Mechanics of Materials

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Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:
J.Wat Oler "Lectures notes on Mechanics od Materials"
Ibrahim A.Assakkaf "Lectures notes on Mechanics od Materials"

Stress and Strain – Axial Loading

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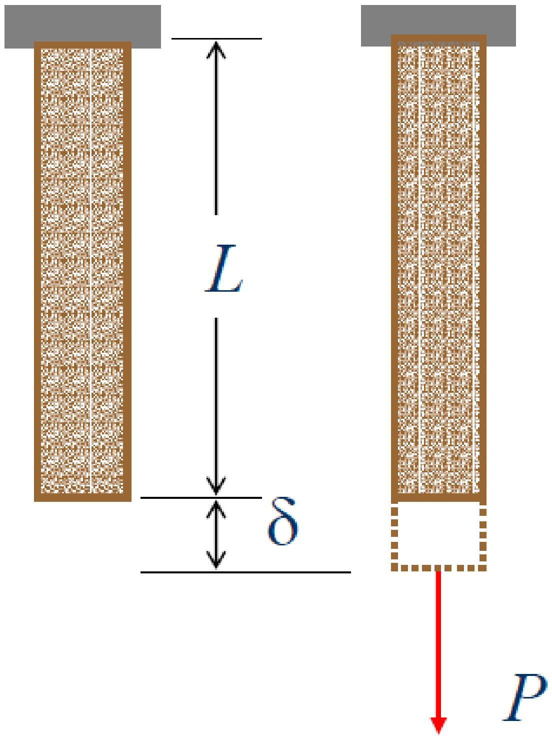
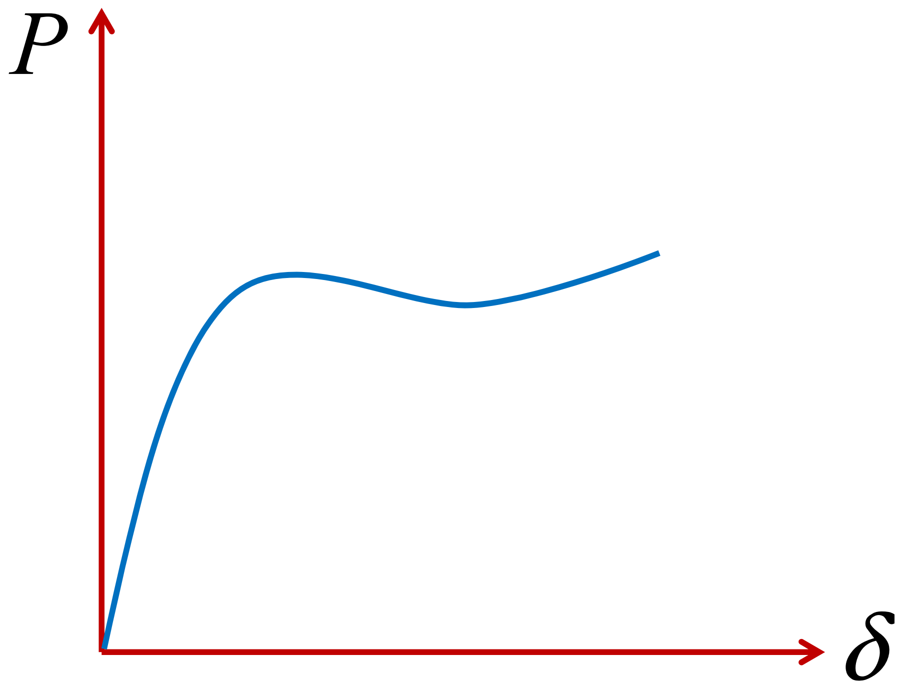
Stress and Strain – Axial Loading

❑ Load - deformation diagram





Plotting the magnitude P of the load against the deformation a, we obtain a certain load-deformation diagram. While this diagram contains information useful to the analysis of the rod under consideration, ***it cannot be used directly to predict the deformation of a rod of the same material but of different dimensions.***

Stress and Strain – Axial Loading

❑ *Deformations of Members under Axial Loading*

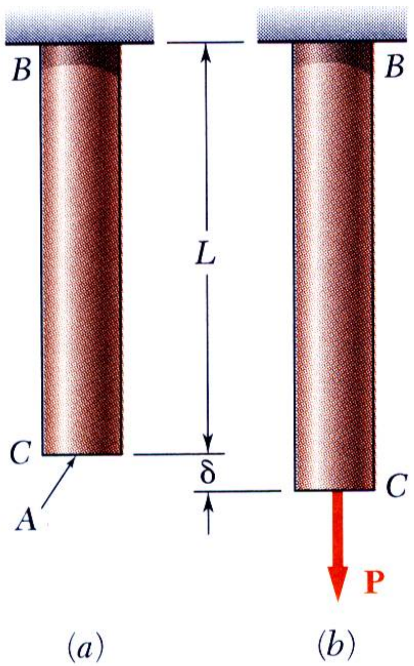


Fig. 2.1

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\frac{\delta}{L} = \text{c}te$$

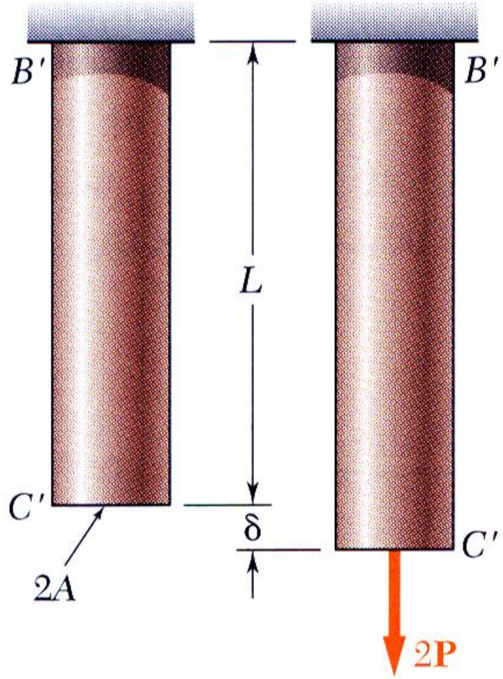


Fig. 2.3

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\frac{\delta}{L} = cte$$

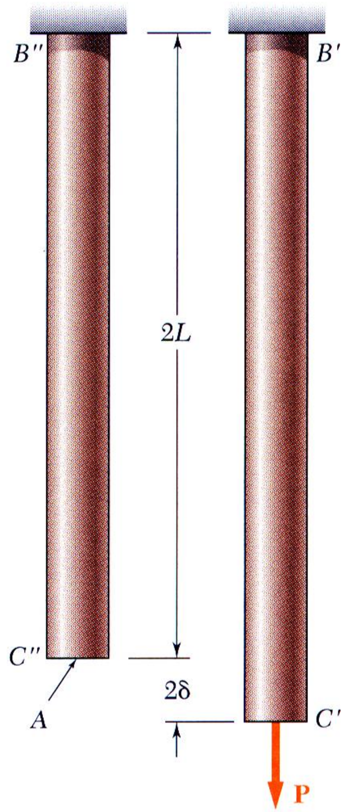


Fig. 2.4

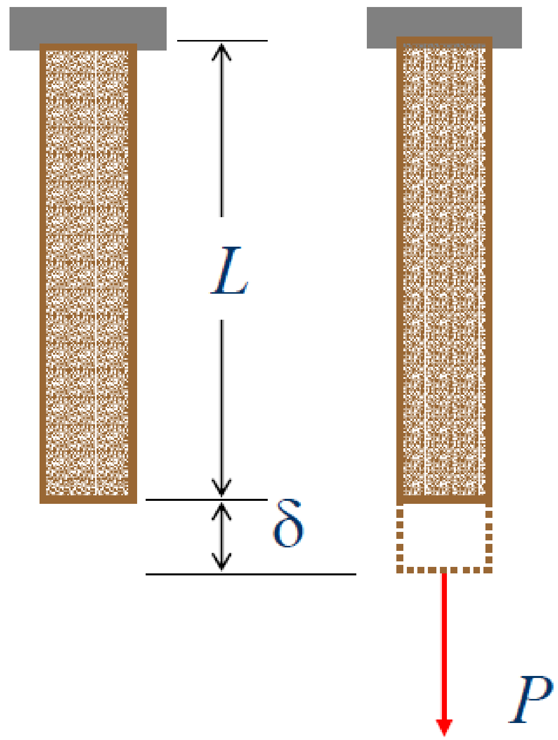
$$\sigma = \frac{P}{A}$$

$$\frac{\delta}{L} = cte$$

Stress and Strain – Axial Loading

❑ *Normal Strain*

We define the normal strain in a rod under axial loading as the ***deformation per unit length*** of that rod.



$$if \quad A = cte \Rightarrow \quad \varepsilon = \frac{\delta}{L}$$

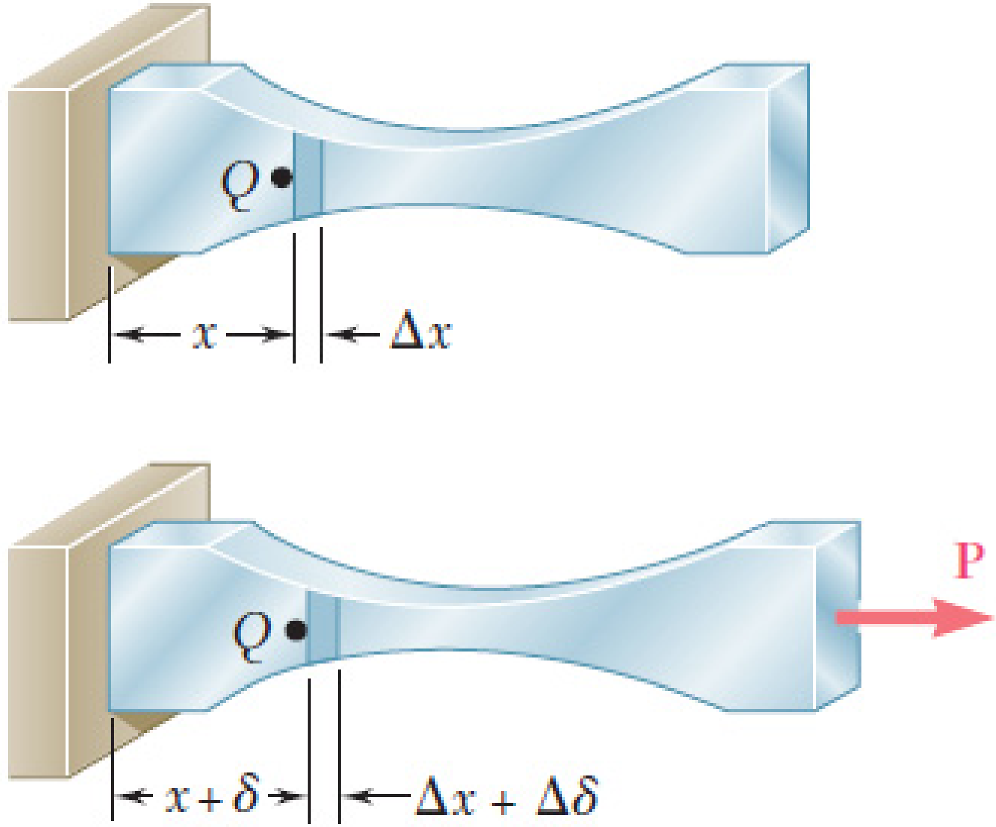
Strain has no dimension

Stress and Strain – Axial Loading

❑ *Normal Strain*

$$if \quad A \neq cte \Rightarrow$$

$$\varepsilon = \lim_{\Delta x \to 0} \frac{\Delta\delta}{\Delta x} = \frac{d\delta}{dx}$$



Stress and Strain – Axial Loading

❑ *Stress-Strain Test*

Photo 2.2 This machine is used to test tensile test specimens, such as those shown in this chapter.

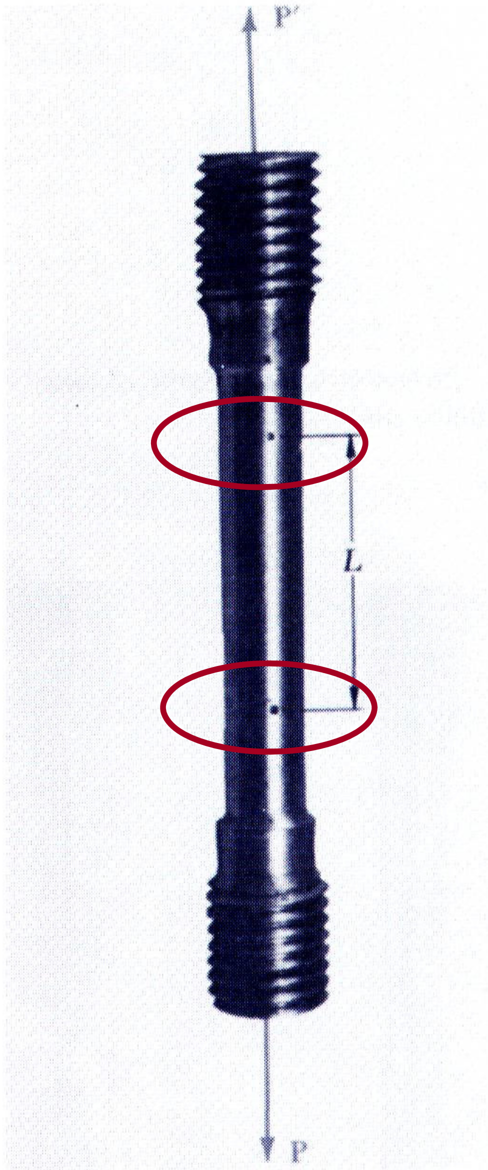
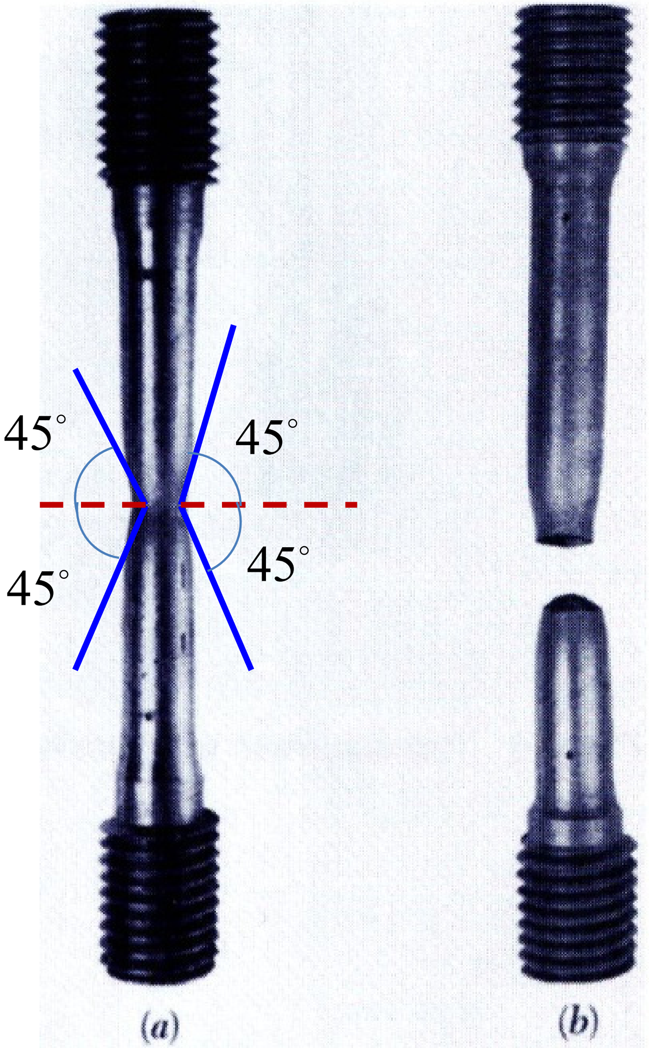
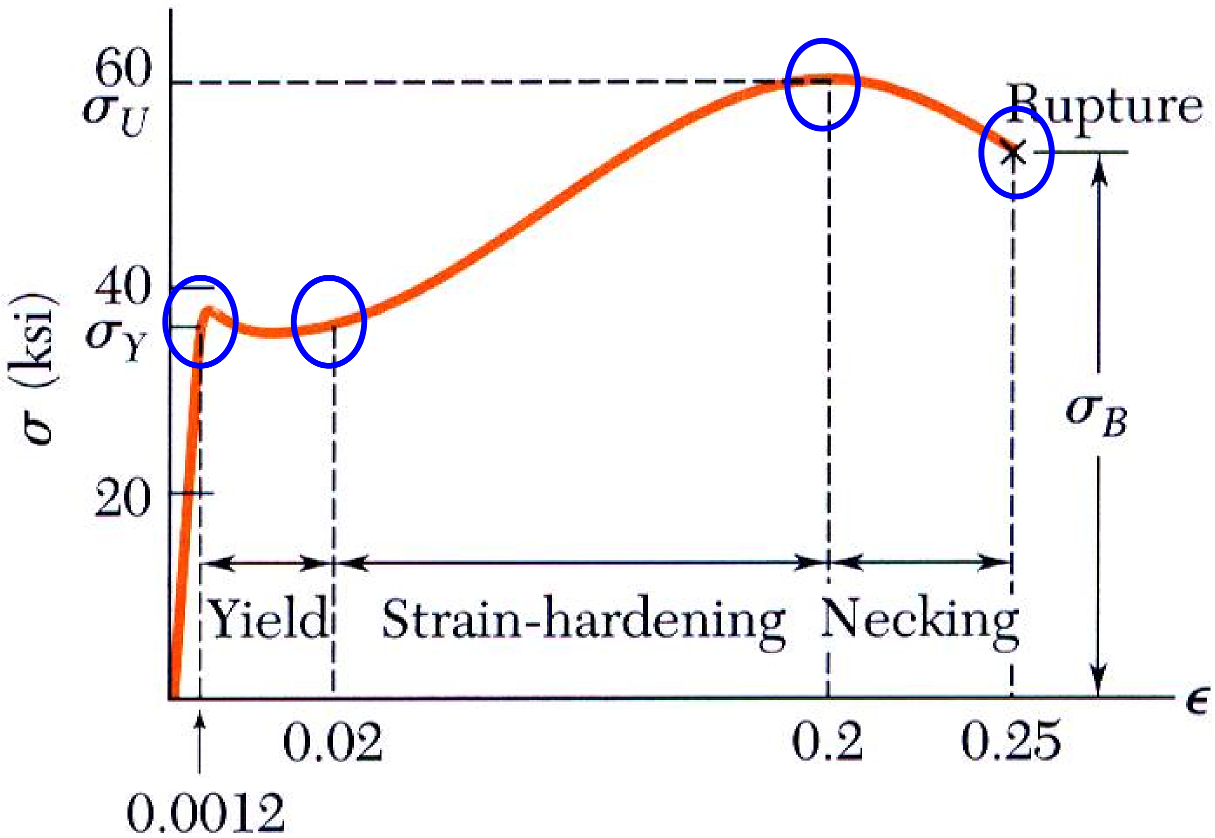


Fig. 2.8 Test specimen with tensile load.

Stress and Strain – Axial Loading

❑ *Stress-Strain Diagram: Ductile Materials*



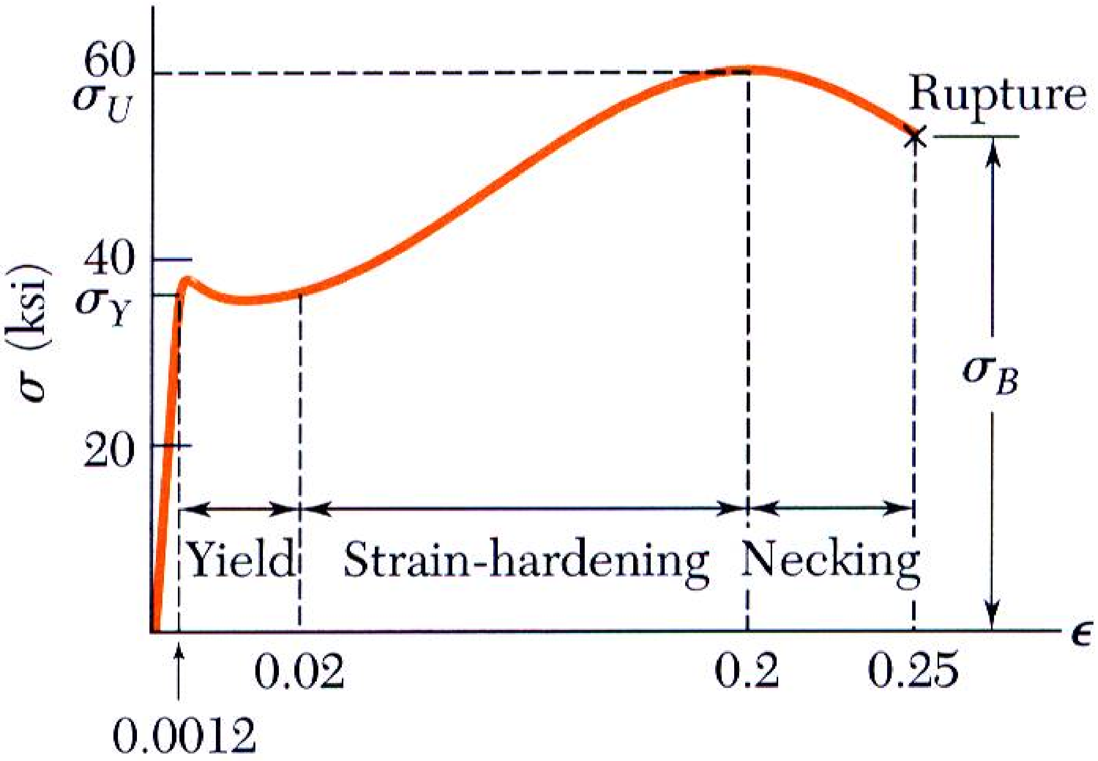


(*a*) Low-carbon steel

Shear stress is primarily responsible for the failure of ductile materials, and confirms the fact that, under an axial load, ***shearing stresses are largest on surfaces forming an angle of 45° with the load***.

Stress and Strain – Axial Loading

❑ *Stress-Strain Diagram: Ductile Materials*



(*a*) Low-carbon steel

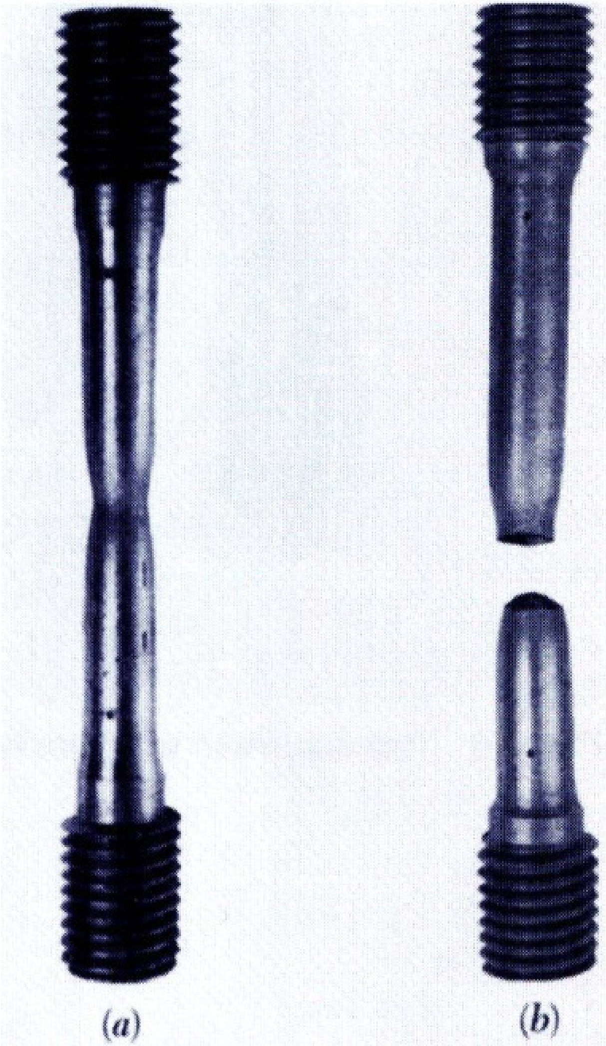
σ_Y : Yield Strength

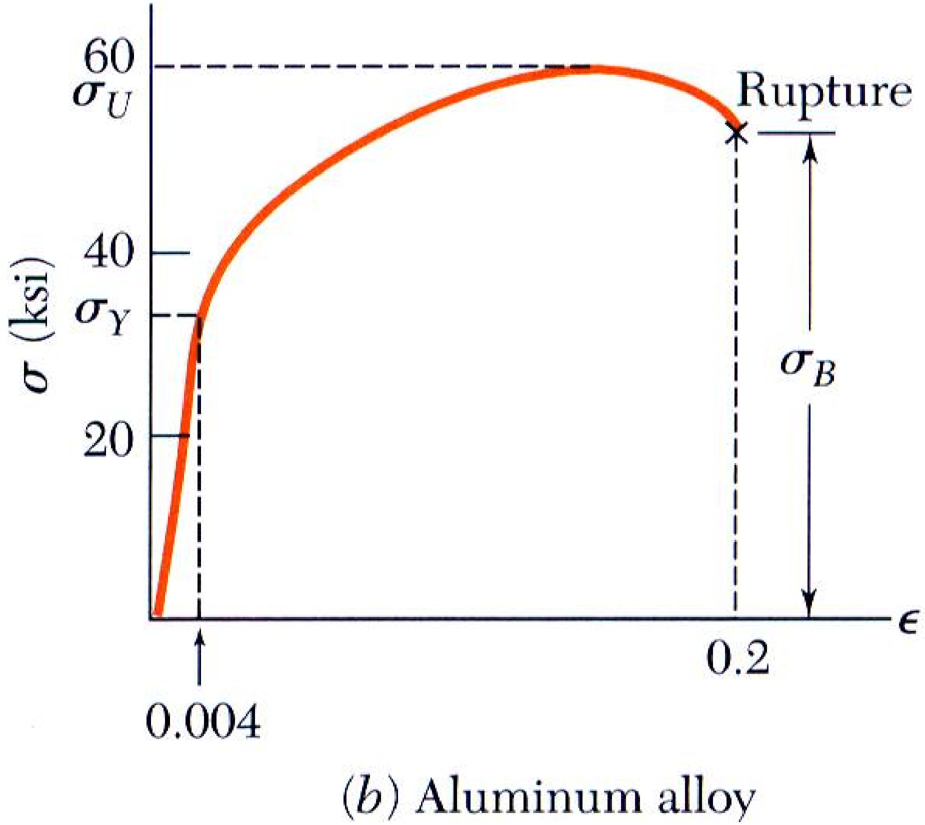
σ_U : Ultimate Strength

σ_B : Breaking Strength

Stress and Strain – Axial Loading

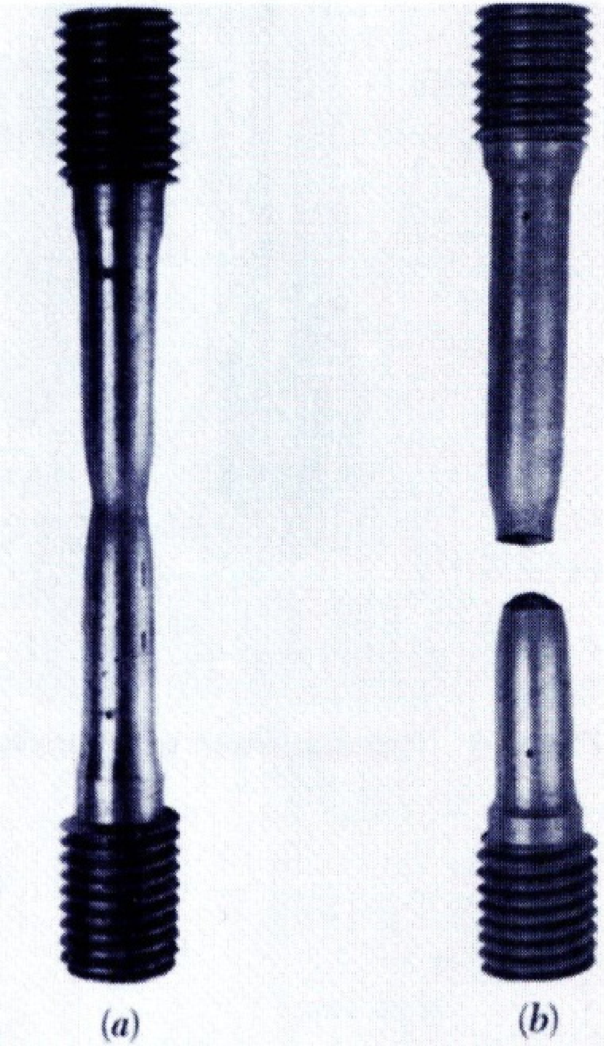
❑ *Stress-Strain Diagram: Ductile Materials*





Stress and Strain – Axial Loading

❑ *Stress-Strain Diagram: Ductile Materials*



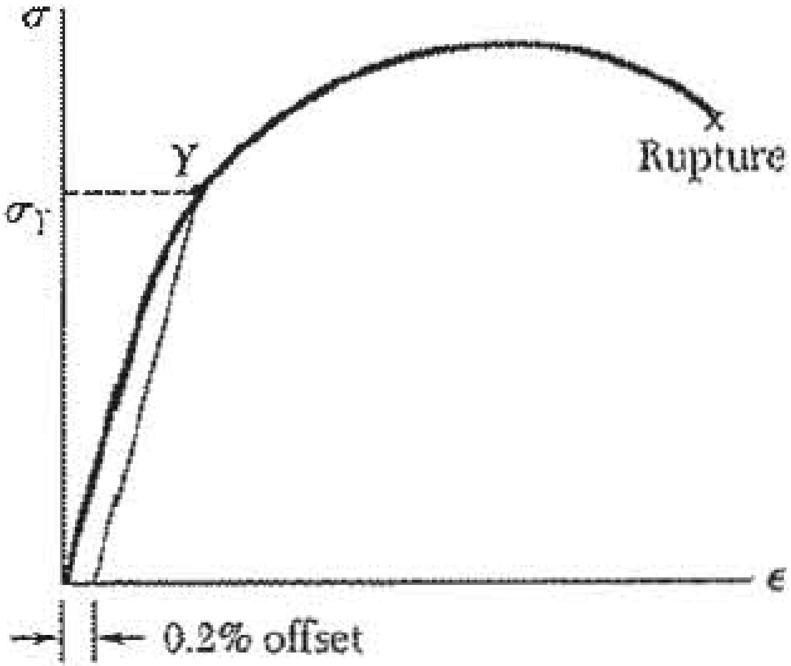
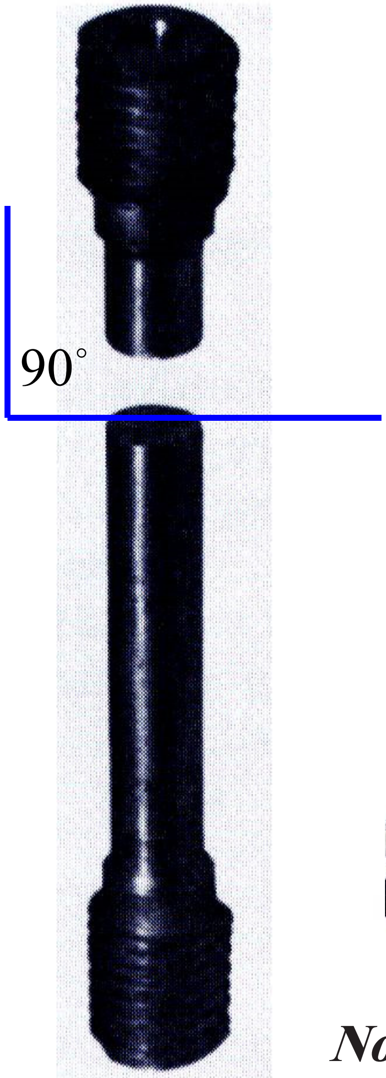
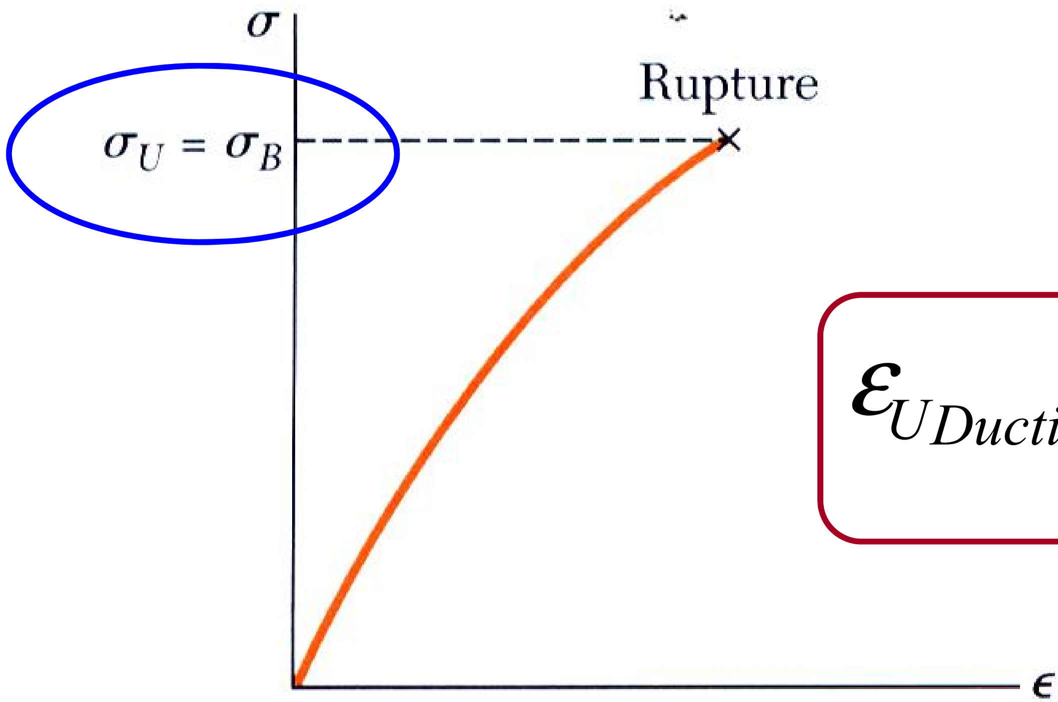


Fig. 2.13 Determination of yield strength by offset method.

