STATICS

- Vector Mechanics for Engineers: Statics, 9th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Statics, 5th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Statics"

Equilibrium of Rigid Bodies

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Equilibrium of Rigid Bodies

Introduction

- For a rigid body in static equilibrium, the external forces and moments are balanced and will impart no translational or rotational motion to the body.

- The ***necessary*** and ***sufficient*** condition for the static equilibrium of a body are that the ***resultant force and couple from all external forces form a system equivalent to zero***,

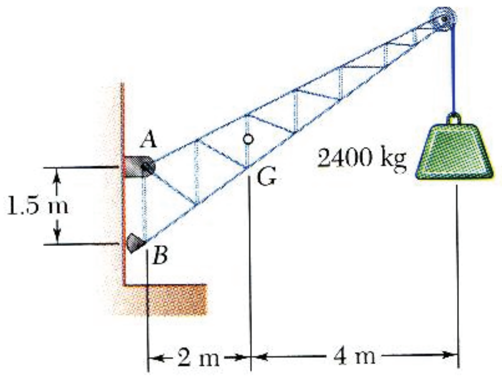
$$\sum \vec{F} = 0 \qquad \sum \vec{M}_O = \sum (\vec{r} \times \vec{F}) = 0$$

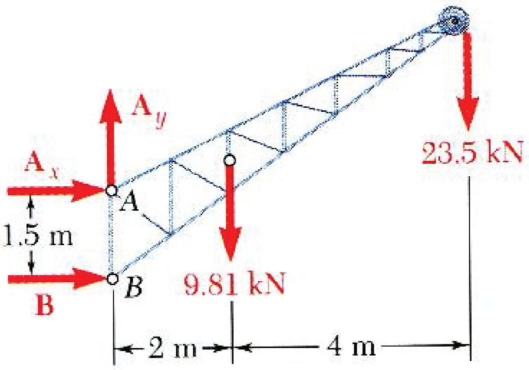
- Resolving each force and moment into its rectangular components leads to 6 scalar equations which also express the conditions for static equilibrium,

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

Equilibrium of Rigid Bodies

❑ Free-Body Diagram





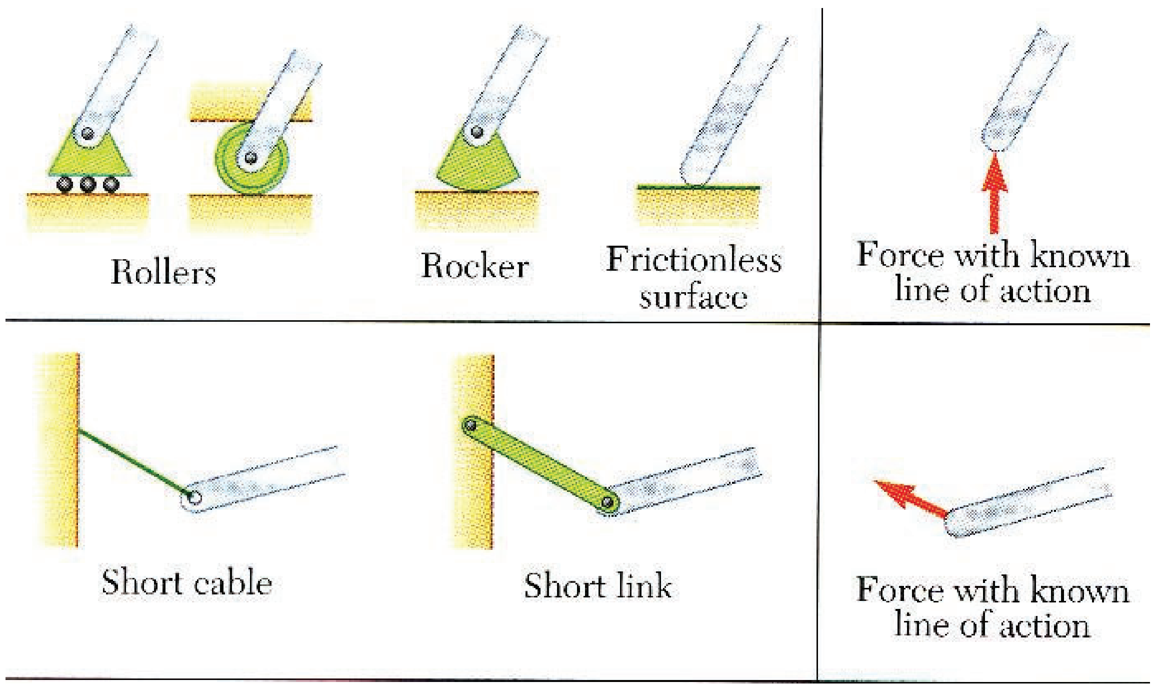
First step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free-body* diagram.

- Select the extent of the free-body and detach it from the ground and all other bodies.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown applied forces. These usually ***consist of reactions*** through which the ground and other bodies oppose the possible motion of the rigid body.
- Include the dimensions necessary to compute the moments of the forces.

Equilibrium of Rigid Bodies

❑ Reactions at Supports and Connections for a Two-Dimensional Structure

- Reactions equivalent to a force with known line of action.



