STATICS

دانشگاه کردستان
University of Kurdistan
زانکۆی کوردستان

- Vector Mechanics for Engineers: Statics, 9th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Statics, 5th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Statics"

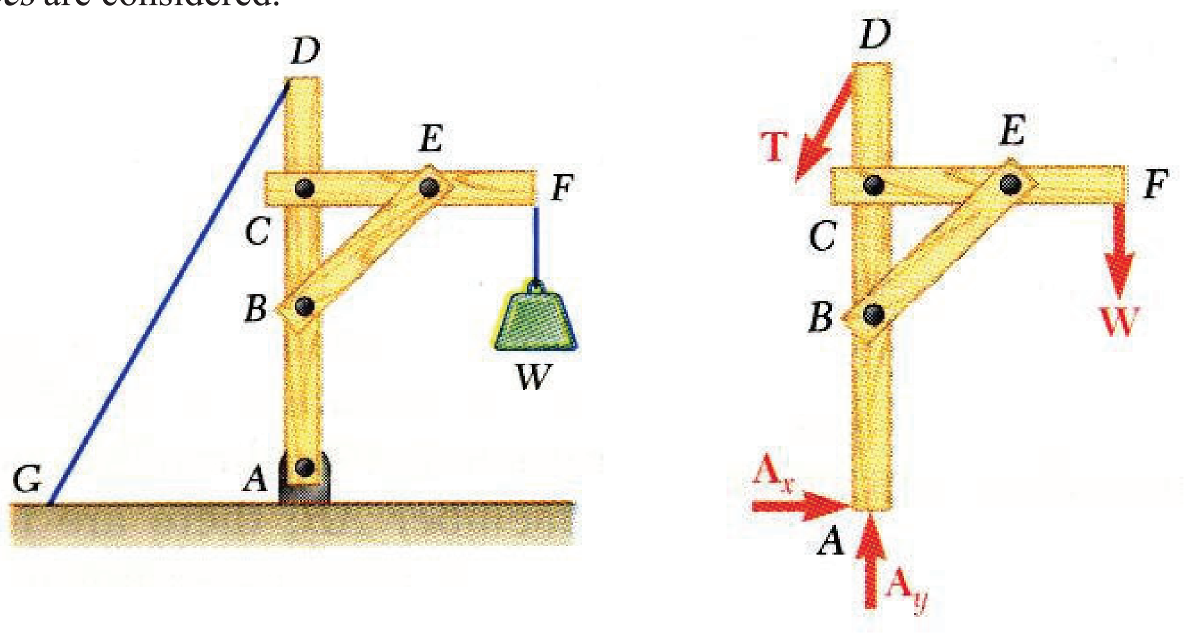
Analysis of Structures

By: Kaveh Karami
Associate Prof. of Structural Engineering
https://prof.uok.ac.ir/Ka.Karami

Analysis of Structures

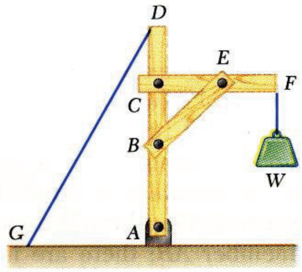
❑ Introduction

- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.

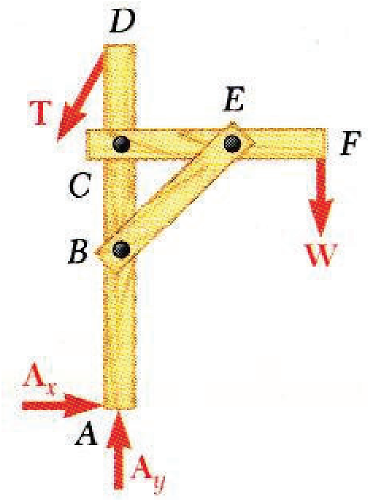


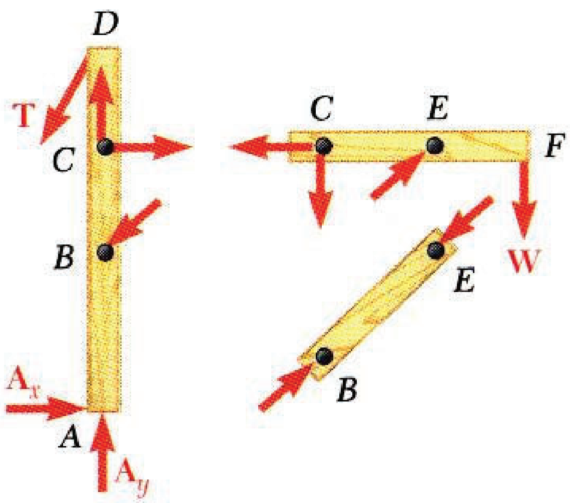
Analysis of Structures

❑ Introduction



- In the interaction between connected parts, Newton's 3rd Law states that ***the forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense.***





Analysis of Structures

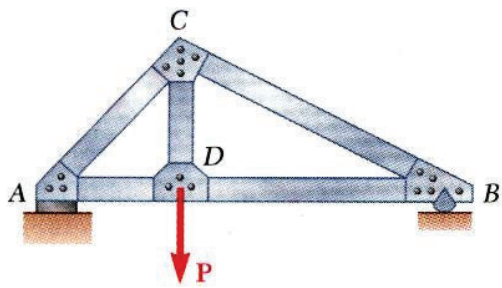
❑ Definition of a Truss

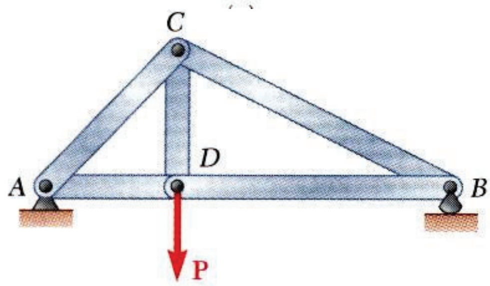
- A truss consists of straight members connected at joints. No member is continuous through a joint.

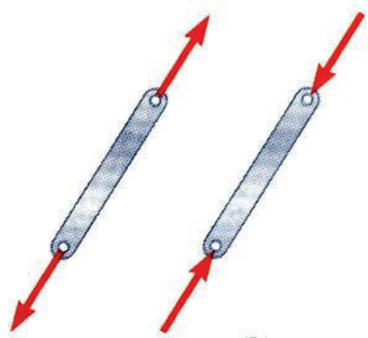
- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.

- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.

- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.







Analysis of Structures

❑ Definition of a Truss

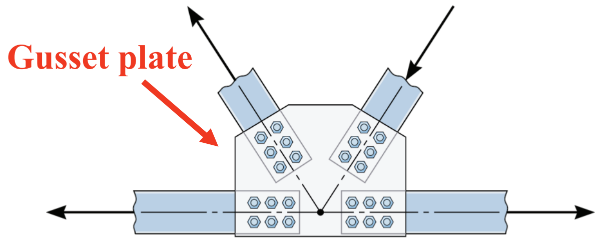


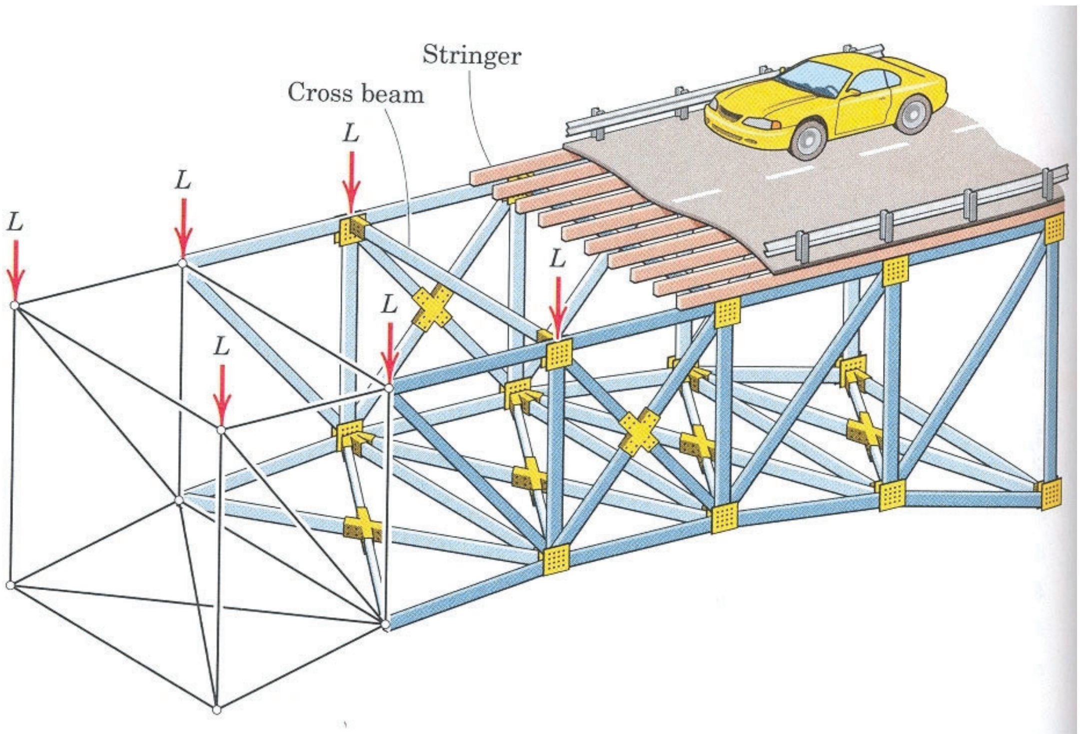
Figure 06.01(a)

Joints are often bolted, riveted, or welded. Gusset plates are also often included to tie the members together. However, the members are designed to support axial loads so assuming that the joints act as if they are pinned is a good approximation.

