 = \tau_A(t_A \Delta x) - \tau_B(t_B \Delta x)$$

$$\tau_A t_A = \tau_B t_B = \tau t$$

$$\Rightarrow q = \tau t = \text{shear flow} = \text{cte}$$

shear stress varies inversely with thickness

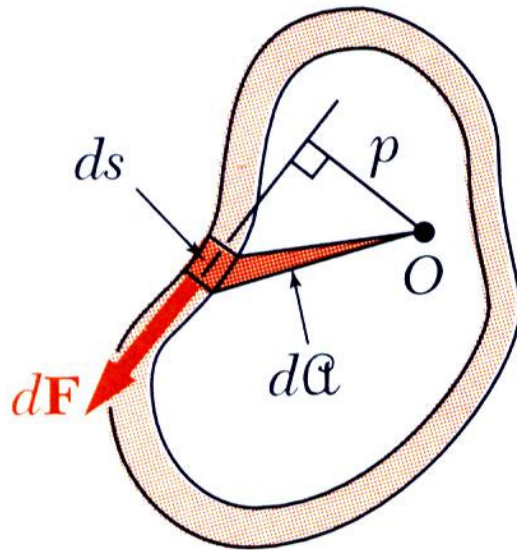
$$\tau = \frac{q}{t} \neq \text{cte}$$



Torsion

□ Torsion of Noncircular Members

Thin-Walled Hollow Shafts

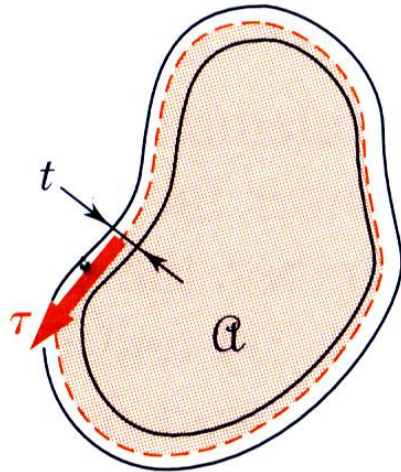


- Compute the shaft torque from the integral of the moments due to shear stress

$$dM_0 = dF \cdot p = \tau t ds \cdot p = q ds \cdot p = q \cdot 2 dA$$

$$T = \oint dM_0 = \oint 2q dA = 2qA_m$$

$$\Rightarrow \tau = \frac{T}{2tA_m}$$



- Angle of twist

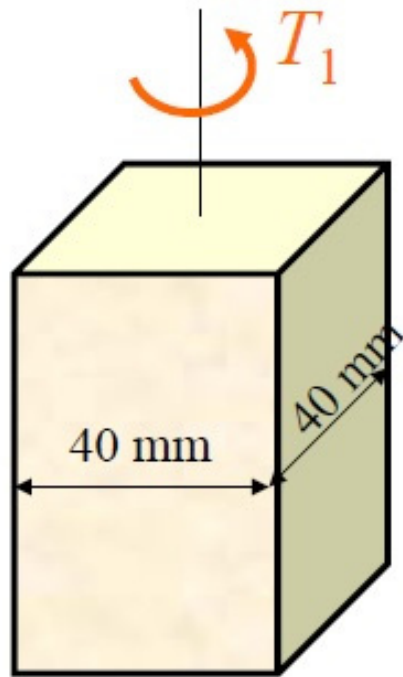
$$\phi = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t}$$