Stress and Strain – Axial Loading

❑ *Stress-Strain Diagram: Brittle Materials*





$$\varepsilon_{UDuctile} >> \varepsilon_{UBritle}$$

Fig. 2.11 Stress-strain diagram for a typical brittle material.

Normal stresses are primarily responsible for the failure of brittle materials

Stress and Strain – Axial Loading

❑ *Hooke's Law: Modulus of Elasticity*

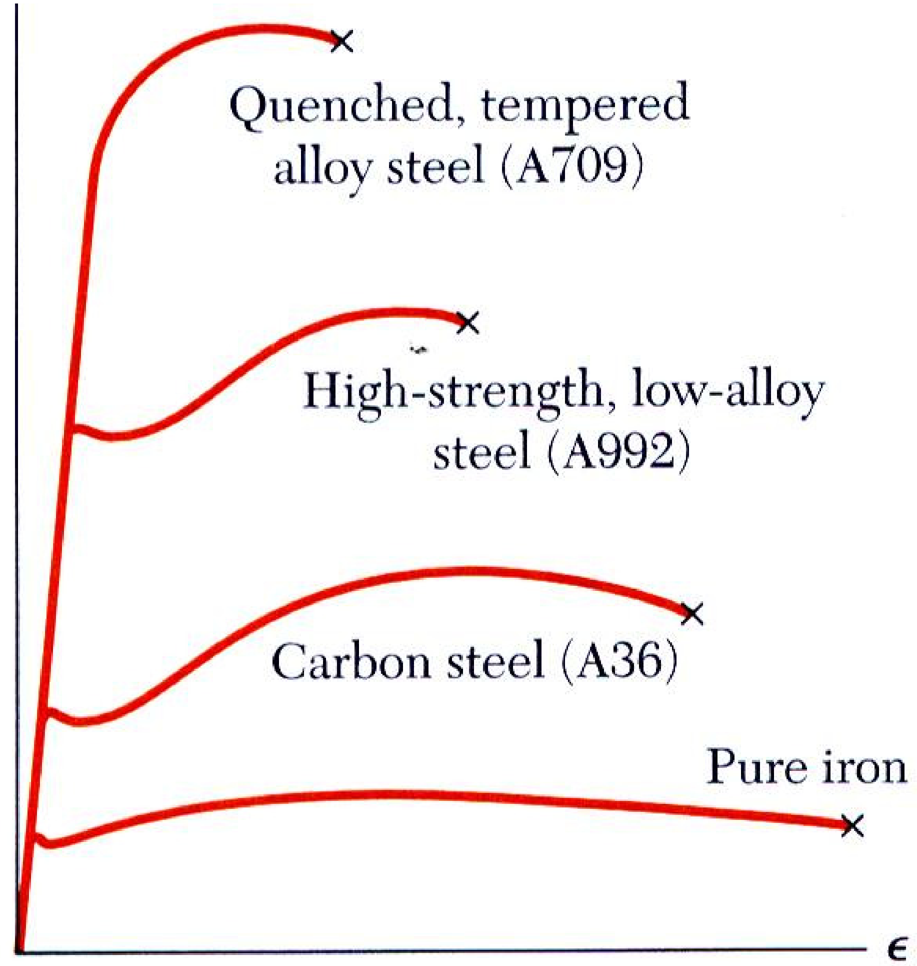


Fig. 2.16 Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

$$\sigma = E\varepsilon$$

E = Youngs Modulus or Modulus of Elasticity

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Stress and Strain – Axial Loading

- ***Stress-Strain Diagram: Brittle Materials***

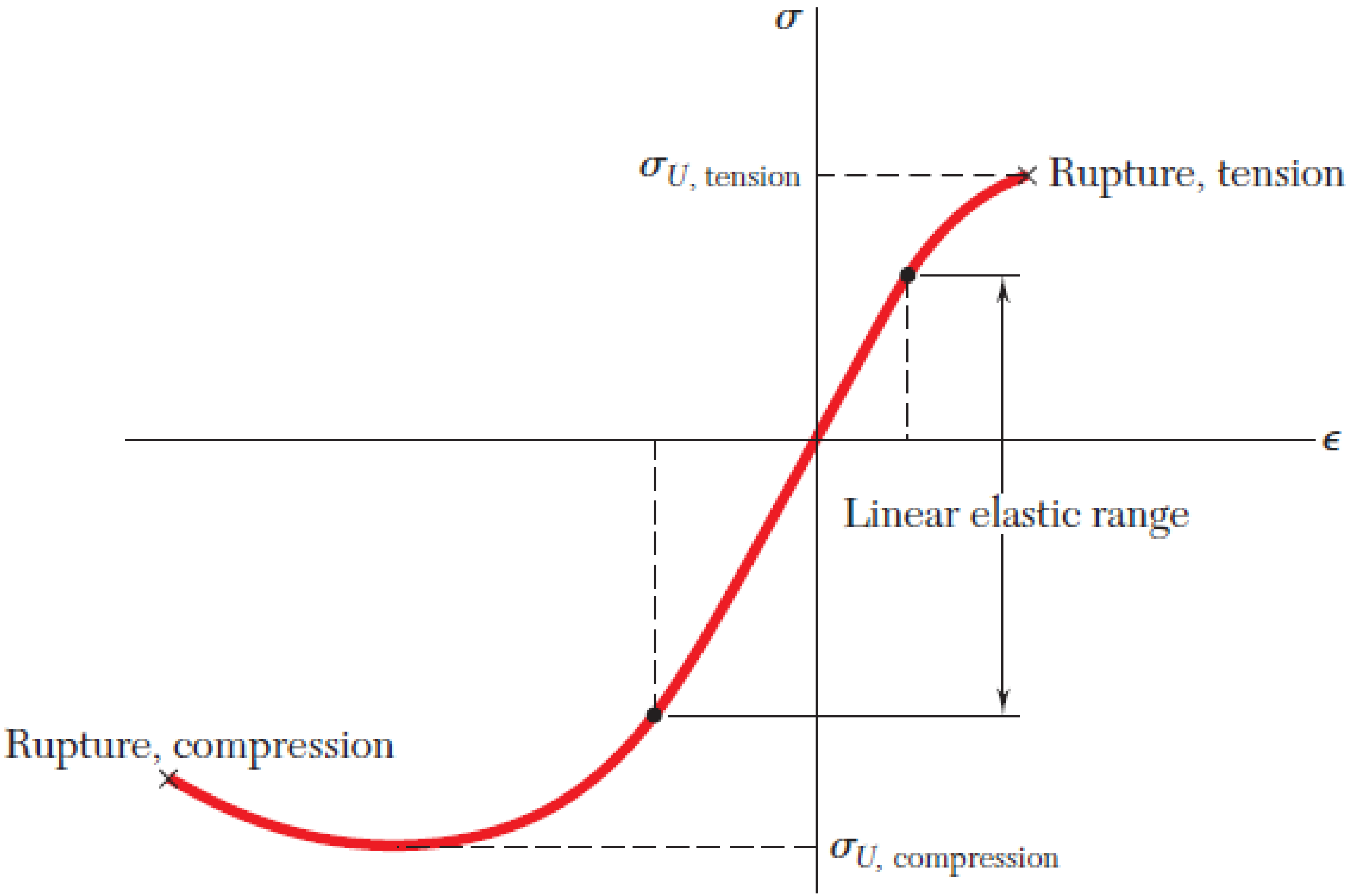
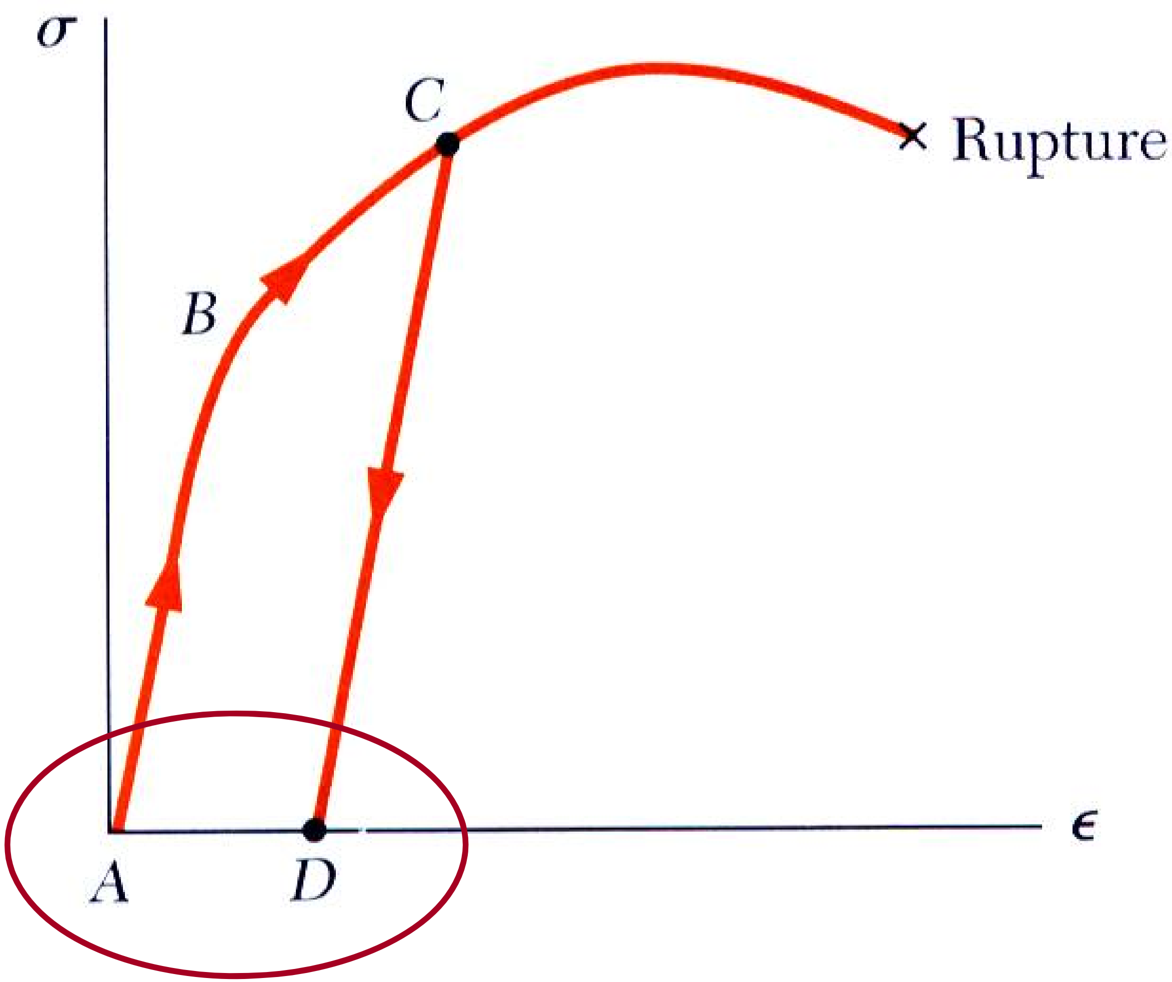


Fig. 2.9 Stress-strain diagram for concrete.

Stress and Strain – Axial Loading

- ***Elastic vs. Plastic Behavior***



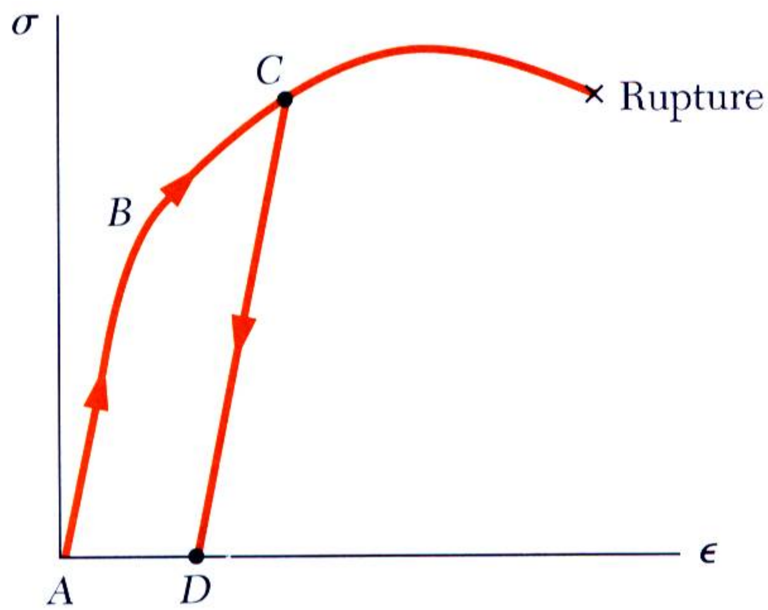
- If the strain disappears when the stress is removed, the material is said to ***behave elastically***.
- The largest stress for which this occurs is called the ***elastic limit***.
- When the strain does not return to zero after the stress is removed, the material is said to ***behave plastically***.

Stress and Strain – Axial Loading

❑ *Elastic vs. Plastic Behavior*

The plastic deformation depends on:

- The maximum value reached by the stress.
- The time elapsed before the load is removed.



The stress-dependent part of the plastic deformation is referred to as ***slip***, and the time-dependent part-which is also influenced by the temperature-as ***creep***.

Stress and Strain – Axial Loading

❑ *Elastic vs. Plastic Behavior*

$$R_1 > R_2$$

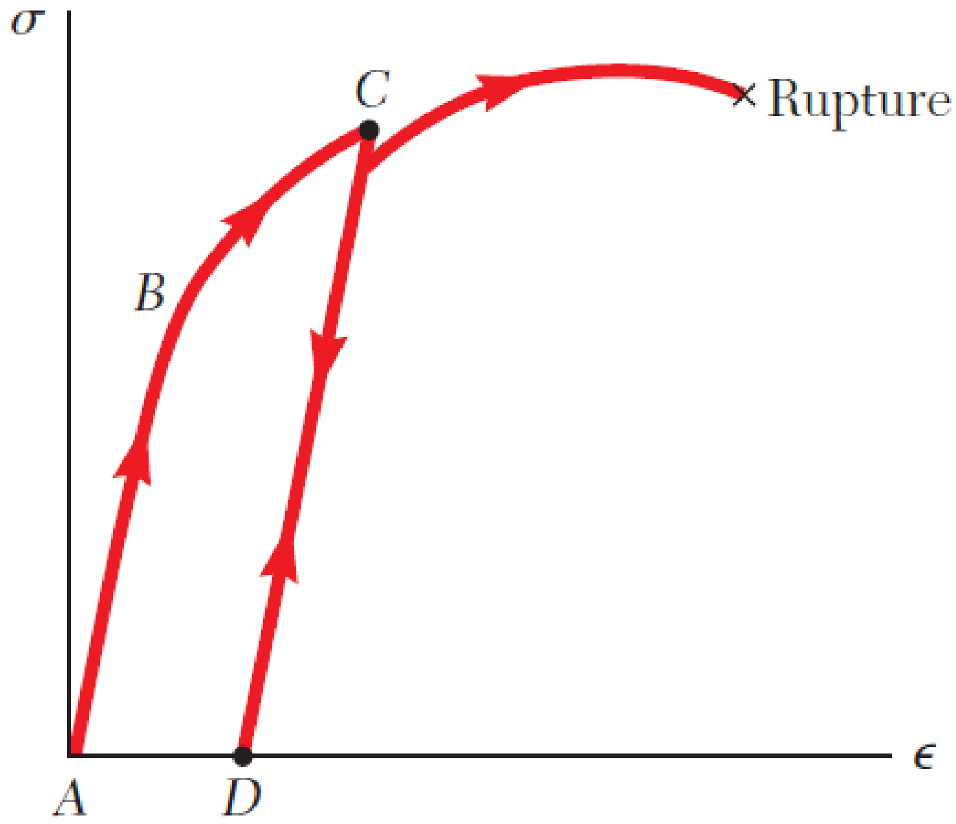
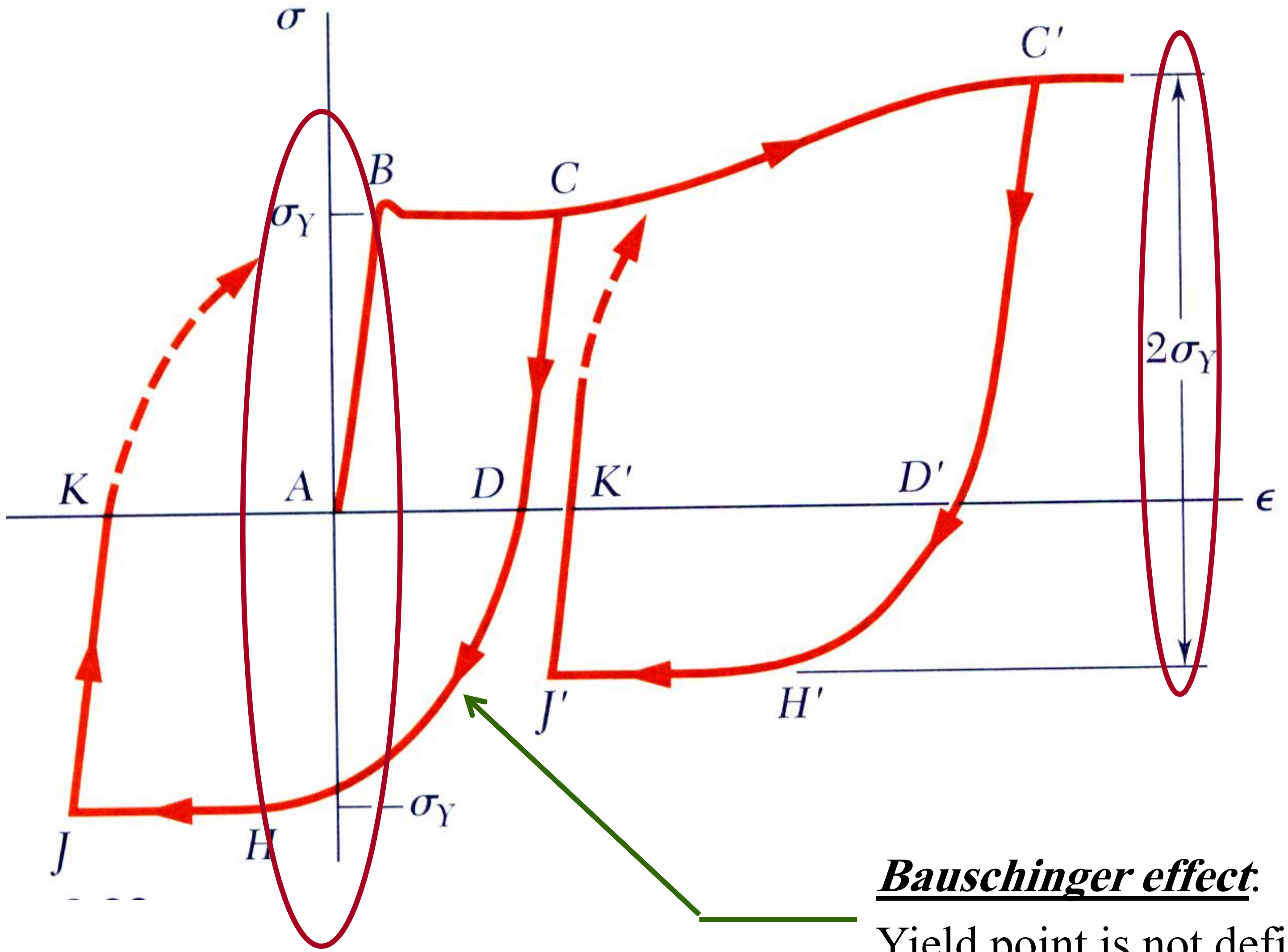


Fig. 2.14 Stress-strain characteristics of ductile material reloaded after prior yielding.

Stress and Strain – Axial Loading

❑ *Elastic vs. Plastic Behavior*



Bauschinger effect:

Yield point is not defined clearly

Stress and Strain – Axial Loading

❑ *Fatigue*

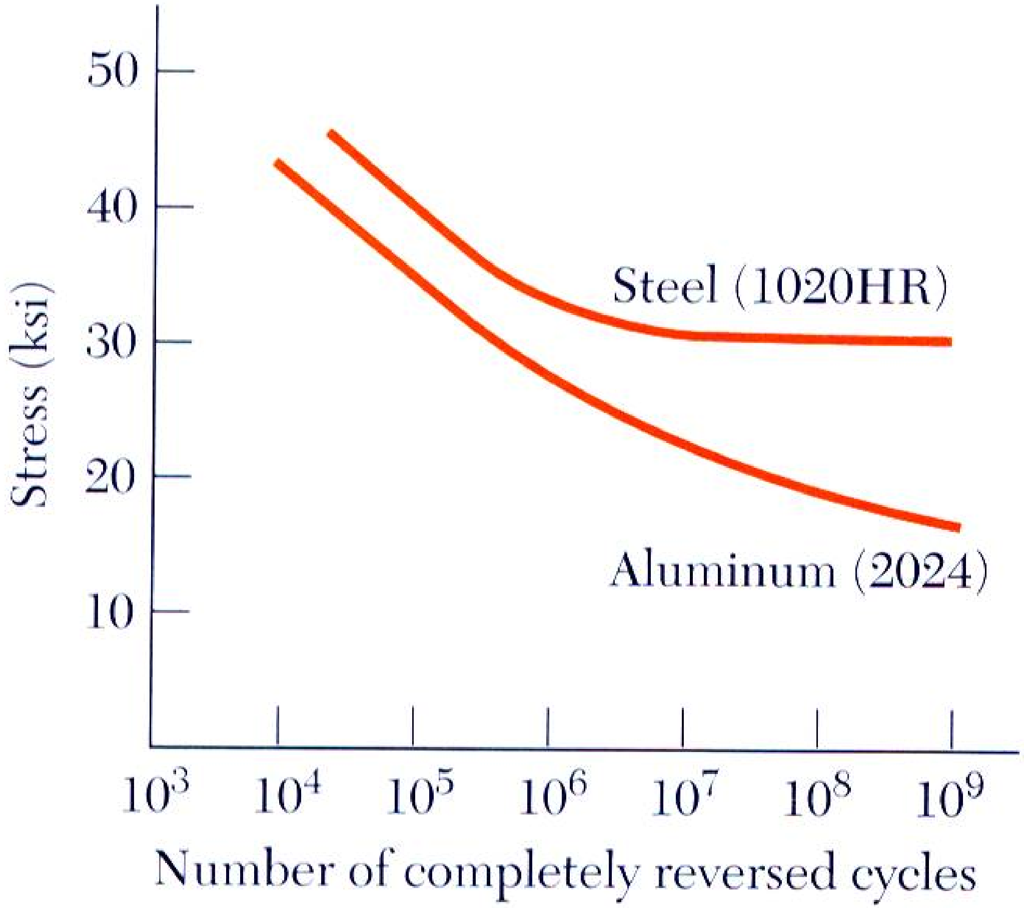


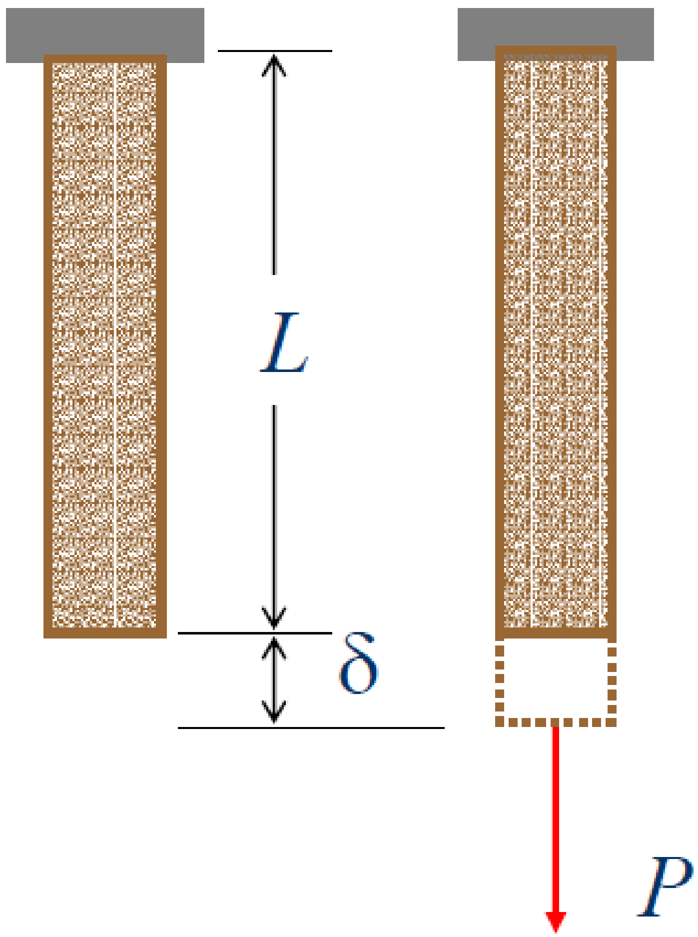
Fig. 2.21

- Fatigue properties are shown on S-N diagrams.
- A member may fail due to *fatigue* at stress levels significantly below the ultimate strength if subjected to many loading cycles.
- When the stress is reduced below the *endurance limit*, fatigue failures do not occur for any number of cycles.

Stress and Strain – Axial Loading

- ***Deformations of Members under Axial Loading***

- Uniform Member



Hooke's law

$$\sigma = E\varepsilon$$

$$\varepsilon = \frac{\delta}{L} \qquad \sigma = \frac{P}{A}$$

σ : Stress

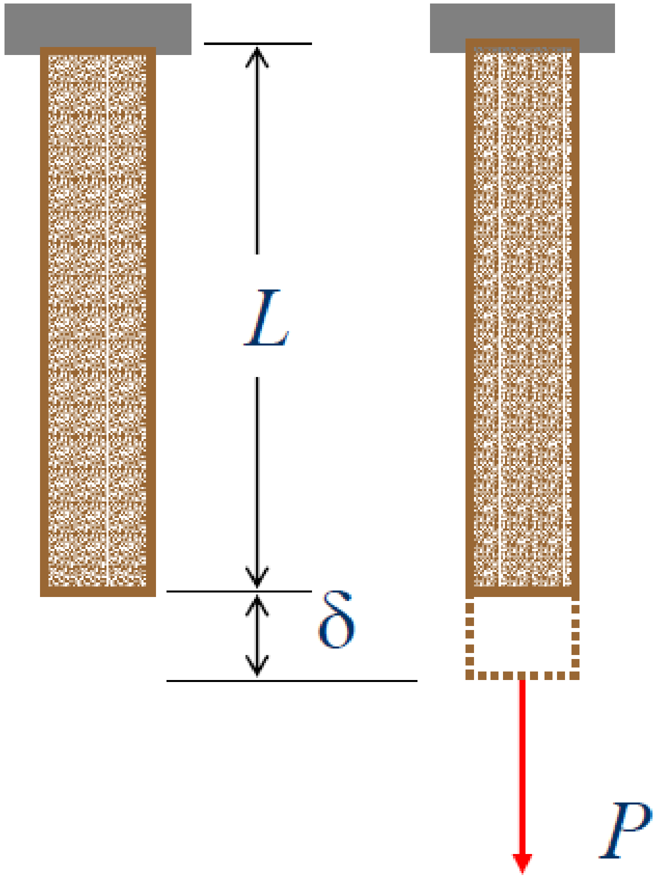
ε : Strain

E : Modulus of Elasticity

Stress and Strain – Axial Loading

❑ *Deformations of Members under Axial Loading*

- Uniform Member



Deformation:

$$\sigma = E\varepsilon \quad \Rightarrow \quad \frac{P}{A} = E\frac{\delta}{L}$$

$$\Rightarrow \quad \boxed{\delta = \frac{PL}{EA}}$$

Axial Stiffness:

$$P = \frac{EA}{L}\delta \quad \approx \quad F = K\Delta x \quad \Rightarrow$$

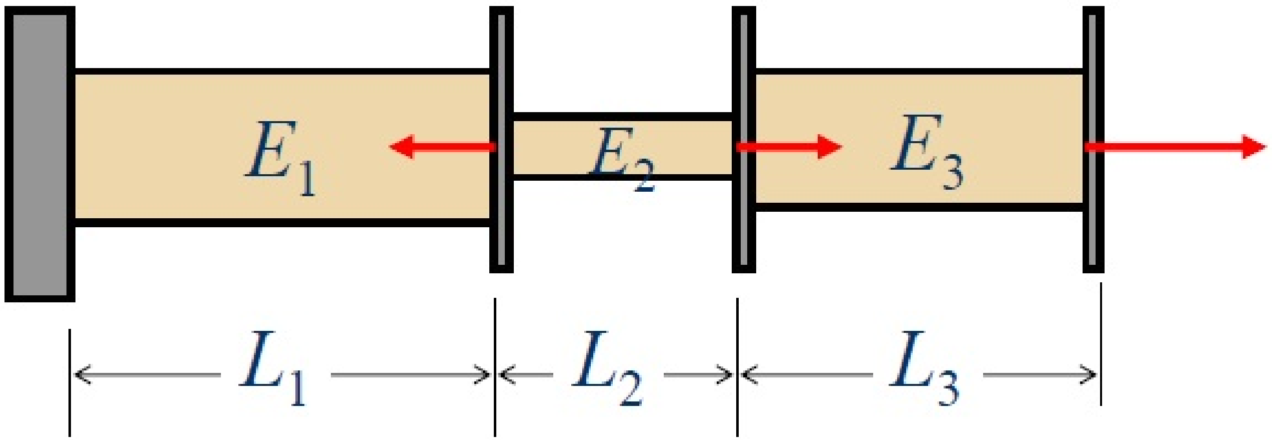
$$\boxed{K = \frac{EA}{L}}$$

❑ ***Deformations of Members under Axial Loading***

Multiple Loads/Sizes

- With variations in loading, cross-section or material properties,

$$\delta = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i}$$



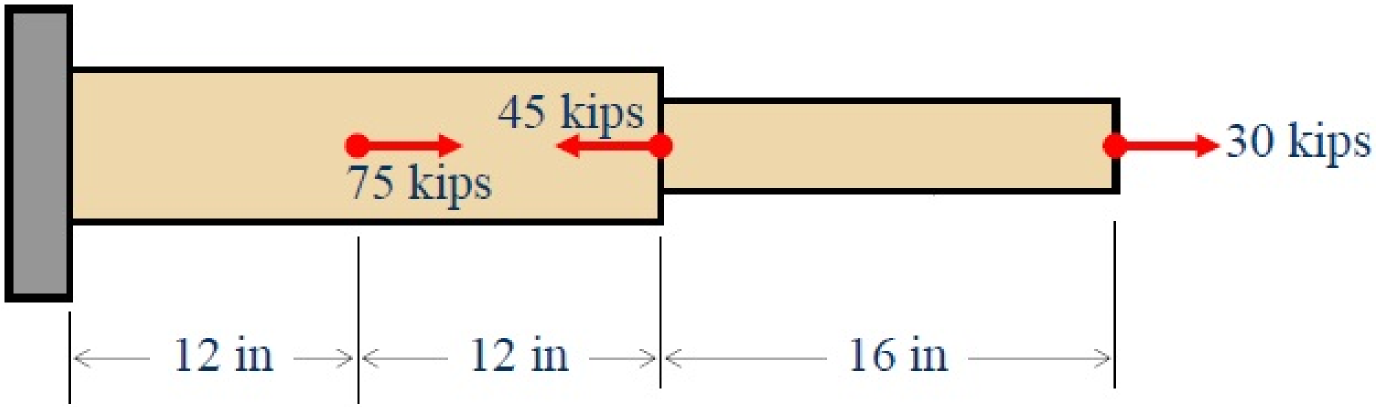
❑ *Deformations of Members under Axial Loading*

Example 1

Determine the deformation of the steel rod shown under the given loads.