Analysis of Structures

❑ Definition of a Truss

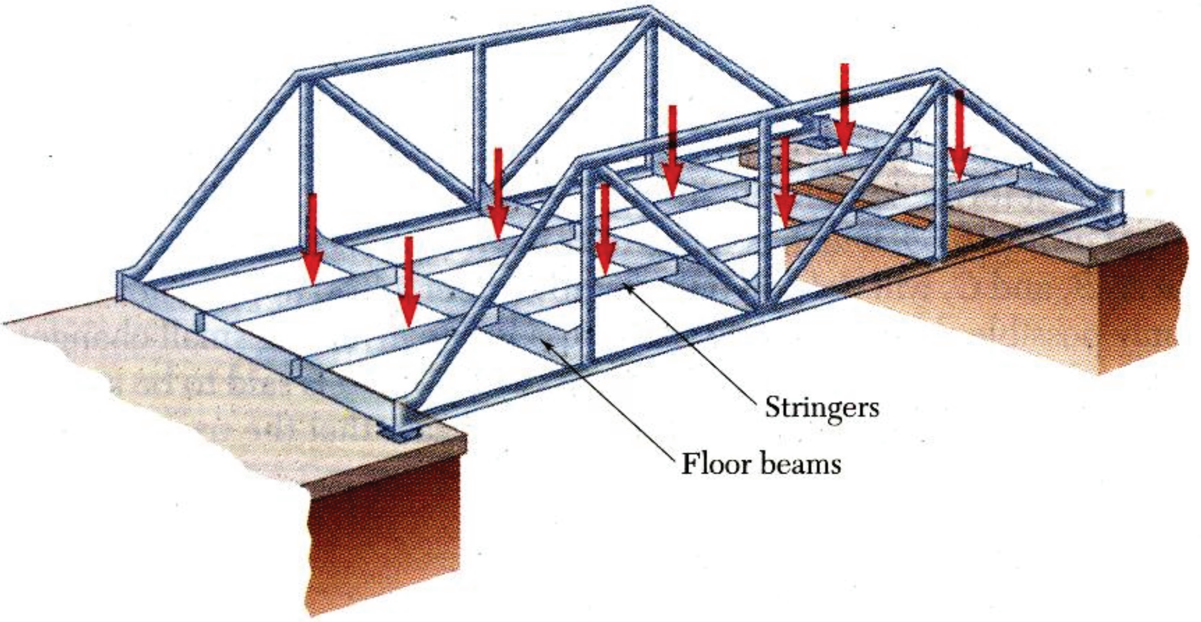
Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.



Analysis of Structures

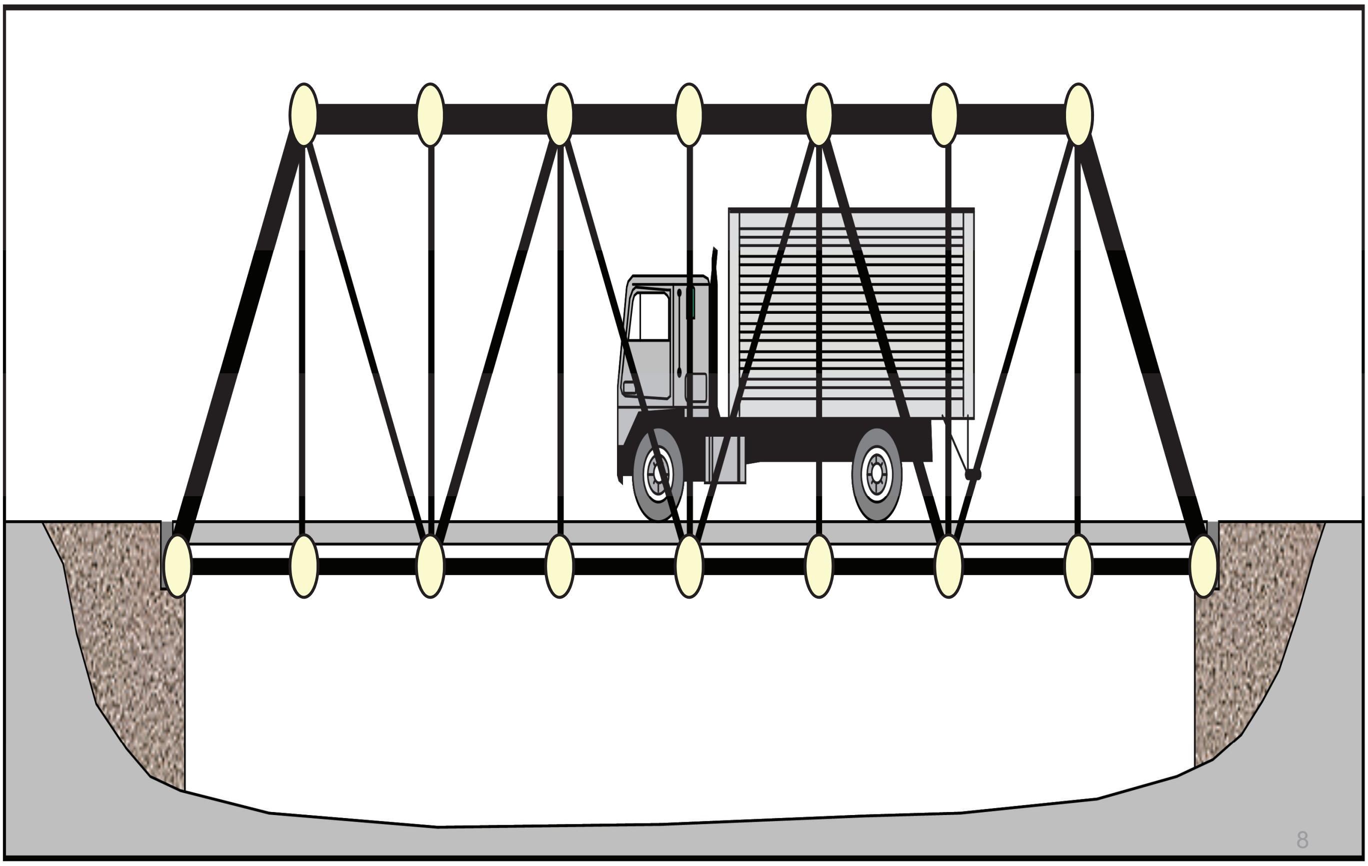
❑ Definition of a Truss

Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.