Torsion

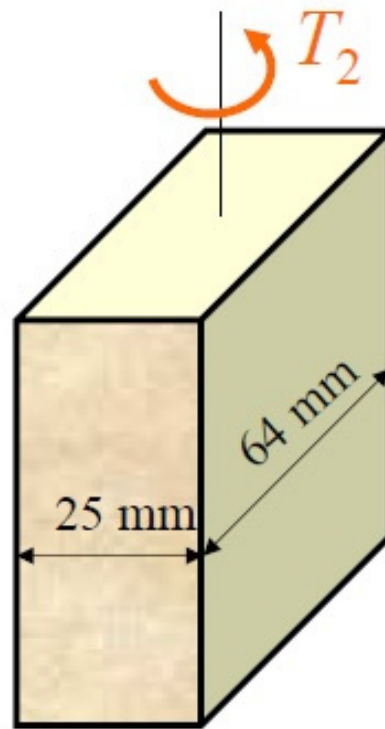
□ Torsion of Noncircular Members

Example 02

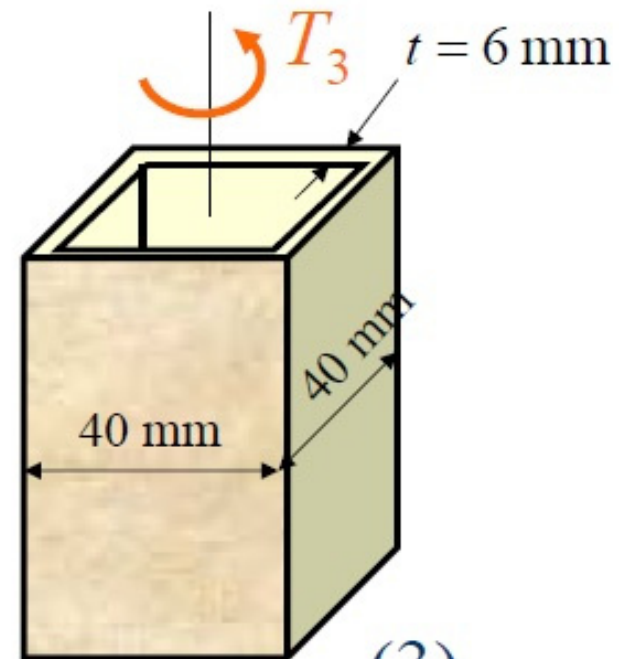
Using $\tau_{all} = 40$ MPa, determine the largest torque which may be applied to each of the brass bars and to the brass tube shown. Note that the two solid bars have the same cross-sectional area, and that the square bar and square tube have the same outside dimensions.



(1)



(2)



(3)

Torsion

□ Torsion of Noncircular Members

Example 02

1. Bar with Square Cross Section:

Torsion

□ Torsion of Noncircular Members

Example 02

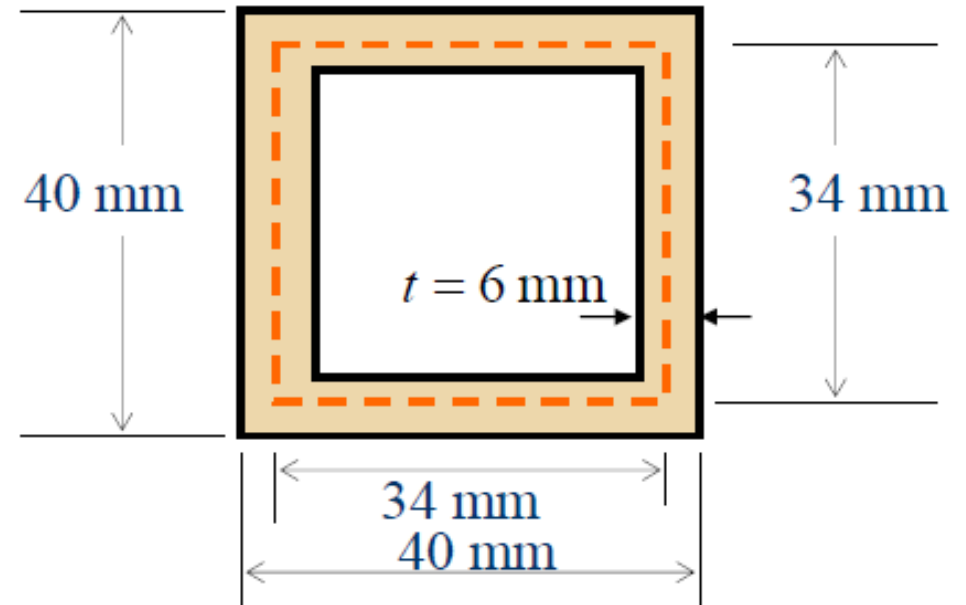
2. Bar with Rectangular Cross Section:

Torsion

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Example 02

3. Square Tube:

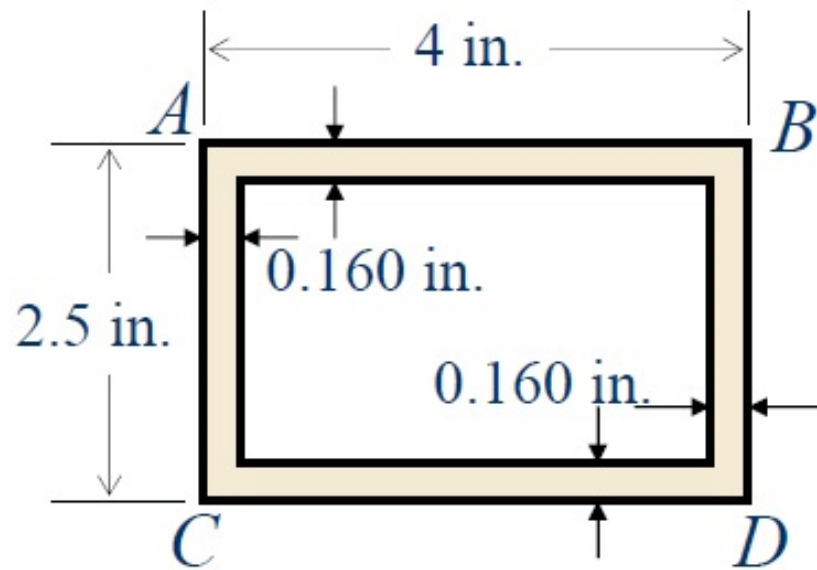


Torsion

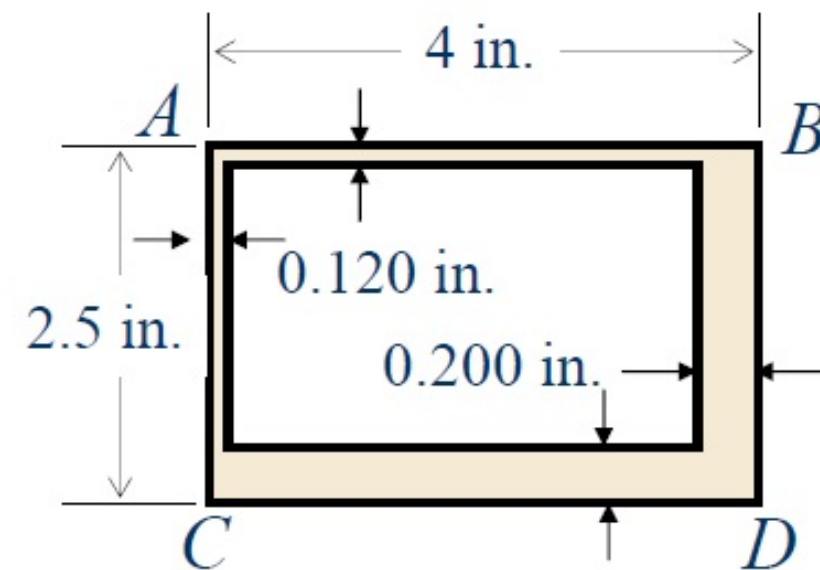
□ Torsion of Noncircular Members

Example 03

Structural aluminum tubing of 2.5×4 -in. rectangular cross section was fabricated by extrusion. Determine the shearing stress in each of the four walls of a portion of such tubing when it is subjected to a torque of $24 \text{ kip}\cdot\text{in.}$, assuming (a) a uniform 0.160 -in. wall thickness, (b) that, as a result of defective fabrication, walls AB and AC are 0.120 -in thick, and walls BD and CD are 0.200 -in thick.