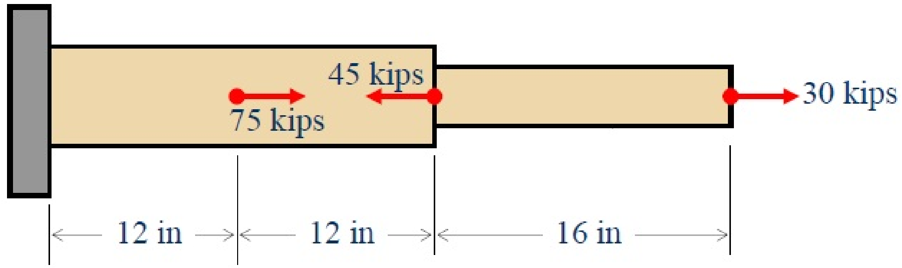
$E = 29\times10^{-6}\,\text{psi}$

$D = 1.07\text{ in.}\quad d = 0.618\text{ in.}$



❑ *Deformations of Members under Axial Loading*

Example 1



$$E = 29\times10^{-6}\,\text{psi}$$

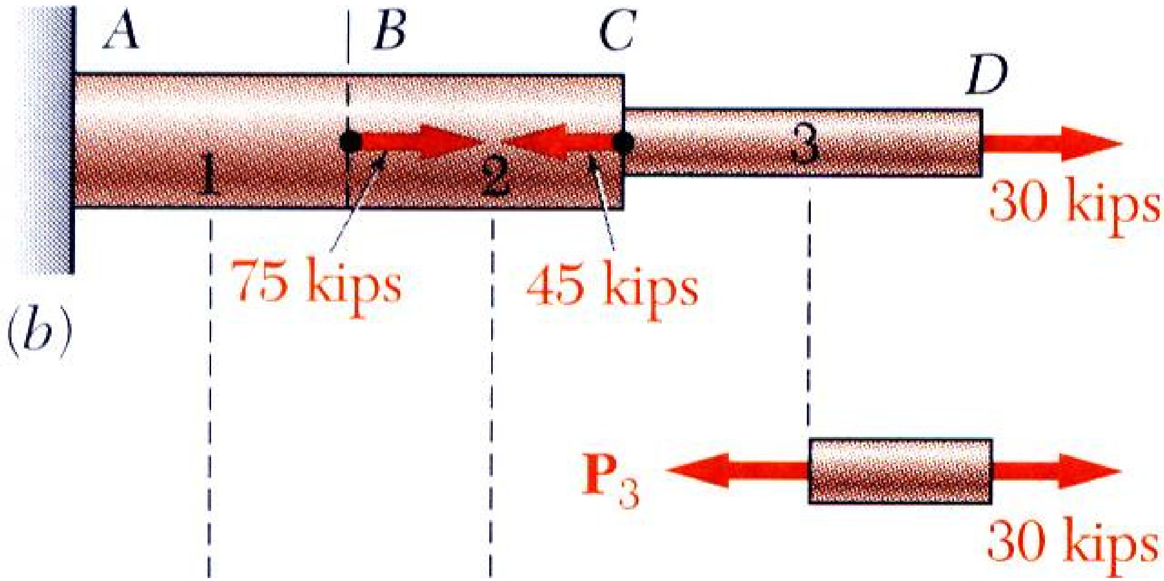
$$D = 1.07\ \text{in.} \quad d = 0.618\ \text{in.}$$

SOLUTION:

- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.

- ***Deformations of Members under Axial Loading***

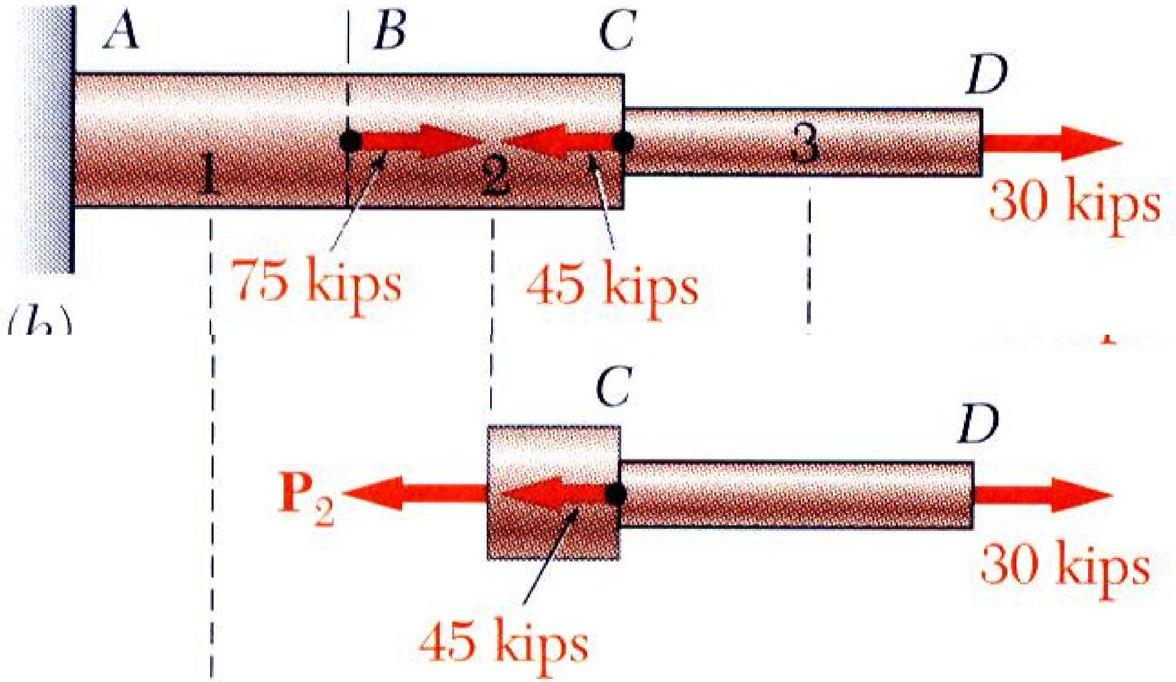
Example 1



Stress and Strain – Axial Loading

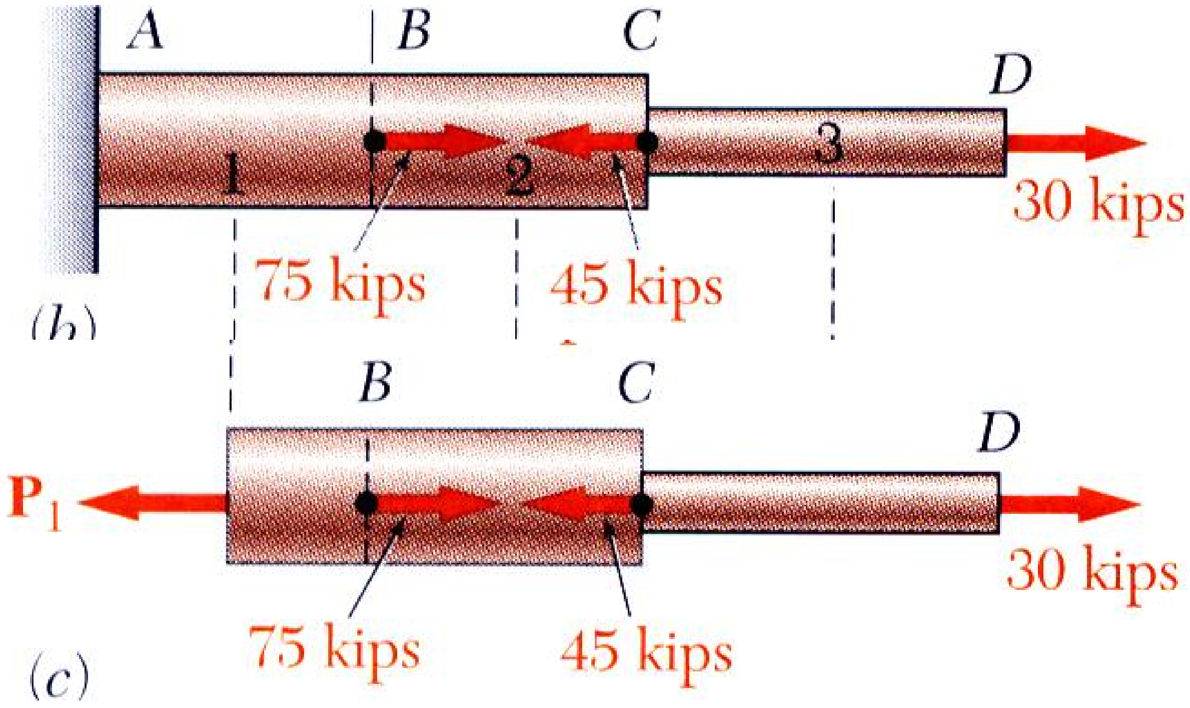
- ***Deformations of Members under Axial Loading***

Example 1



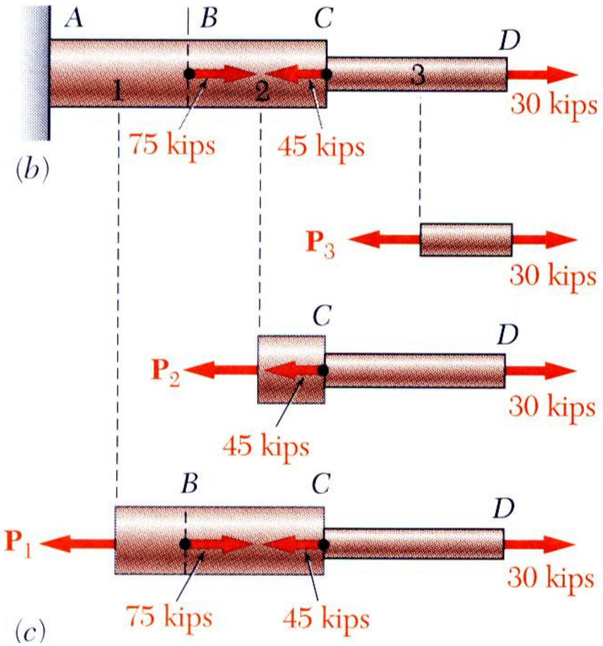
❑ ***Deformations of Members under Axial Loading***

Example 1



❑ *Deformations of Members under Axial Loading*

Example 1



$L_1 = L_2 = 12\text{ in.}$ $\quad L_3 = 16\text{ in.}$

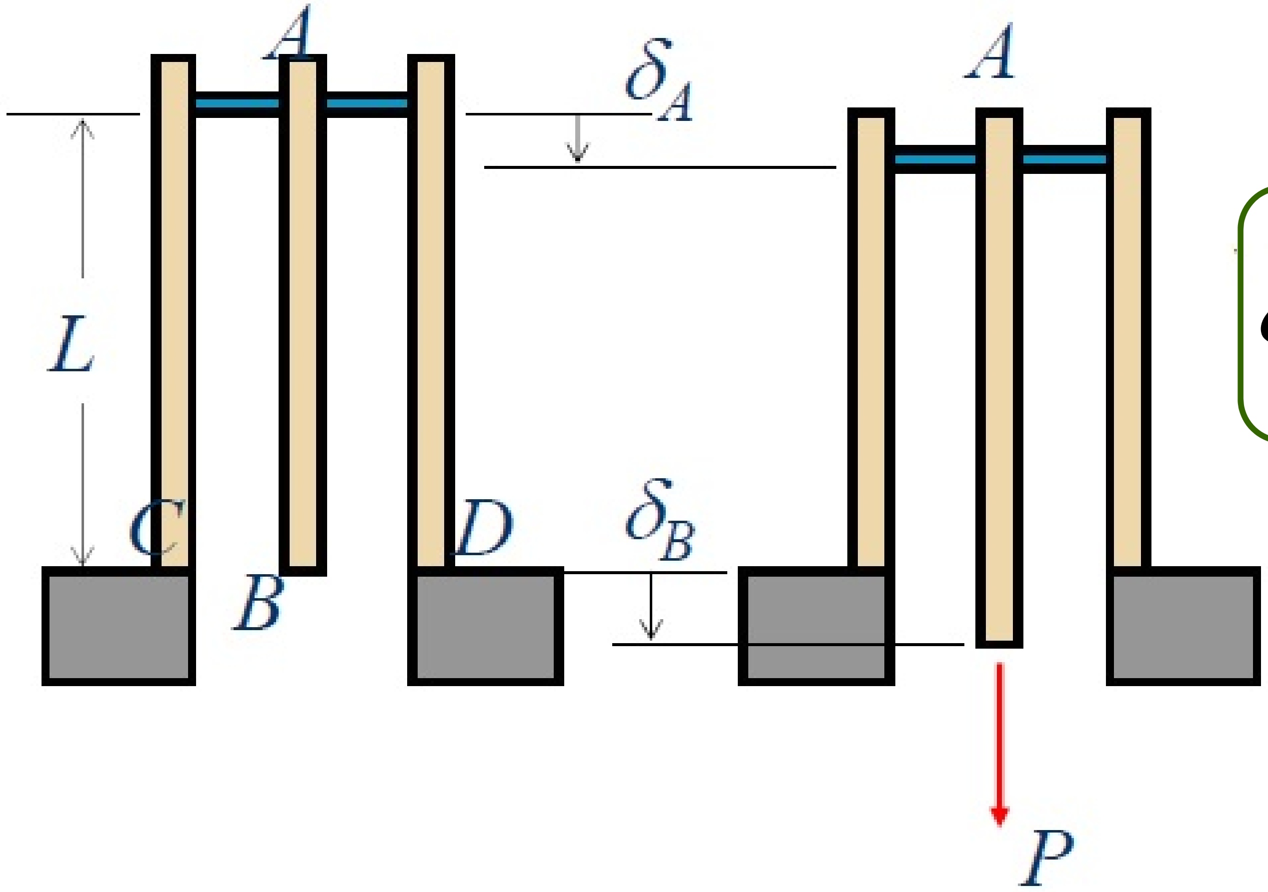
$A_1 = A_2 = 0.9\text{ in}^2$ $\quad A_3 = 0.3\text{ in}^2$

- Evaluate total deflection,

Stress and Strain – Axial Loading

❑ *Deformations of Members under Axial Loading*

▪ Relative Deformation



$$\delta_{B/A} = \delta_B - \delta_A = \frac{PL}{AE}$$

❑ *Deformations of Members under Axial Loading*

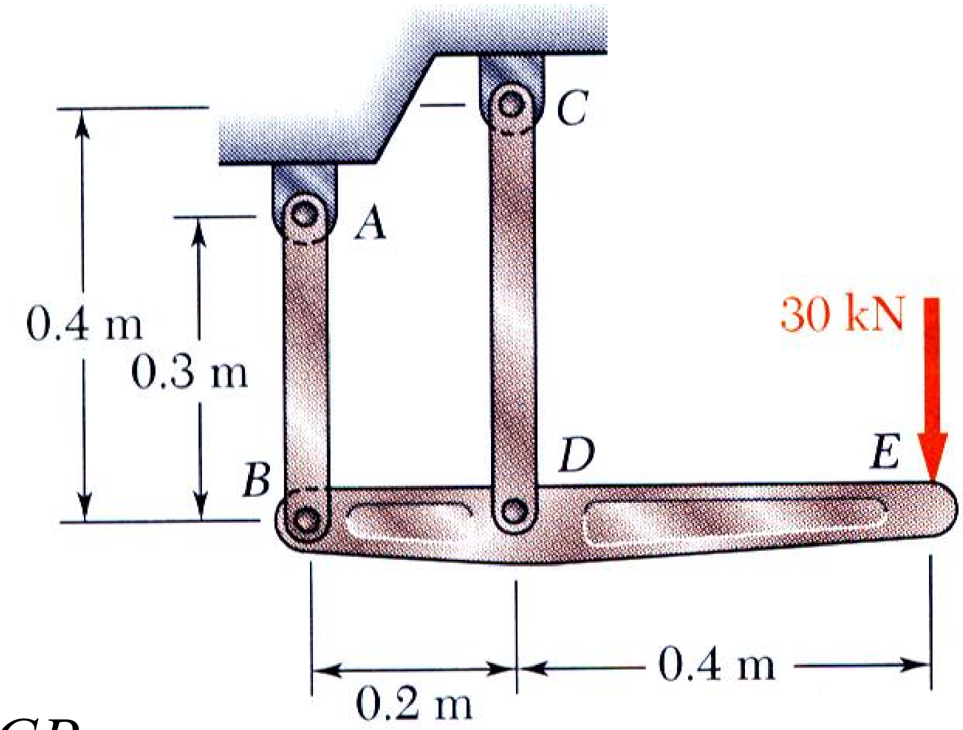
Example 2

The rigid bar BDE is supported by two links AB and CD. For the 30-kN force shown, determine the deflection

a) of B

b) of D

c) of E.



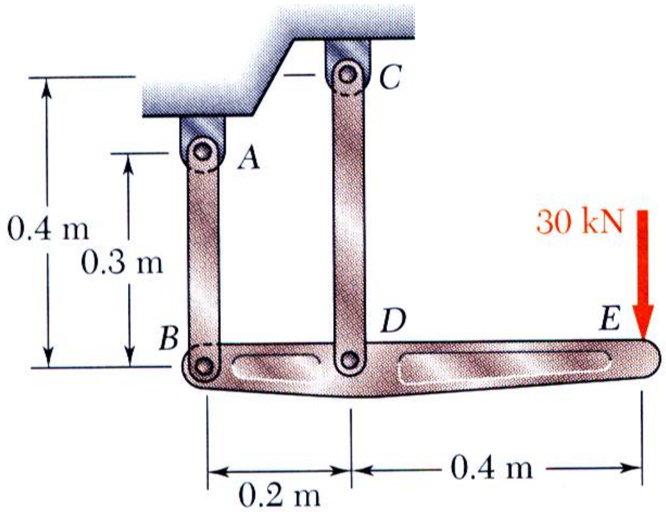
$$E_{AB} = 70\ GPa \qquad E_{CD} = 200\ GPa$$

$$A_{AB} = 500\ mm^2 \qquad A_{CD} = 600\ mm^2$$

Stress and Strain – Axial Loading

❑ *Deformations of Members under Axial Loading*

Example 2



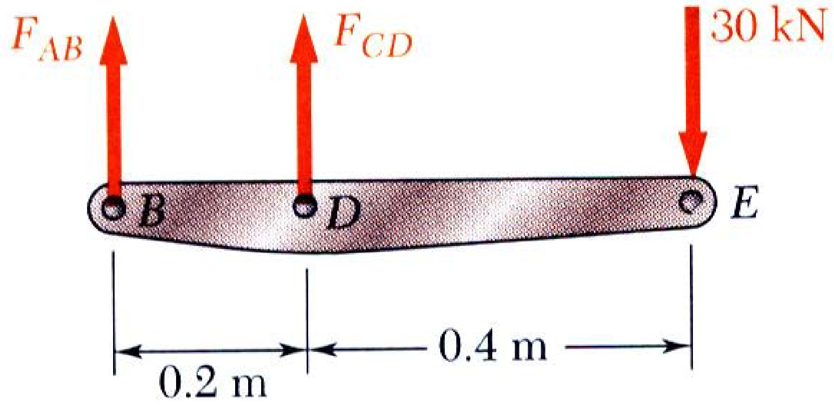
SOLUTION:

- Apply a free-body analysis to the bar *BDE* to find the forces exerted by links *AB* and *DC*.
- Evaluate the deformation of links *AB* and *DC* or the displacements of *B* and *D*.
- Work out the geometry to find the deflection at E given the deflections at B and D.

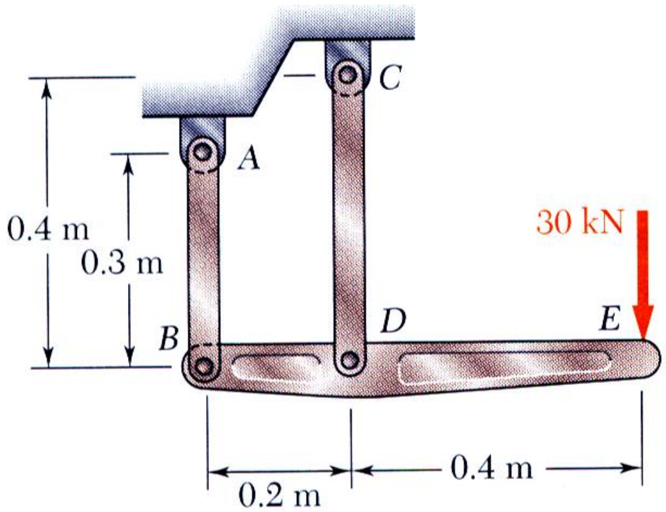
Deformations of Members under Axial Loading

Example 2

Free body: Bar *BDE*



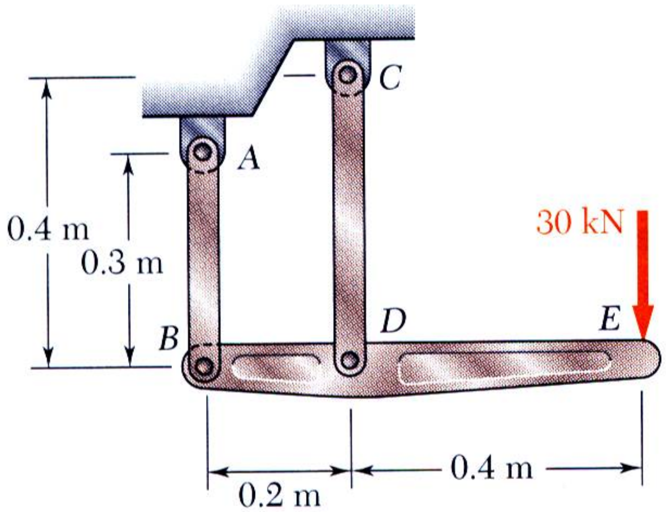
SOLUTION:



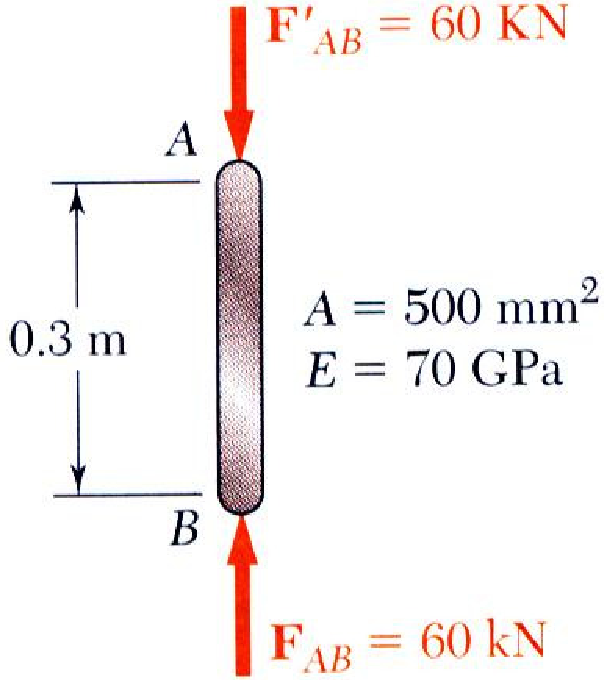
❑ *Deformations of Members under Axial Loading*

Example 2

SOLUTION:



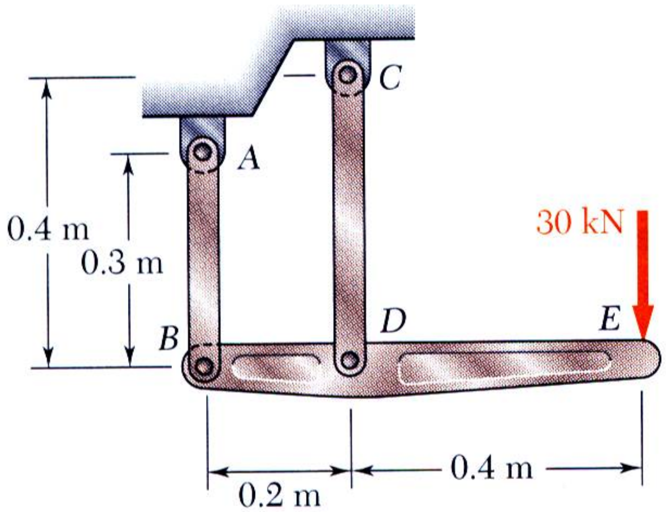
Displacement of B:



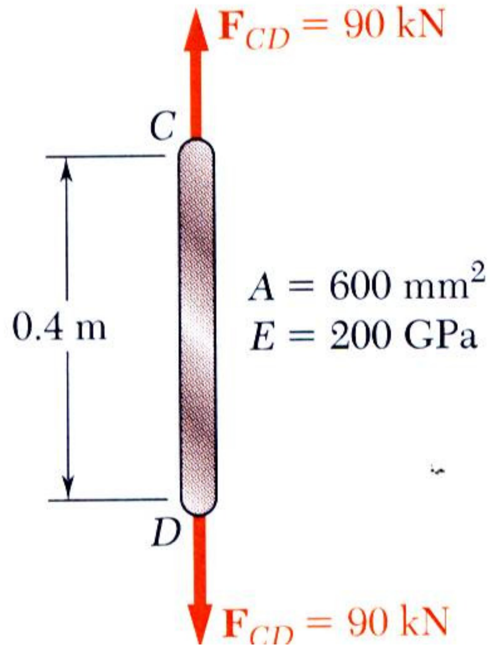
❑ *Deformations of Members under Axial Loading*

Example 2

SOLUTION:



Displacement of *D*:

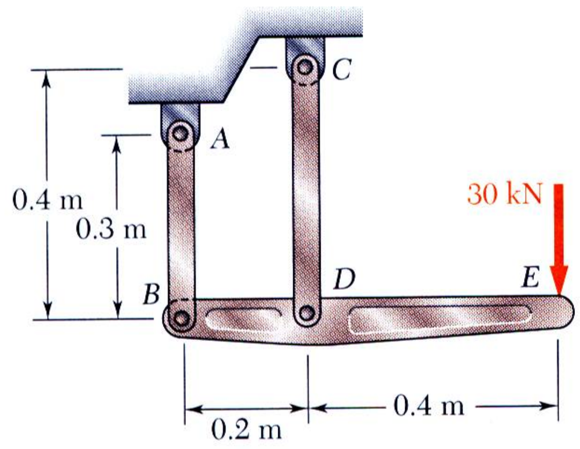


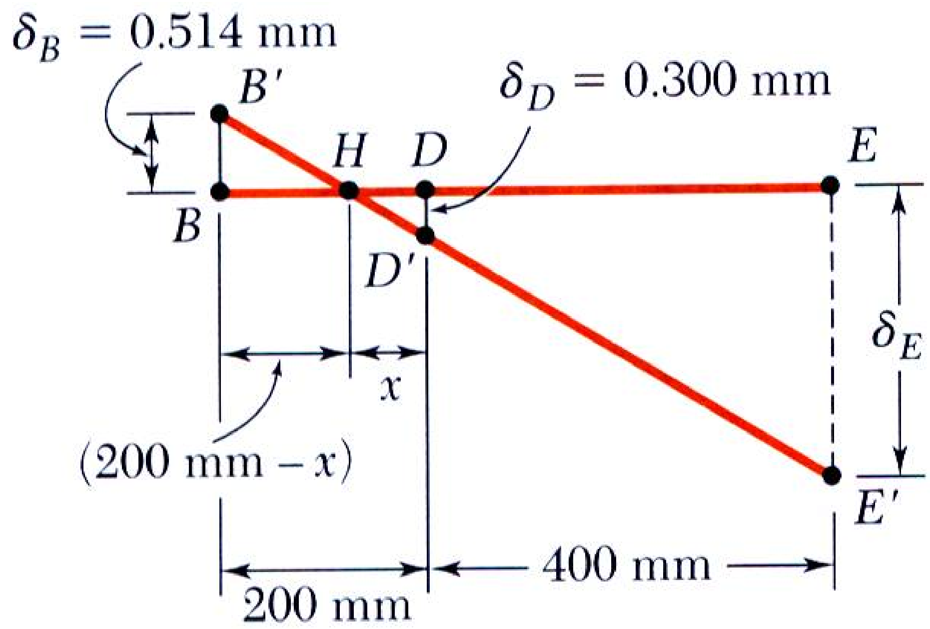
Stress and Strain – Axial Loading

❑ *Deformations of Members under Axial Loading*

Example 2

SOLUTION:

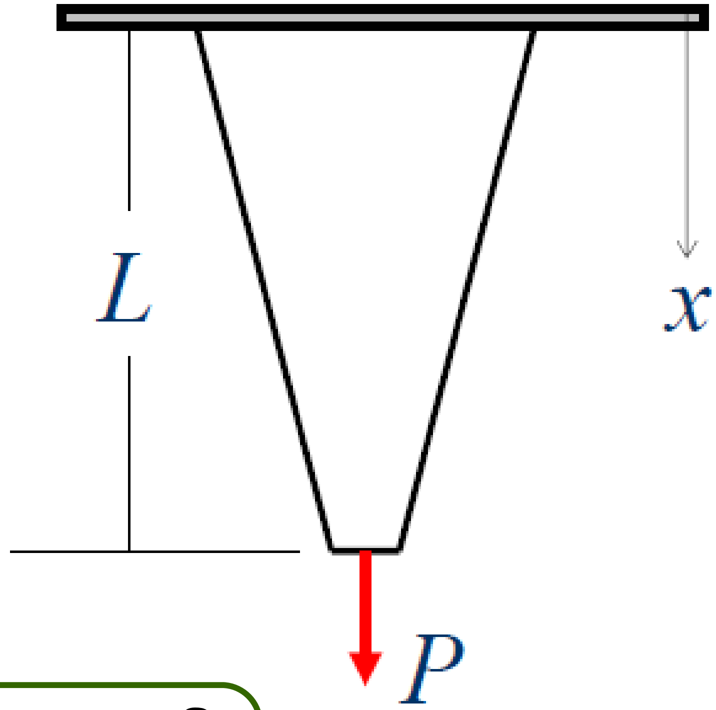




Displacement of E:

Deformations of Members under Axial Loading

▪ Nonuniform Deformation



$$\varepsilon = \frac{\sigma}{E} \quad \& \quad \sigma = \frac{P}{A} \quad \& \quad d\delta = \varepsilon\, dx \quad \Rightarrow$$

$$d\delta = \frac{\sigma}{E}\, dx = \frac{P}{EA}\, dx \quad \Rightarrow$$

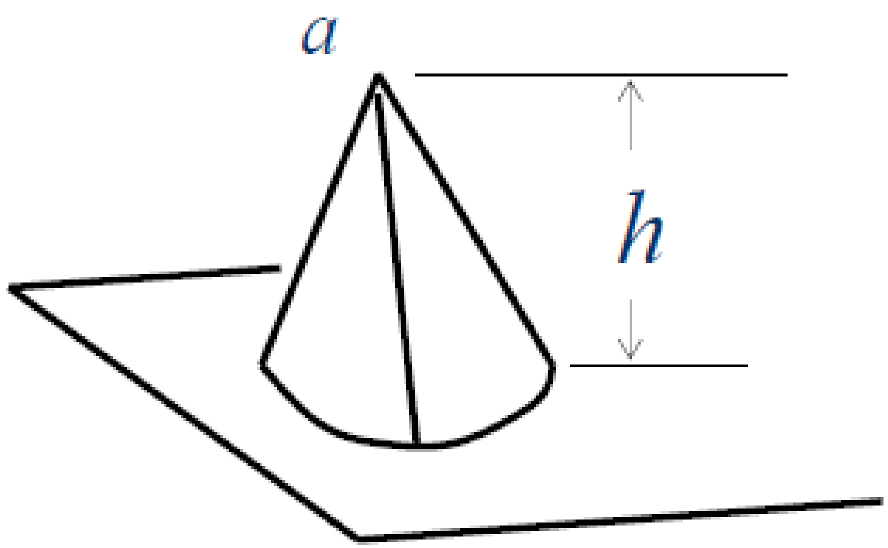
$$\varepsilon = \frac{d\delta}{dx}$$

$$\delta = \int_0^l \frac{P_{(x)}}{E_{(x)} A_{(x)}}\, dx$$

❑ *Deformations of Members under Axial Loading*

Example 3

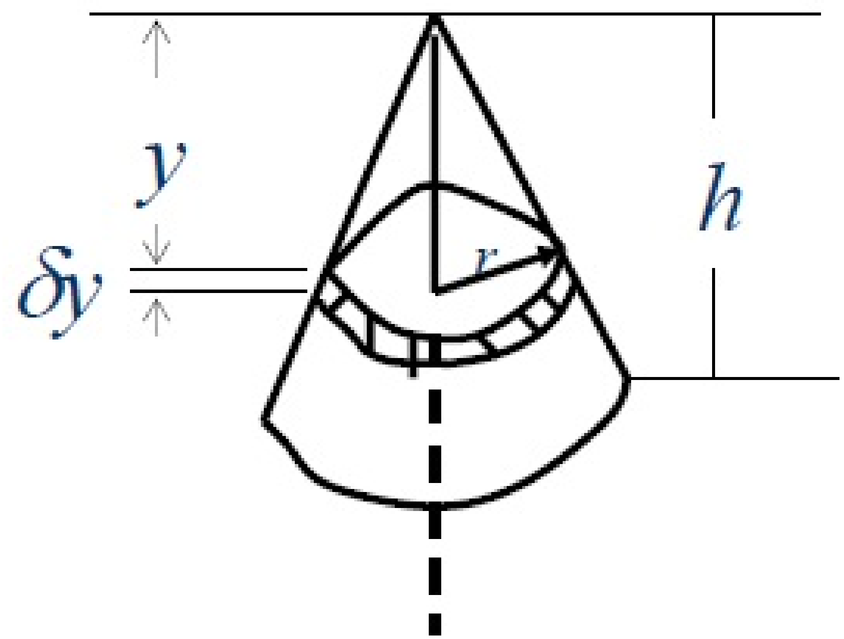
– Determine the deflection of point a of a homogeneous circular cone of height h, density ρ, and modulus of elasticity E due to its own weight.