Analysis of Structures

❑ Definition of a Truss



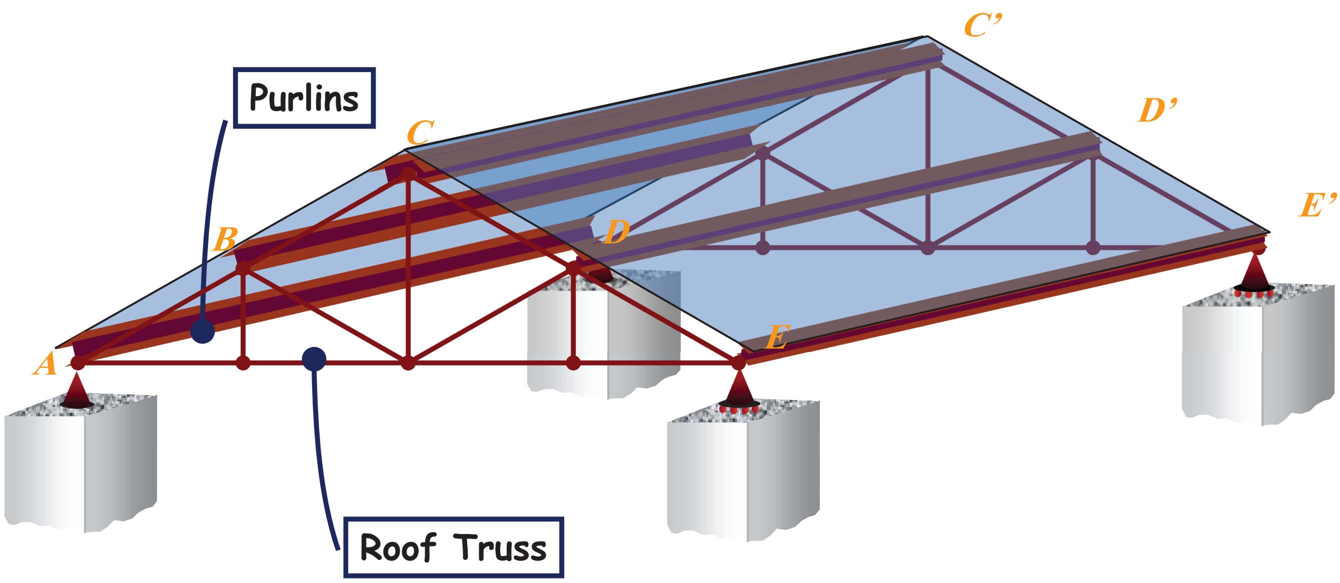
Analysis of Structures

❑ Definition of a Truss

Roof trusses – Safeco Field in Seattle

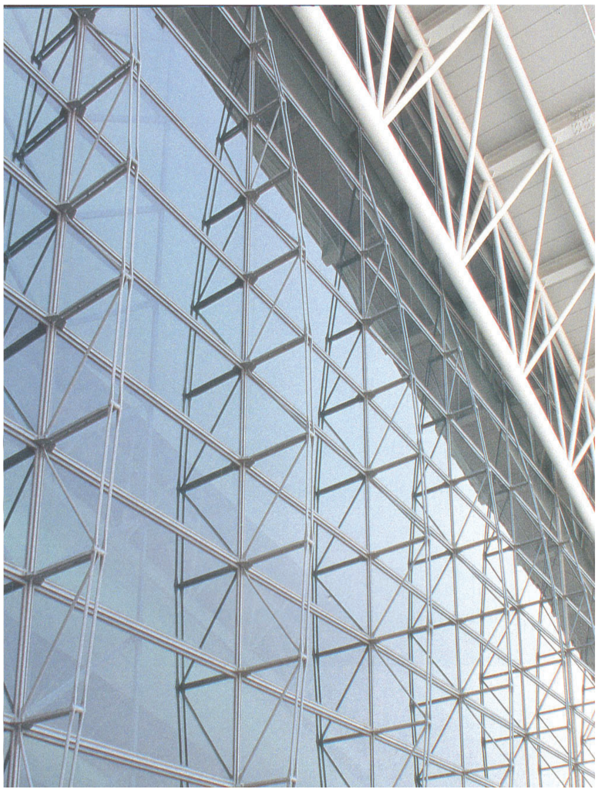
Analysis of Structures

❑ Definition of a Truss



Analysis of Structures

- Definition of a Truss

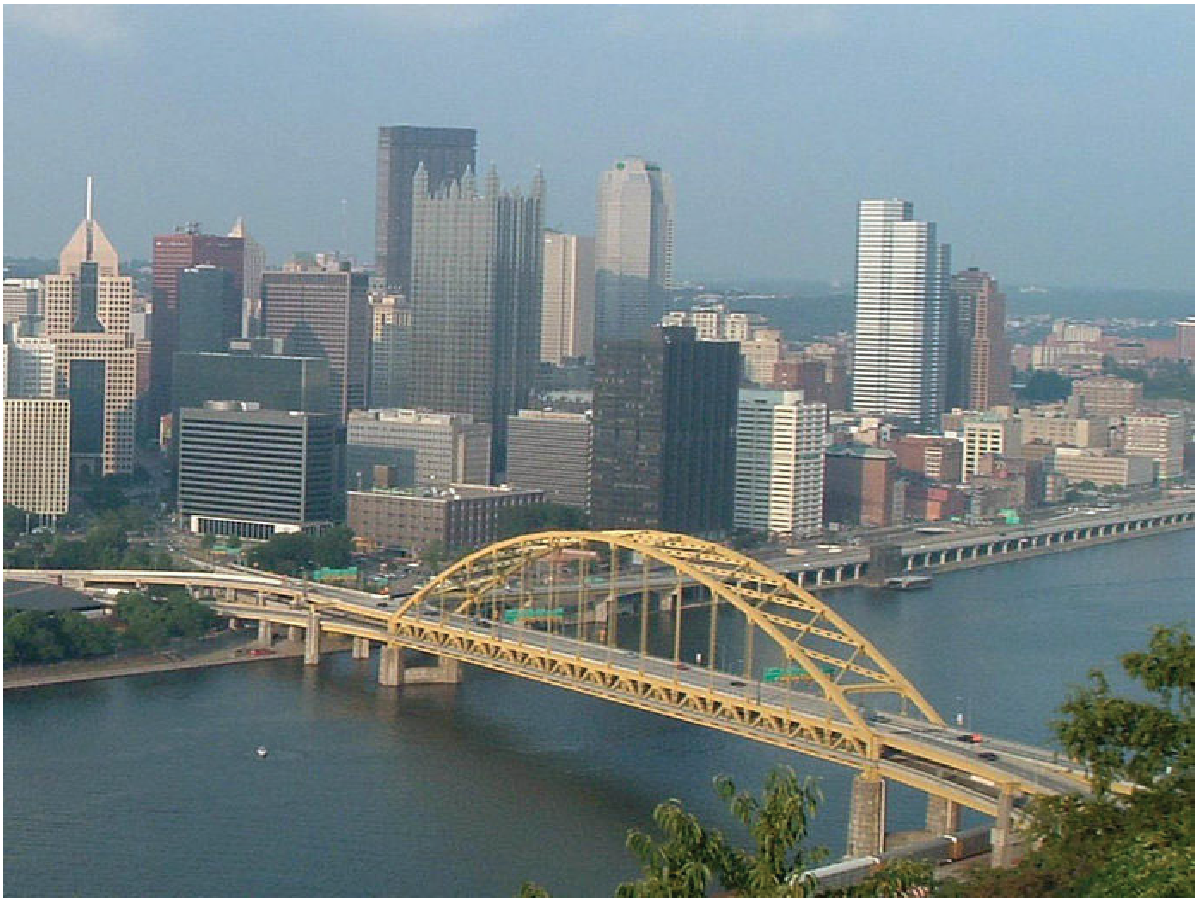


Analysis of Structures

- Definition of a Truss

Analysis of Structures

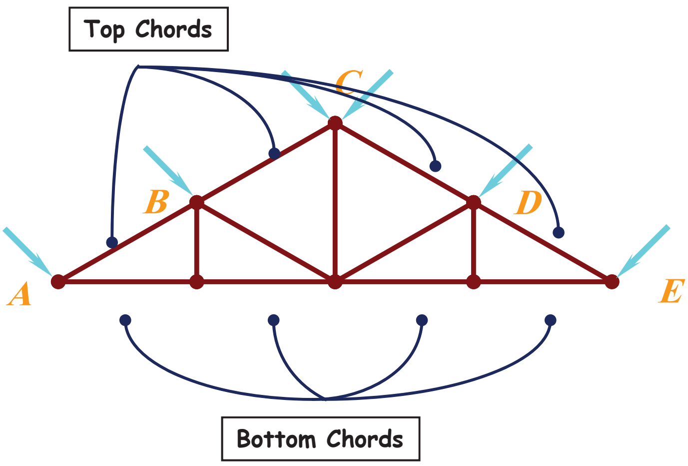
❑ Definition of a Truss

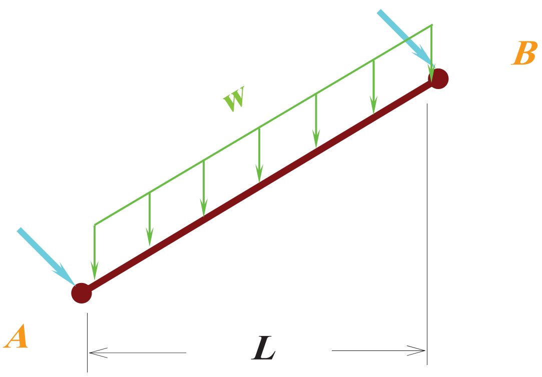


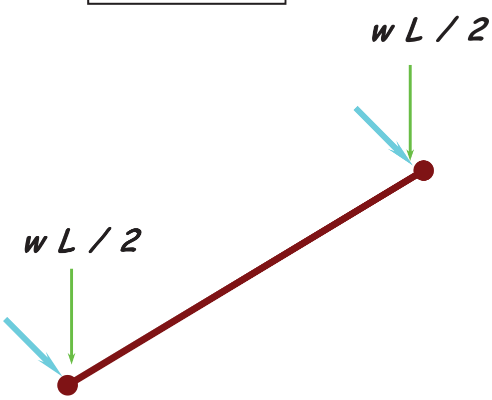
Analysis of Structures

❑ Definition of a Truss

As we can see,
external loads are applied directly to the joints. Member self weights, on the other hand, are assumed to be equally divided at the end joints of each member and added to the externally applied loads.





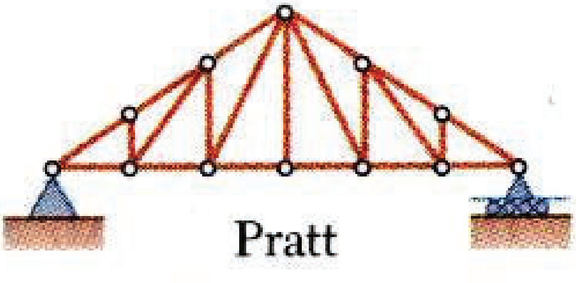


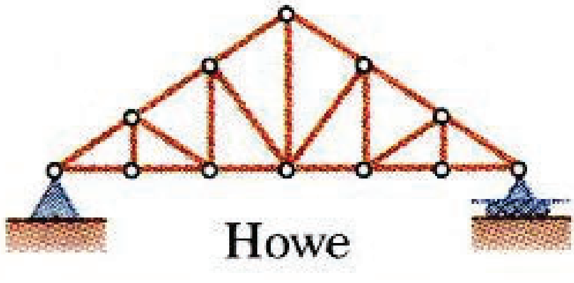
The analysis error made in this process is negligible

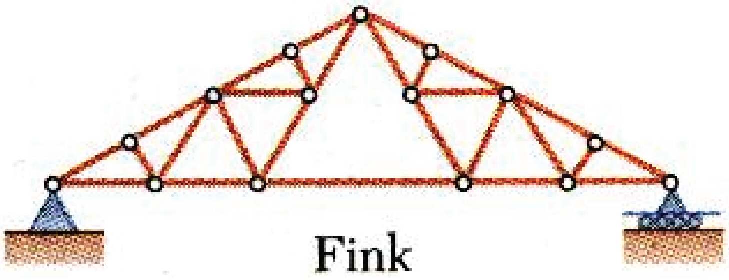
Analysis of Structures

❑ Definition of a Truss

<u>Typical Roof Trusses</u>



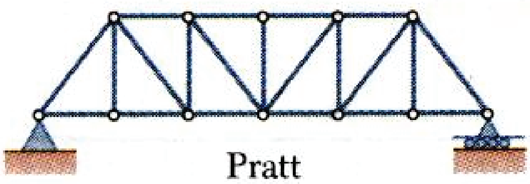


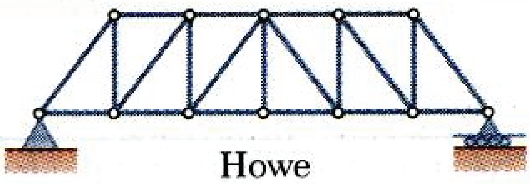


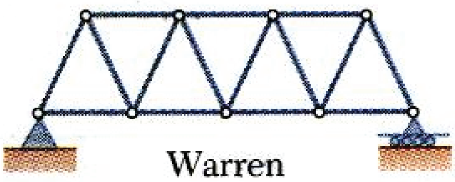
Analysis of Structures

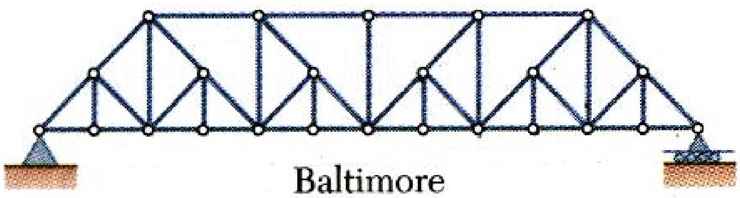
❑ Definition of a Truss

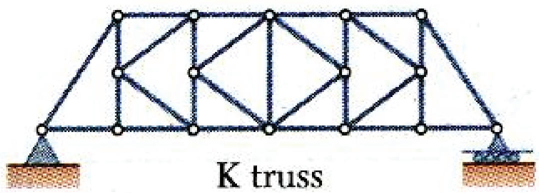
<u>Typical Bridge Trusses</u>







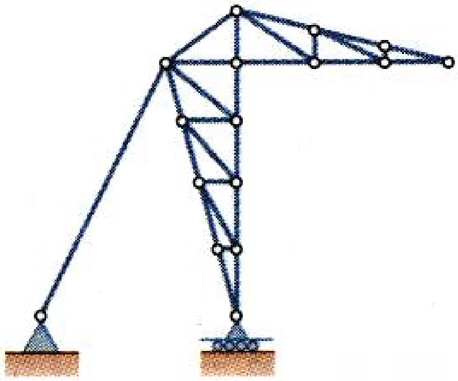




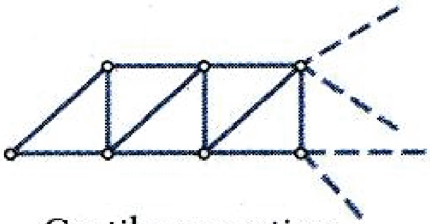
Analysis of Structures

❑ Definition of a Truss

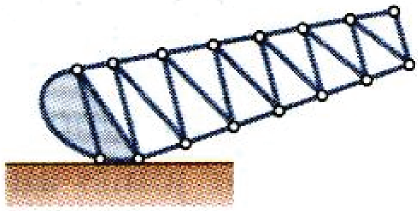
<u>Other Type of Trusses</u>



Stadium



Cantilever portion of a truss

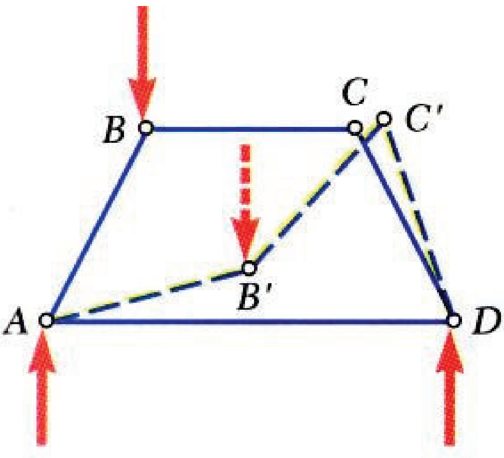


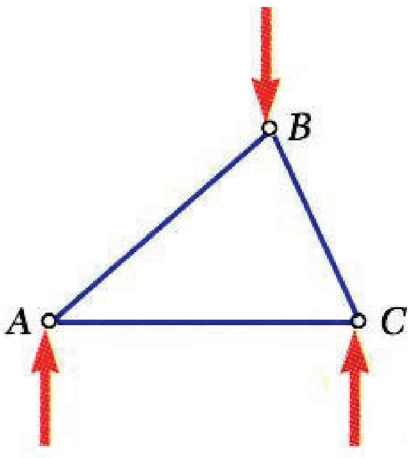
Bascule

Analysis of Structures

❑ Simple Trusses

- A *rigid truss* will not collapse under the application of a load.