Equilibrium of Rigid Bodies

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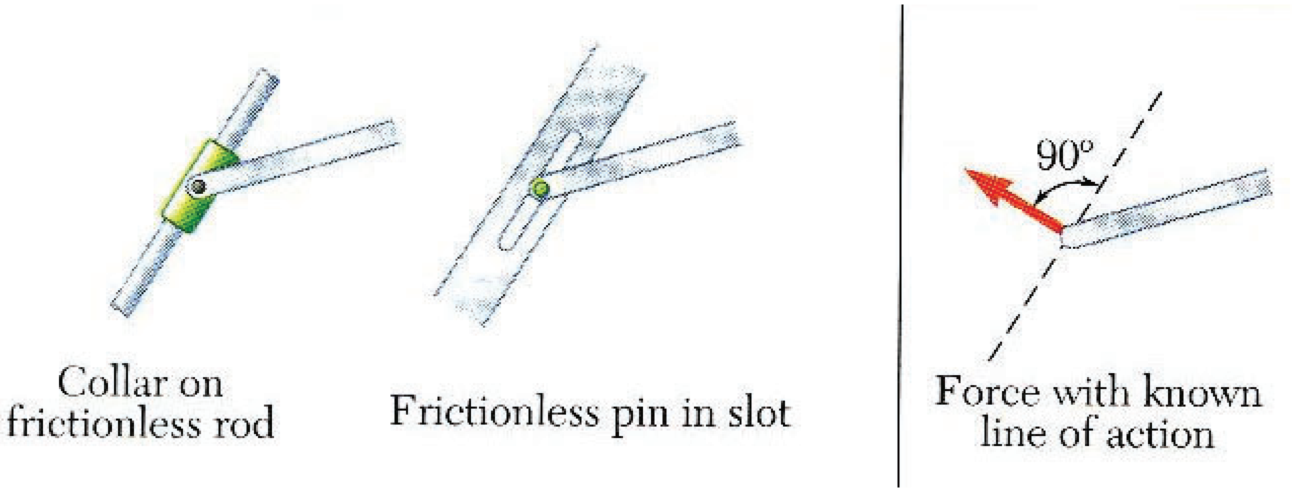
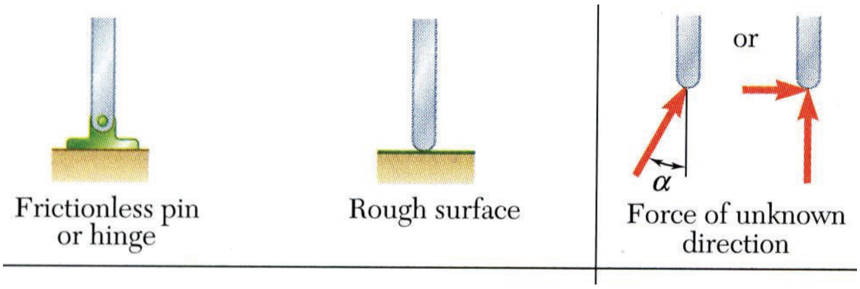


Equilibrium of Rigid Bodies

❑ Reactions at Supports and Connections for a Two-Dimensional Structure

- Reactions equivalent to a force of unknown direction and magnitude.



- Reactions equivalent to a force of unknown direction and magnitude and a couple.of unknown magnitude

Equilibrium of Rigid Bodies

❑ Equilibrium of a Rigid Body in Two Dimensions

- For all forces and moments acting on a two-dimensional structure,

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

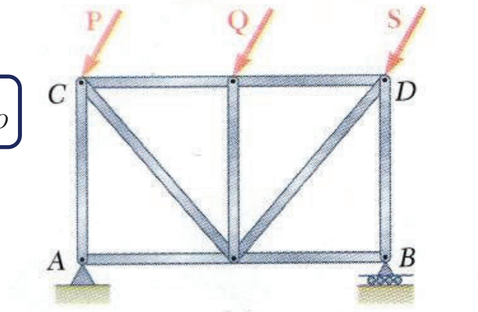
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

where A is any point in the plane of the structure.

- The 3 equations can be solved for no more than 3 unknowns.

- The 3 equations can not be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$



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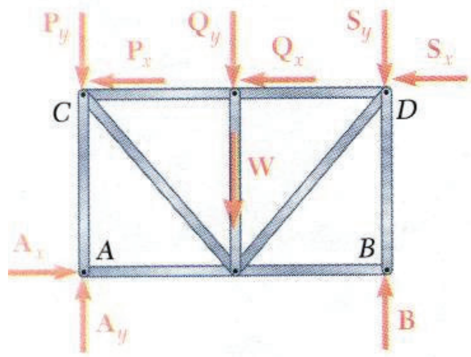
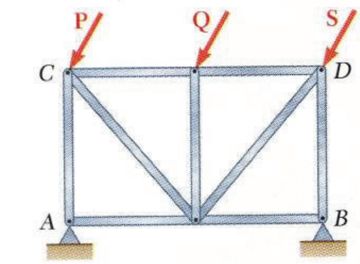


(b)

Equilibrium of Rigid Bodies

❑ Statically Indeterminate Reactions



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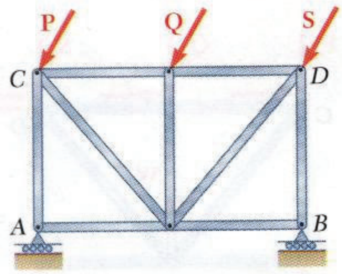