❑ *Deformations of Members under Axial Loading*

Example 3

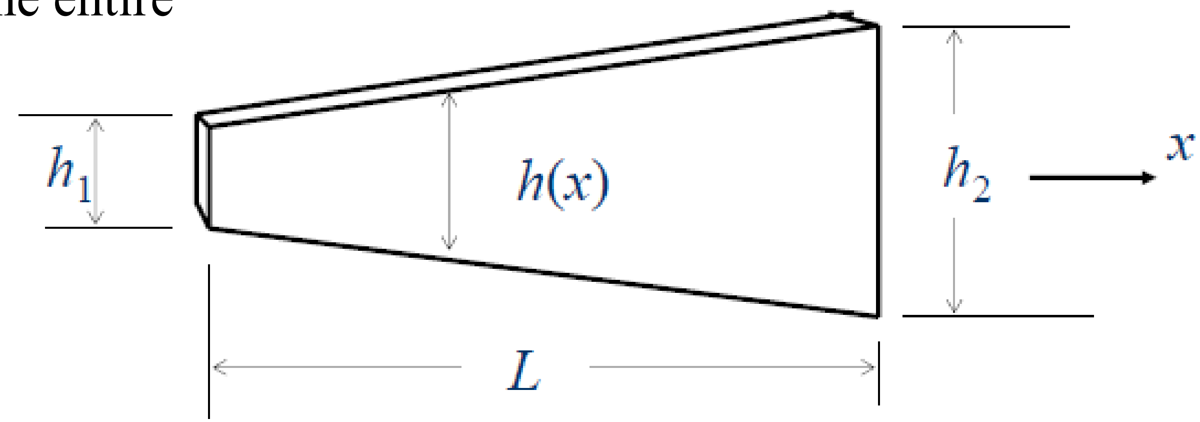
Consider a slice of thickness dy
P = weight of above slice



❑ *Deformations of Members under Axial Loading*

▪ Normal Stresses in Tapered Bar

– Consider the following tapered bar with a ***thickness t that is constant*** along the entire length of the bar.



$$h_{(x)} = h_1 + (h_2 - h_1)\frac{x}{L}$$

$$A_{(x)} = t \cdot h_{(x)} = t \cdot \left[h_1 + (h_2 - h_1)\frac{x}{L} \right]$$

Stress and Strain – Axial Loading

❑ *Deformations of Members under Axial Loading*

▪ Normal Stresses in Tapered Bar

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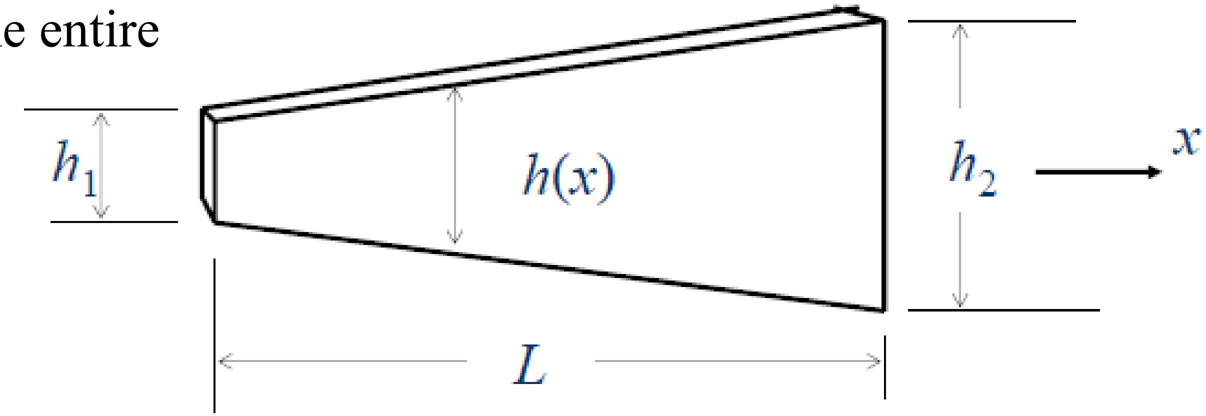


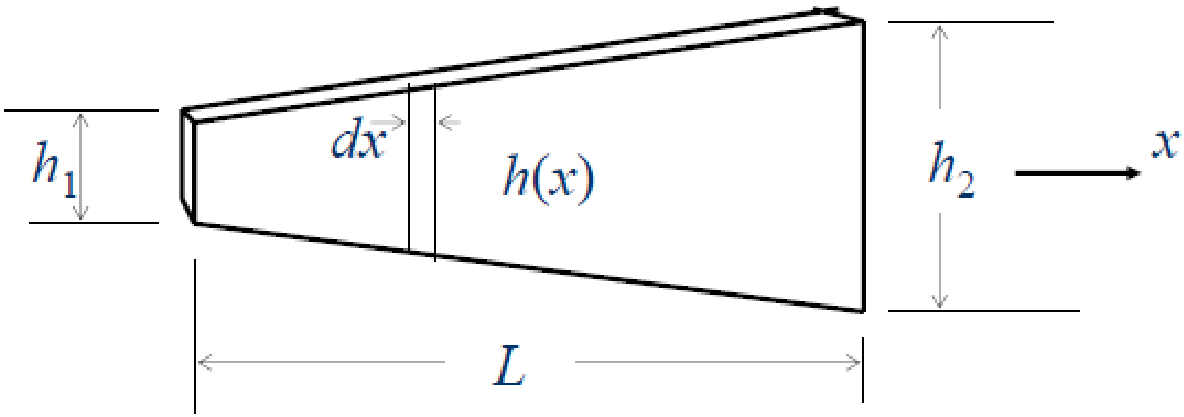
$$\sigma_{(x)} = \frac{P}{A_{(x)}} = \frac{P}{t \cdot \left[h_1 + (h_2 - h_1)\dfrac{x}{L} \right]}$$

Stress and Strain – Axial Loading

- ***Deformations of Members under Axial Loading***

▪ Deflection of Tapered Bar

– Consider the following tapered bar with a ***thickness t that is constant*** along the entire length of the bar.



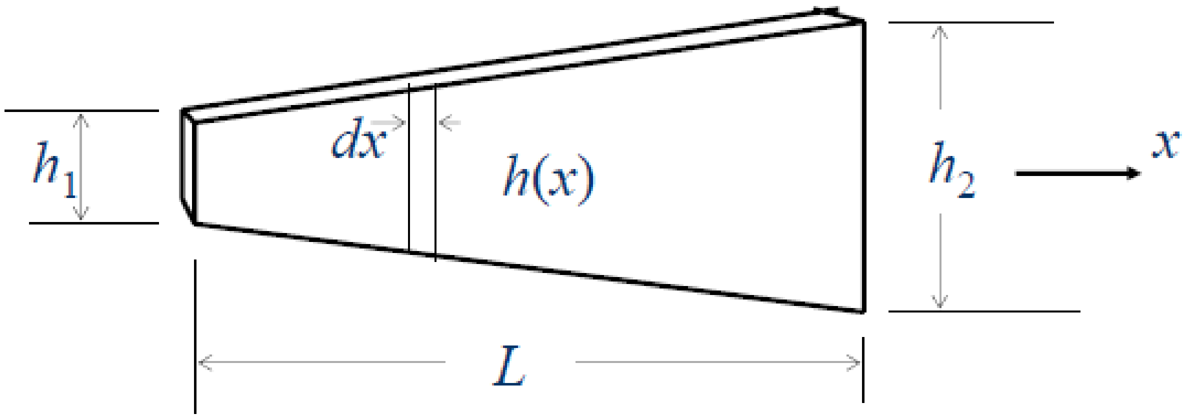
$$\delta = \int_0^l \frac{P_{(x)}}{E_{(x)} A_{(x)}} dx = \frac{PL}{Et} \int_0^L \frac{1}{h_1 L + (h_2 - h_1) x} dx$$

Stress and Strain – Axial Loading

❑ *Deformations of Members under Axial Loading*

▪ Deflection of Tapered Bar

– Consider the following tapered bar with a ***thickness t that is constant*** along the entire length of the bar.



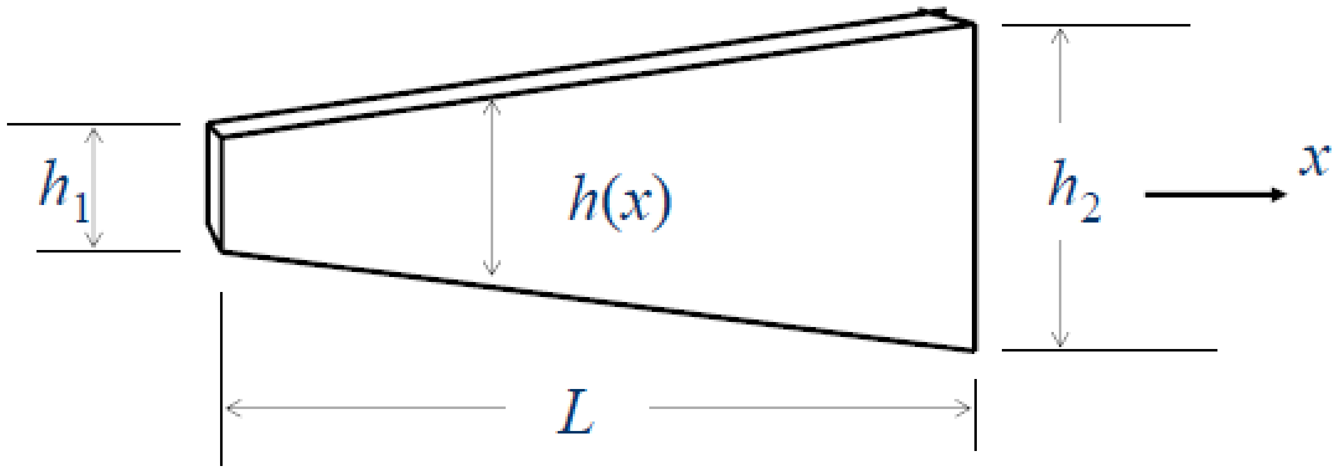
$$\delta = \frac{PL}{Et}\left(\frac{1}{h_2 - h_1}\right)\ln\left[(h_2 - h_1)L\right]$$

Stress and Strain – Axial Loading

Deformations of Members under Axial Loading

Example 4

- Determine the normal stress as a function of x along the length of the tapered bar shown if
- h1 = 2 in
- h2 = 6 in
- t = 3 in, and
- L = 36 in
- P = 5,000 lb



Deformations of Members under Axial Loading

Example 4

x (in)	σ (psi)
0	833.3
3	714.3
6	625.0
9	555.6
12	500.0
15	454.5
18	416.7
21	384.6
24	357.1
27	333.3
30	312.5
33	294.1
36	277.8

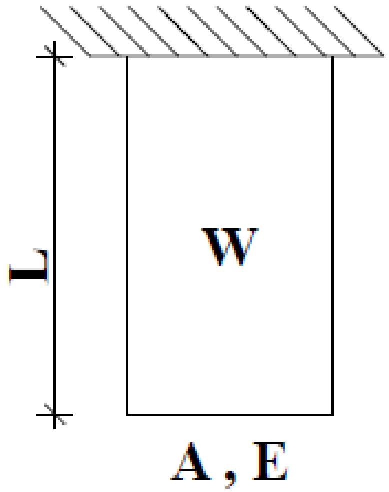
❑ *Deformations of Members under Axial Loading*

Example 5

– Determine the displacement at the end of the cylindrical bar under it's weight W.

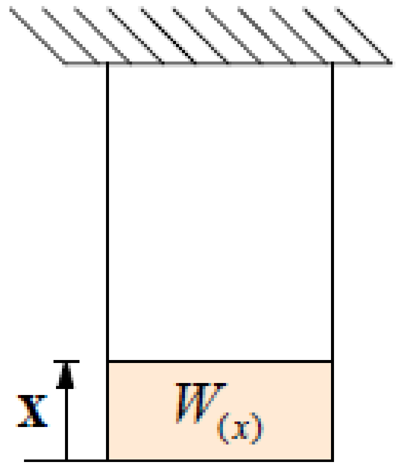
(A: Cross Section)

(E: Modulus of Elasticity)

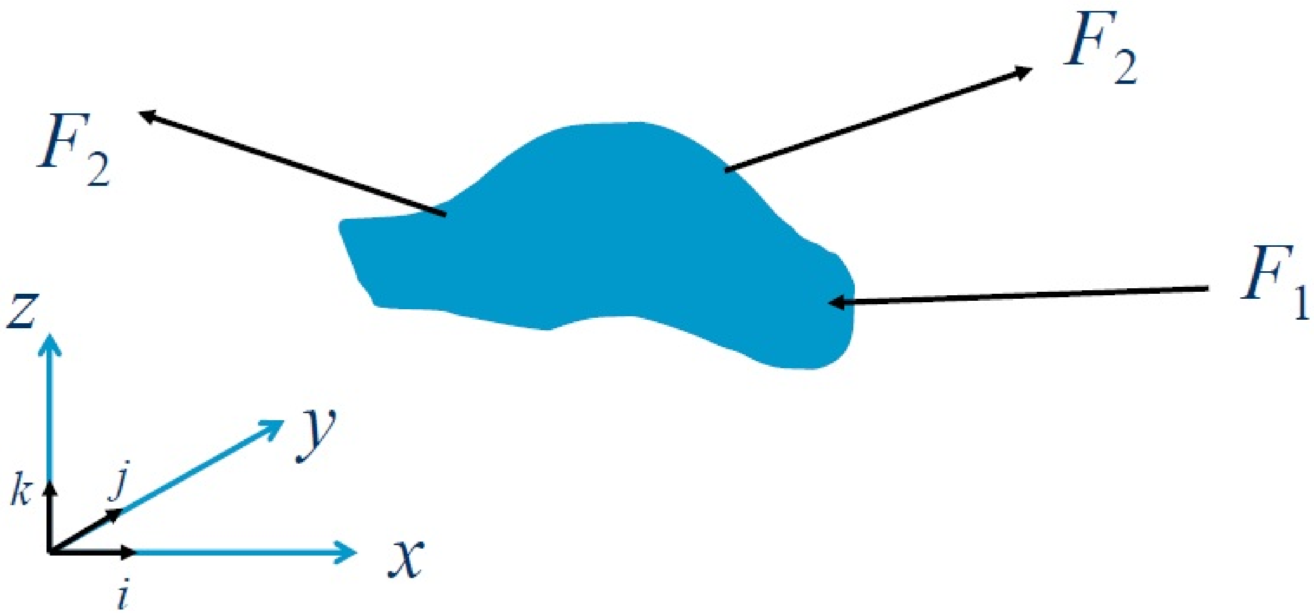


- ***Deformations of Members under Axial Loading***

Example 5



❑ *Statically Indeterminate Structures*



$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

❑ *Statically Indeterminate Structures*

- **Statically Determinate Member**

When equations of equilibrium are sufficient to determine the forces and stresses in a structural member, we say that the problem is ***statically determinate***

❑ *Statically Indeterminate Structures*

- **Statically Indeterminate Member**

When the equilibrium equations alone are not sufficient to determine the loads or stresses, then such problems are referred to as ***statically indeterminate*** problems.

❑ *Statically Indeterminate Structures*

▪ Determinacy of Beams

For a coplanar (two-dimensional) structure, there are at most three equilibrium equations for each part, so that if there is a total of ***n parts*** and ***r reactions***, we have

$$r = 3n \quad \Rightarrow \quad \text{statically determinate}$$

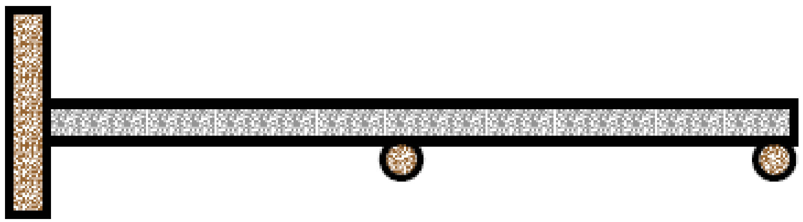
$$r > 3n \quad \Rightarrow \quad \text{statically indeterminate}$$

❑ *Statically Indeterminate Structures*

Example 6

– Classify each of the beams shown as statically determinate or statically indeterminate.

I



II

III

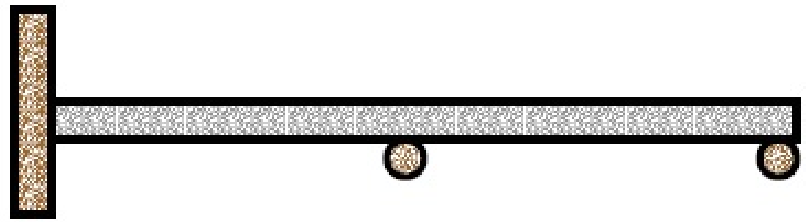
❑ *Statically Indeterminate Structures*

Example 6

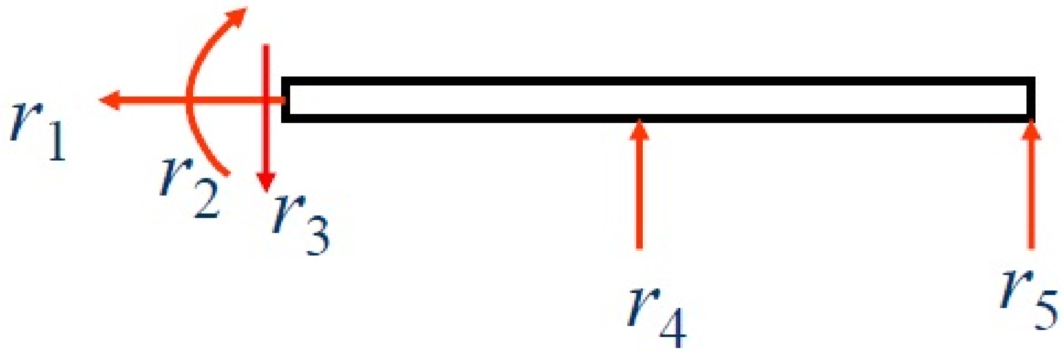
-For part I:

❑ *Statically Indeterminate Structures*

Example 5



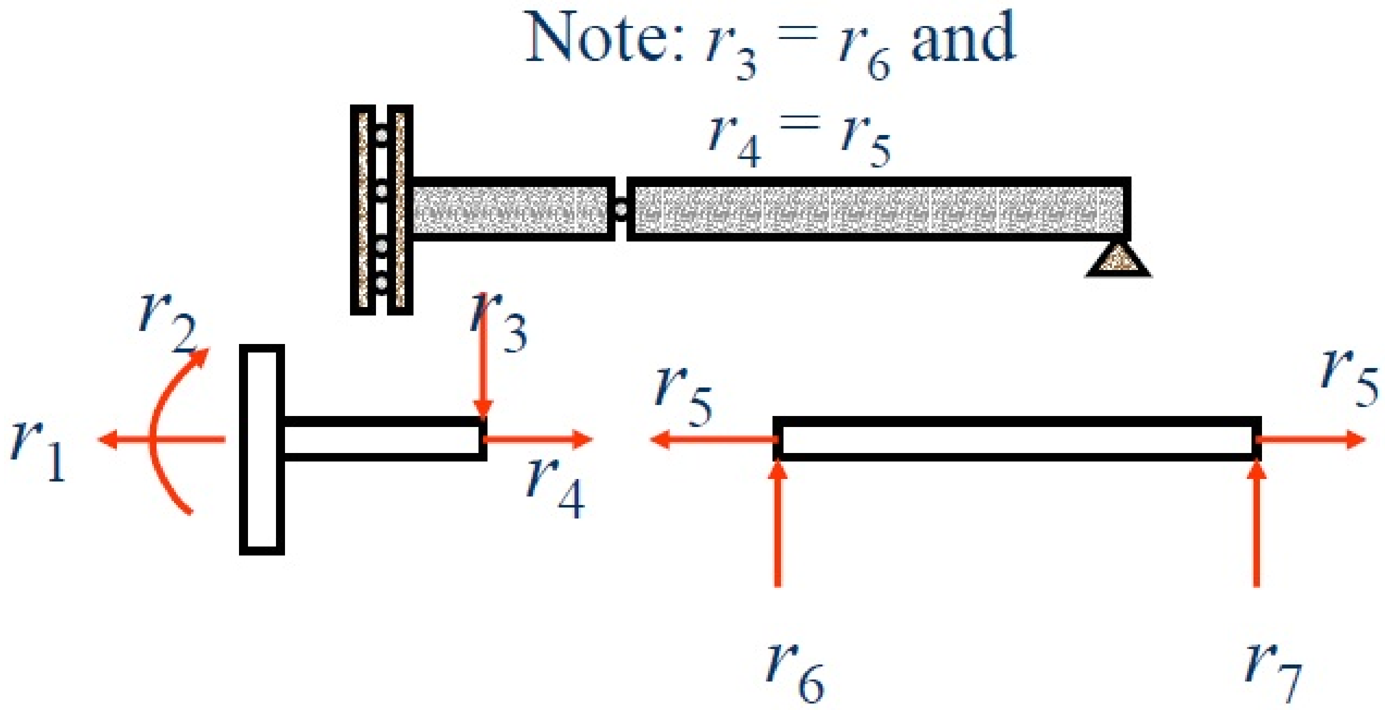
-For part II:



❑ *Statically Indeterminate Structures*

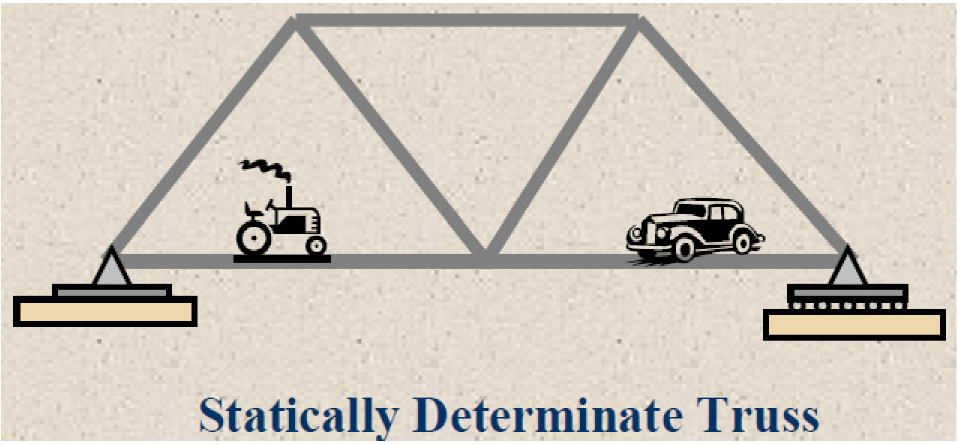
Example 5

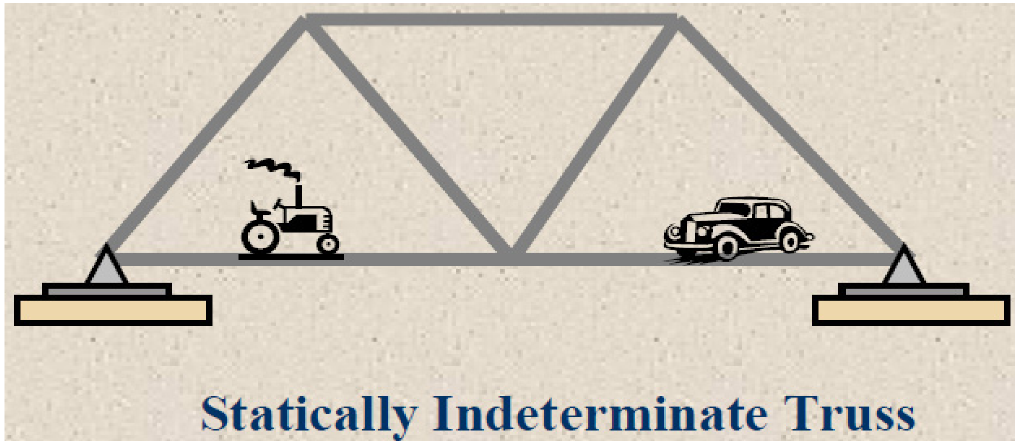
-For part III:



❑ *Statically Indeterminate Structures*

Determinacy of Trusses





$$m + r = 2n \quad \Rightarrow \quad \text{statically determinate}$$

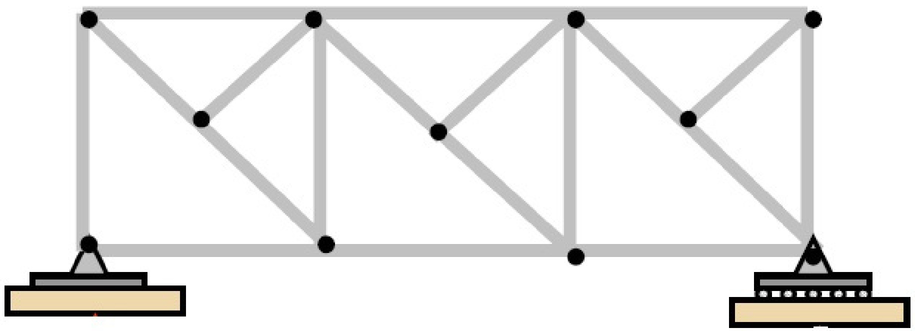
$$m + r > 2n \quad \Rightarrow \quad \text{statically indeterminate}$$

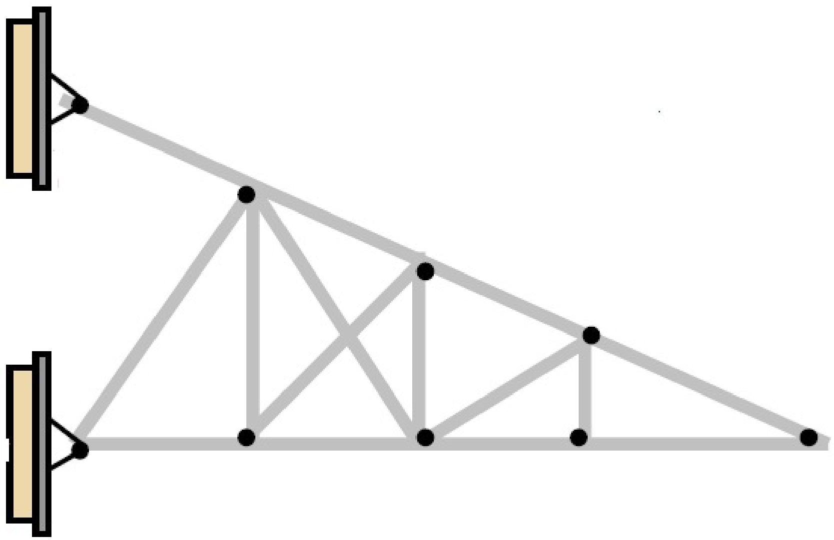
m: members   n: nodes   r: reactions of supports

❑ *Statically Indeterminate Structures*

Example 7

– Classify each of the trusses shown as statically determinate or statically indeterminate.