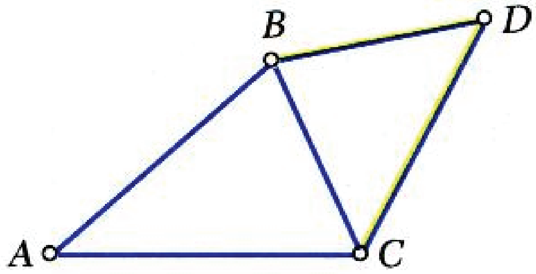




Analysis of Structures

❑ Simple Trusses

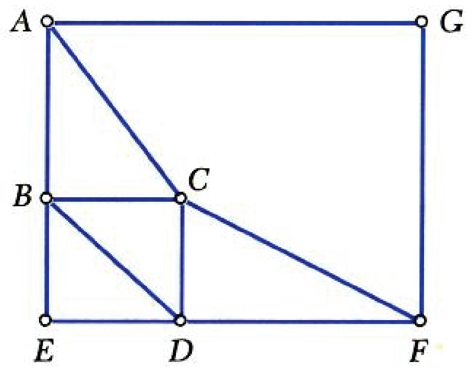
- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.



- In a simple truss,

$$m = 2n - 3$$

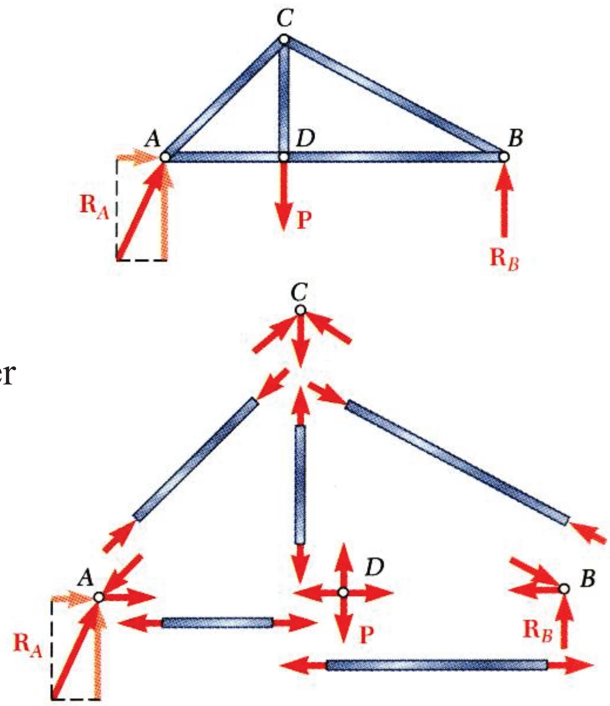
where m is the total number of members and n is the number of joints.



Analysis of Structures

❑ Analysis of Trusses by the Method of Joints

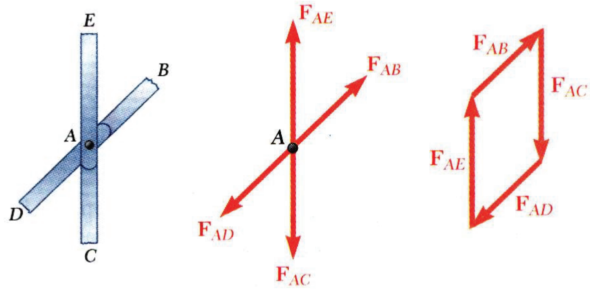
- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.



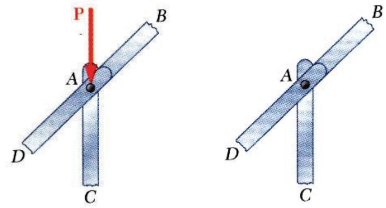
Analysis of Structures

❑ Joints Under Special Loading Conditions

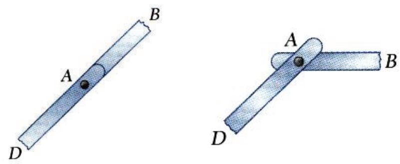
- Forces in opposite members intersecting in two straight lines at a joint are equal.



- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).



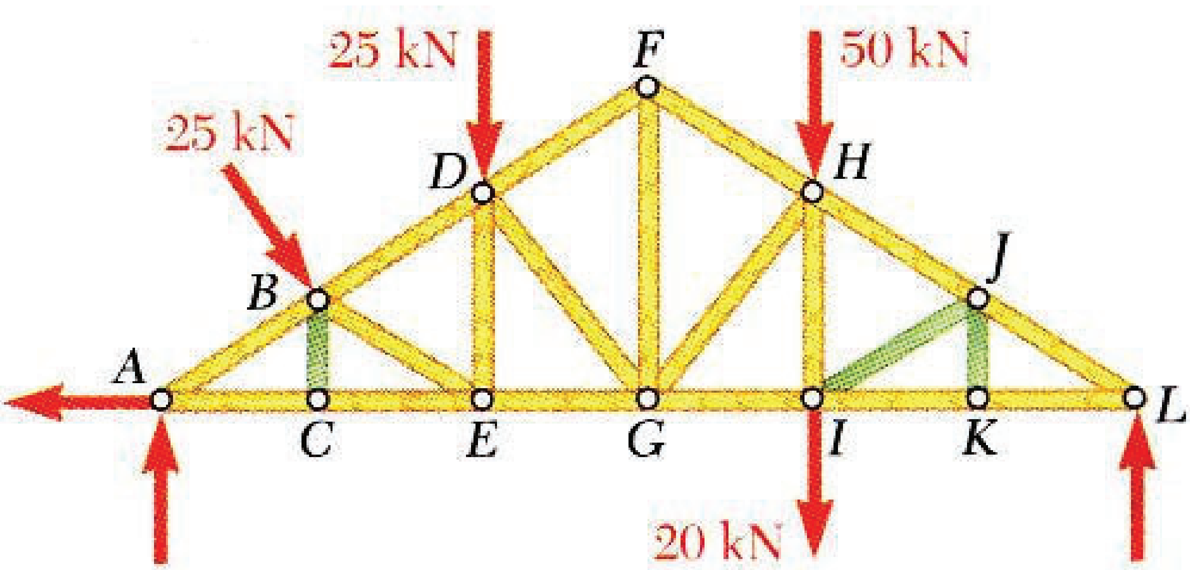
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.



Analysis of Structures

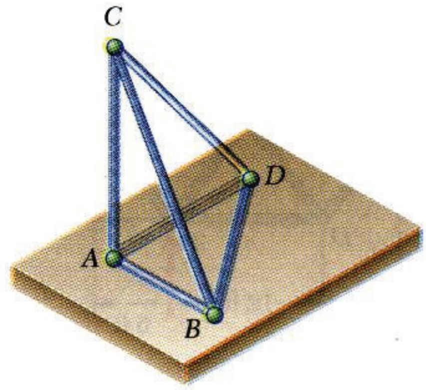
❑ Joints Under Special Loading Conditions

- Recognition of joints under special loading conditions simplifies a truss analysis.



Analysis of Structures

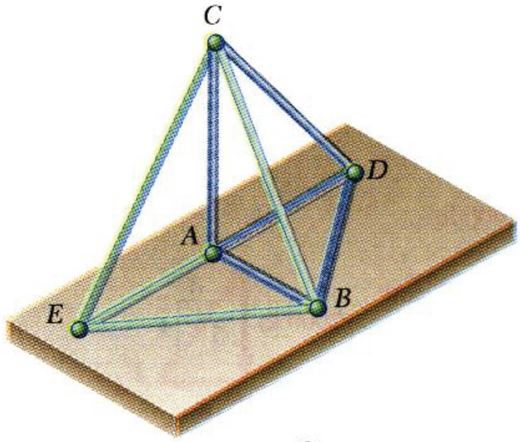
❑ Space Trusses



- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss,

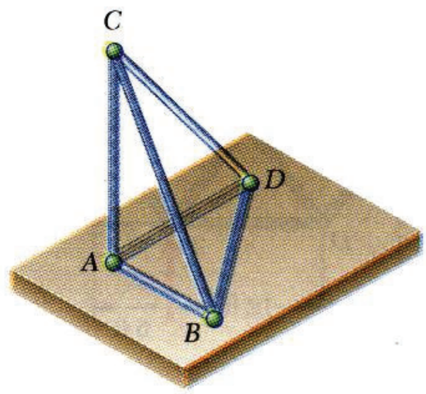
$$m = 3n - 6$$

where m is the number of members and n is the number of joints.

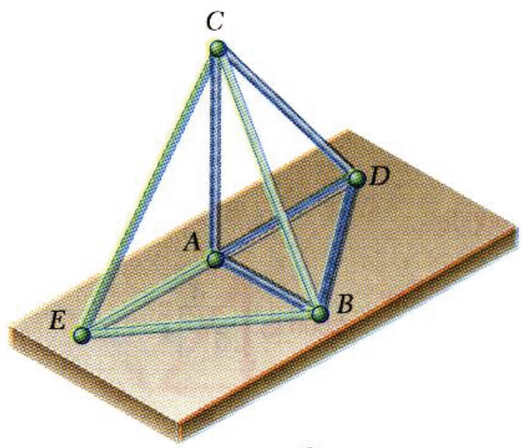


Analysis of Structures

❑ Space Trusses



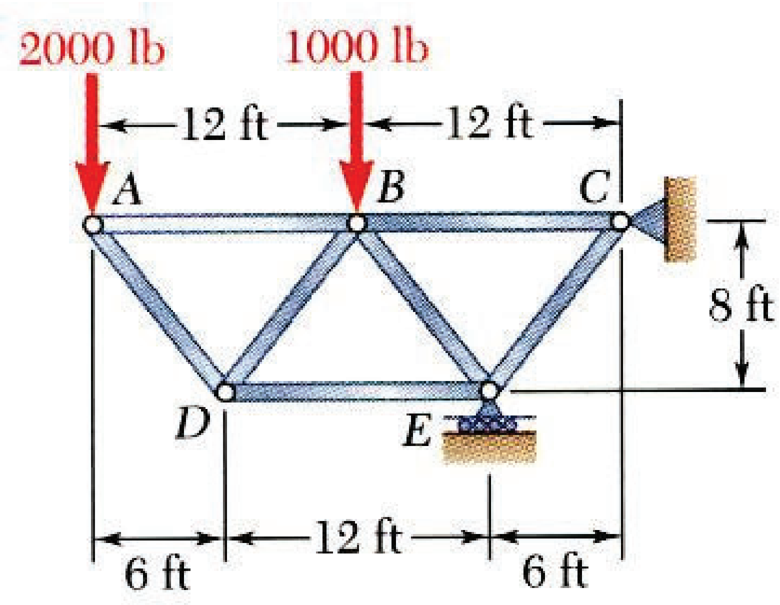
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.