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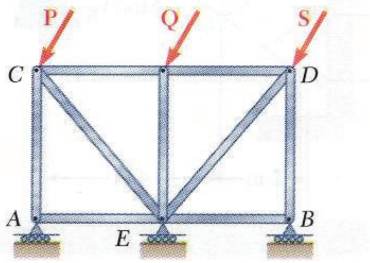

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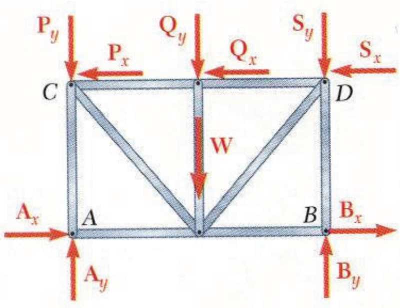

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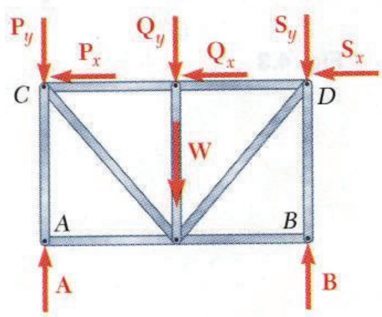

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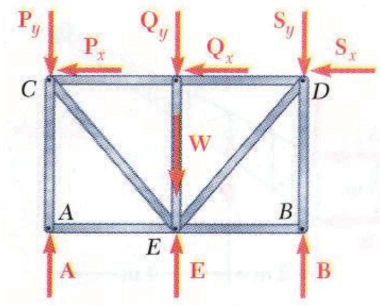


(b)

- More unknowns than equations

- Fewer unknowns than equations, partially constrained

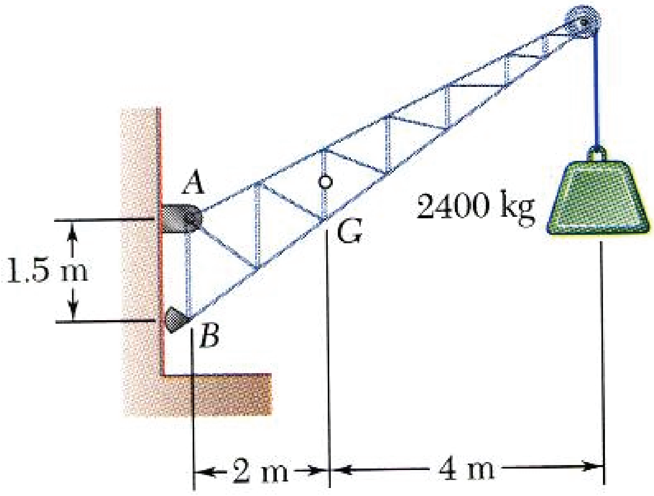
- Equal number unknowns and equations but improperly constrained

Equilibrium of Rigid Bodies

❑ Sample Problem 01

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the reactions at *A* and *B*.

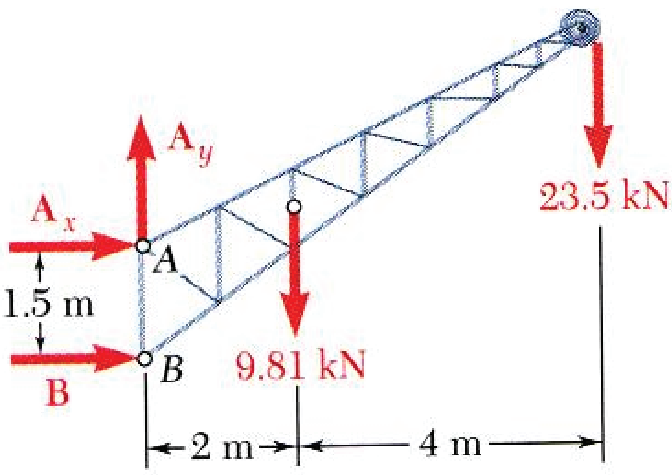


Equilibrium of Rigid Bodies

❑ Sample Problem 01

SOLUTION:

- Create the free-body diagram.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*.



$$\Rightarrow \boxed{B = +107.1 \ \text{(kN)}}$$

- Determine the reactions at *A* by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\Rightarrow \boxed{A_x = -107.1 \ \text{(kN)}}$$

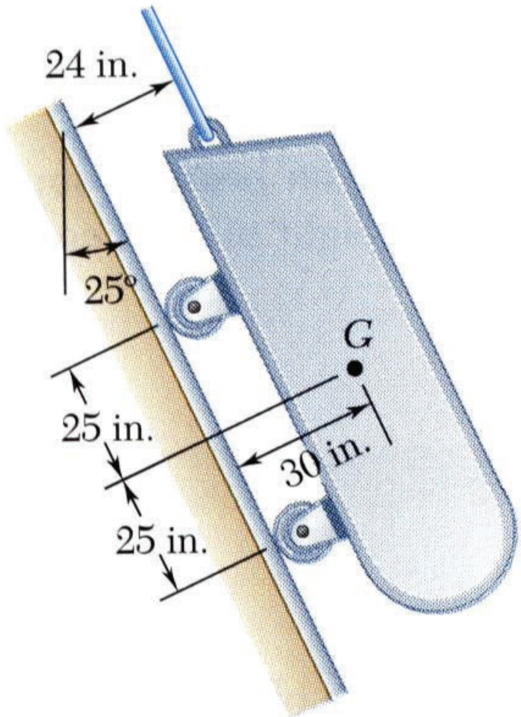
$$\Rightarrow \boxed{A_y = +33.3 \ \text{(kN)}}$$

Equilibrium of Rigid Bodies

❑ Sample Problem 02

A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb, and it is applied at at *G*. The cart is held in position by the cable.