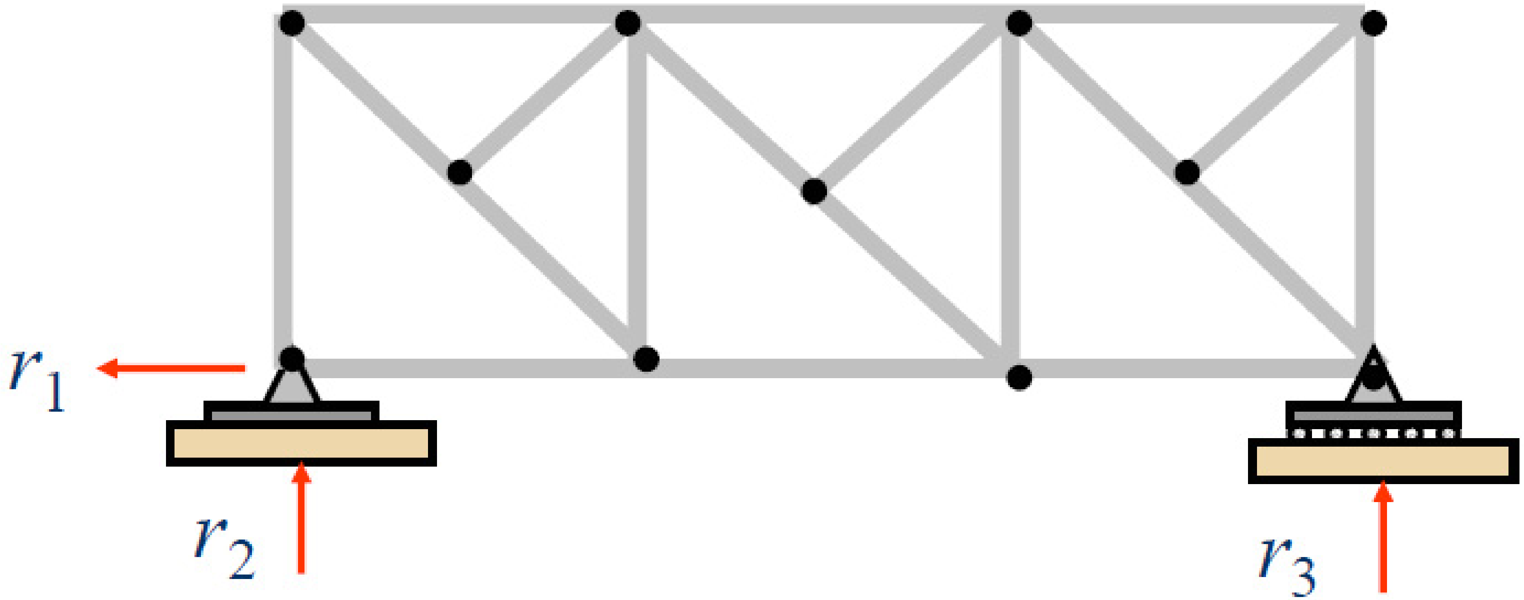




❑ *Statically Indeterminate Structures*

Example 7

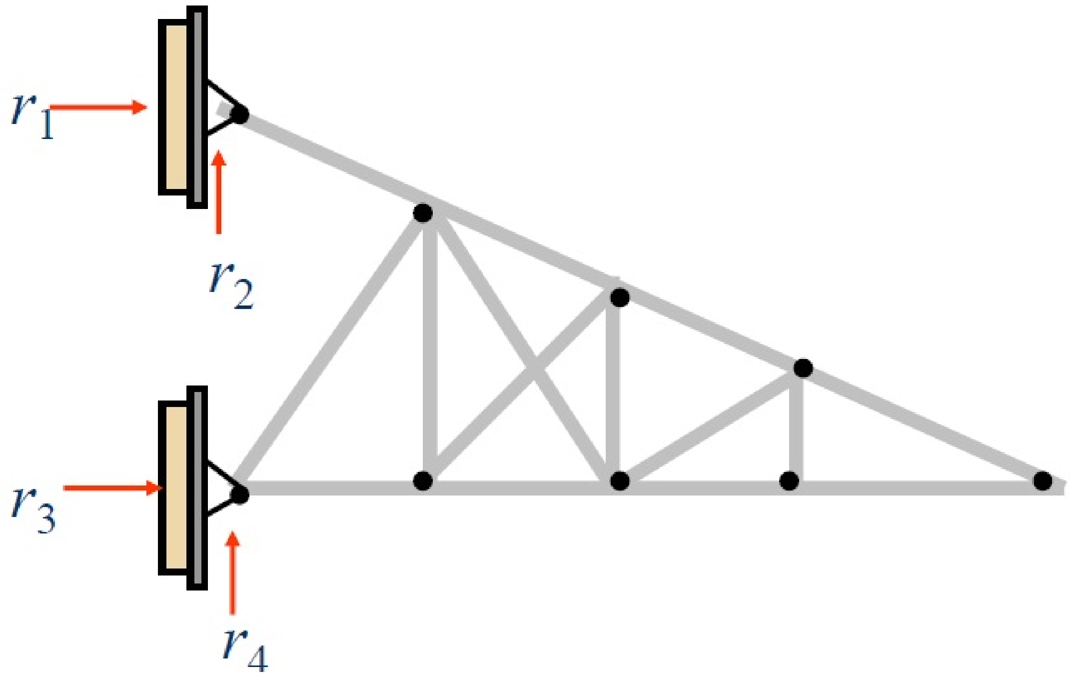
-For part I:



❑ *Statically Indeterminate Structures*

Example 7

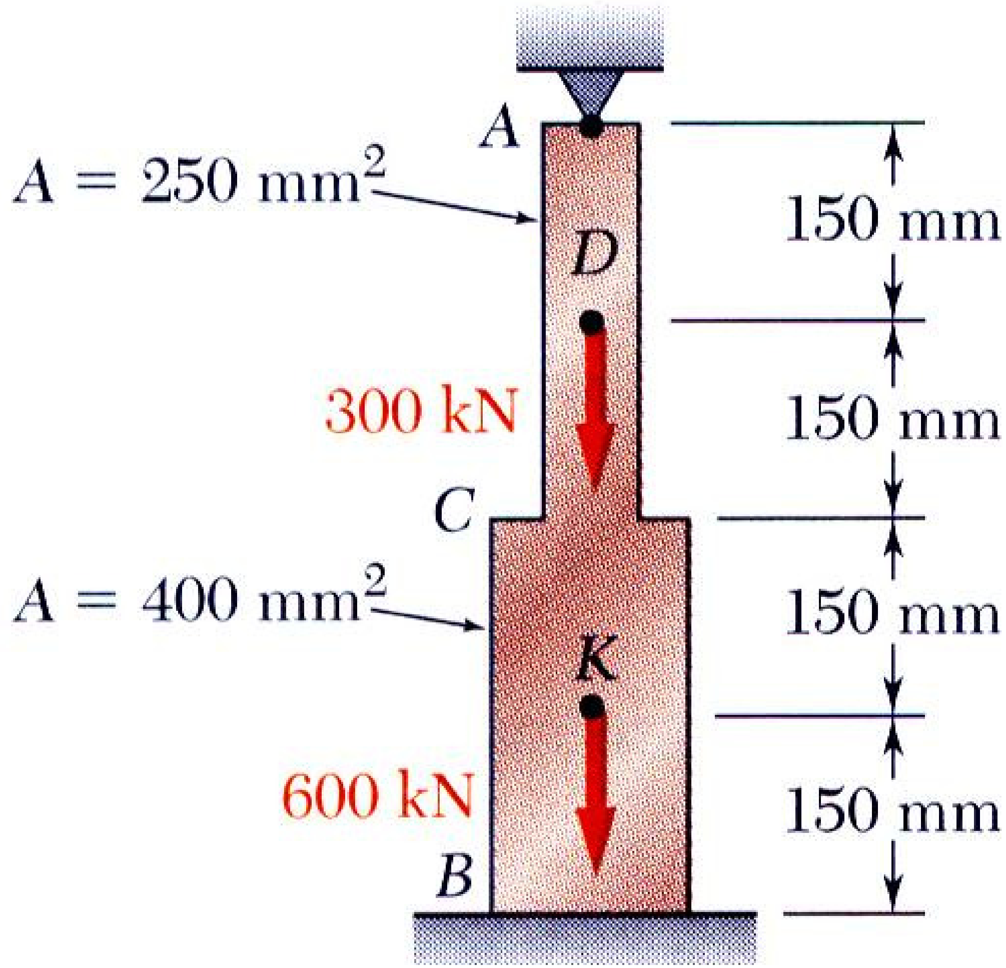
-For part II:



❑ *Statically Indeterminate Structures*

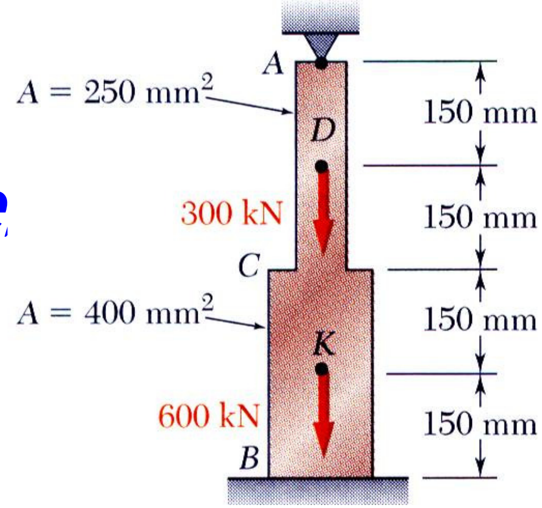
Example 8

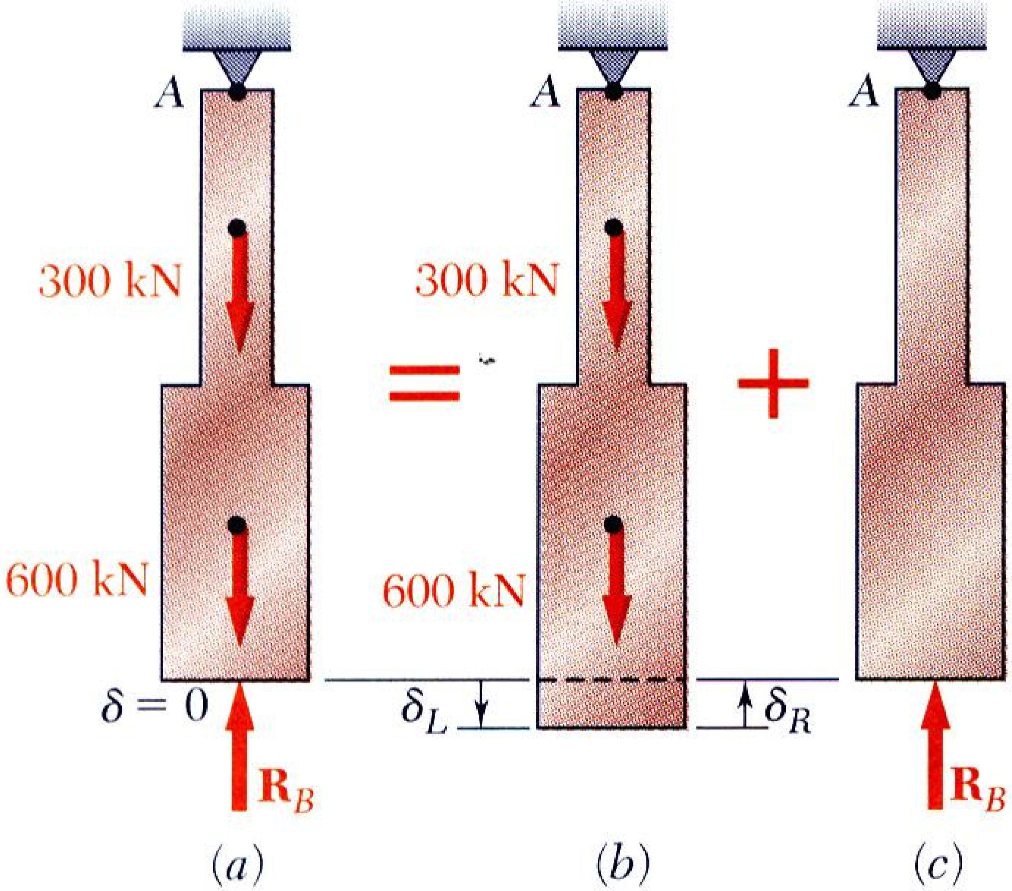
Determine the reactions at A and B for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.



❑ *Statically Indeterminate Structures*

Example 8





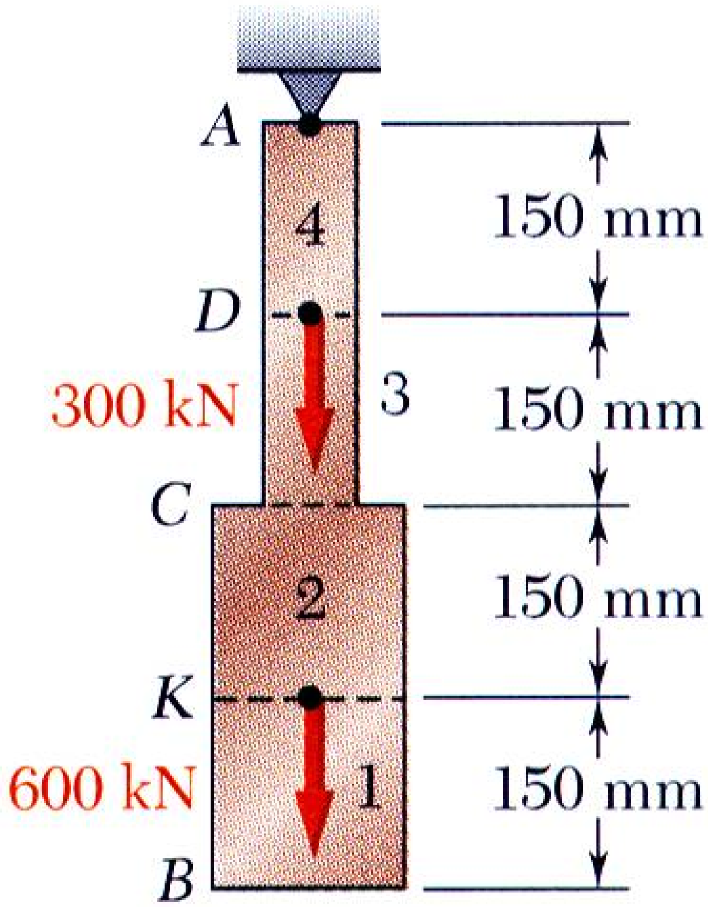
SOLUTION:

- Consider the reaction at *B* as redundant, release the bar from that support, and solve for the displacement at *B* due to the applied loads.

- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.

❑ *Statically Indeterminate Structures*

Example 8

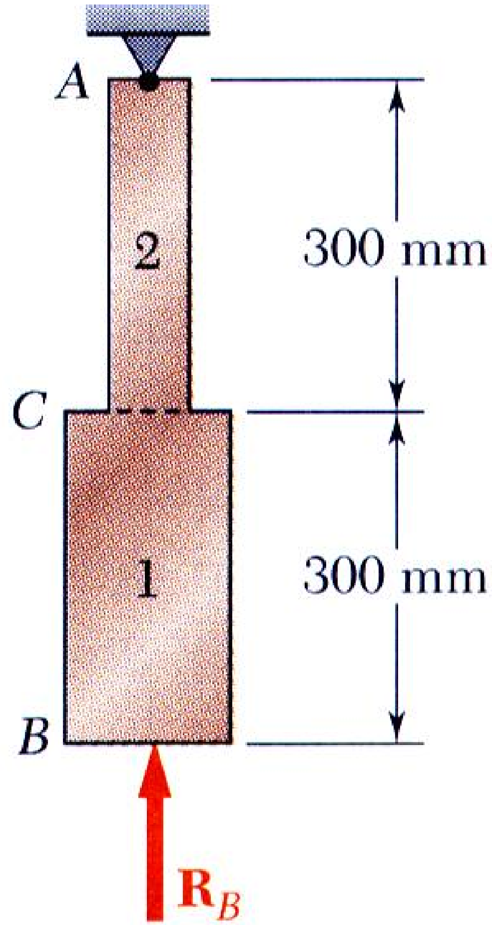


SOLUTION:

- Solve for the displacement at B due to the applied loads with the redundant constraint released,

❑ *Statically Indeterminate Structures*

Example 8

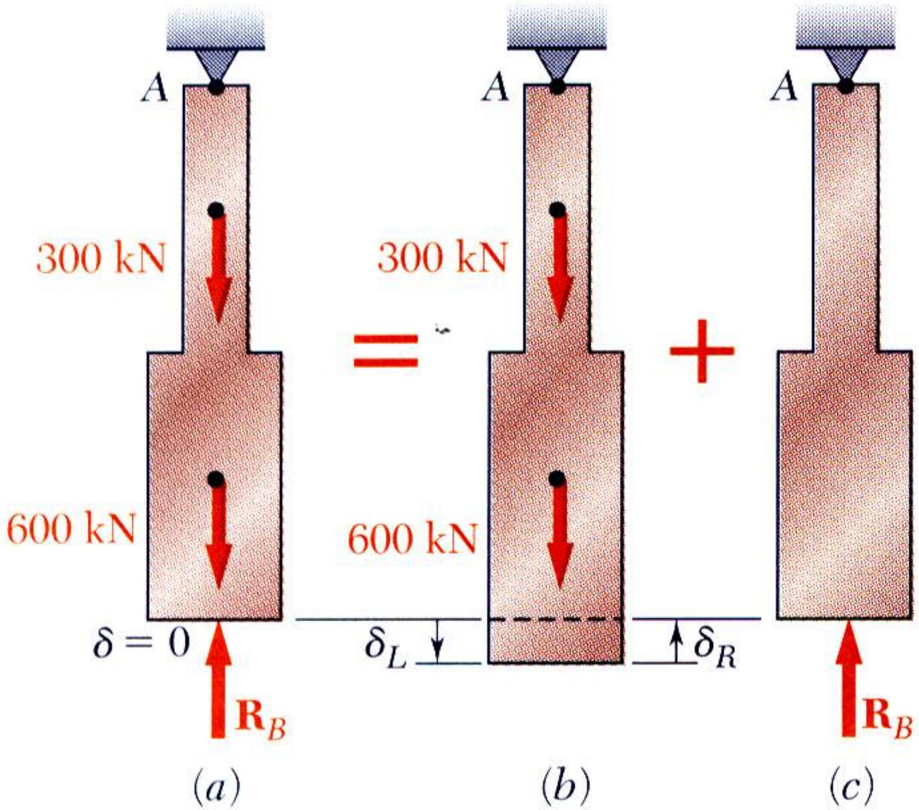


- Solve for the displacement at *B* due to the redundant constraint,

❑ *Statically Indeterminate Structures*

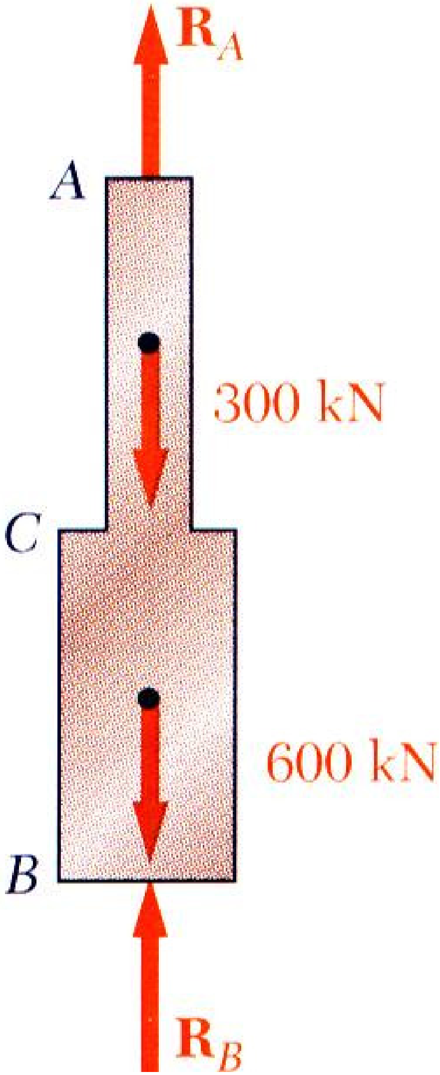
Example 8

- Require that the displacements due to the loads and due to the redundant reaction be compatible,



❑ *Statically Indeterminate Structures*

Example 8



- Find the reaction at *A* due to the loads and the reaction at *B*

❑ *Statically Indeterminate Structures*

Example 9

A rigid plate C is used to transfer a 20-kip load P to rod A and pipe B, as shown. The supports at the top of the rod and bottom of the pipe are rigid and there are no stresses in the rod or pipe before the load P applied.

$$E_A = 30000\ ksi \quad A_A = 0.8\ in^2$$

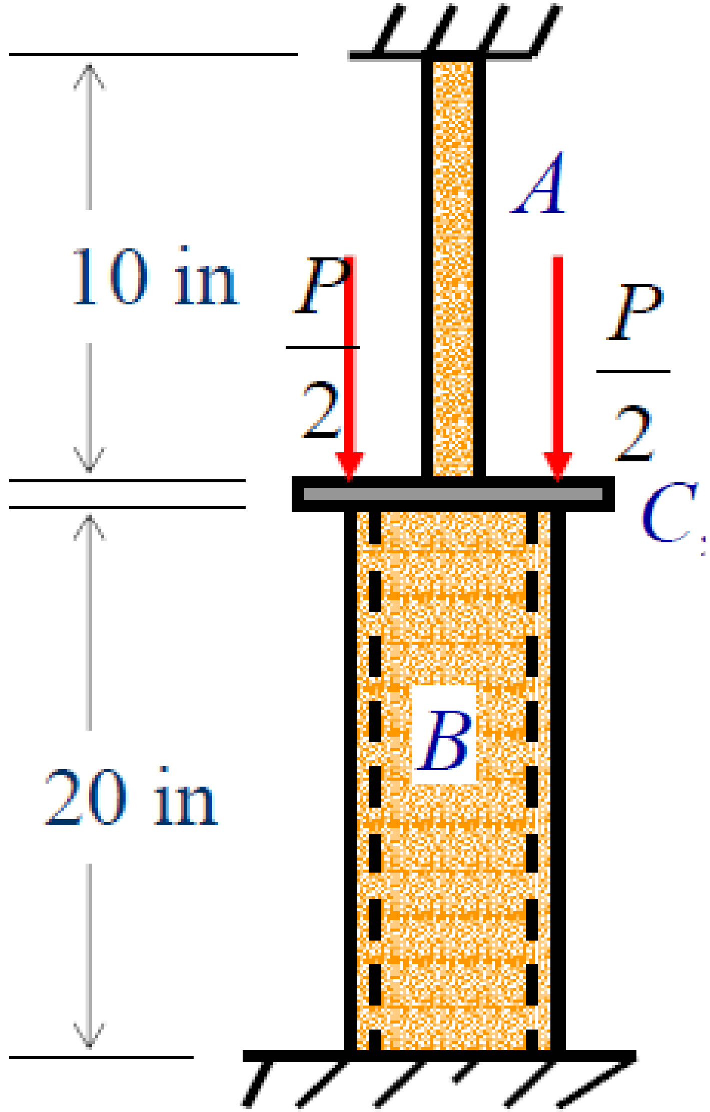
$$E_B = 10000\ ksi \quad A_B = 3.0\ in^2$$

Determine

(a) The axial stresses in rod A and pipe B.

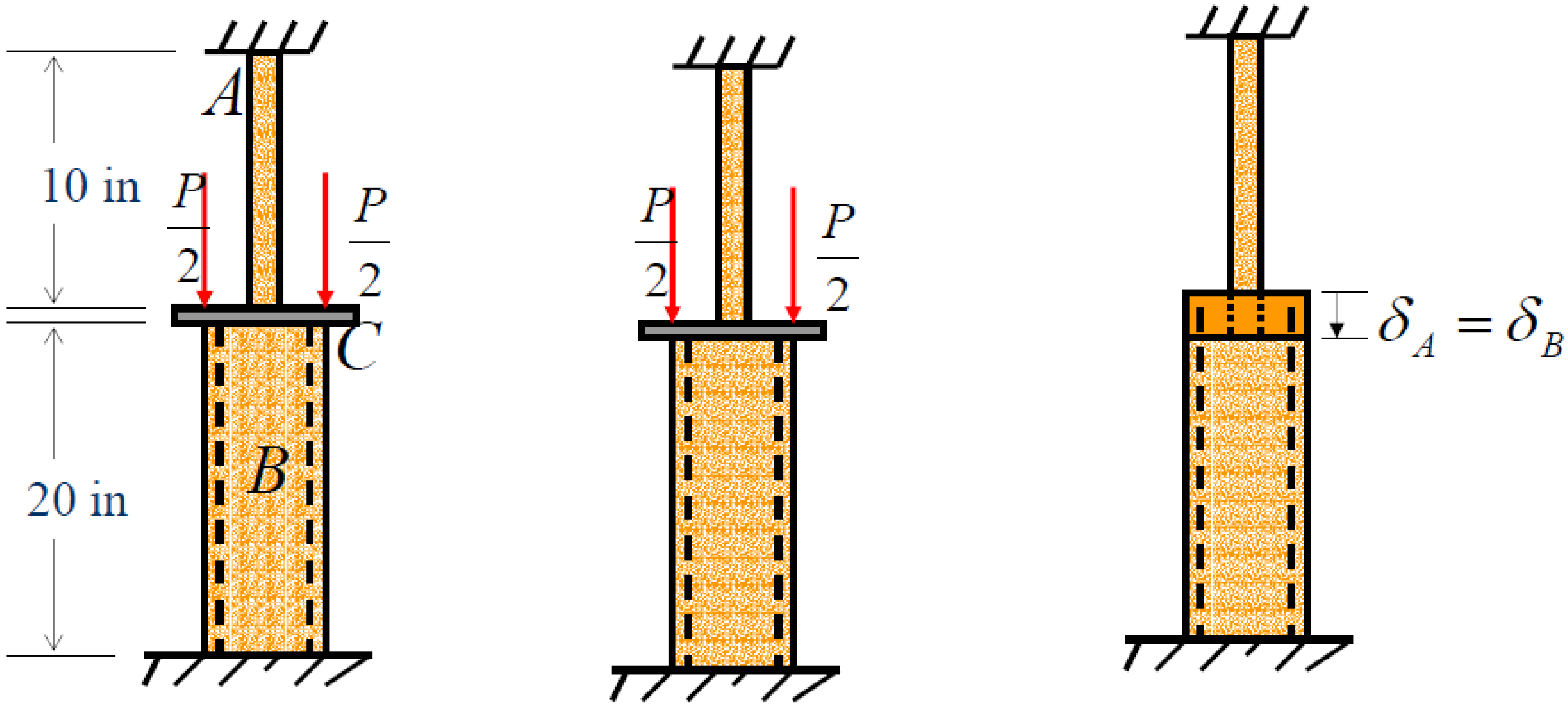
(b) The displacement of plate C.

(c) The reactions.



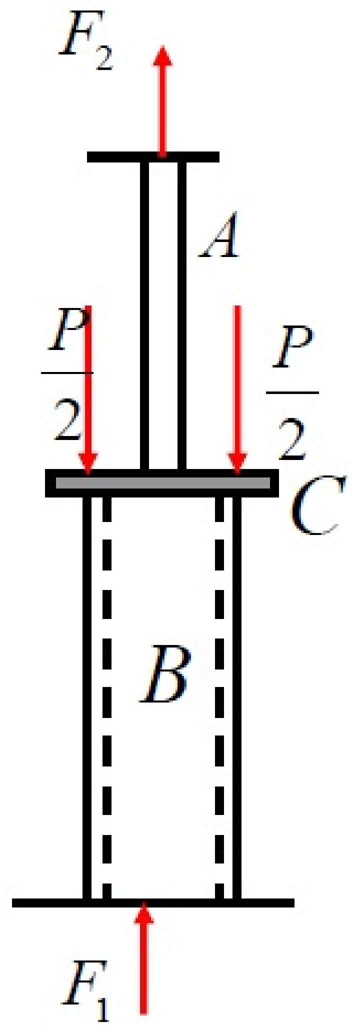
❑ *Statically Indeterminate Structures*

Example 9



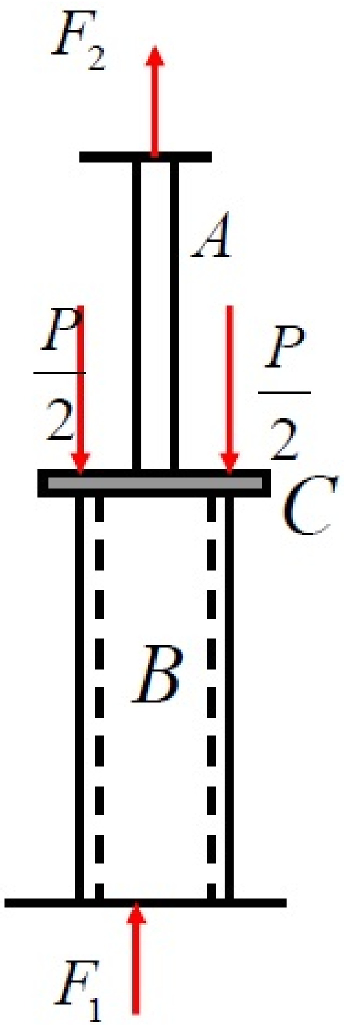
❑ *Statically Indeterminate Structures*

Example 9



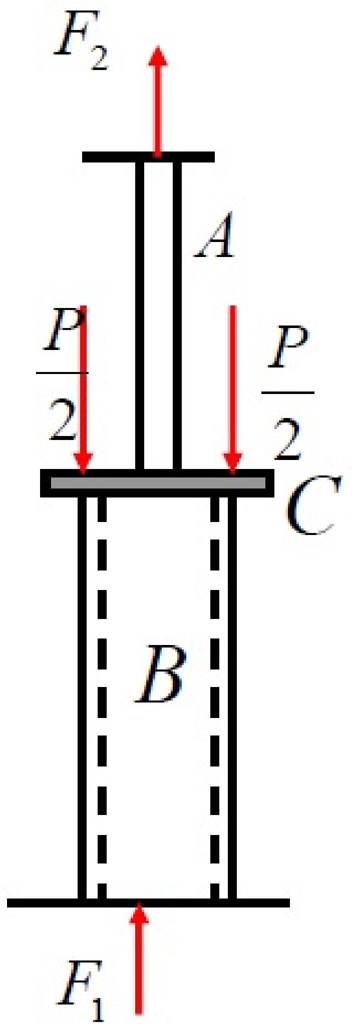
❑ *Statically Indeterminate Structures*

Example 9



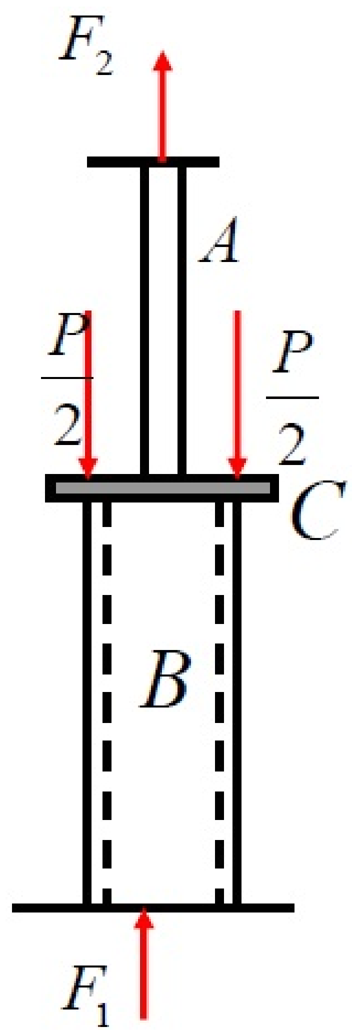
❑ *Statically Indeterminate Structures*

Example 9



❑ *Statically Indeterminate Structures*

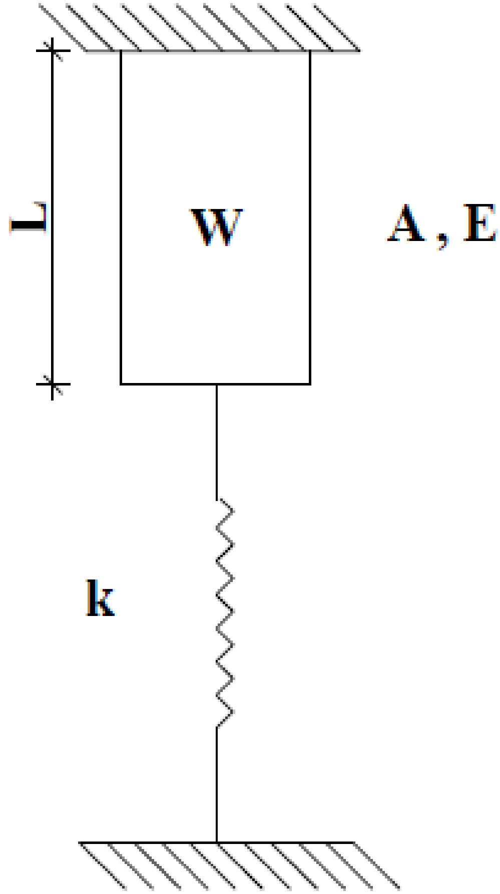
Example 9



❑ *Statically Indeterminate Structures*

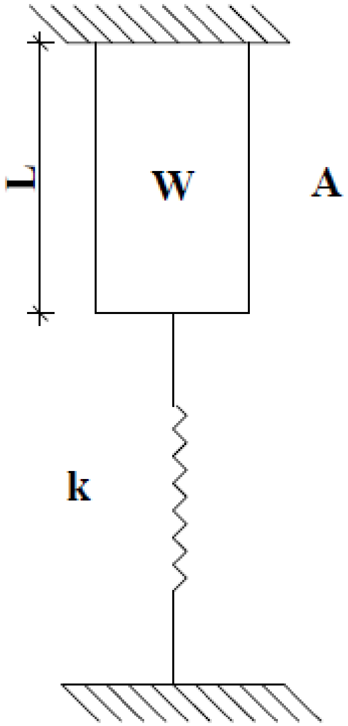
Example 10

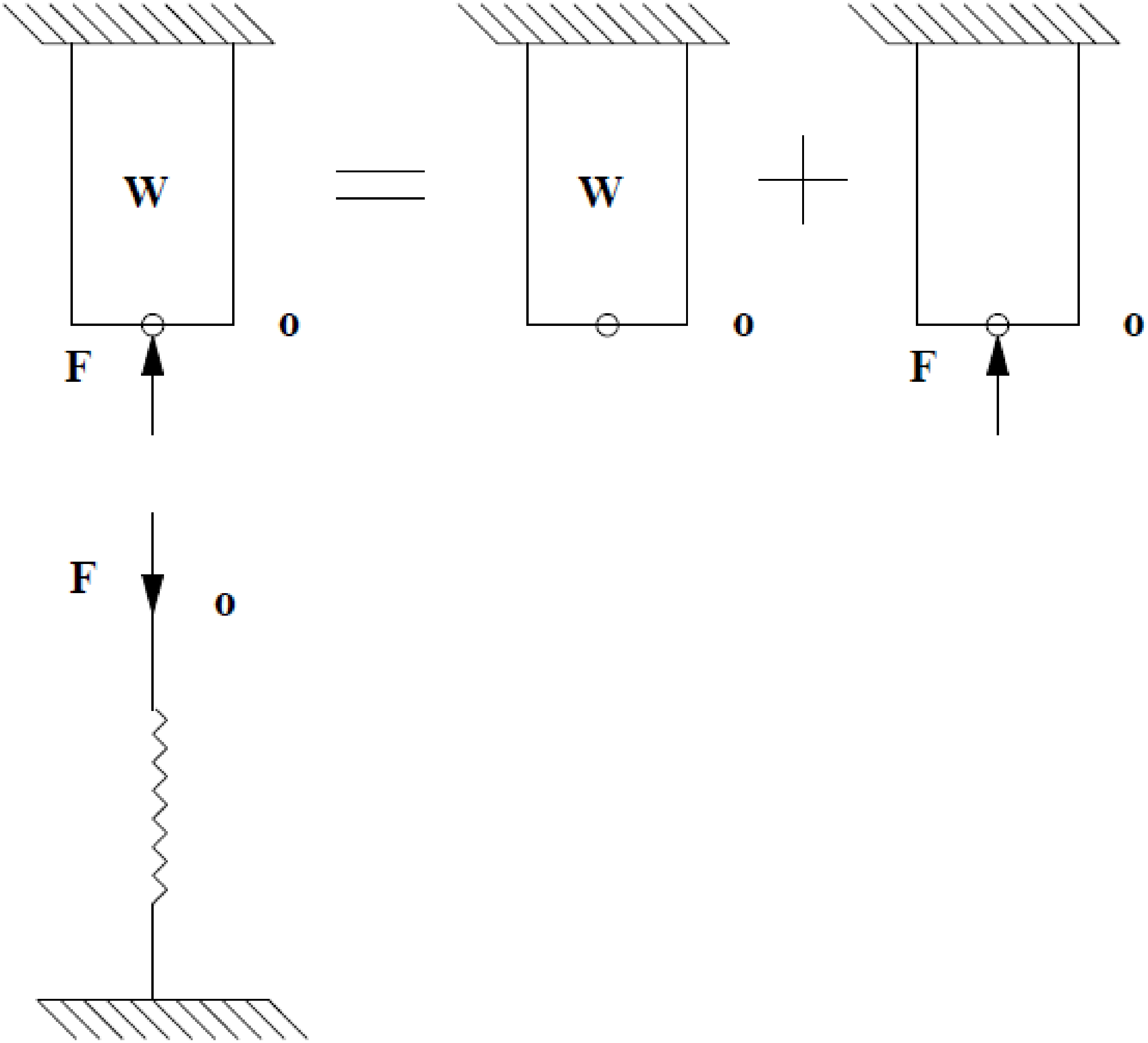
Determine the deformation of the spring.