Analysis of Structures

❑ Sample Problem 01

Using the method of joints, determine the force in each member of the truss.

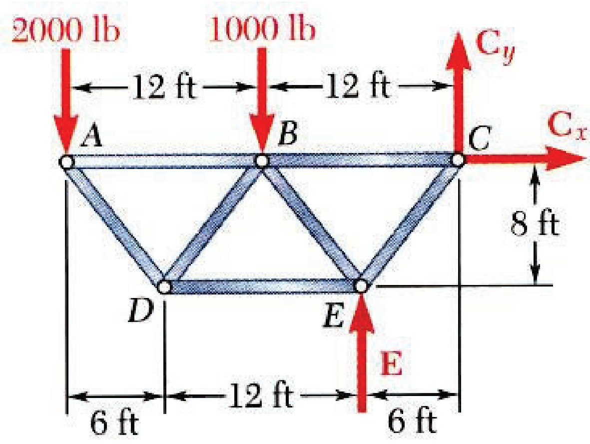


Analysis of Structures

❑ Sample Problem 01

SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at E and C.



$E = 10000\ (lb) \uparrow$

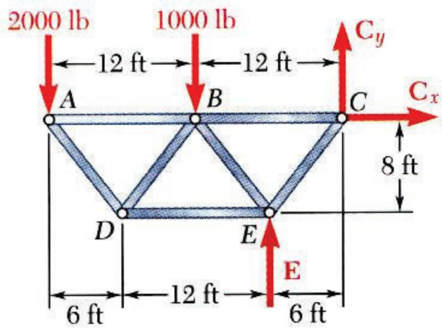
$C_x = 0$

$C_y = 7000\ (lb) \downarrow$

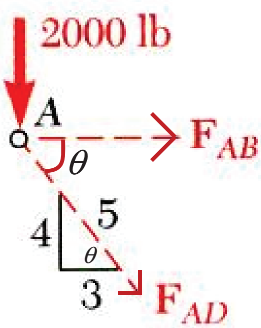
Analysis of Structures

❑ Sample Problem 01

SOLUTION:



- Joint A is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.



$$\Rightarrow \boxed{F_{AB} + F_{AD}\left(\frac{3}{5}\right) = 0} \quad (I)$$

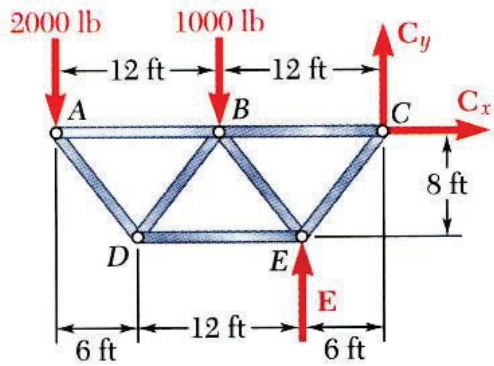
$$\Rightarrow \boxed{-2000 - F_{AD}\left(\frac{4}{5}\right) = 0} \quad (II)$$

$$(I)\ \&\ (II) \quad \Rightarrow \boxed{\begin{cases} F_{AB} = 1500\text{ (lb)} & T \\ F_{AD} = -2500\text{ (lb)} & C \end{cases}}$$

Analysis of Structures

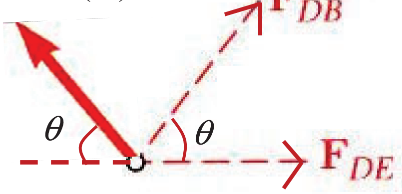
❑ Sample Problem 01

SOLUTION:



- There are now only two unknown member forces at joint D.

$F_{AD} = -2500\text{ (lb)}$



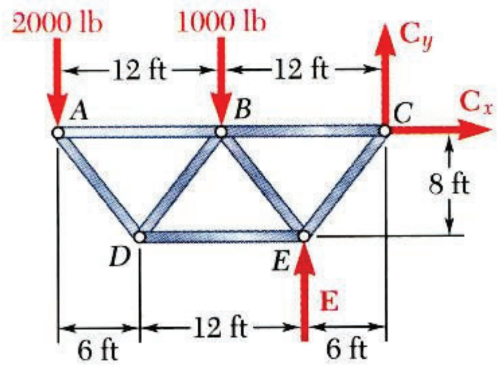
$$\Rightarrow \boxed{+2500\left(\frac{3}{5}\right) + F_{DB}\left(\frac{3}{5}\right) + F_{DE} = 0} \quad (I)$$

$$\Rightarrow \boxed{-2500\left(\frac{4}{5}\right) + F_{DB}\left(\frac{4}{5}\right) = 0} \quad (II)$$

$$(I)\ \&\ (II) \quad \Rightarrow \boxed{\begin{cases} F_{DB} = 2500\text{ (lb)} & T \\ F_{DE} = -3000\text{ (lb)} & C \end{cases}}$$

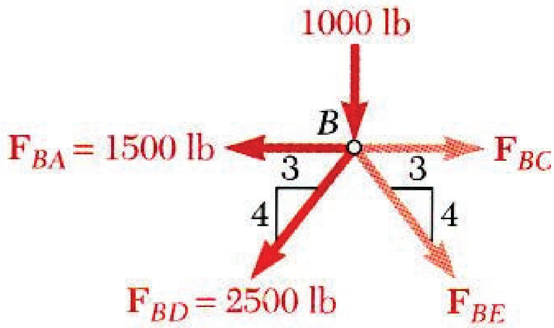
Analysis of Structures

❑ Sample Problem 01



SOLUTION:

- There are now only two unknown member forces at joint B. Assume both are in tension.



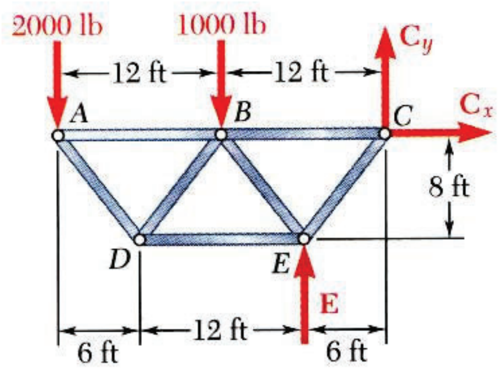
$$\Rightarrow \boxed{-1000-(2500)\left(\frac{4}{5}\right)-F_{BE}\left(\frac{4}{5}\right)=0} \quad (I)$$

$$\Rightarrow \boxed{F_{BC}-1500-2500\left(\frac{3}{5}\right)+F_{BE}\left(\frac{3}{5}\right)=0} \quad (II)$$

$$(I)\ \&\ (II) \Rightarrow \boxed{\begin{cases} F_{BC}=5250\ (\text{lb}) & T \\ F_{BE}=-3750\ (\text{lb}) & C \end{cases}}$$

Analysis of Structures

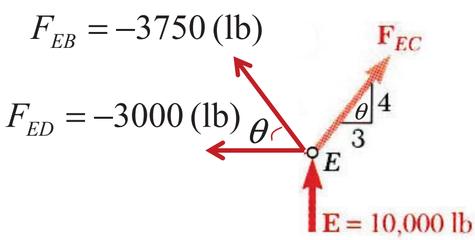
❑ Sample Problem 01



SOLUTION:

- There is one unknown member force at joint *E*. Assume the member is in tension.

$$\Rightarrow \boxed{+10000+(-3750)\left(\frac{4}{5}\right)+F_{EC}\left(\frac{4}{5}\right)=0} \quad (I)$$



$$\Rightarrow \boxed{-(-3000)-(-3750)\left(\frac{3}{5}\right)+F_{EC}\left(\frac{3}{5}\right)=0} \quad (II)$$

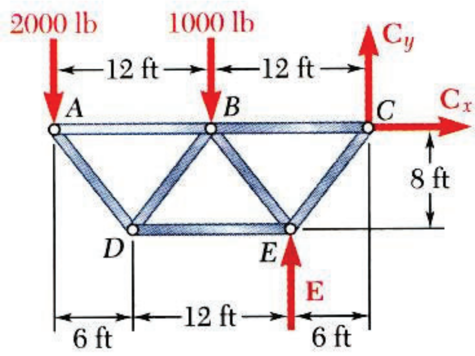
$$(I)\ or\ (II) \Rightarrow \boxed{F_{EC}=-8750\ (\text{lb})\ \ C}$$

Analysis of Structures

❑ Sample Problem 01

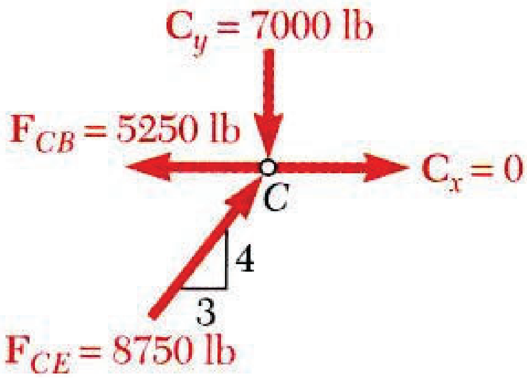
SOLUTION:

- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.



$$\sum F_x = 0 \quad \Rightarrow \quad -5250 + 8750\left(\frac{3}{5}\right) = 0 \quad \text{(OK)}$$

$$\sum F_y = 0 \quad \Rightarrow \quad -7000 + 8750\left(\frac{4}{5}\right) = 0 \quad \text{(OK)}$$



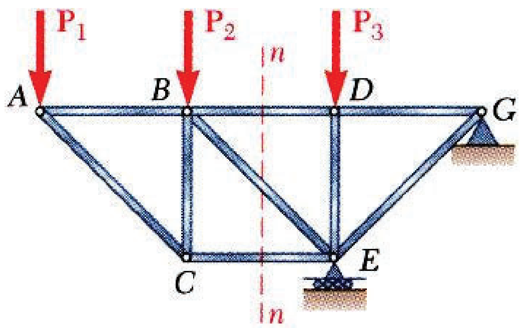
Analysis of Structures

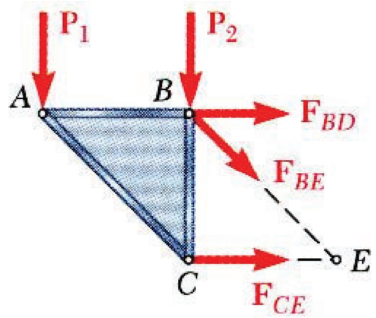
❑ Analysis of Trusses by the Method of Sections

- When the force in only one member or the forces in a very few members are desired, the ***method of sections*** works well.

