STATICS

- Vector Mechanics for Engineers: Statics, 9th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Statics, 5th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Statics"

Statics of Particles

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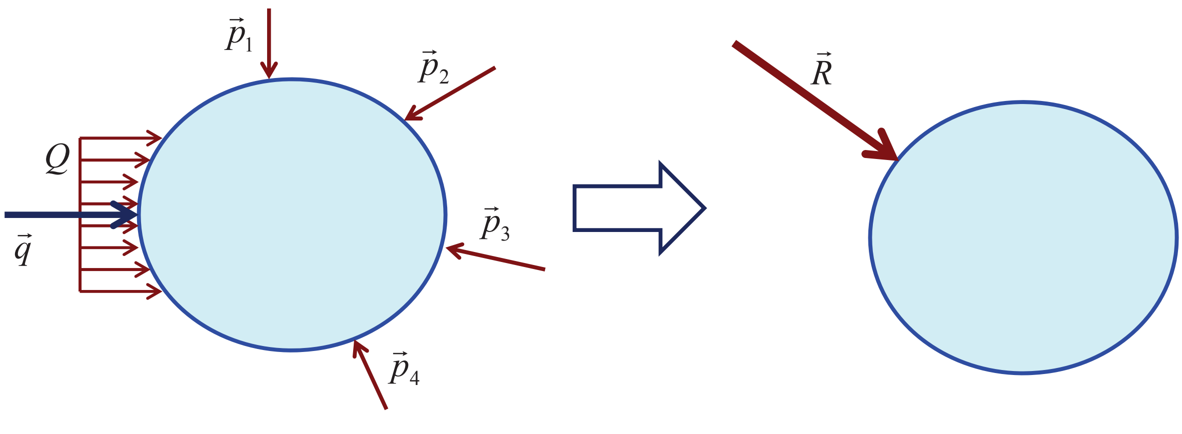
Statics of Particles

❑ Introduction

The objective for the current chapter is to investigate the ***effects of forces on particles***:

a) Replacing multiple forces acting on a particle with a single equivalent or ***resultant force***.

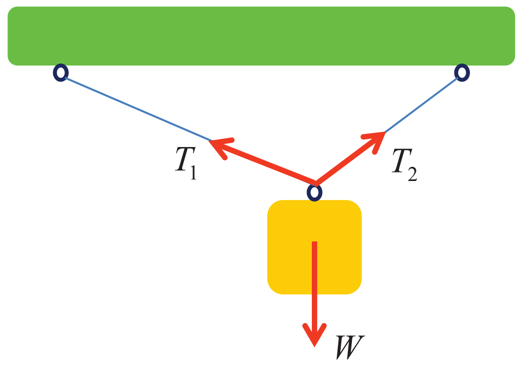


Statics of Particles

❑ Introduction

b) relations between forces acting on a particle that is in a state of ***equilibrium***.

$$T_1 = f(W)$$
$$T_2 = g(W)$$

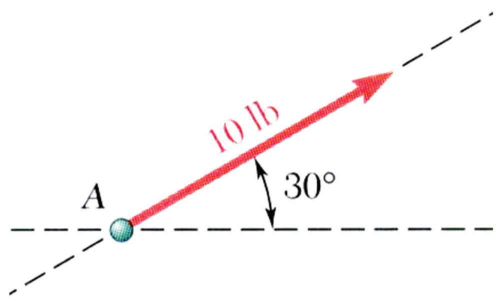


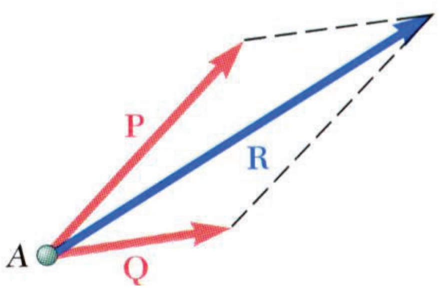
- The focus on *particles* does not imply a restriction to miniscule bodies. Rather, the study is restricted to analyses in which ***the size and shape of the bodies is not significant so that all forces may be assumed to be applied at a single point.***

Statics of Particles

❑ Resultant of Two Forces

- force: action of one body on another; characterized by its ***point of application***, ***magnitude***, ***line of action***, and *sense*.
- Force is a ***vector quantity***.

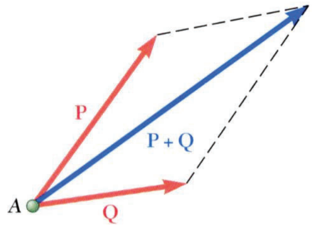




- Experimental evidence shows that the combined effect of two forces may be represented by a ***single resultant force***.
- The ***resultant*** is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.