❑ *Statically Indeterminate Structures*

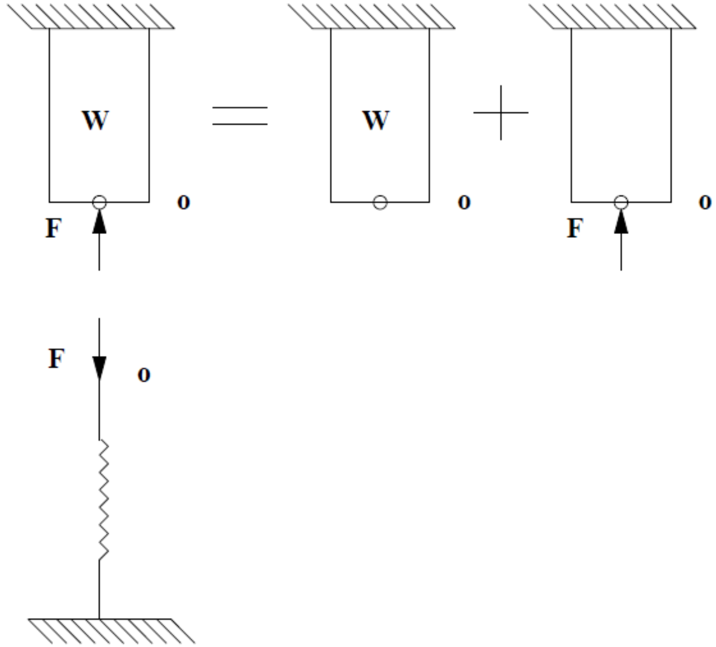
Example 10





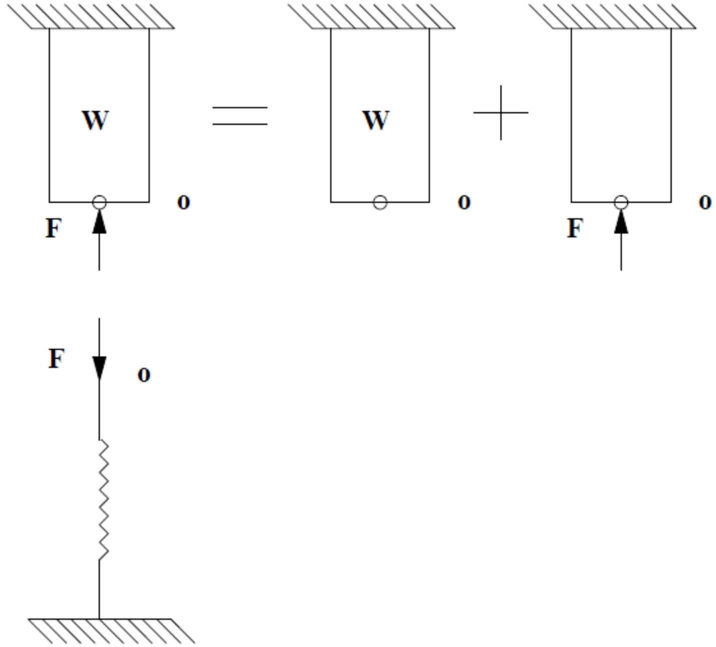
❑ *Statically Indeterminate Structures*

Example 10



❑ *Statically Indeterminate Structures*

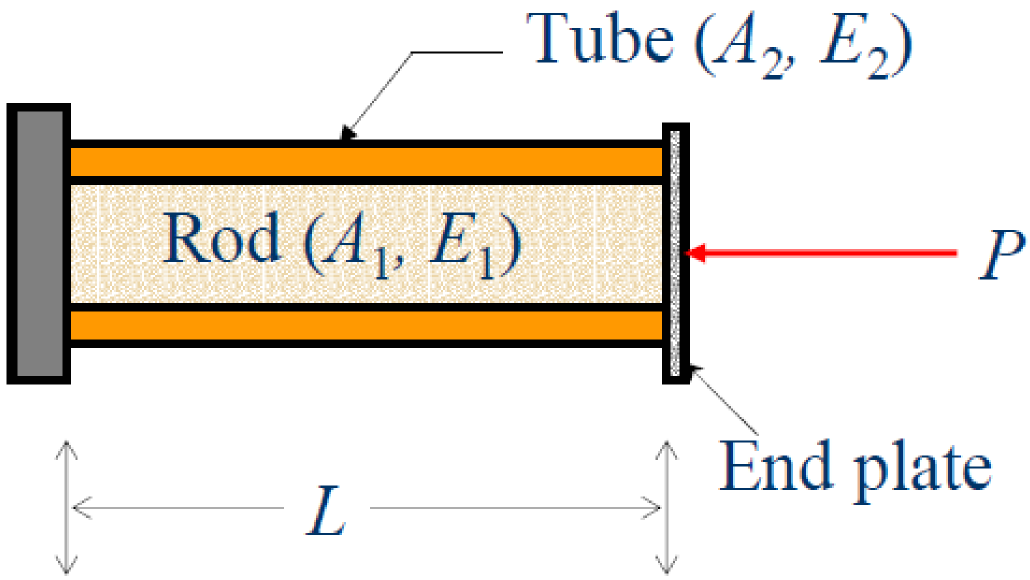
Example 10



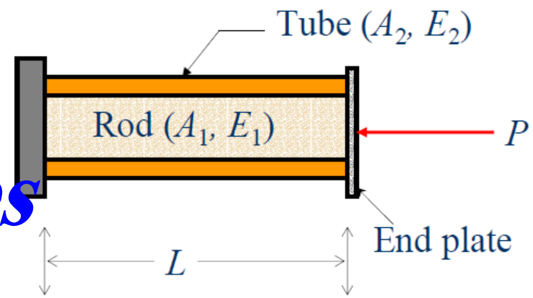
❑ *Statically Indeterminate Structures*

Example 11

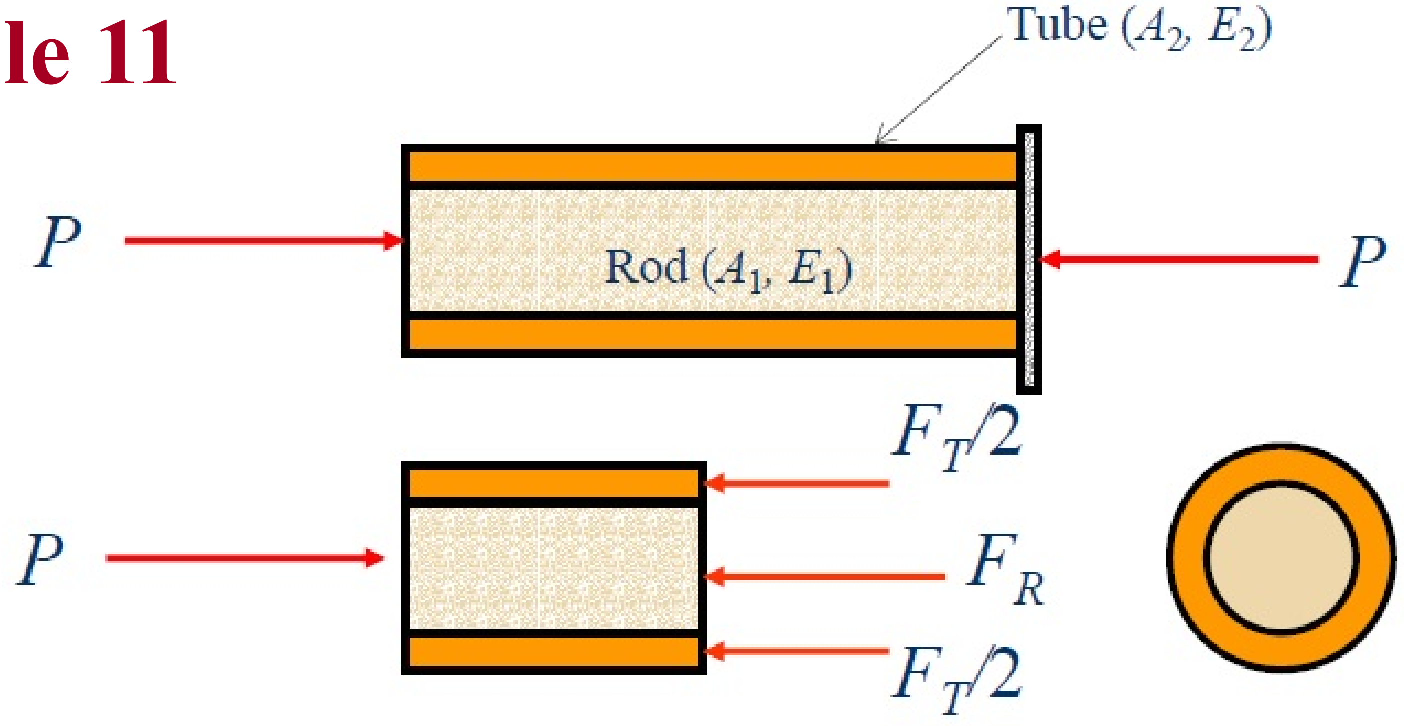
A rod of length L, cross-sectional area A1, and modulus of elasticity E1, has been placed inside a tube of the same length L, but of cross-sectional area A2 and modulus of elasticity E2. **What is the deformation of the rod and tube when a force P is exerted on a rigid end plate as shown? What are the internal forces in the rod and the tube?**



❑ *Statically Indeterminate Structures*

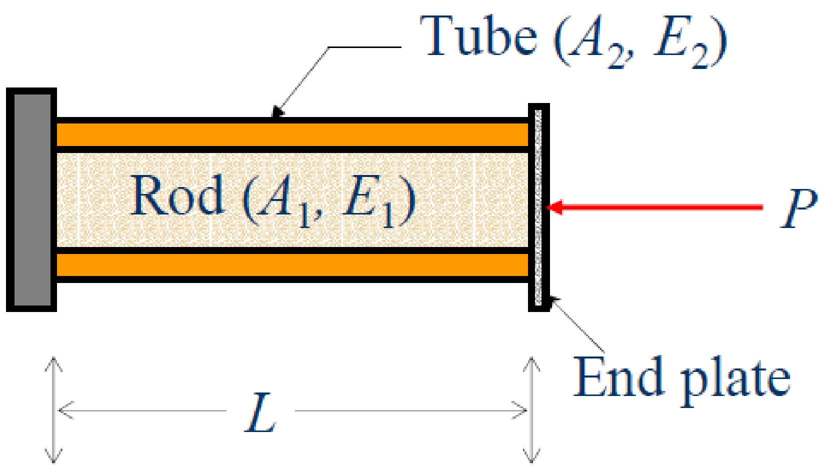


Example 11



❑ *Statically Indeterminate Structures*

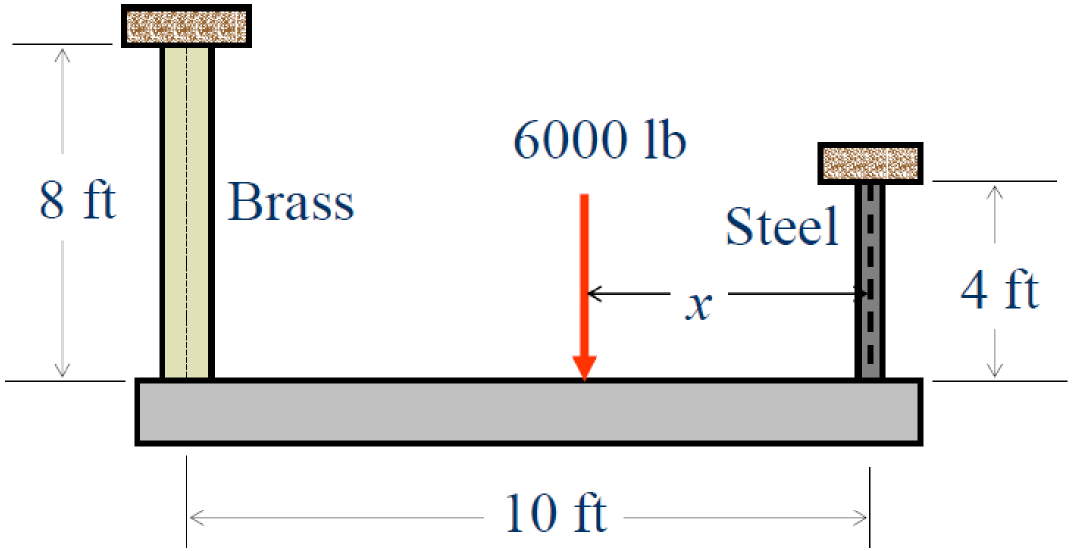
Example 11



❑ *Statically Indeterminate Structures*

Example 12

A very stiff bar of negligible weight is suspended horizontally by two vertical rods as shown. One of the rods is of steel, and is ½-in in diameter and 4 ft long; the other is of brass and is 7/8-in in diameter and 8 ft long. If a vertical load of 6000 lb is applied to the bar, where must be placed in order that the bar will remain horizontal? Also find the stresses in the brass and steel rods.

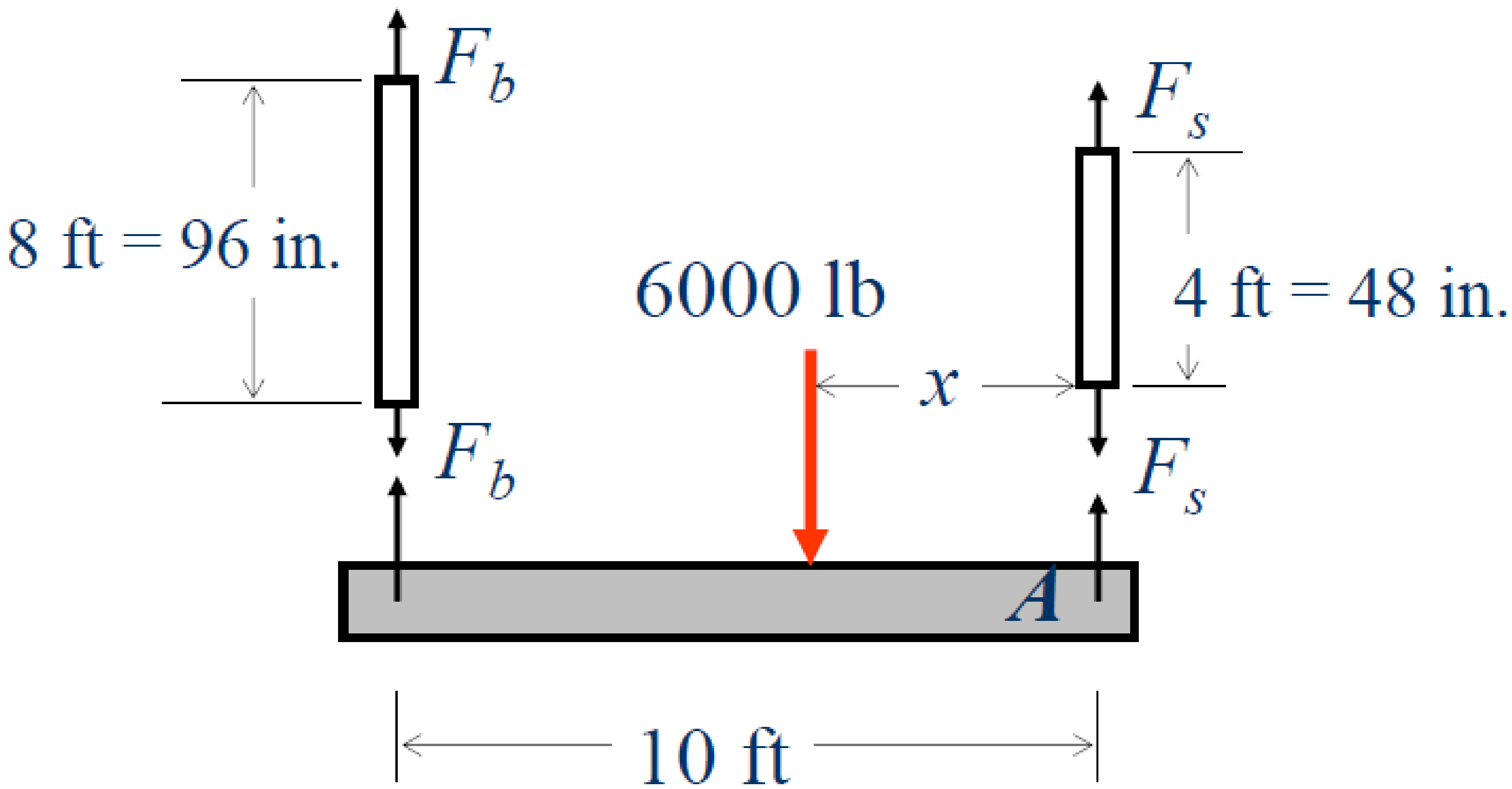


$$E_s = 30\times10^6 \ psi \qquad d_s = \frac{1}{2}\ in$$

$$E_b = 14\times10^6 \ psi \qquad d_b = \frac{7}{8}\ in$$

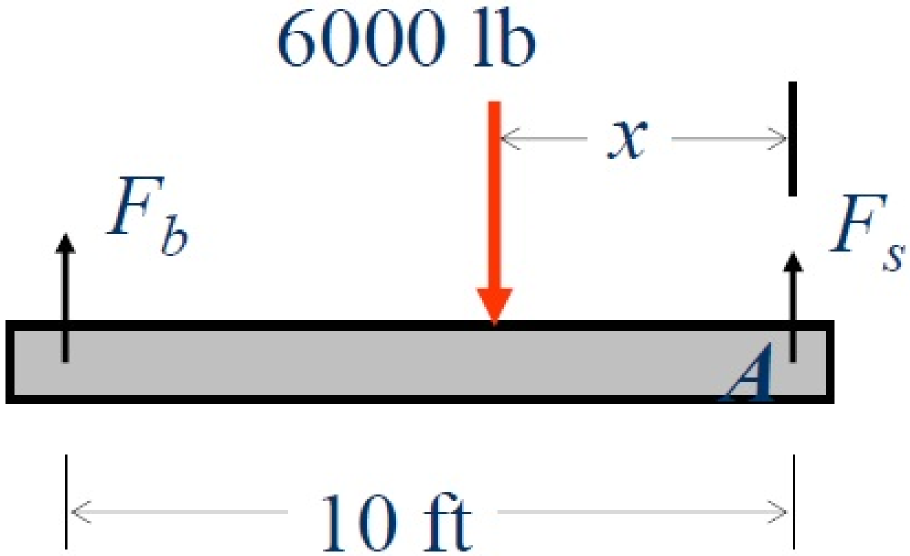
❑ *Statically Indeterminate Structures*

Example 12



❑ *Statically Indeterminate Structures*

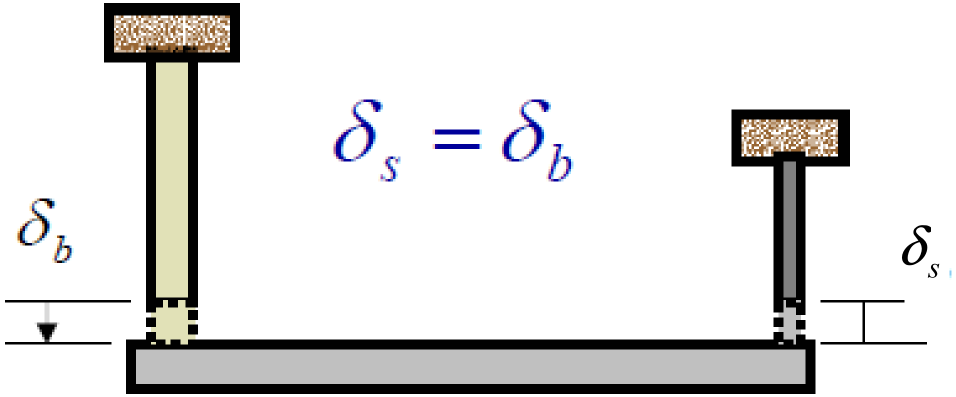
Example 12



$F_s\ ,F_b\ ,x=?$

❑ *Statically Indeterminate Structures*

Example 12

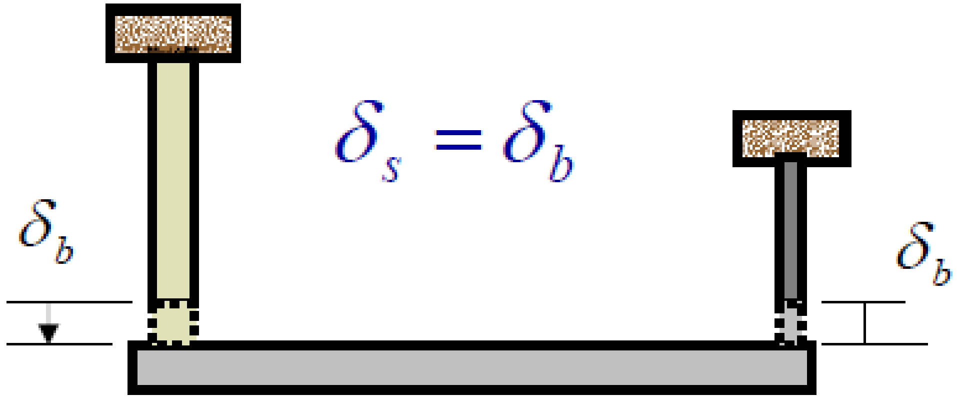


One additional independent equation is needed. The problem requires that the bar remain horizontal. Therefore, the rods must undergo equal elongations, that is

$$\delta_s = \delta_b$$

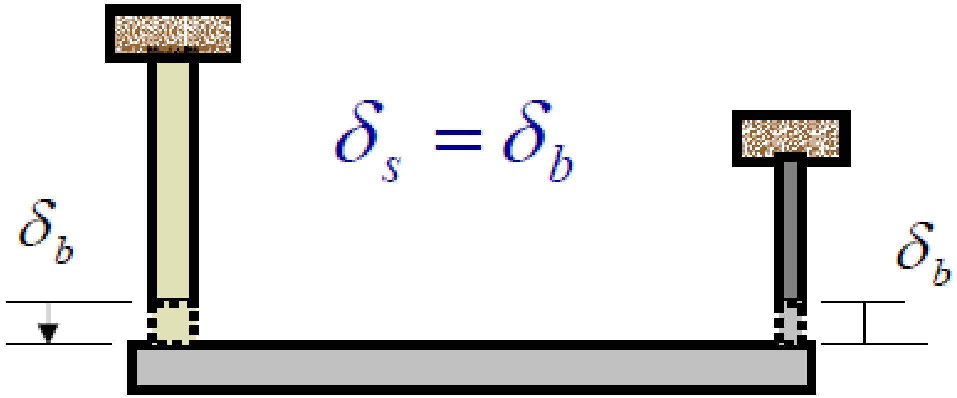
❑ *Statically Indeterminate Structures*

Example 12



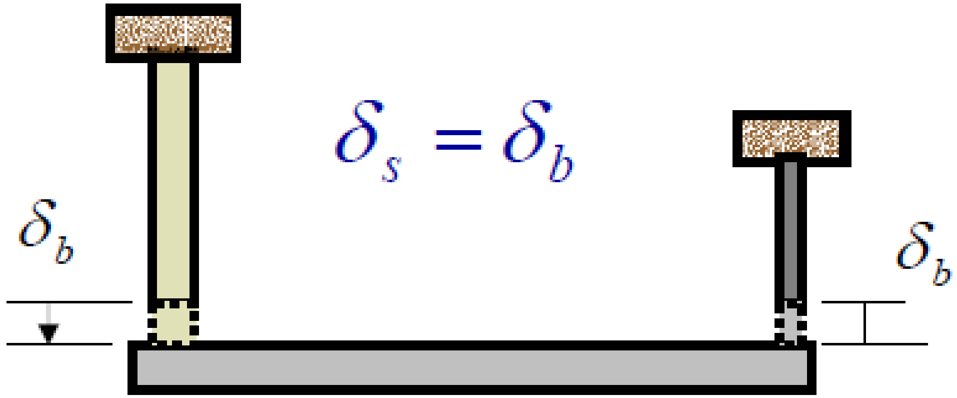
❑ *Statically Indeterminate Structures*

Example 12



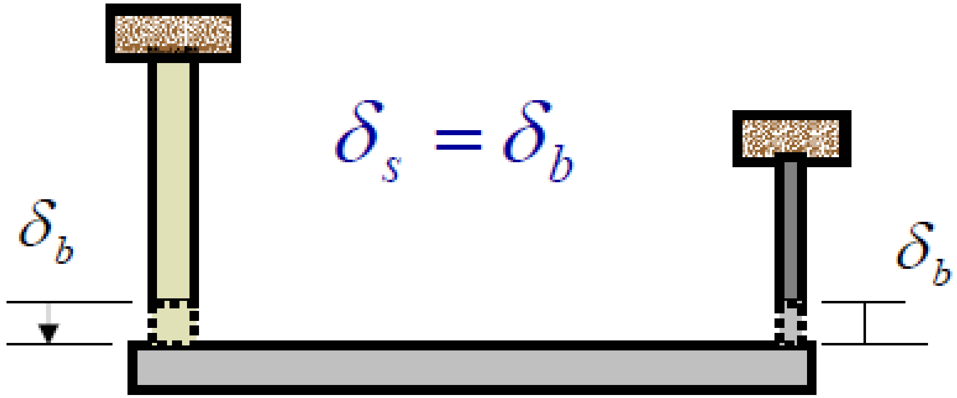
❑ *Statically Indeterminate Structures*

Example 12



❑ *Statically Indeterminate Structures*

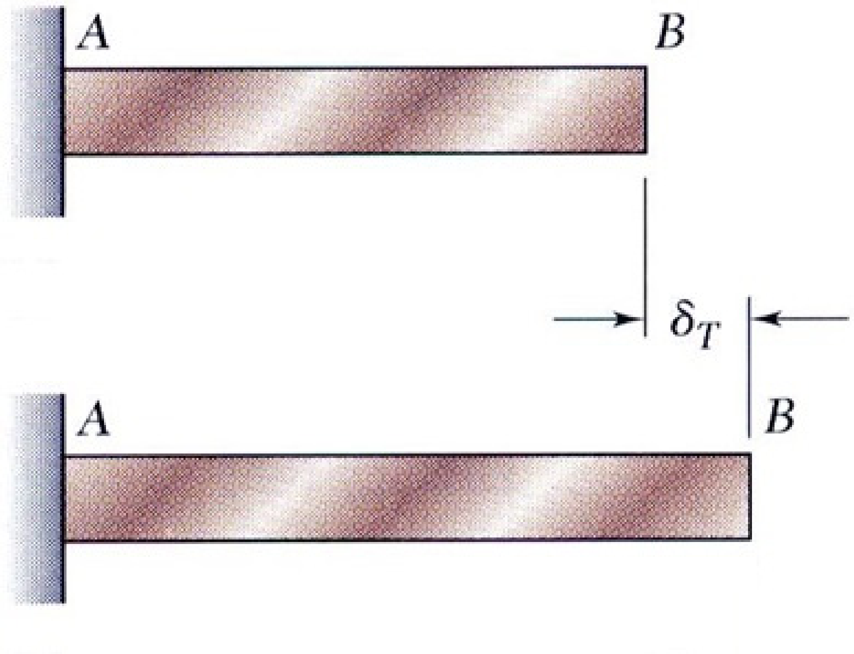
Example 12



Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

Most materials when unstrained expand when heated and contract when cooled.



$$\delta_T = \alpha(\Delta T)L$$

$$\varepsilon = \frac{\delta_T}{L} \Rightarrow \varepsilon = \alpha(\Delta T)$$

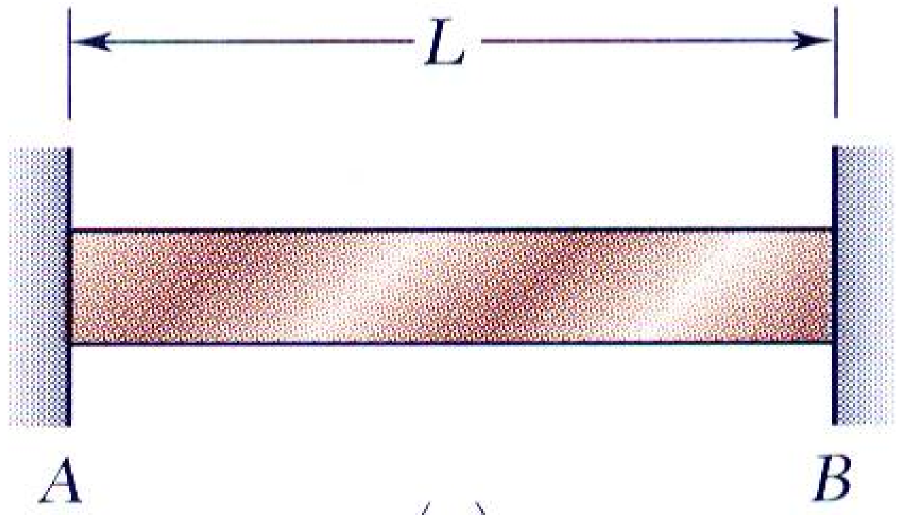
α = thermal expansion coef.

$$\sigma = E\varepsilon \Rightarrow \sigma = E\alpha(\Delta T)$$

Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

Example 13



- Determine the axial force and normal stress due to temperature changing in the following beam.