- To determine the force in member *BD, pass a section* through the truss as shown and create a free body diagram for the left side.

- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}.

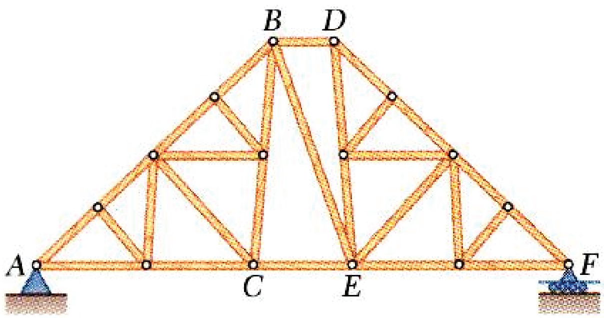


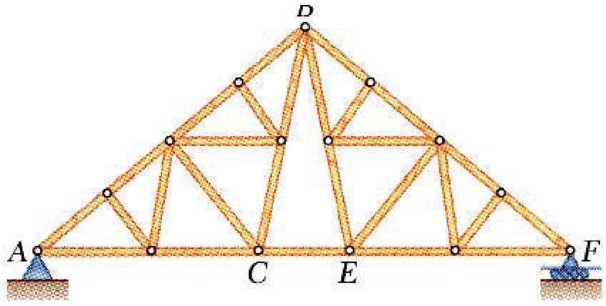


Analysis of Structures

❑ Trusses Made of Several Simple Trusses

- *Compound trusses* are statically ***determinant***, ***rigid***, and ***completely constrained***.





Unknown: $m + r$

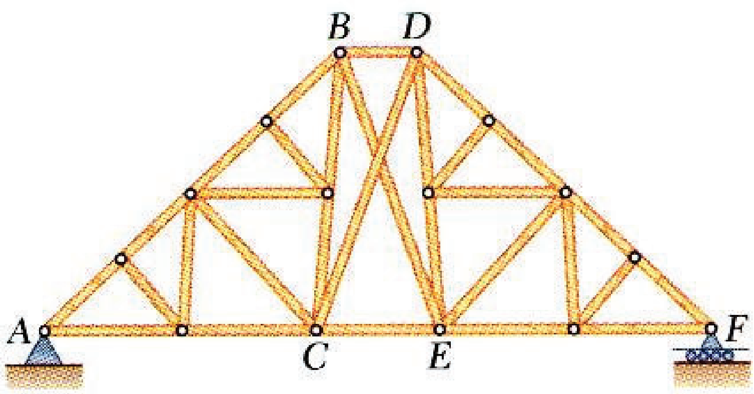
Equations: $2n$

$$\Rightarrow \boxed{m + r = 2n}$$

Analysis of Structures

❑ Trusses Made of Several Simple Trusses

- Truss contains a ***redundant member*** and is ***statically indeterminate***.



Unknown: $m + r$

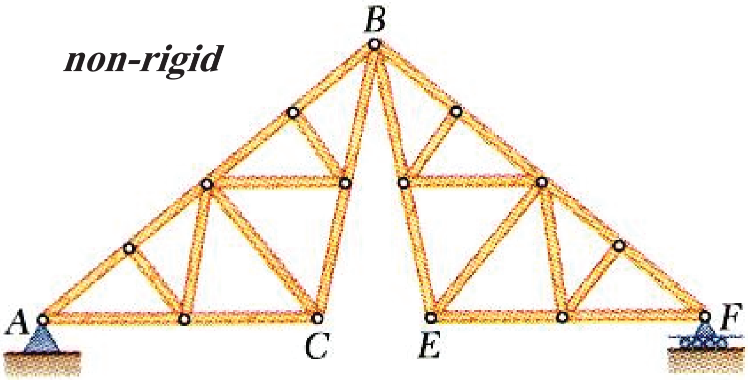
Equations: $2n$

$$\Rightarrow \boxed{m + r > 2n}$$

Analysis of Structures

❑ Trusses Made of Several Simple Trusses

- Additional reaction forces may be necessary for a rigid truss.



Unknown: $m+r$

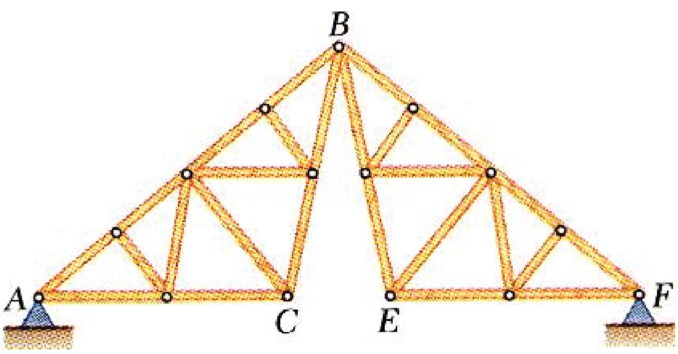
Equations: $2n$

$\Rightarrow$ $m+r<2n$

Analysis of Structures

❑ Trusses Made of Several Simple Trusses

- Necessary but insufficient condition for a compound truss to be statically determinant, rigid, and completely constrained,



Unknown: $m+r$

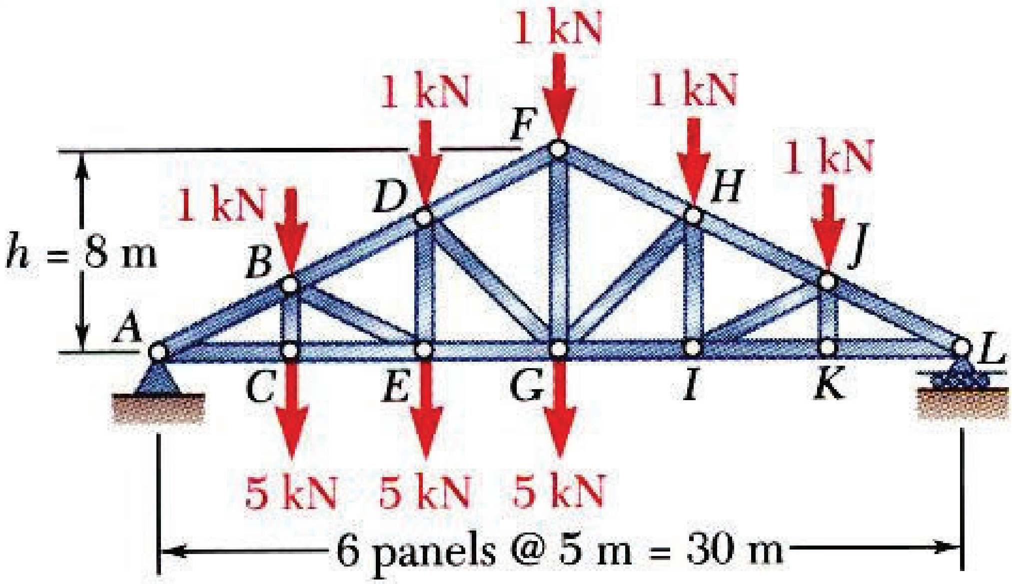
Equations: $2n$

$\Rightarrow$ $m+r<2n$

Analysis of Structures

❑ Sample Problem 02

Determine the force in members *FH*, *GH*, and *GI*.

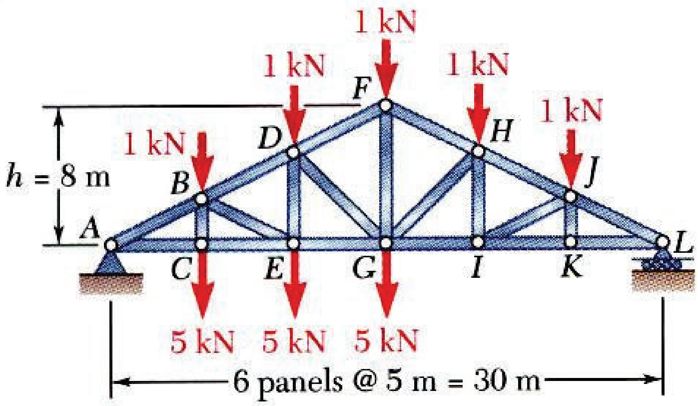


Analysis of Structures

❑ Sample Problem 02

SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at *A* and *L*.



$$\Rightarrow \boxed{L = 7.5\,(\text{kN}) \uparrow}$$

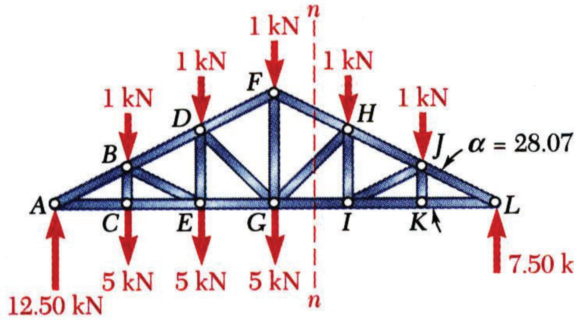
$$\boxed{A_y = 12.5\,(\text{kN}) \uparrow}$$

$$\boxed{A_x = 0}$$

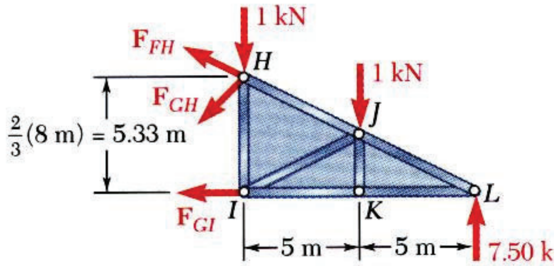
Analysis of Structures

❑ Sample Problem 02

SOLUTION:



- Pass a section through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.