(a)



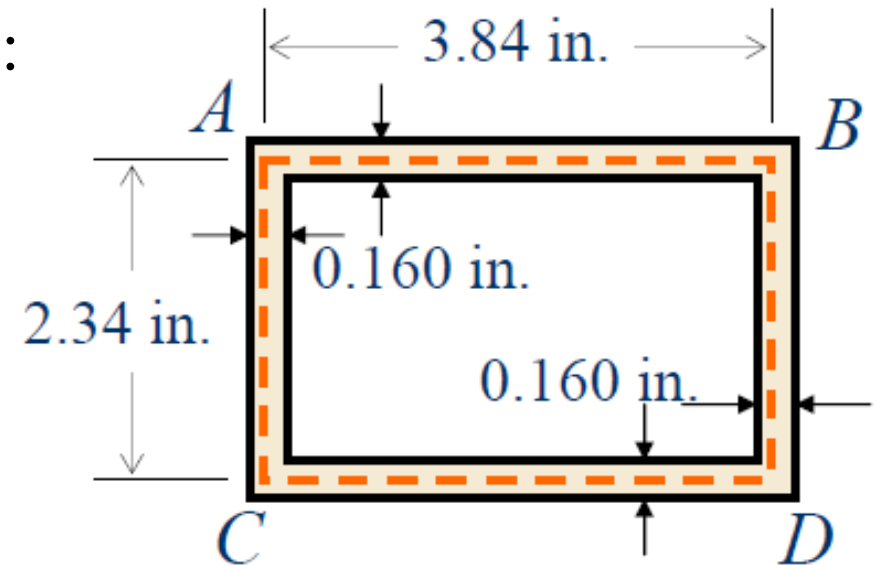
(b)

Torsion

□ Torsion of Noncircular Members

Example 03

(a) Tubing of Uniform Wall Thickness:



Torsion

□ Torsion of Noncircular Members

Example 03

(b) Tubing with Variable Wall Thickness:

Observing that the area A_m bounded by the center line is the same as in Part a

Torsion

□ Torsion of Noncircular Members

R: Torsional resistance

$$\tau_{\max} = \frac{T}{R}$$

Rectangular section:

$$\tau_{\max} = \frac{T}{c_1 ab^2} \Rightarrow R = c_1 ab^2$$

Opened thin-Walled Member:

$$\tau_{\max} = \frac{3Tb_{\max}}{\sum_{i=1}^n a_i b_i^3} \Rightarrow R = \frac{\sum_{i=1}^n a_i b_i^3}{3b_{\max}}$$

closed thin-Walled Member:

$$\tau_{\max} = \frac{T}{2t_{\min} A_m} \Rightarrow R = 2t_{\min} A_m$$

Torsion

□ Torsion of Noncircular Members

K: Torsional Rigidity

$$\phi = \frac{T}{K}$$

Rectangular section:

$$\phi = \frac{TL}{c_2 ab^3 G} \Rightarrow K = \frac{c_2 ab^3 G}{L}$$

Opened thin-Walled Member:

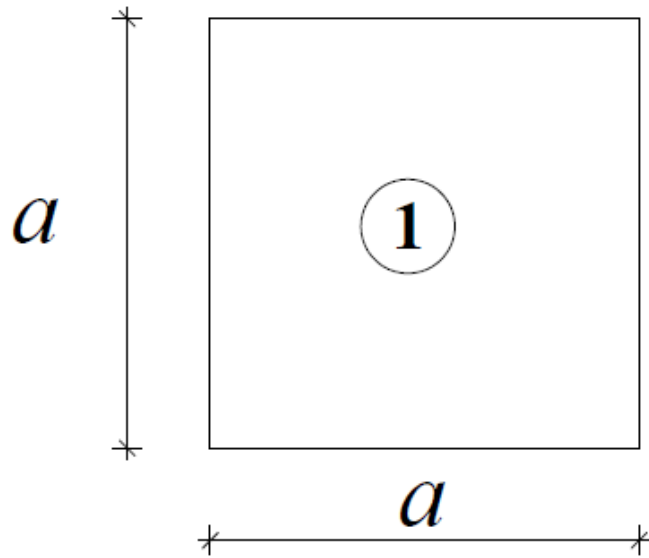
$$\phi = \frac{3TL}{G \sum_{i=1}^n a_i b_i^3} \Rightarrow K = \frac{G \sum_{i=1}^n a_i b_i^3}{3L}$$

closed thin-Walled Member:

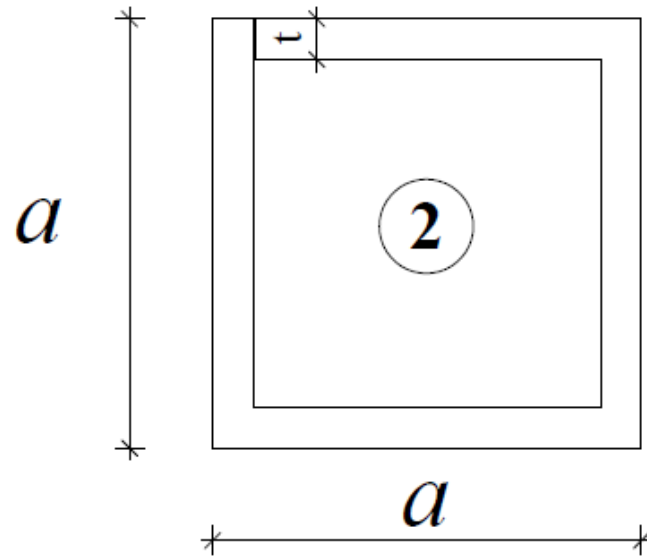
$$\phi = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t} \Rightarrow K = \frac{4A_m^2 G}{L \oint \frac{ds}{t}}$$