Determine the tension in the cable and the reaction at each pair of wheels.



Equilibrium of Rigid Bodies

❑ Sample Problem 02

SOLUTION:

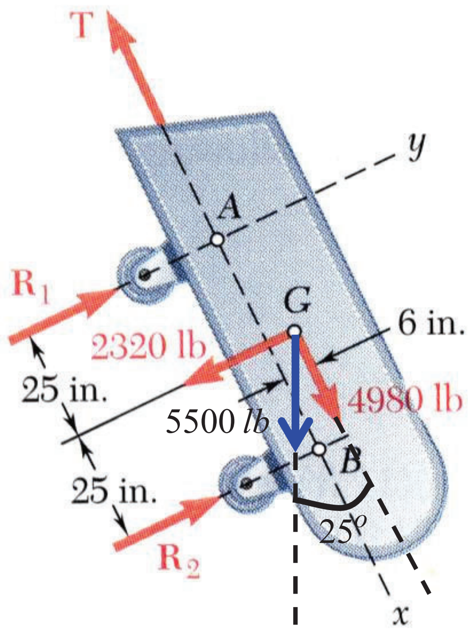
- Create a free-body diagram

$$W_x = 4980 \text{ (lb)}$$
$$W_y = -2320 \text{ (lb)}$$

- Determine the reactions at the wheels.

$$\Rightarrow R_2 = 1758 \text{ (lb)}$$

$$\Rightarrow R_1 = 562 \text{ (lb)}$$



- Determine the cable tension.

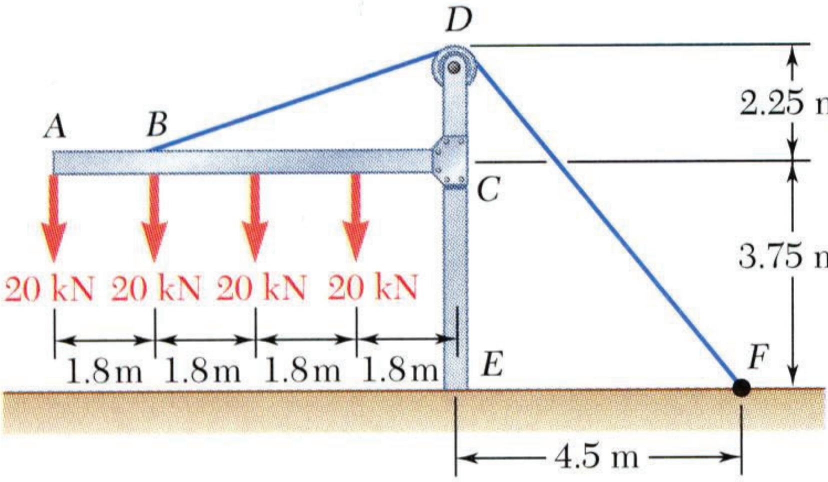
$$\Rightarrow T = 4980 \text{ (lb)}$$

Equilibrium of Rigid Bodies

❑ Sample Problem 03

The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

Determine the reaction at the fixed end *E*.



Equilibrium of Rigid Bodies

❑ Sample Problem 03

SOLUTION:

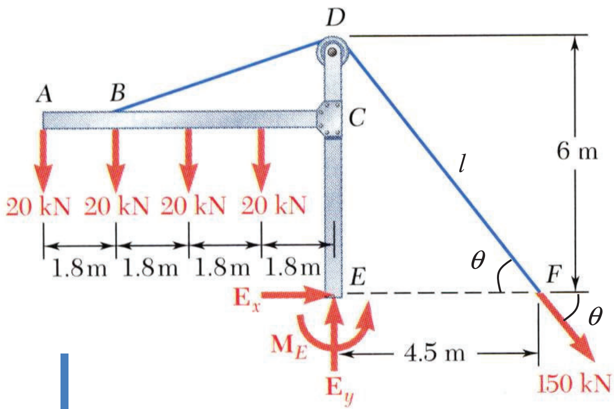
- Create a free-body diagram for the frame and cable.

$$l = 7.5\ (m)$$

- Solve 3 equilibrium equations for the reaction force components and couple.