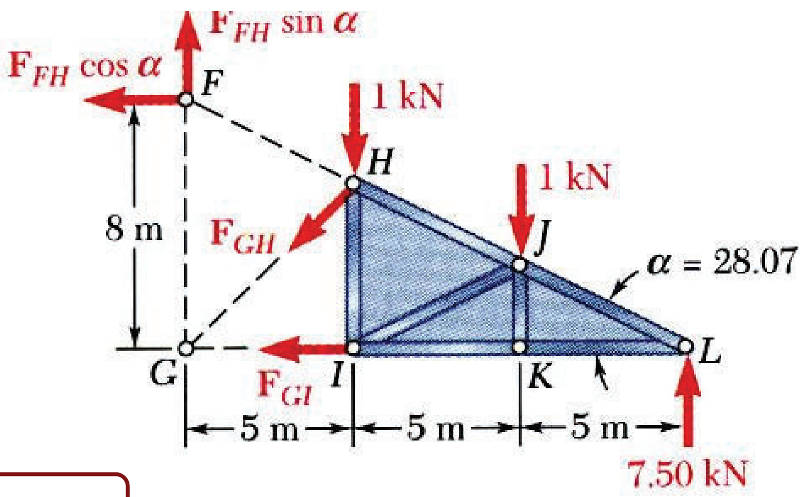
- Apply the conditions for static equilibrium to determine the desired member forces.

$$\Rightarrow \boxed{F_{GI} = +13.13\,(\text{kN}) \quad \text{T}}$$

Analysis of Structures

❑ Sample Problem 02

SOLUTION:



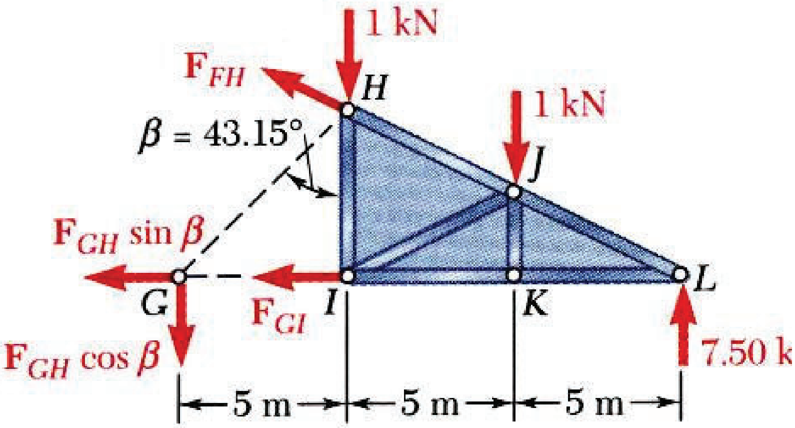
$$\boxed{\alpha = 28.07°}$$

$$\Rightarrow \boxed{F_{FH} = -13.82\,(\text{kN}) \quad \text{C}}$$

Analysis of Structures

❑ Sample Problem 02

SOLUTION:



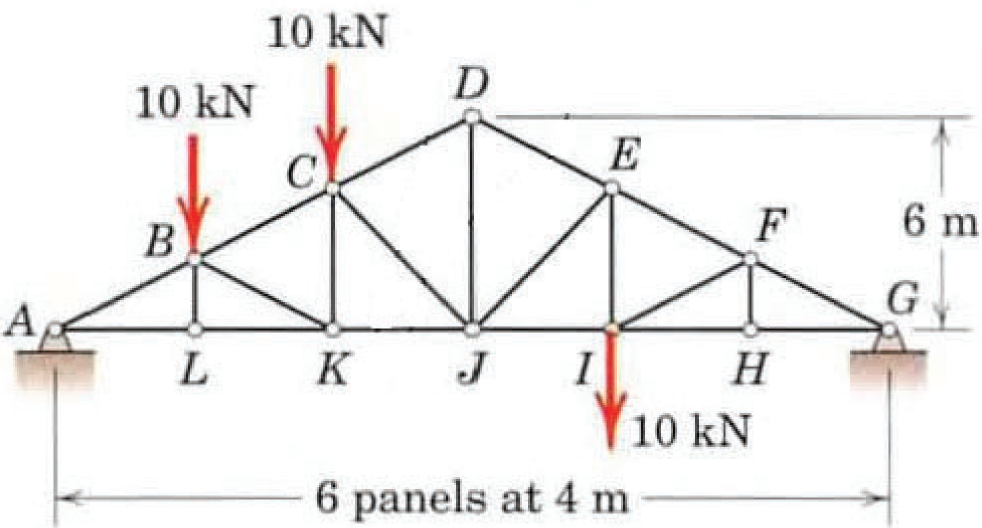
$\beta = 43.15°$

$\Rightarrow$ $F_{GH} = -1.371\,(\text{kN})$ C

Analysis of Structures

❑ Sample Problem 03

Calculate the force in member DJ of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.

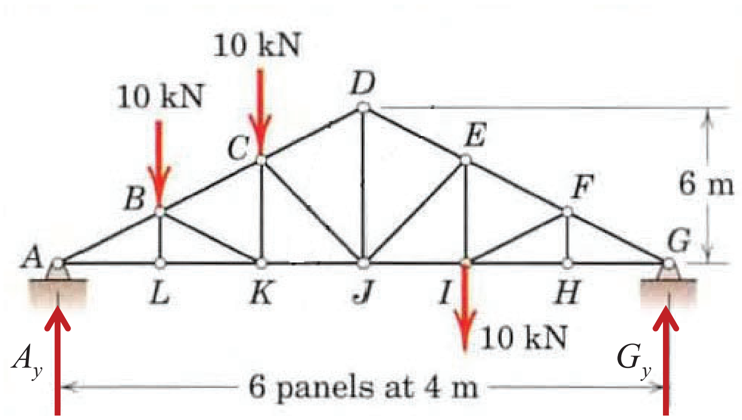


Analysis of Structures

❑ Sample Problem 03

SOLUTION:

- Determine the Unknown Vertical reactions



$$G_y = 11.67 \text{ (kN)} \uparrow$$

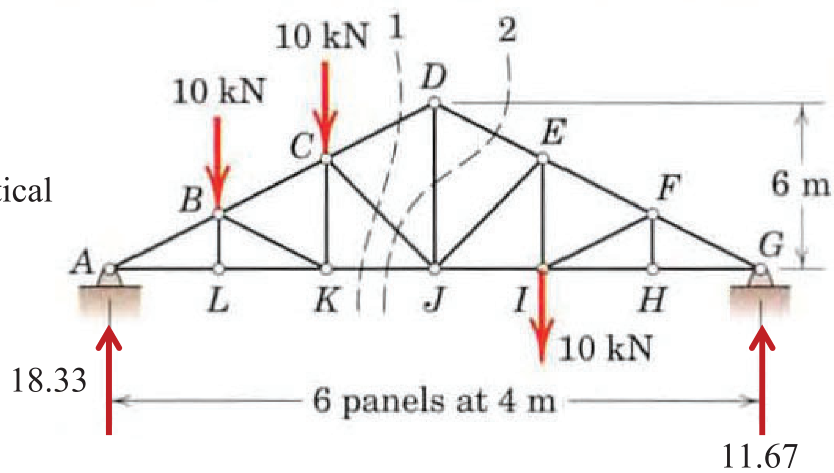
$$A_y = 18.33 \text{ (kN)} \uparrow$$

Analysis of Structures

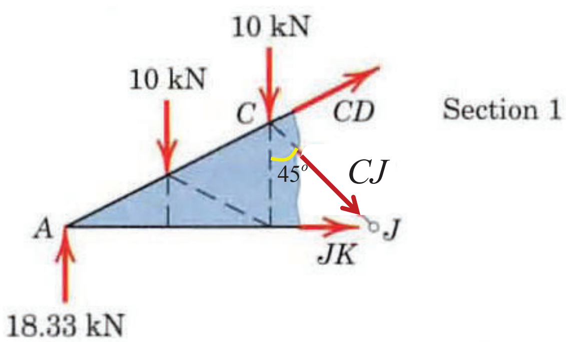
❑ Sample Problem 03

SOLUTION:

- Determine the Unknown Vertical reactions



$$CJ = -14.14 \text{ (kN)}$$

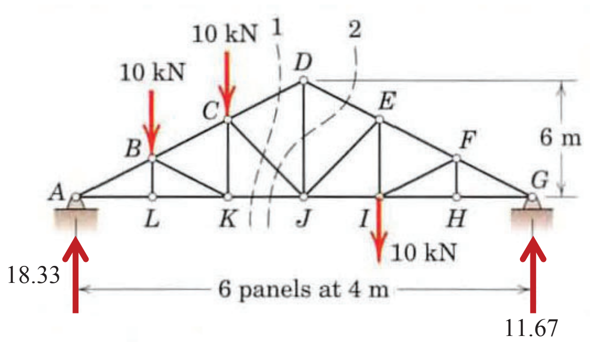


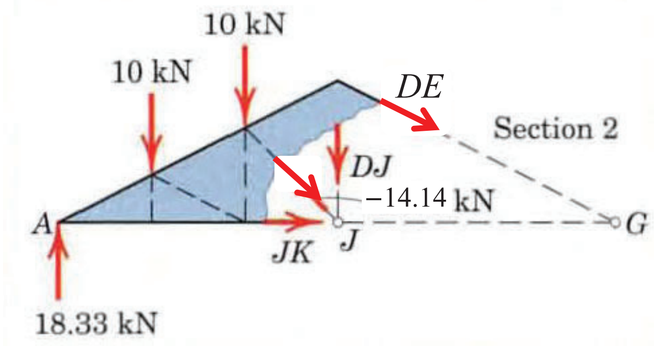
Analysis of Structures

❑ Sample Problem 03

SOLUTION:

- Determine the Unknown Vertical reactions

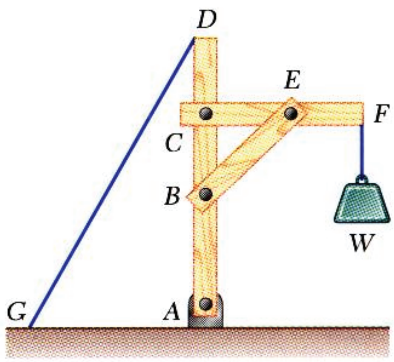




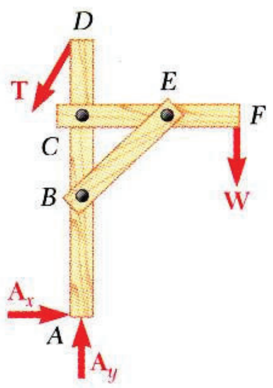
$$DJ = 16.67 \text{ (kN)}$$

Analysis of Structures

❑ Analysis of Frames



- *Frames* and *machines* are structures with at least one *multiforce* member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and are designed to transmit and modify forces.

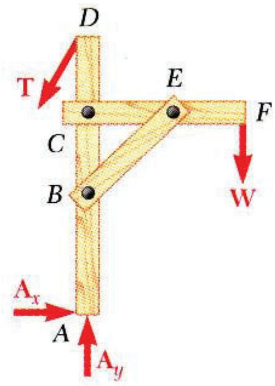


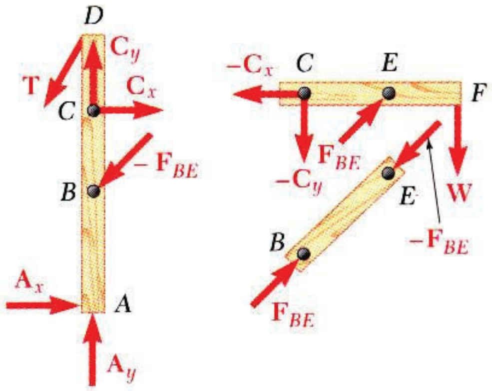
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.