Torsion

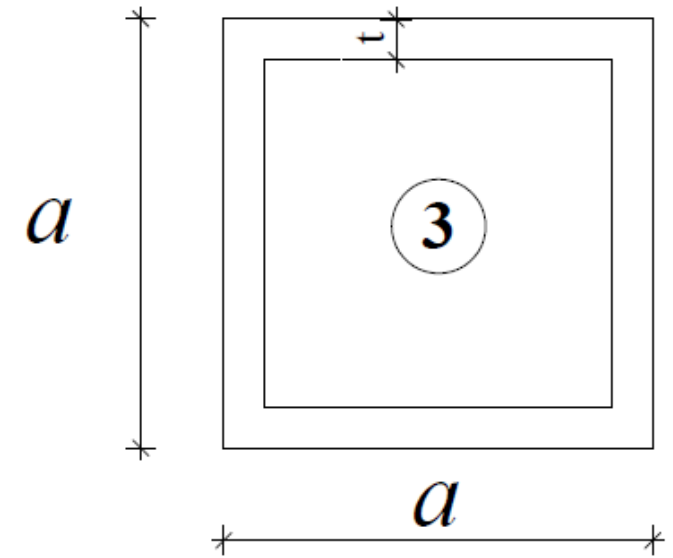
□ Torsion of Noncircular Members



$$R_1 = 0.208a^3$$



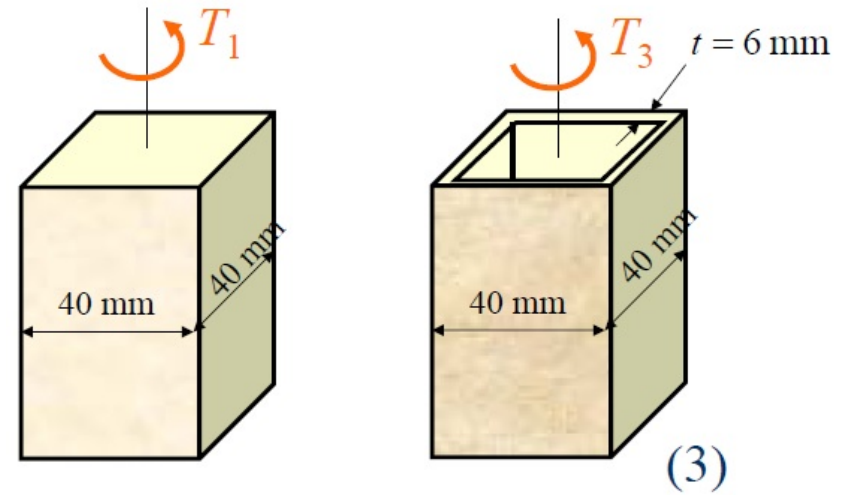
$$R_2 = \frac{4}{3}(a-t)t^2$$



$$R_3 = 2(a-t)^2 t$$

Torsion

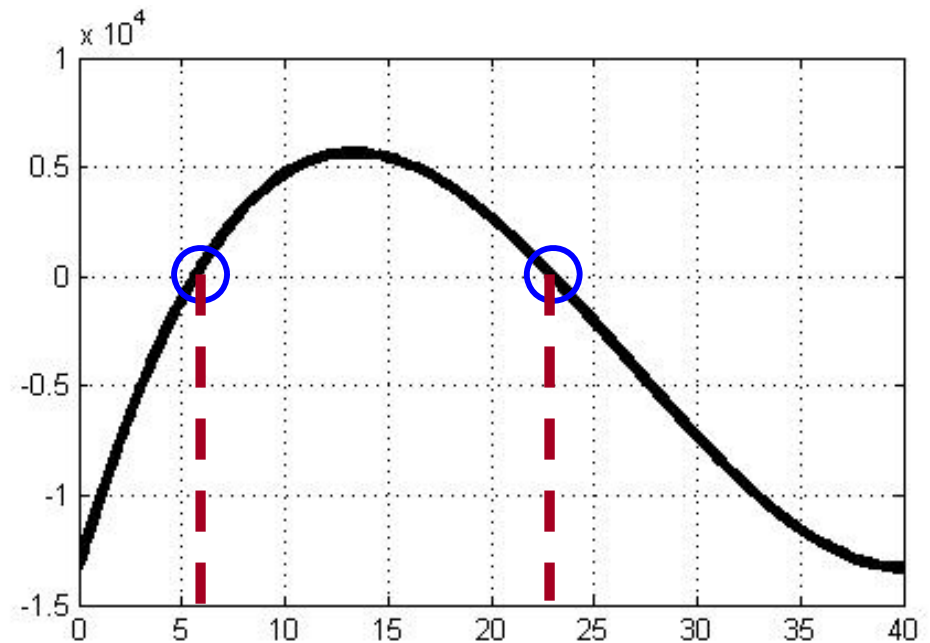
□ Torsion of Noncircular Members



$$\frac{R_3}{R_1} > 1 \Rightarrow \frac{2(a-t)^2 t}{0.208a^3} > 1$$

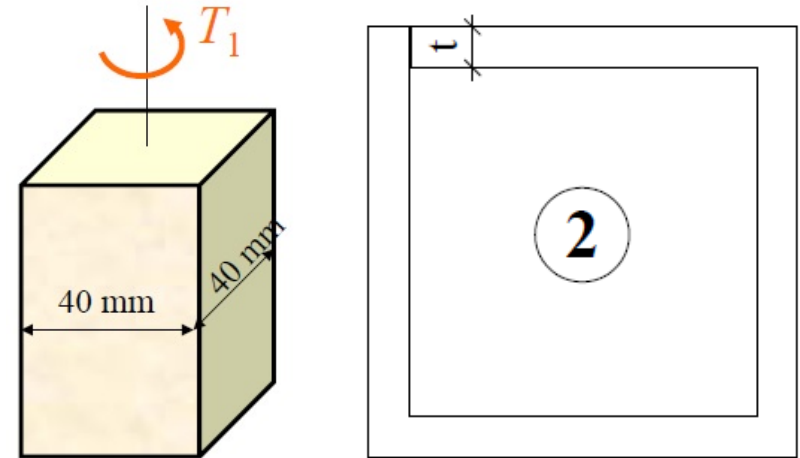
$$\Rightarrow 2(a-t)^2 t - 0.208a^3 > 0$$

$$\text{If } a = 40\text{mm} \Rightarrow 2(40-t)^2 t - 0.208(40)^3 > 0$$



Torsion

□ Torsion of Noncircular Members



$$\frac{R_2}{R_1} > 1 \Rightarrow \frac{\frac{4}{3}(a-t)t^2}{0.208a^3} > 1$$

$$\Rightarrow \frac{4}{3}(a-t)t^2 - 0.208a^3 > 0$$

$$\text{If } a = 40\text{mm} \Rightarrow \frac{4}{3}(40-t)t^2 - 0.208(40)^3 > 0$$