$$\Rightarrow E_x = -90.0\ \text{(kN)}$$

$$\Rightarrow E_y = 200\ \text{(kN)}$$

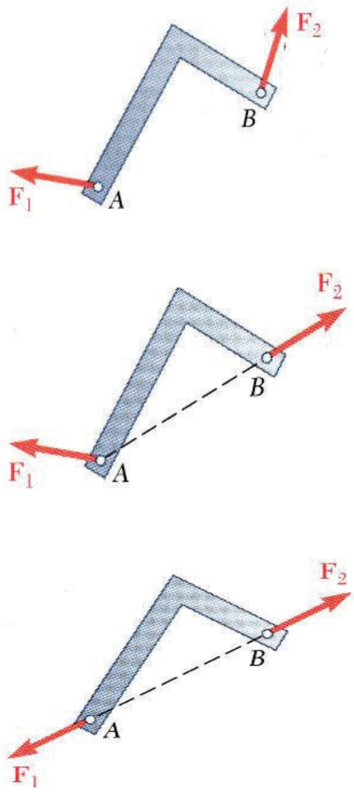


$$\Rightarrow M_E = 180\ \text{(kN·m)}$$

Equilibrium of Rigid Bodies

❑ Equilibrium of a Two-Force Body

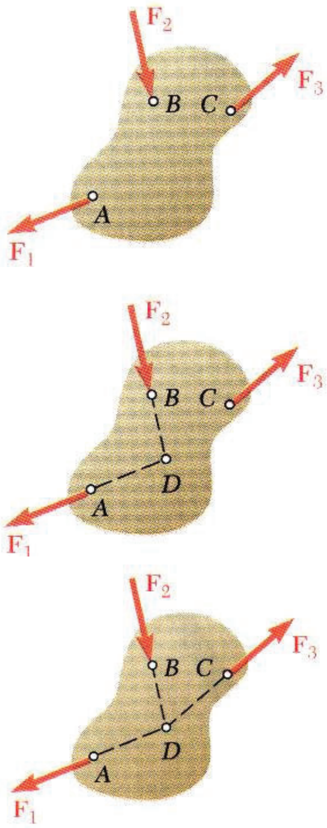
- Consider a plate subjected to two forces F_1 and F_2
- For static equilibrium, the sum of moments about *A* must be zero. The moment of F_2 must be zero. It follows that the line of action of F_2 must pass through *A*.
- Similarly, the line of action of F_1 must pass through *B* for the sum of moments about *B* to be zero.
- Requiring that the sum of forces in any direction be zero leads to the conclusion that F_1 and F_2 must have ***equal magnitude but opposite sense.***



Equilibrium of Rigid Bodies

❑ Equilibrium of a Three-Force Body

- Consider a rigid body subjected to forces acting at only 3 points.
- Assuming that their lines of action intersect, the moment of F_1 and F_2 about the point of intersection represented by *D* is zero.
- Since the rigid body is in equilibrium, the sum of the moments of F_1, F_2, and F_3 about any axis must be zero. It follows that the moment of F_3 about *D* must be zero as well and that the line of action of F_3 must pass through *D*.
- ***The lines of action of the three forces must be concurrent.***

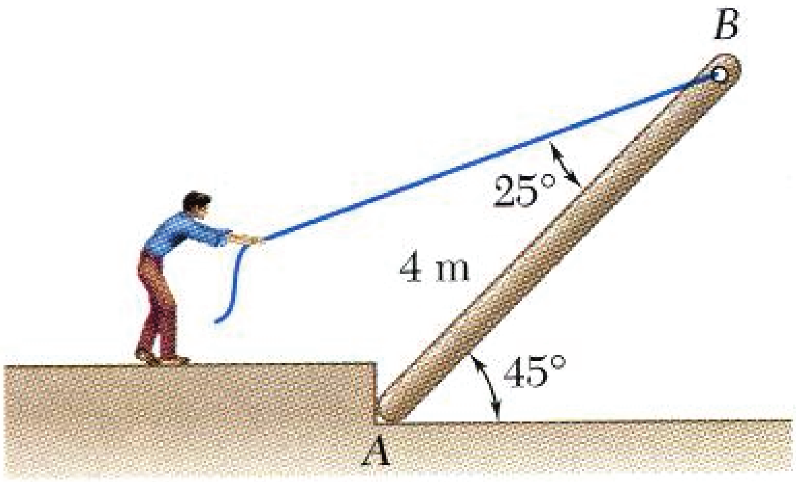


Equilibrium of Rigid Bodies

❑ Sample Problem 04

A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at A.



Equilibrium of Rigid Bodies

❑ Sample Problem 04

SOLUTION:

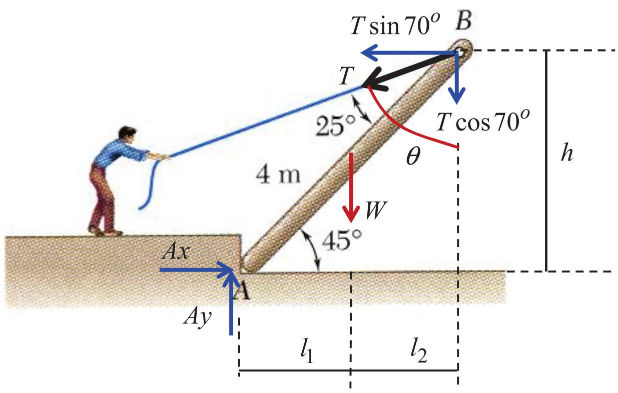
- Create a free-body diagram of the joist.