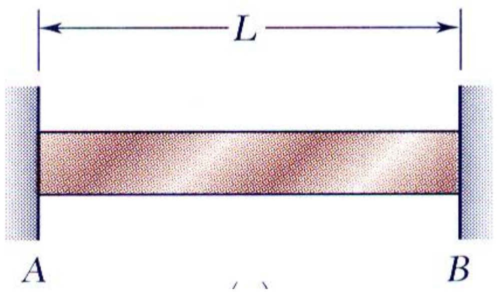
$$\alpha = \text{thermal expansion coef.}$$

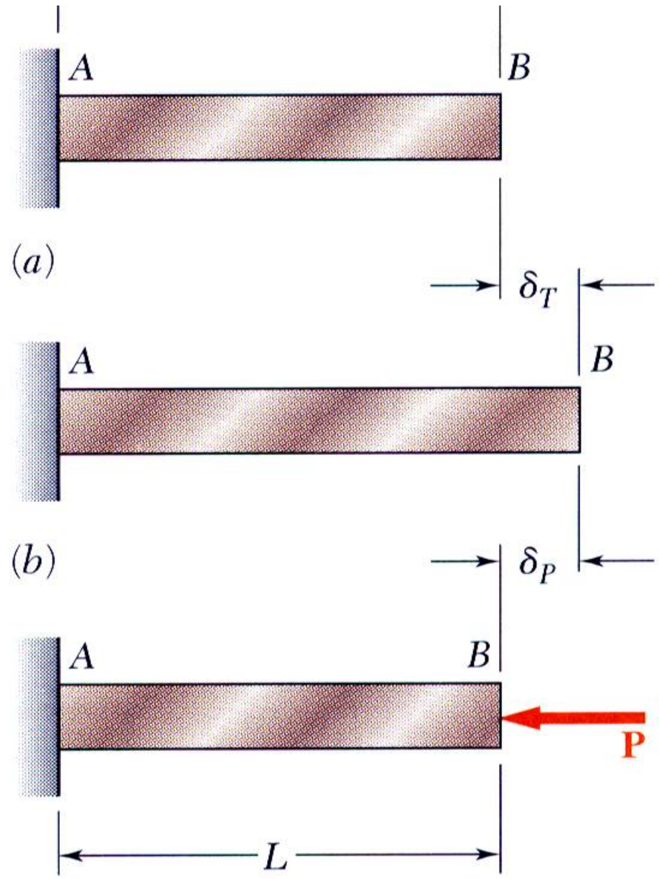
Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

Example 13

- A temperature change results in a change in length or *thermal strain*. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.

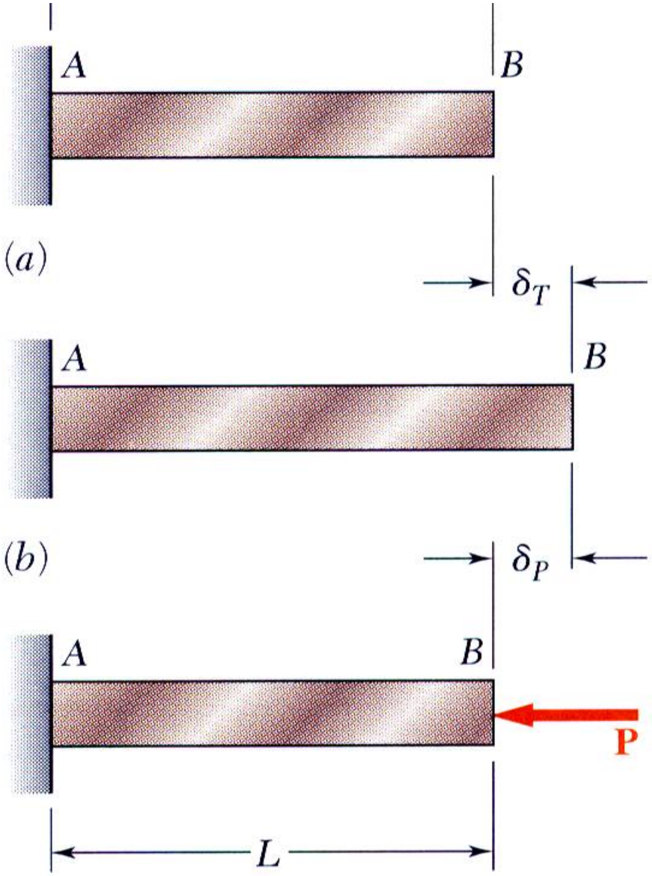




Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

Example 13



- Treat the additional support as redundant and apply the principle of superposition.

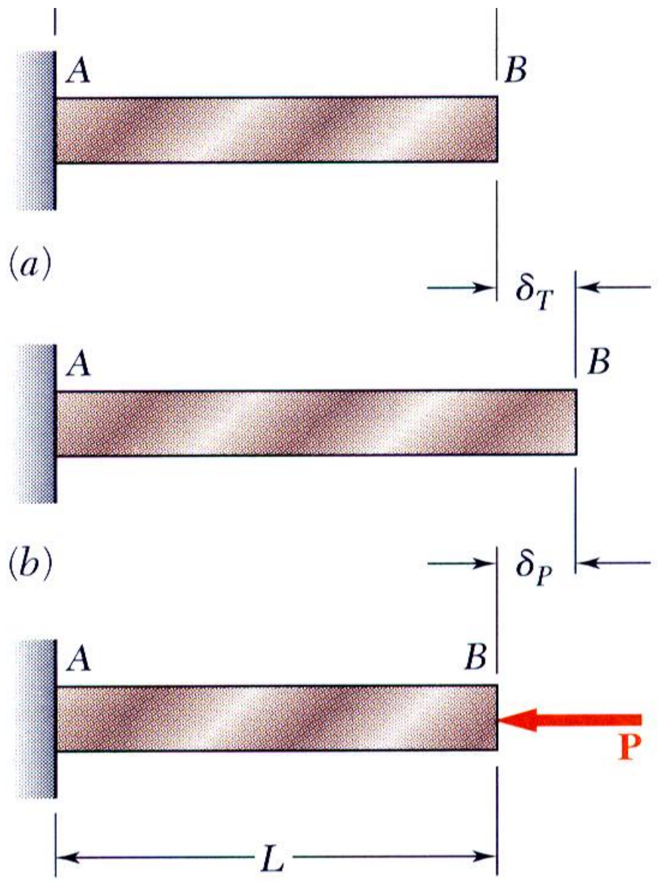
$$\delta_T = \alpha(\Delta T)L$$

$$\delta_P = \frac{PL}{AE}$$

Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

Example 13



- The thermal deformation and the deformation from the redundant support must be compatible.

$$\delta = \delta_T + \delta_P = 0 \quad \Rightarrow \quad \alpha(\Delta T)L + \frac{PL}{AE} = 0$$

$$\Rightarrow \boxed{P = -AE\alpha(\Delta T)} \quad \& \quad \boxed{\sigma = \frac{P}{A} = -E\alpha(\Delta T)}$$

Statically Indeterminate Axially Loaded Members

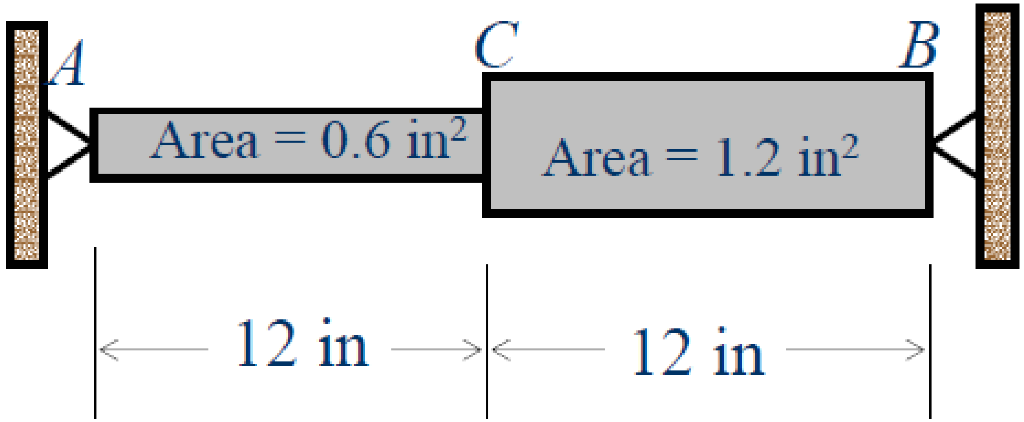
❑ Thermal Stress

Example 14

Determine the values of the stress in portion *AC* and *CB* of the steel bar shown when a the temperature of the bar is –50^{0}F, knowing that a close fit exists at both of the rigid supports when the temperature is +75^{0}F.

$$E = 29\times10^6\ psi$$

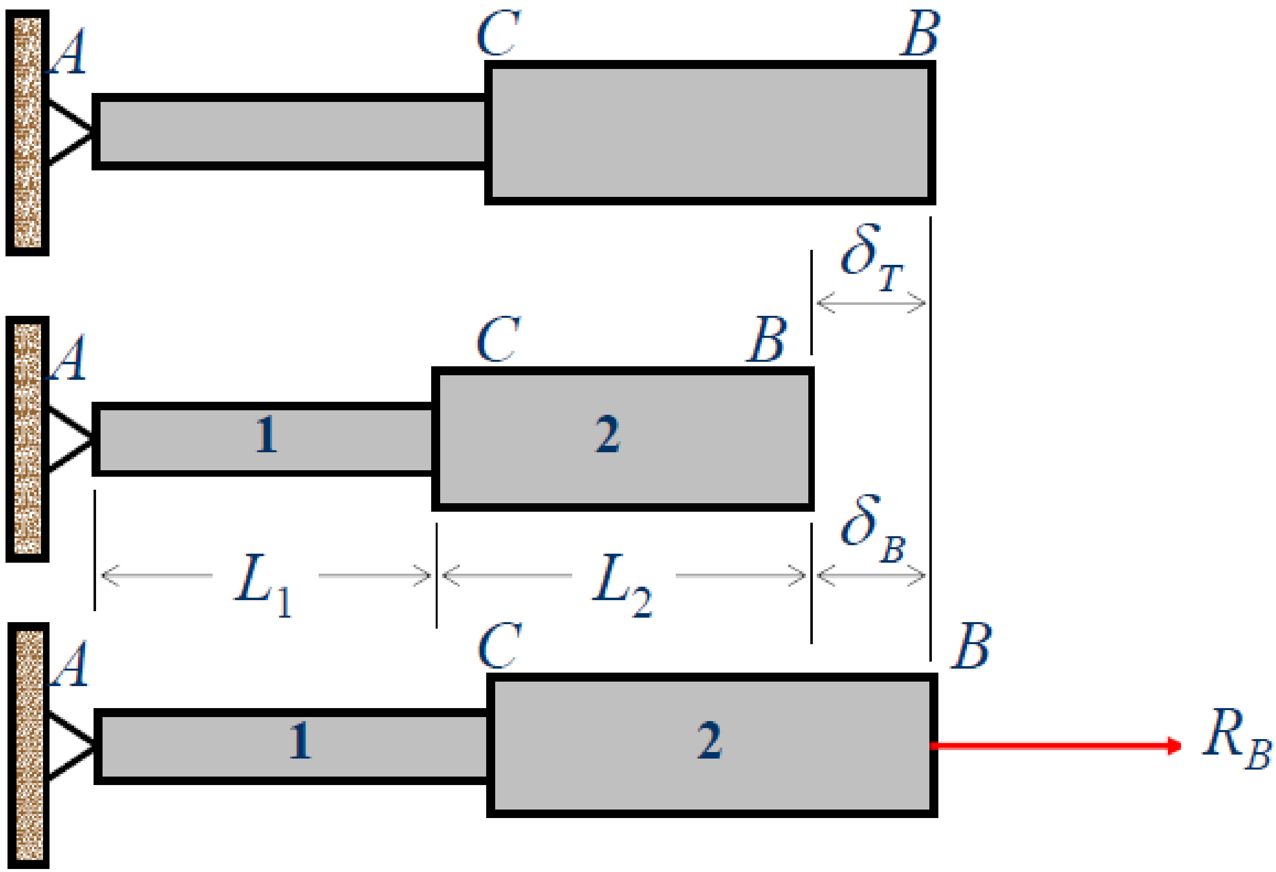
$$\alpha = 6.5\times10^{-6}\ \frac{1}{F^o}$$



Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

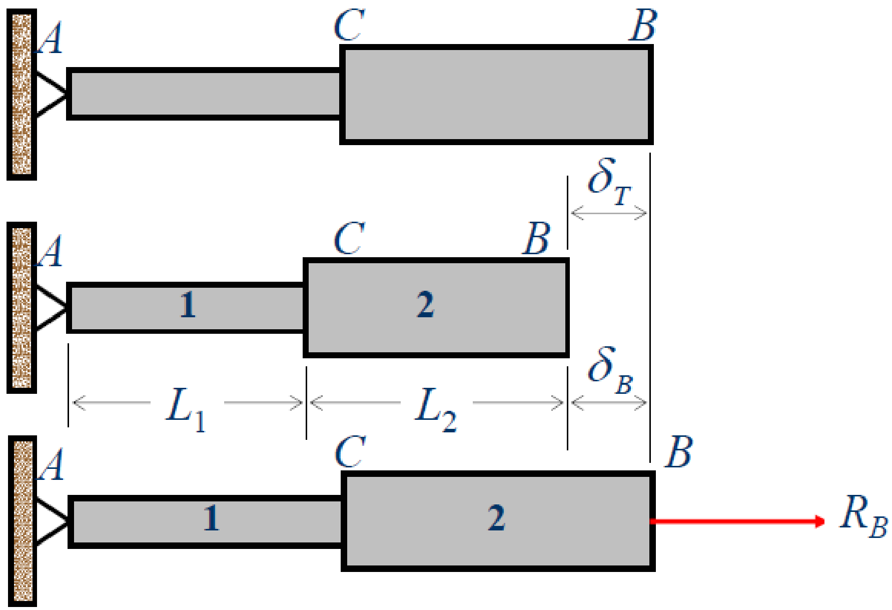
Example 14



Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

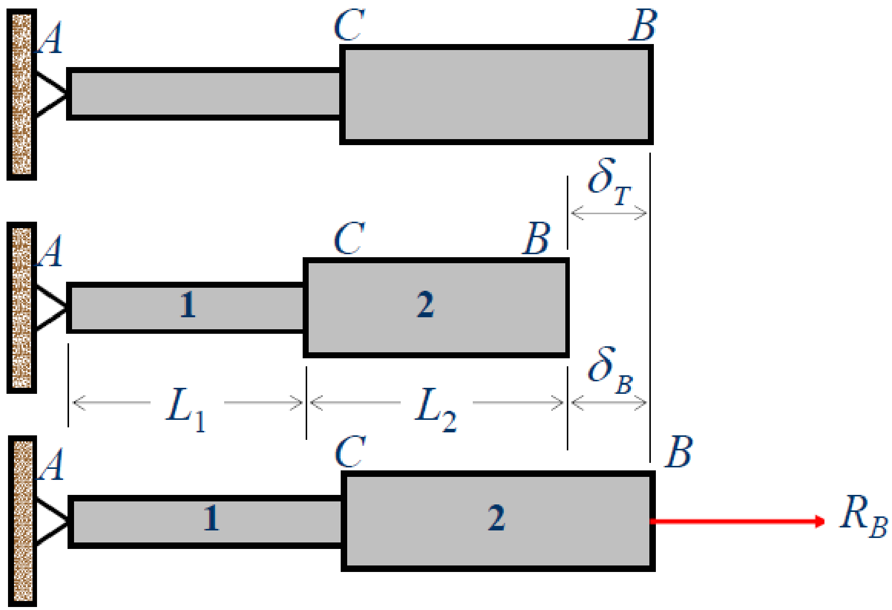
Example 14



Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

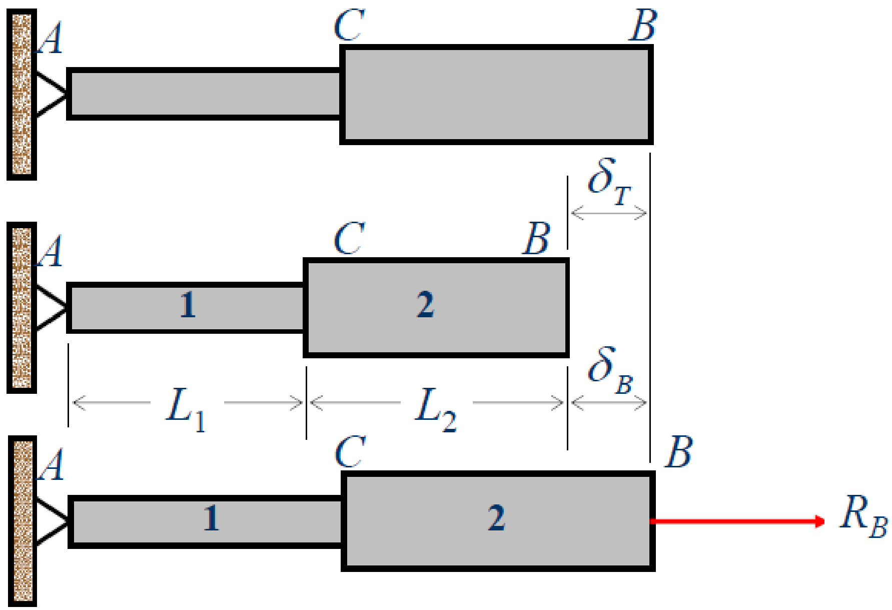
Example 14



Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

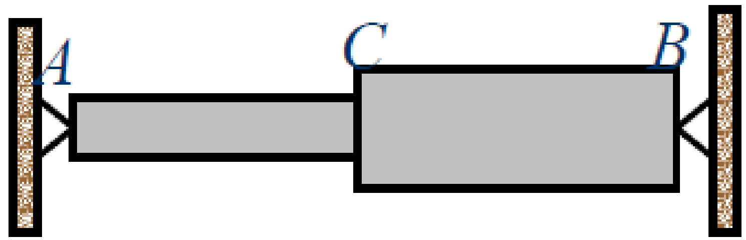
Example 14



Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

Example 14

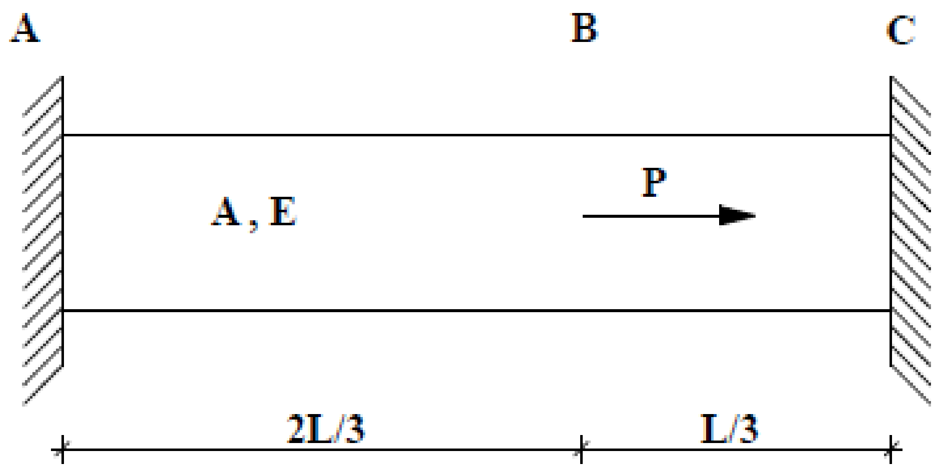


Statically Indeterminate Axially Loaded Members

Thermal Stress

Example 15

- Determine the temperature that we have no tensile in the shown beam.



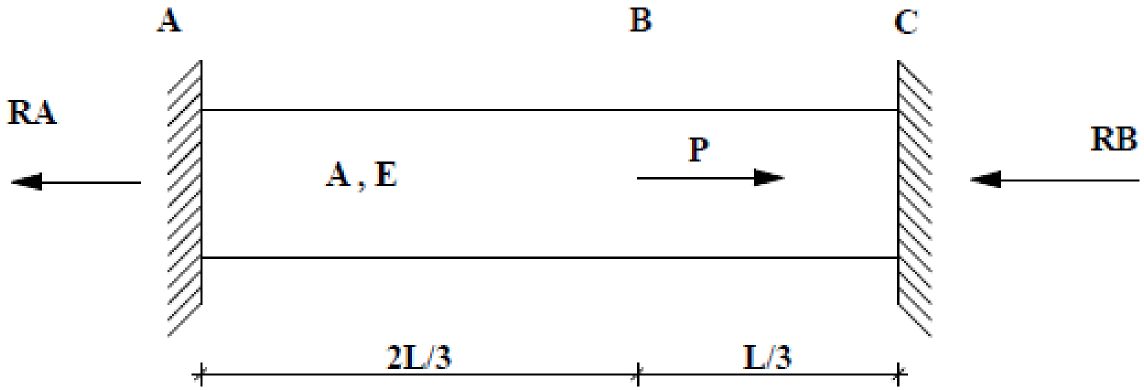
α = thermal expansion coef.

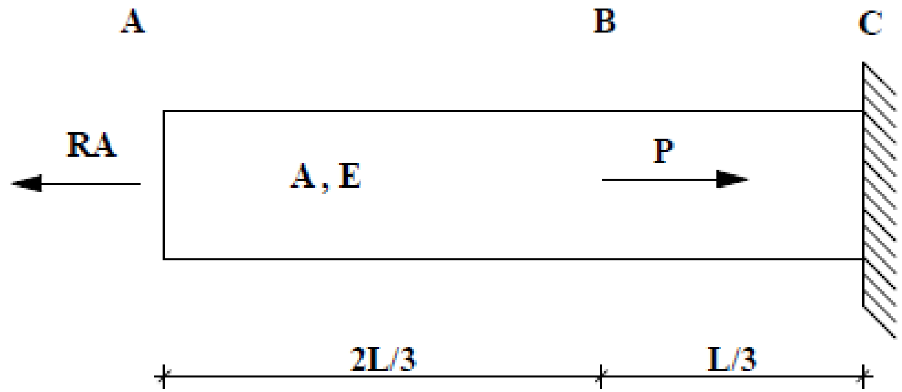
Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

Example 15

- There is only tensile stress in the part AB due to axial force P.



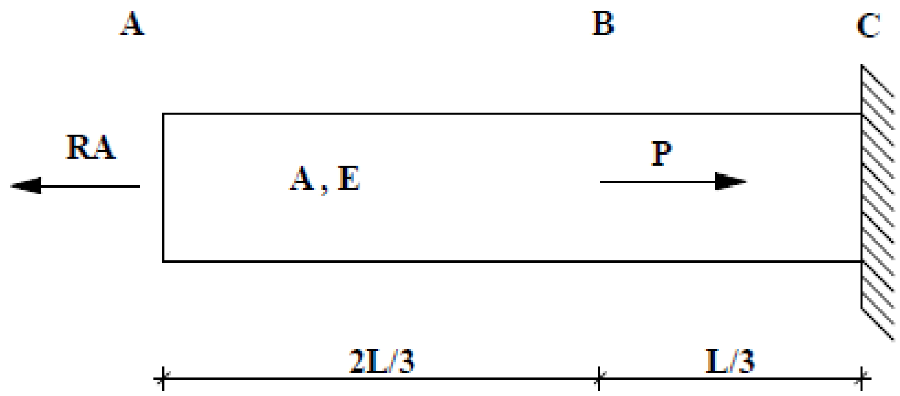


Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

Example 15

- There is only tensile stress in the part AB due to axial force P.

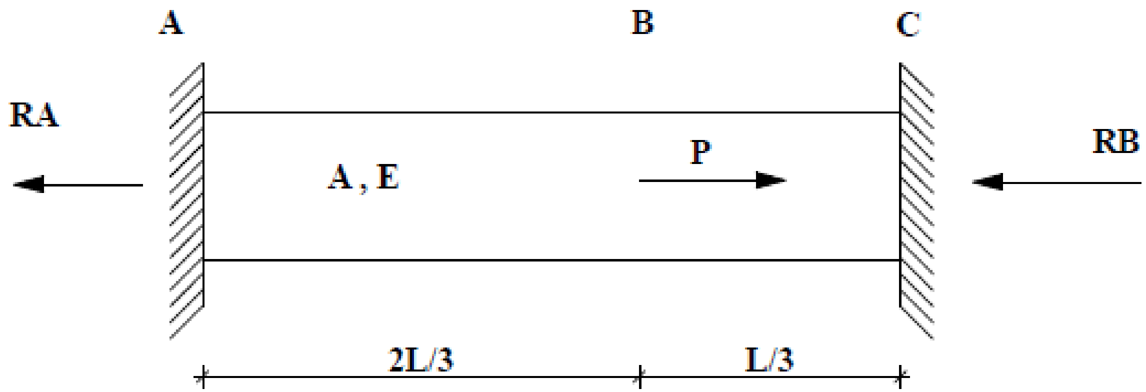


Statically Indeterminate Axially Loaded Members

❑ Thermal Stress

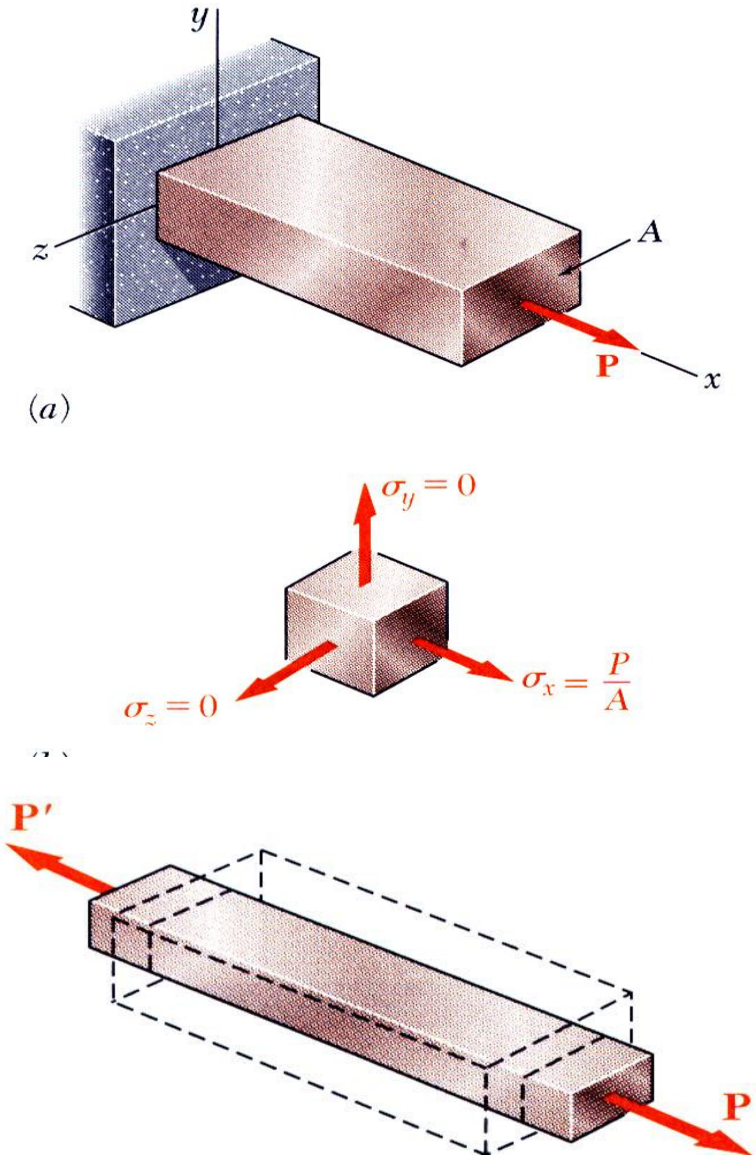
Example 15

- There is only tensile stress in the part AB due to axial force P.