Analysis of Structures

❑ Analysis of Frames

- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense.
- Forces on multiforce members have unknown magnitude and line of action. They must be represented with two unknown components.
- Forces between connected components are equal, have the same line of action, and opposite sense.

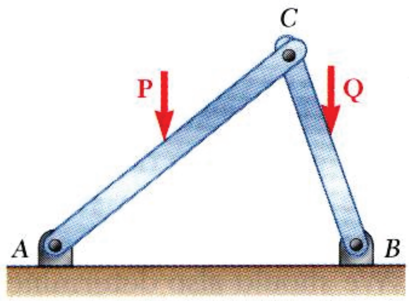




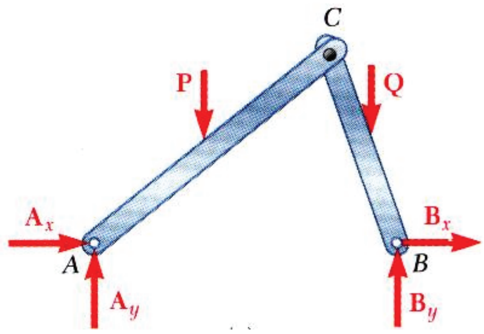
Analysis of Structures

❑ Frames Which Cease To Be Rigid When Detached From Their Supports

- Some frames may collapse if removed from their supports. Such frames can not be treated as rigid bodies.



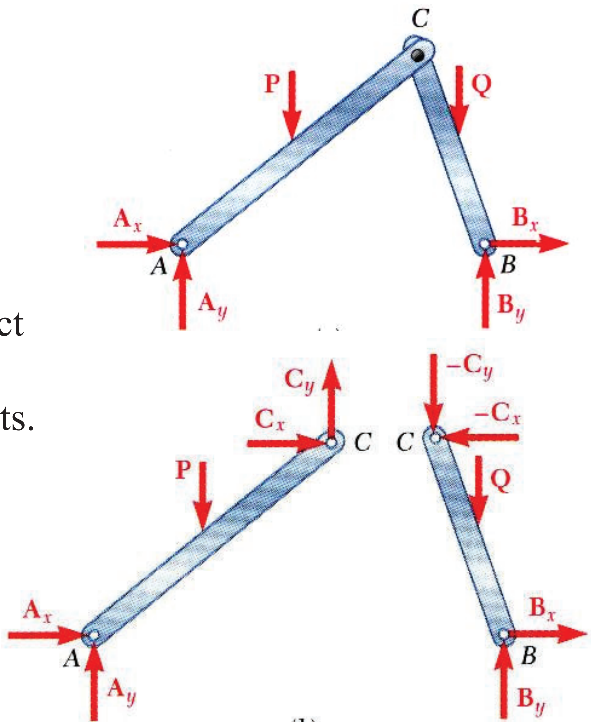
- A free-body diagram of the complete frame indicates four unknown force components which can not be determined from the three equilibrium conditions.



Analysis of Structures

❑ Frames Which Cease To Be Rigid When Detached From Their Supports

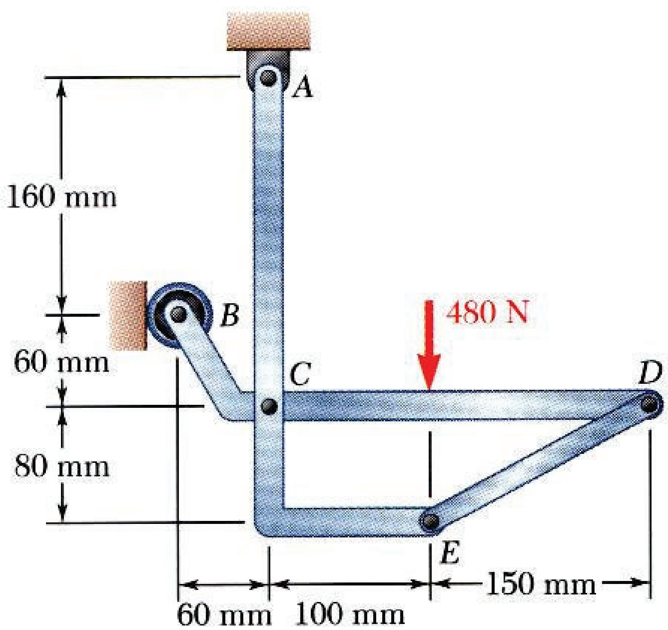
- The frame must be considered as two distinct, but related, rigid bodies.
- With equal and opposite reactions at the contact point between members, the two free-body diagrams indicate 6 unknown force components.
- Equilibrium requirements for the two rigid bodies yield 6 independent equations.



Analysis of Structures

❑ Sample Problem 04

Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

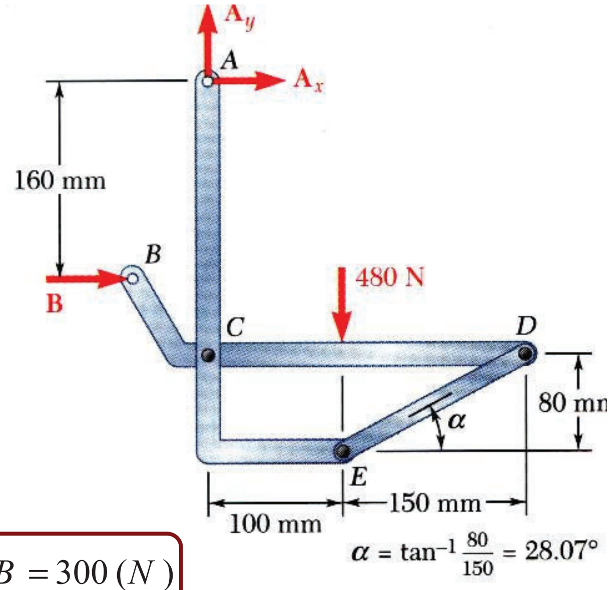


Analysis of Structures

❑ Sample Problem 04

SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.



$$A_y = 480\ (N)$$

$$B = 300\ (N)$$

$$A_x = -300\ (N)$$

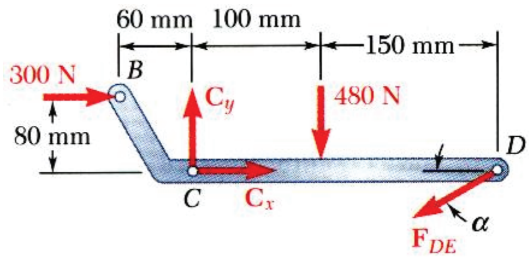
$$\alpha = 28.07°$$

Analysis of Structures

❑ Sample Problem 04

SOLUTION:

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.



$$F_{DE} = -561\ \text{N}$$

- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

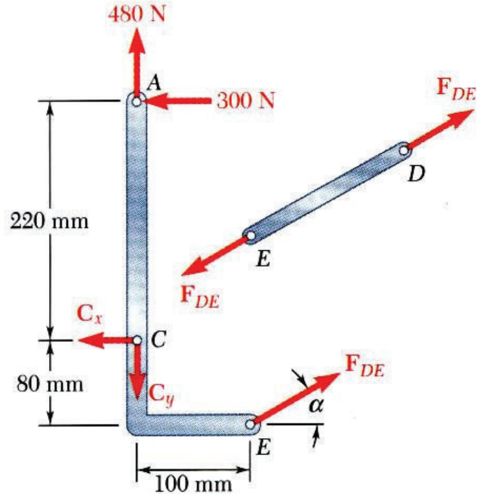
$$\Rightarrow C_x = -795\ (N)$$

$$\Rightarrow C_y = 216\ (N)$$

Analysis of Structures

❑ Sample Problem 04

SOLUTION:



- With member *ACE* as a free-body, check the solution by summing moments about *A*.

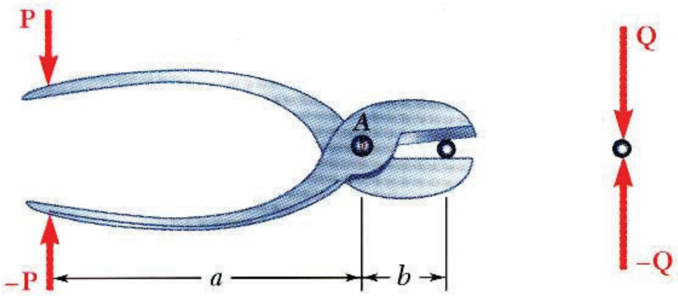
$$\Rightarrow \quad (-561\cos 28.07^\circ)(300) + (-561\sin 28.07^\circ)(100) - (-795)(220) = 0$$

(OK)

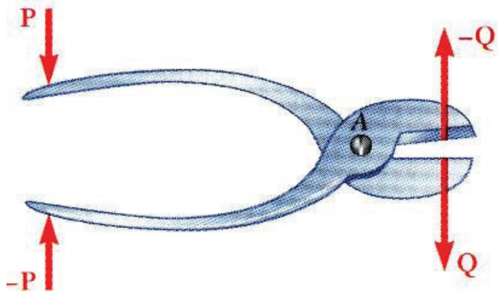
Analysis of Structures

❑ Machines

- Machines are structures designed to transmit and modify forces. Their main purpose is to transform *input forces* into *output forces*.



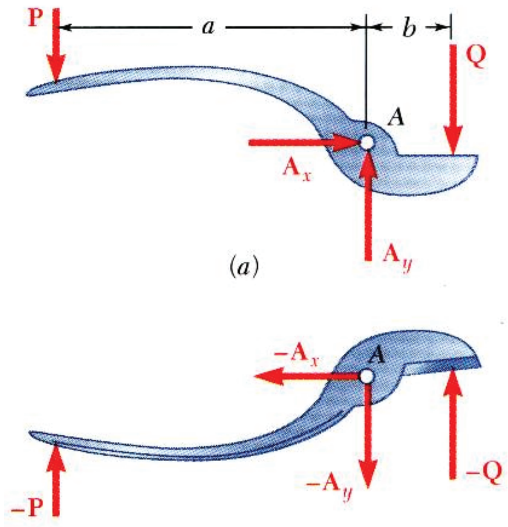
- Given the magnitude of ***P***, determine the magnitude of ***Q***.



Analysis of Structures

❑ Machines

- Create a free-body diagram of the complete machine, including the reaction that the wire exerts.



- The machine is a nonrigid structure. Use one of the components as a free-body.

- Taking moments about A,

$$\sum M_A = 0 \quad \Rightarrow \quad aP - bQ = 0 \quad \Rightarrow \quad \boxed{Q = \frac{a}{b}P}$$