$W = 98.1\ (N)$

$l_1 = l_2 = \sqrt{2}\ (m)$

$h = 2\sqrt{2}\ (m)$

$\theta = 70^o$



$$\Rightarrow\quad (T\sin 70^o)(2\sqrt{2}) - (98.1)(\sqrt{2}) - (T\cos 70^o)(\sqrt{2}+\sqrt{2}) = 0$$

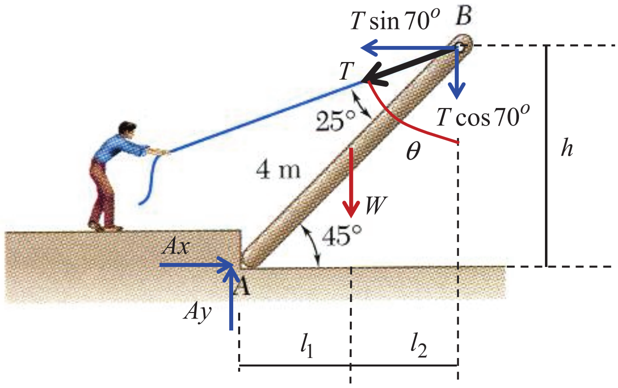
$T = 82.07\ (N)$

Equilibrium of Rigid Bodies

❑ Sample Problem 04

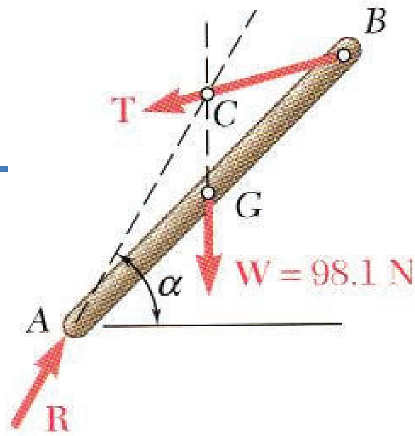
SOLUTION:

- Create a free-body diagram of the joist.



$$\Rightarrow \boxed{A_x = 77.12 \text{ (N)}}$$

$$\Rightarrow \boxed{A_y = 125.83 \text{ (N)}}$$



$$\boxed{R = 147.58\ (N)}$$

$$\boxed{\alpha = 58.5^o}$$

Equilibrium of Rigid Bodies

❑ Equilibrium of a Rigid Body in Three Dimensions

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

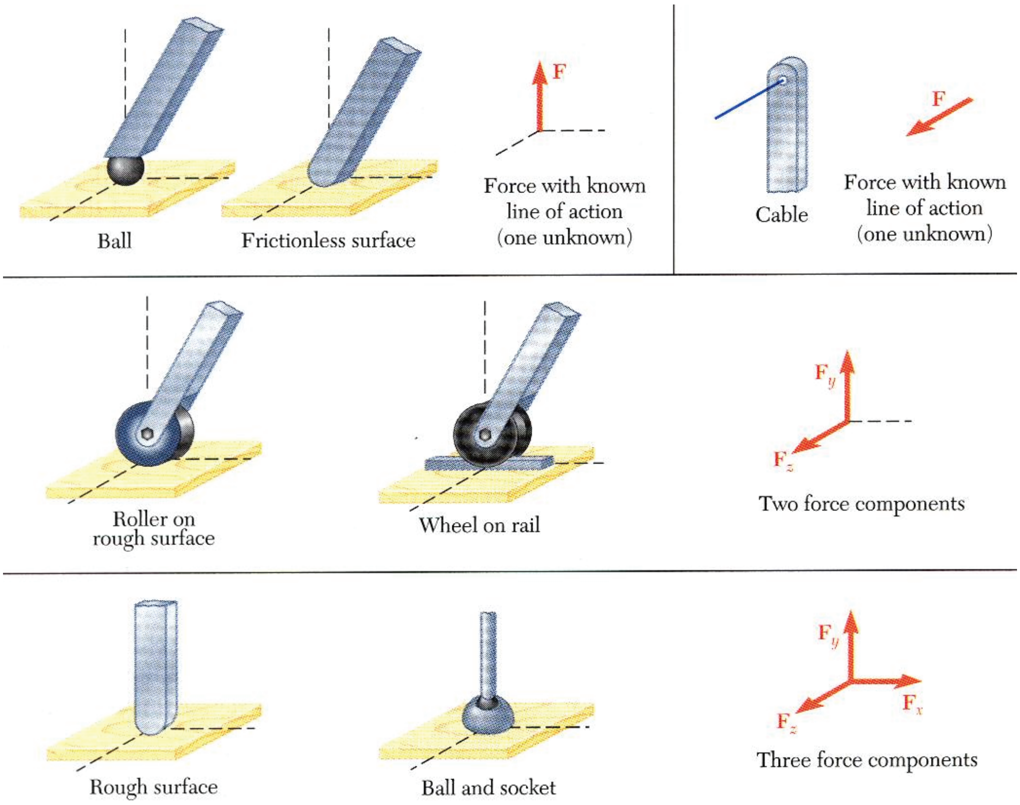
- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.

- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum (\vec{r} \times \vec{F}) = 0$$