Stress and Strain – Axial Loading

Poisson's Ratio



- For a slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \sigma_y = \sigma_z = 0$$

- The elongation in the x-direction is accompanied by a contraction in the other directions. Assuming that the material is isotropic (no directional dependence),

$$\varepsilon_y = \varepsilon_z \neq 0$$

- Poisson's ratio is defined as

$$\nu = \left| \frac{\text{lateral strain}}{\text{axial strain}} \right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

Stress and Strain – Axial Loading

Generalized Hooke's law

$$\nu = \left|\frac{\text{lateral strain}}{\text{axial strain}}\right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\Rightarrow \quad \begin{cases} \varepsilon_y = -\nu\varepsilon_x \\ \varepsilon_z = -\nu\varepsilon_x \end{cases} \quad \Rightarrow \quad \begin{cases} \varepsilon_y = -\nu(\dfrac{\sigma_x}{E}) \\ \varepsilon_z = -\nu(\dfrac{\sigma_x}{E}) \end{cases}$$

Stress and Strain – Axial Loading

❑ Generalized Hooke's law

- For an element subjected to multi-axial loading, the normal strain components resulting from the stress components may be determined from the *principle of superposition*. This requires:

1) strain is linearly related to stress
2) deformations are small

Stress	X: Direction	Y: Direction	Z: Direction
σ_x	$\varepsilon_x = \frac{\sigma_x}{E}$	$\varepsilon_y = -\nu \frac{\sigma_x}{E}$	$\varepsilon_z = -\nu \frac{\sigma_x}{E}$
σ_y	$\varepsilon_x = -\nu \frac{\sigma_y}{E}$	$\varepsilon_y = \frac{\sigma_y}{E}$	$\varepsilon_z = -\nu \frac{\sigma_y}{E}$
σ_z	$\varepsilon_x = -\nu \frac{\sigma_z}{E}$	$\varepsilon_y = -\nu \frac{\sigma_z}{E}$	$\varepsilon_z = \frac{\sigma_z}{E}$

Stress and Strain – Axial Loading

❑ Generalized Hooke's law

General State of Strain

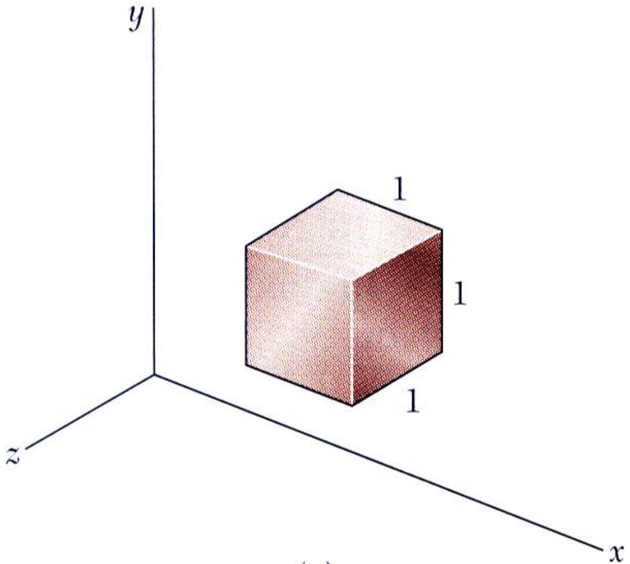
$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

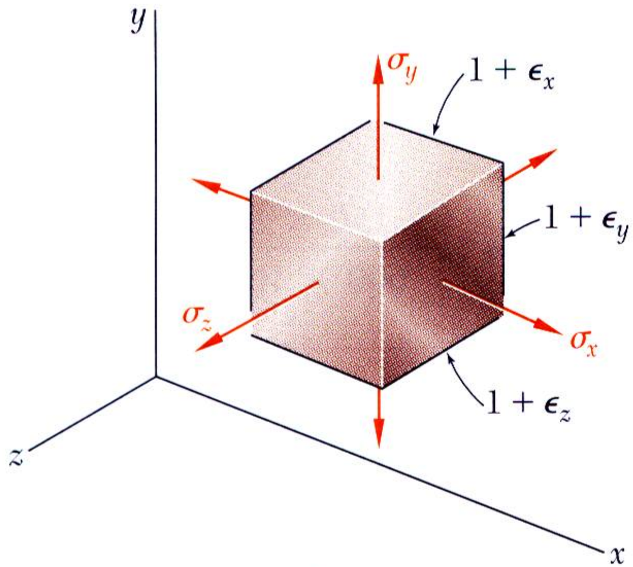
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

Stress and Strain – Axial Loading

❑ Dilatation: Bulk Modulus



(a)



(b)

- Relative to the unstressed state, the change in volume is

$$e = V_2 - V_1 = [(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)] - 1$$

$$= [1 + \varepsilon_x + \varepsilon_y + \varepsilon_z] - 1$$

$$\Rightarrow \quad e = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$e = \frac{1-2\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

Stress and Strain – Axial Loading

❑ Dilatation: Bulk Modulus

- For element subjected to uniform hydrostatic pressure,

$$\sigma_x = \sigma_y = \sigma_z = -P$$

$$e = \frac{1-2\nu}{E}\left(-P-P-P\right) = -P\frac{3(1-2\nu)}{E}$$

$$k = \frac{E}{3(1-2\nu)} \quad : Bulk\ Modulus$$

$$\Rightarrow \quad e = -\frac{P}{k}$$

- Subjected to uniform pressure, dilatation must be negative, therefore

$$\frac{E}{3(1-2\nu)} > 0 \Rightarrow 1-2\nu > 0 \quad \Rightarrow \quad 0 < \nu < \tfrac{1}{2}$$

Stress and Strain – Axial Loading

Shearing Strain

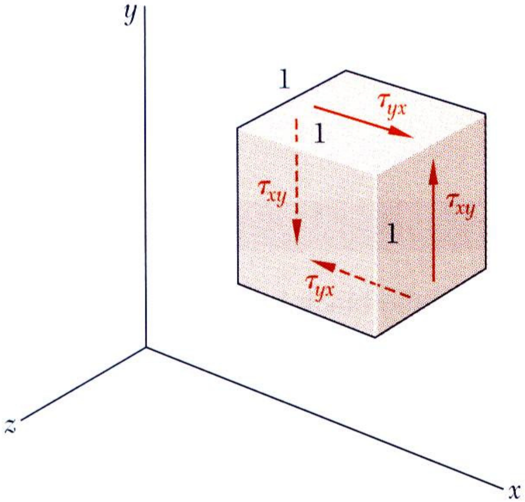


Fig. 2.46

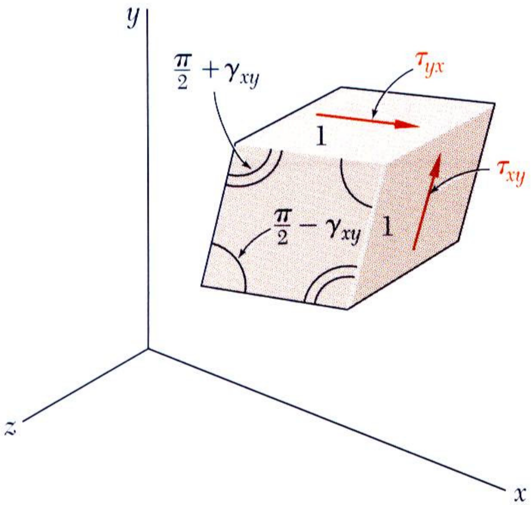


Fig. 2.47

- A cubic element subjected to a shear stress will deform into a rhomboid. The corresponding *shear* strain is quantified in terms of the change in angle between the sides,

$$\tau_{xy} = f(\gamma_{xy})$$

- A plot of shear stress vs. shear strain is similar the previous plots of normal stress vs. normal strain except that the strength values are approximately half. For small strains,

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

where ***G* is the modulus of rigidity or shear modulus.**

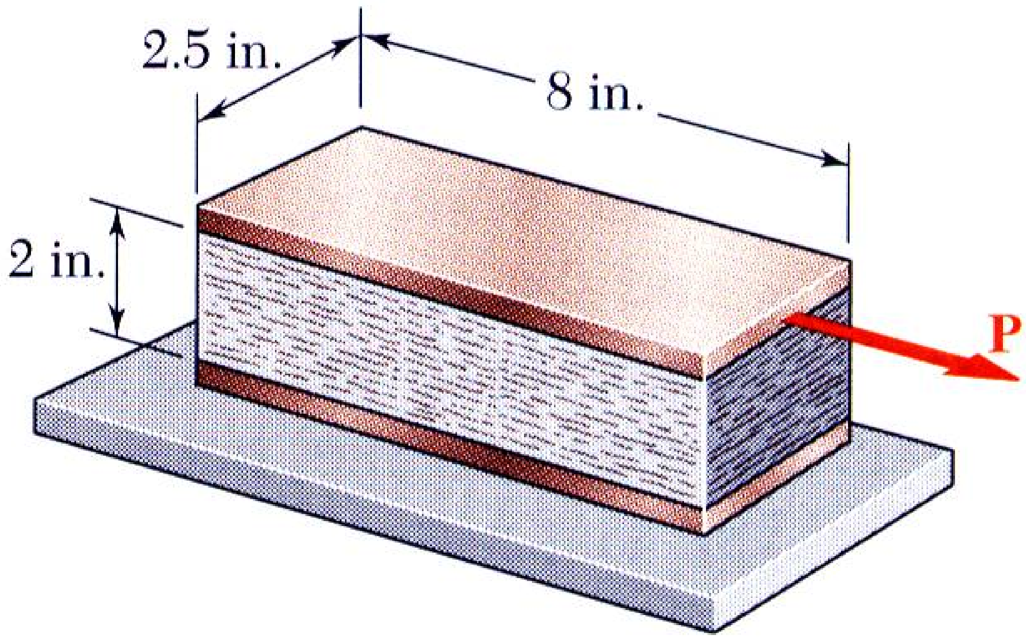
Relation Among *E*, ν, and *G*

$$\frac{E}{2G} = (1+\nu)$$

Stress and Strain – Axial Loading

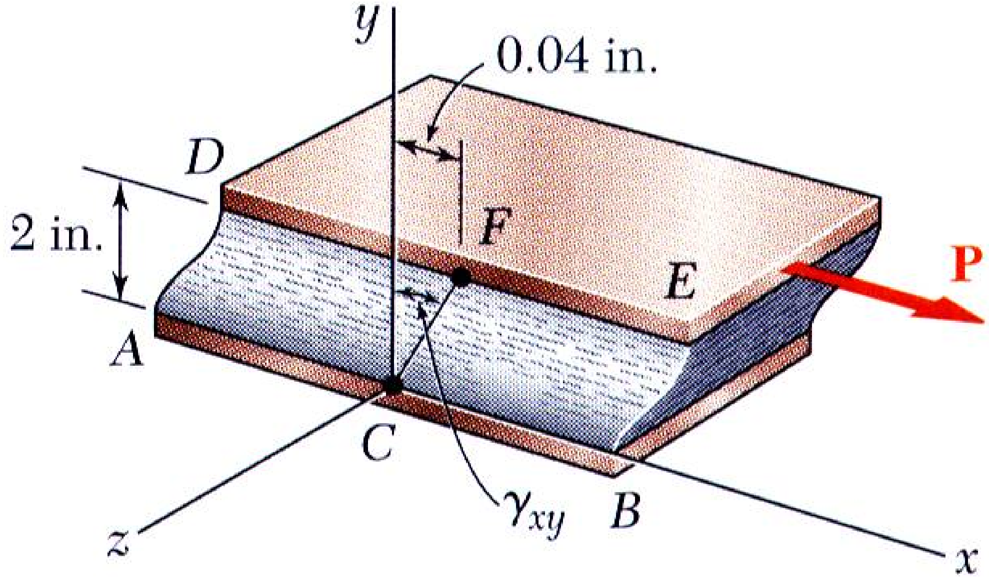
Example 16

A rectangular block of material with modulus of rigidity $G = 90$ ksi is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force *P*. Knowing that the upper plate moves through 0.04 in. under the action of the force, determine a) the average shearing strain in the material, and b) the force *P* exerted on the plate.



Stress and Strain – Axial Loading

Example 16



- Determine the average angular deformation or shearing strain of the block.

Stress and Strain – Axial Loading

Example 17

A circle of diameter $d = 9$ in. is scribed on an unstressed aluminum plate of thickness $t = 3/4$ in. Forces acting in the plane of the plate later cause normal stresses

$\sigma_x = 12$ ksi and $\sigma_z = 20$ ksi.

For $E = 10\text{x}10^6$ psi and $\nu = 1/3$,

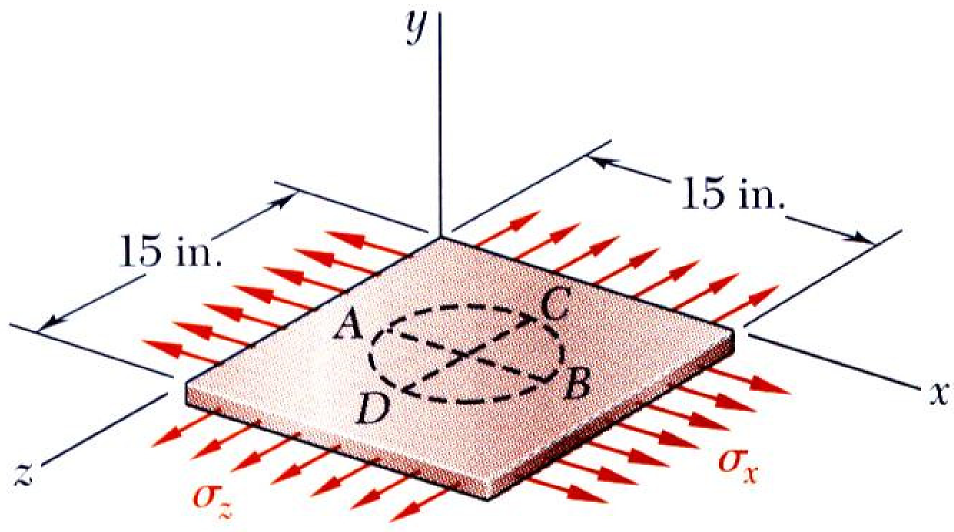
determine the change in:

a) the length of diameter *AB*,

b) the length of diameter *CD*,

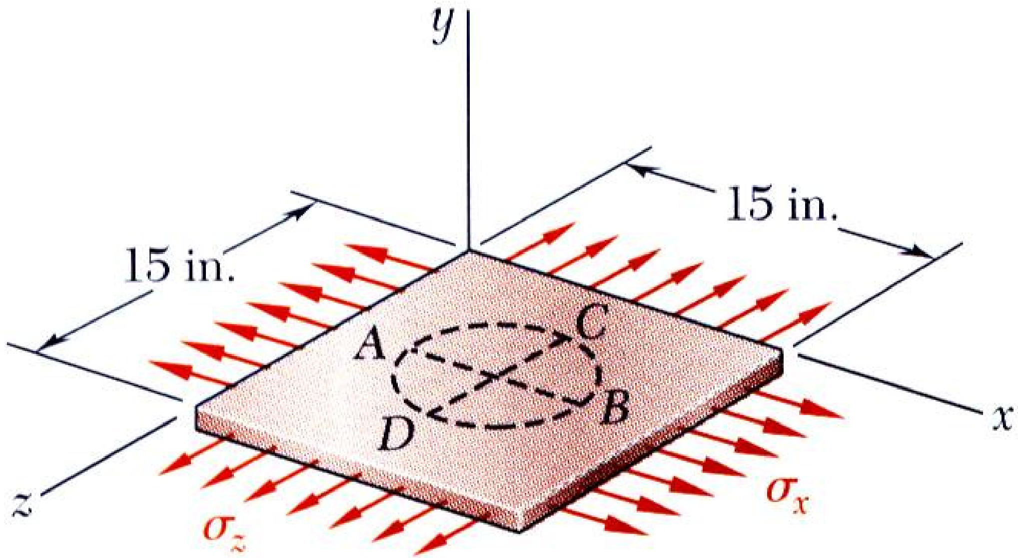
c) the thickness of the plate, and

d) the volume of the plate.



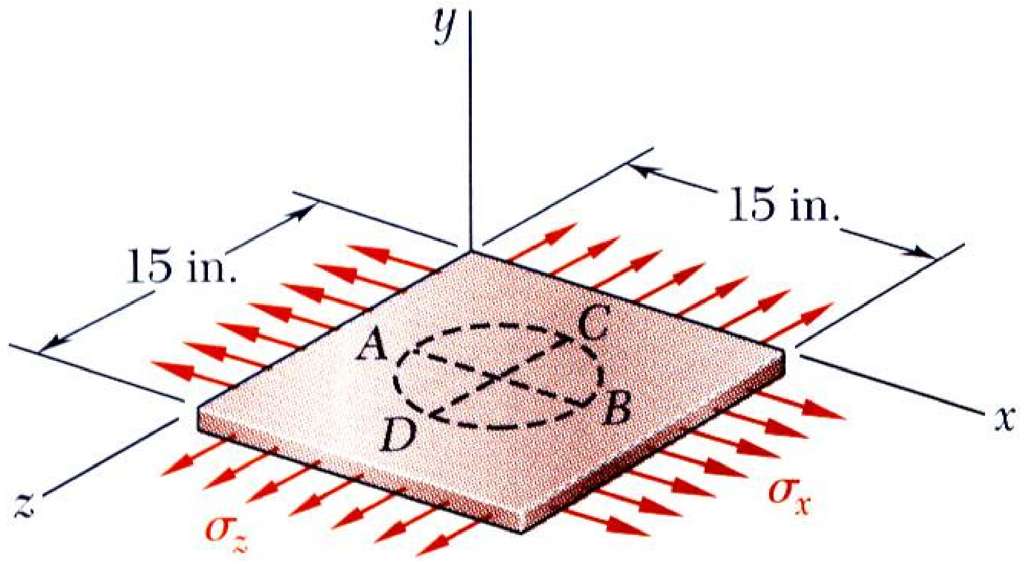
Stress and Strain – Axial Loading

Example 17



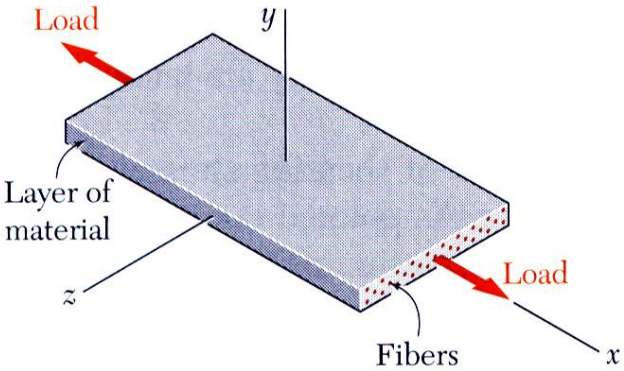
Stress and Strain – Axial Loading

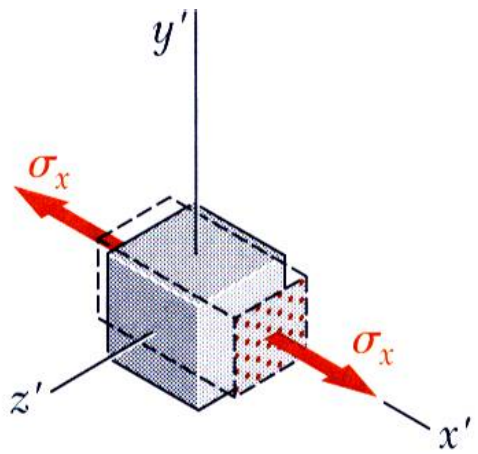
Example 17



Stress and Strain – Axial Loading

Composite Materials





- *Fiber-reinforced composite materials* are formed from *lamina* of fibers of graphite, glass, or polymers embedded in a resin matrix.

- Normal stresses and strains are related by Hooke's Law but with directionally dependent moduli of elasticity,

$$E_x = \frac{\sigma_x}{\varepsilon_x} \quad E_y = \frac{\sigma_y}{\varepsilon_y} \quad E_z = \frac{\sigma_z}{\varepsilon_z}$$

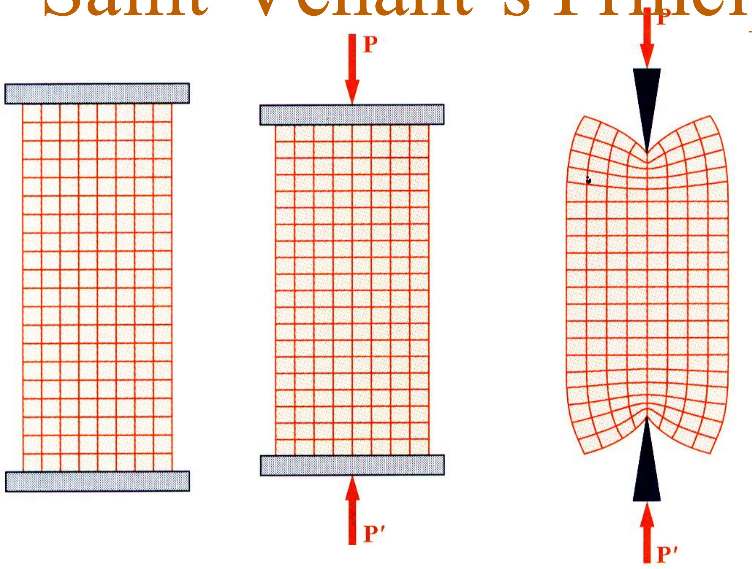
- Transverse contractions are related by directionally dependent values of Poisson's ratio, e.g.,

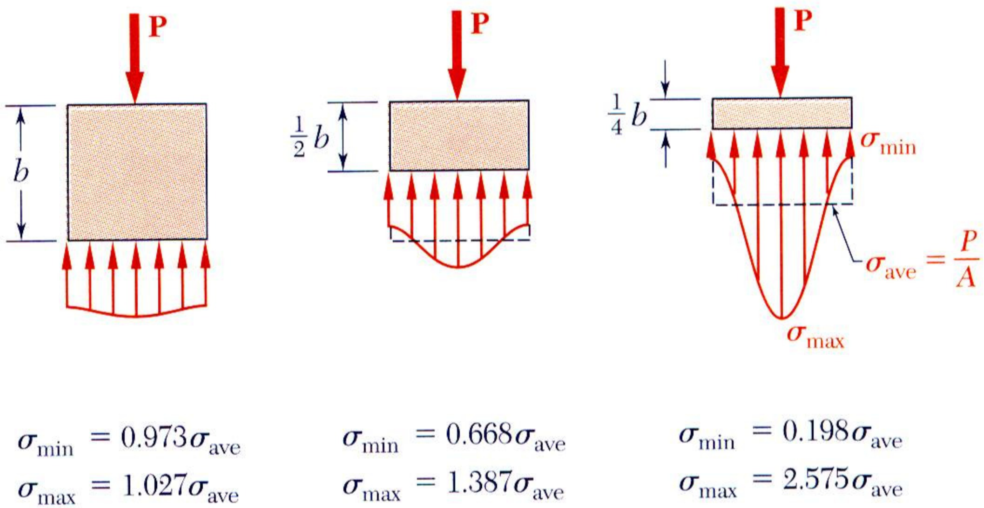
$$\nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} \quad \nu_{xz} = -\frac{\varepsilon_z}{\varepsilon_x}$$

- Materials with directionally dependent mechanical properties are *anisotropic.*

Stress and Strain – Axial Loading

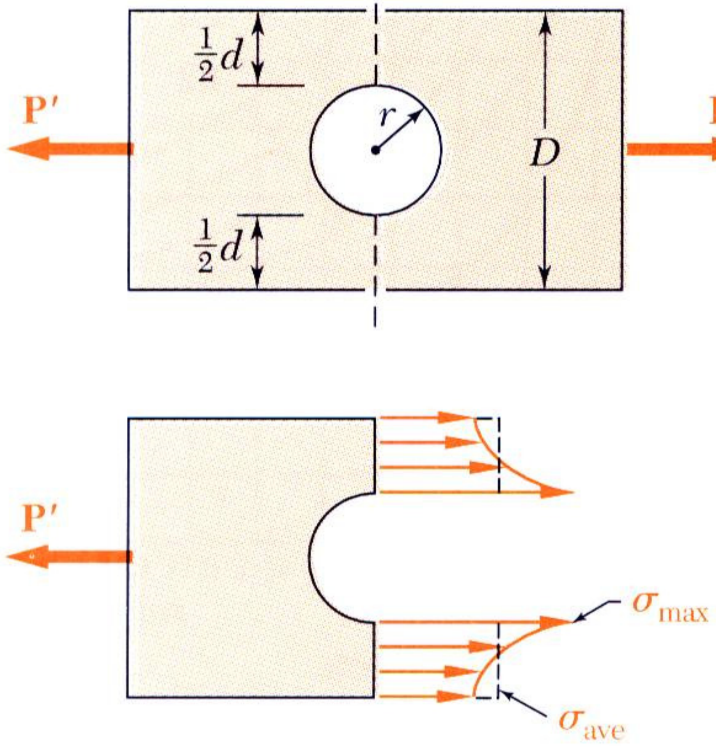
❑ Saint-Venant's Principle

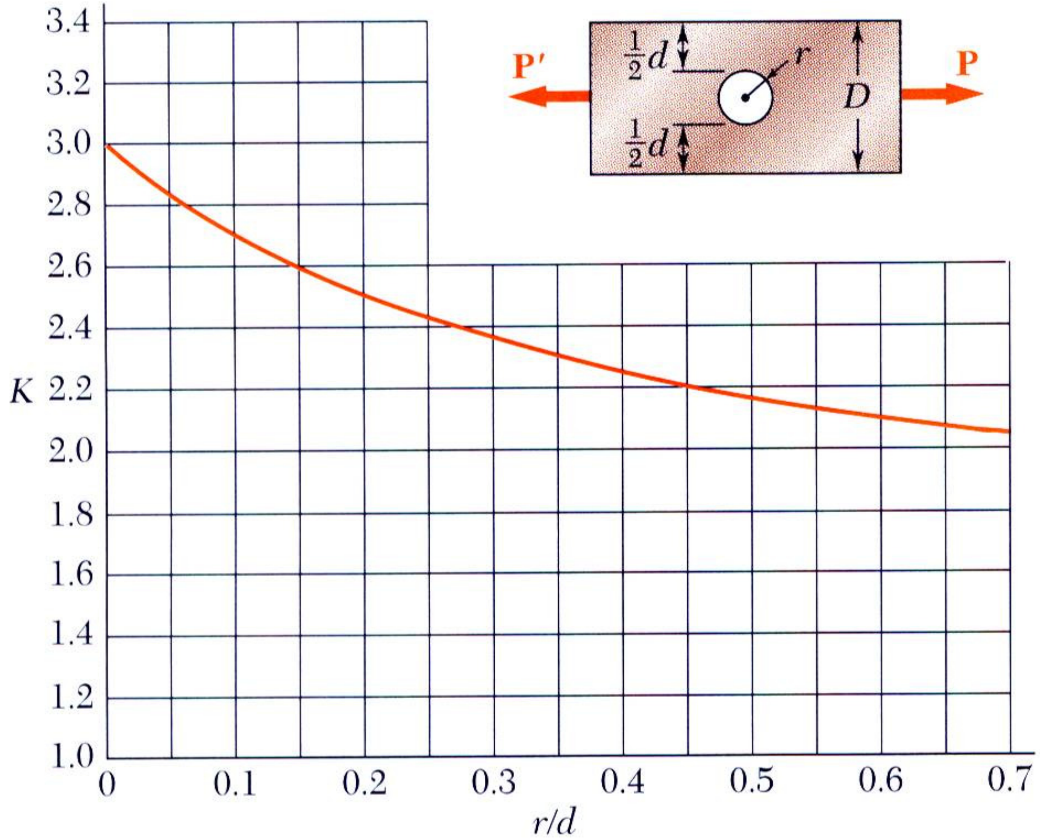




- Loads transmitted through rigid plates result in uniform distribution of stress and strain.
- Concentrated loads result in large stresses in the vicinity of the load application point.
- Stress and strain distributions become uniform at a relatively short distance from the load application points.
- **Saint-Venant's Principle:** Stress distribution may be assumed independent of the mode of load application except in the immediate vicinity of load application points.

Stress Concentration: Hole





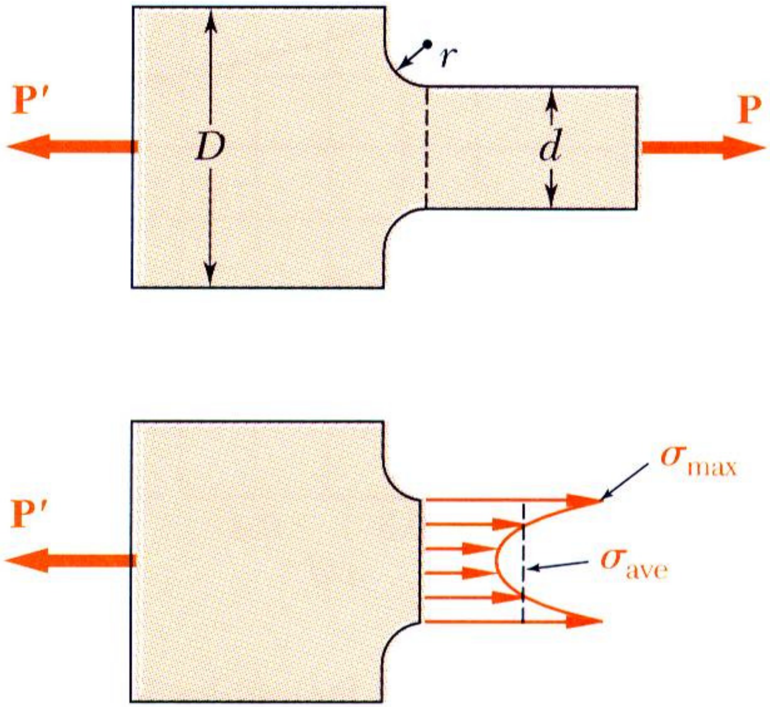
(*a*) Flat bars with holes

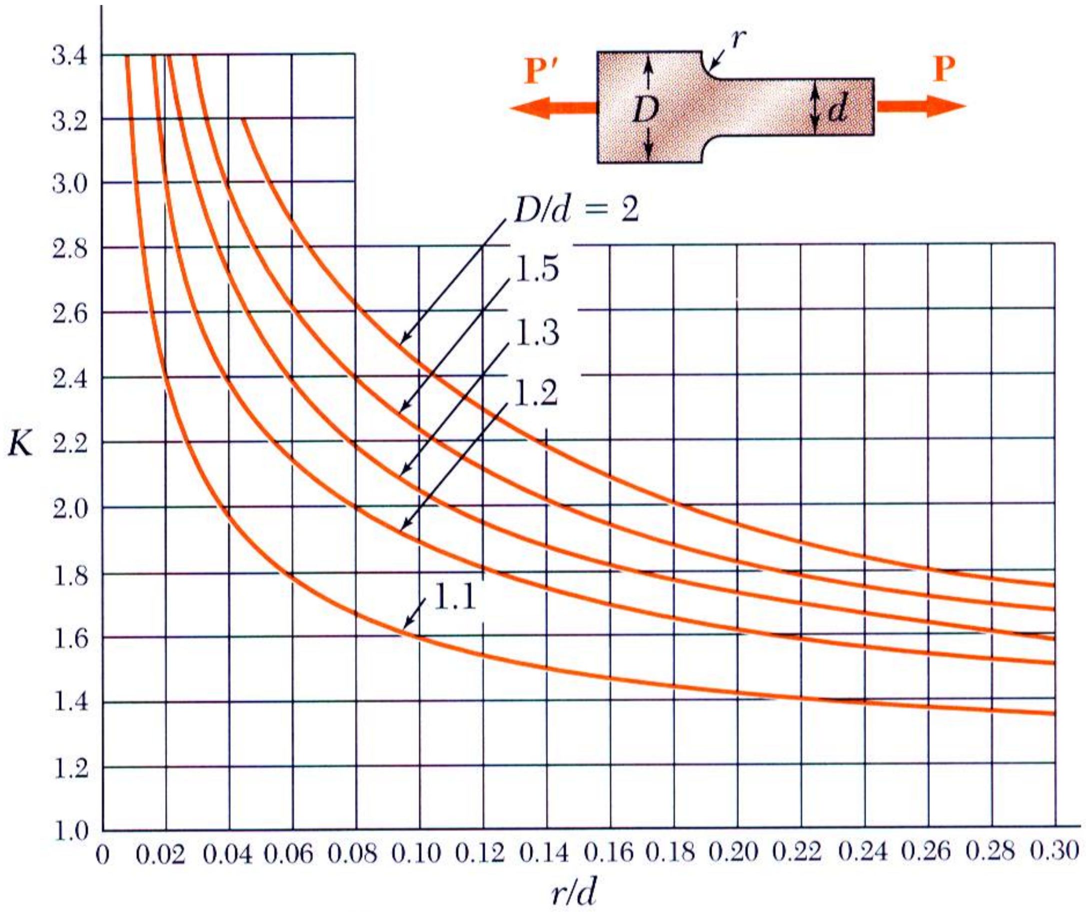
Discontinuities of cross section may result in high localized or *concentrated* stresses.

$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

Stress and Strain – Axial Loading

❑ Stress Concentration: Fillet





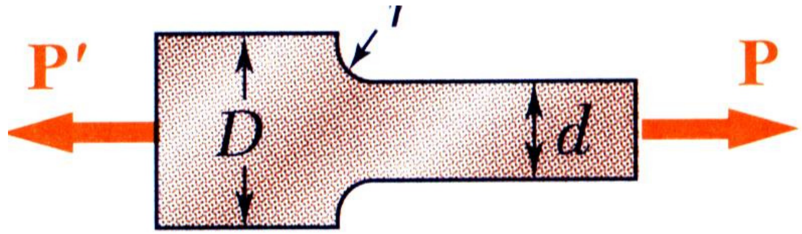
(*b*) Flat bars with fillets

Stress and Strain – Axial Loading

❑ Stress Concentration

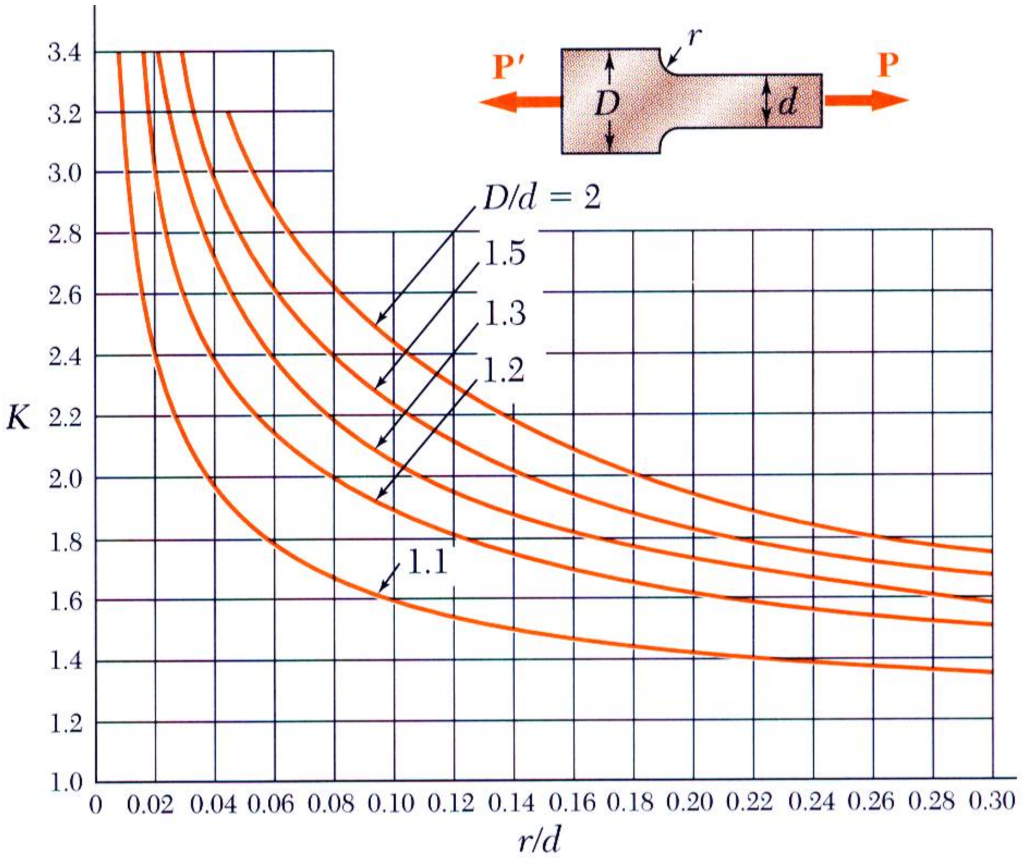
Example 18

Determine the largest axial load P that can be safely supported by a flat steel bar consisting of two portions, both 10 mm thick, and respectively 40 and 60 mm wide, connected by fillets of radius $r = 8$ mm. Assume an allowable normal stress of 165 MPa.



❑ Stress Concentration

Example 18



(*b*) Flat bars with fillets