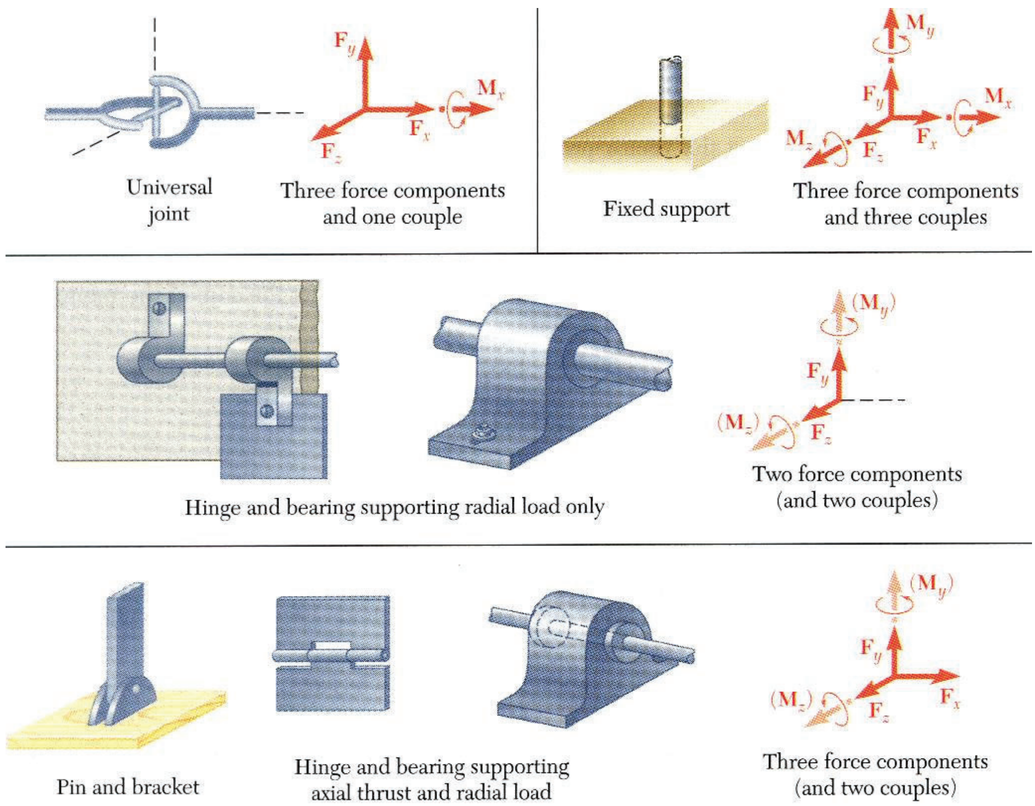
Equilibrium of Rigid Bodies

- **Reactions at Supports and Connections for a Three-Dimensional Structure**



Equilibrium of Rigid Bodies

- **Reactions at Supports and Connections for a Three-Dimensional Structure**

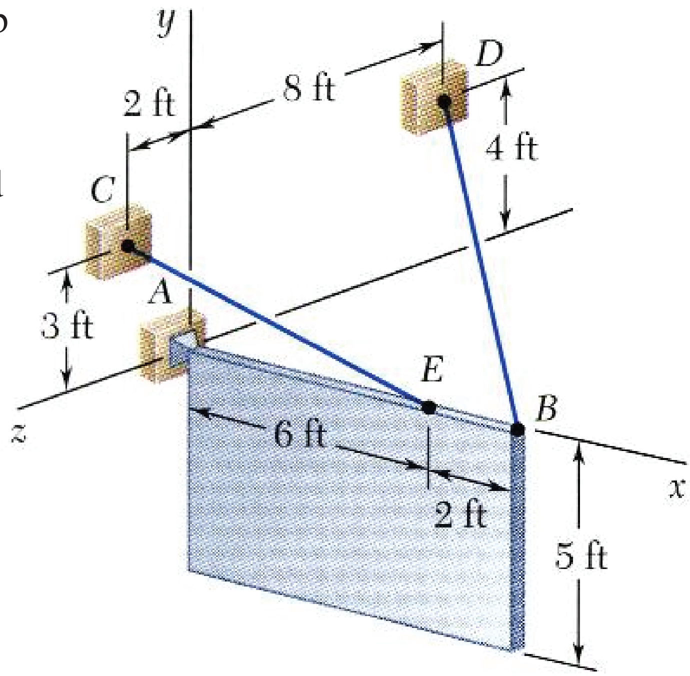


Equilibrium of Rigid Bodies

❑ Sample Problem 05

A sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at *A* and by two cables.

Determine the tension in each cable and the reaction at *A*.



Equilibrium of Rigid Bodies

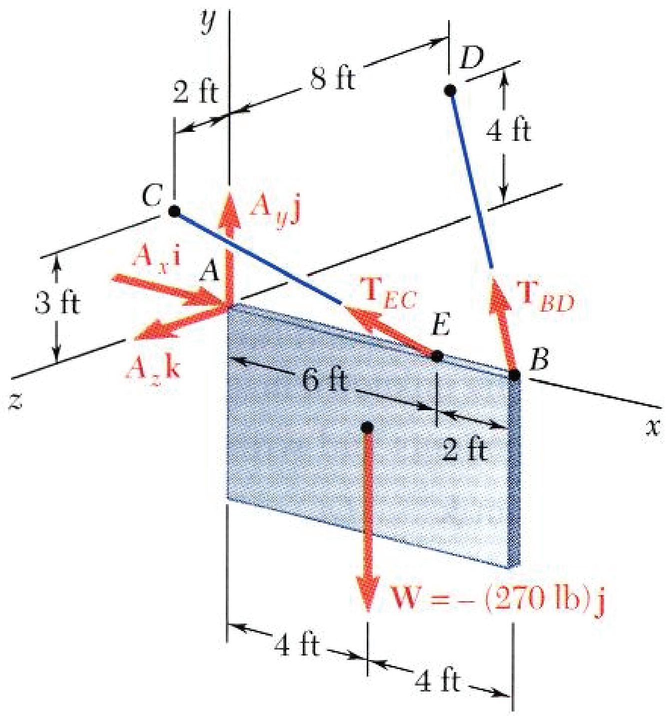
❑ Sample Problem 05

SOLUTION:

- Create a free-body diagram for the sign.

$$\vec{T}_{BD} = T_{BD}\left(-\frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}\right)$$

$$\vec{T}_{EC} = T_{EC}\left(-\frac{6}{7}\vec{i} + \frac{3}{7}\vec{j} + \frac{2}{7}\vec{k}\right)$$

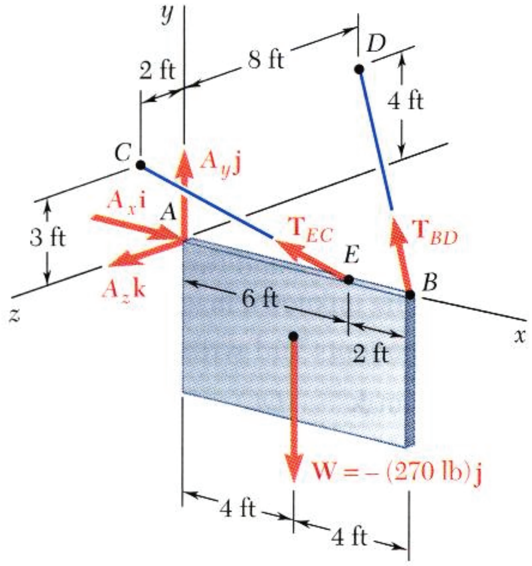


Equilibrium of Rigid Bodies

❑ Sample Problem 05

SOLUTION:

- Apply the conditions for static equilibrium to develop equations for the unknown reactions.



$$\sum \vec{F} = \vec{A} + \vec{T}_{BD} + \vec{T}_{EC} + \vec{W} = 0$$

$$\vec{i}: \quad \sum F_x = 0 \quad \Rightarrow \quad A_x - \frac{2}{3}T_{BD} - \frac{6}{7}T_{EC} = 0$$

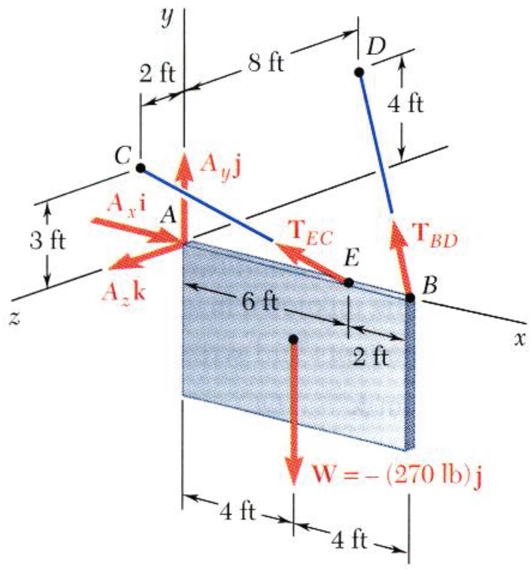
$$\vec{j}: \quad \sum F_y = 0 \quad \Rightarrow \quad A_y + \frac{1}{3}T_{BD} + \frac{3}{7}T_{EC} - 270\text{ lb} = 0 \qquad (I)$$

$$\vec{k}: \quad \sum F_z = 0 \quad \Rightarrow \quad A_z - \frac{2}{3}T_{BD} + \frac{2}{7}T_{EC} = 0$$

Equilibrium of Rigid Bodies

❑ Sample Problem 05

SOLUTION:



$$\vec{M}_{\vec{T}_{BD}/A} = \left(\frac{16}{3}T_{BD}\right)\vec{j} + \left(\frac{8}{3}T_{BD}\right)\vec{k}$$

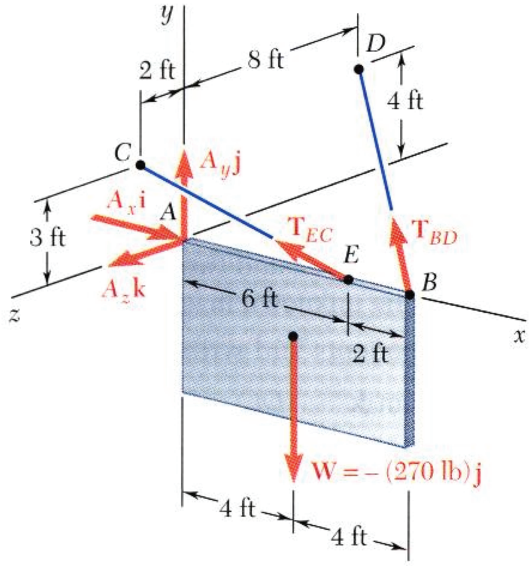
$$\vec{M}_{\vec{T}_{EC}/A} = \left(-\frac{12}{7}T_{EC}\right)\vec{j} + \left(\frac{18}{7}T_{EC}\right)\vec{k}$$

$$\vec{M}_{\vec{W}/A} = (-1080)\,\vec{k}$$

Equilibrium of Rigid Bodies

❑ Sample Problem 05

SOLUTION:



(I) & (II) $\Rightarrow$ Solve the 5 equations for the 5 unknowns,

$$T_{BD} = 101.3\,(\text{lb})$$
$$T_{EC} = 315\,(\text{lb})$$
$$\vec{A} = (338\ \text{lb})\vec{i} + (101.2\ \text{lb})\vec{j} - (22.5\ \text{lb})\vec{k}$$