Statics of Particles

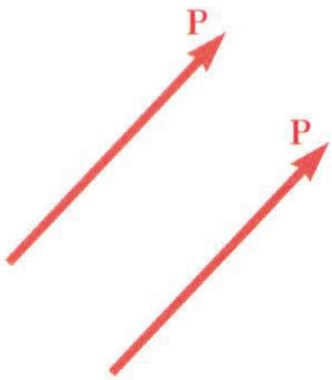
❑ Vectors

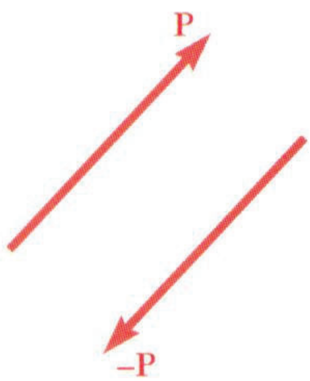


- ***Vector***: defined as mathematical expressions possessing **magnitude** and **direction** which add according to the parallelogram law. Examples: ***displacements***, ***velocities***, ***accelerations***.
- ***Scalar***: parameters possessing magnitude but not direction. Examples: ***mass***, ***volume***, ***temperature***
- ***Vector classifications:***
  - ***Fixed or bound vectors*** have well defined points of application that cannot be changed without affecting an analysis. Examples: Reaction Support
  - ***Free vectors*** may be freely moved in space without changing their effect on an analysis. Examples: Couples
  - ***Sliding vectors*** may be applied anywhere along their line of action without affecting an analysis.

Statics of Particles

❑ Vectors



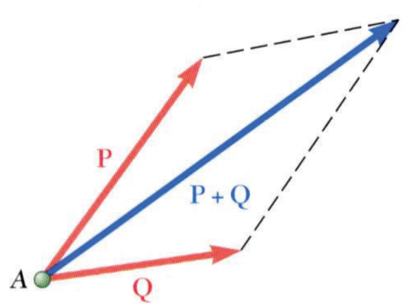


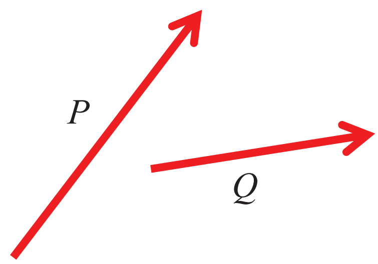
- ***Equal vectors*** have the same magnitude and direction.
- ***Negative vector*** of a given vector has the same magnitude and the opposite direction.

Statics of Particles

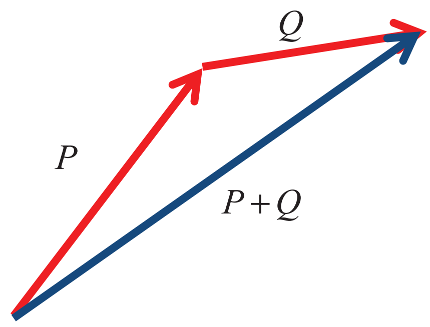
❑ Addition of Vectors

- ***Trapezoid*** rule for vector addition





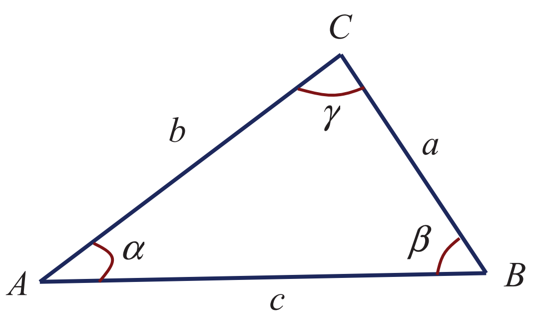
- ***Triangle*** rule for vector addition



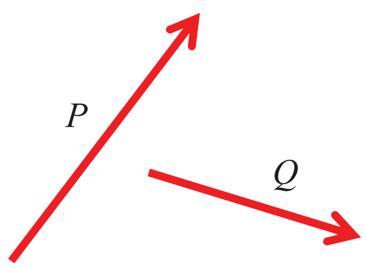
Statics of Particles

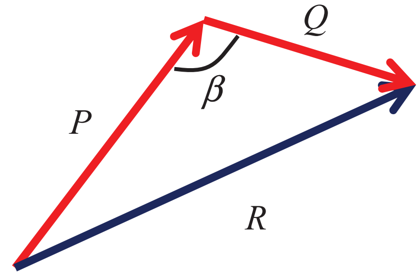
❑ Addition of Vectors

- ***Law of cosines,***



$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$b^2 = c^2 + a^2 - 2ac\cos\beta$$
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$



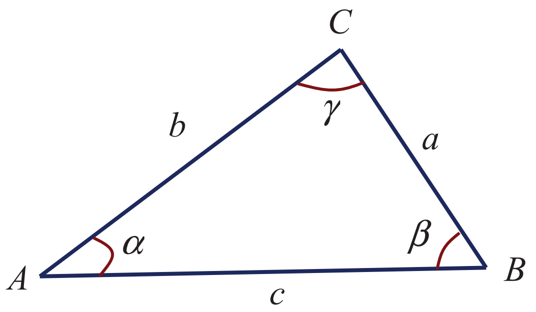


$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

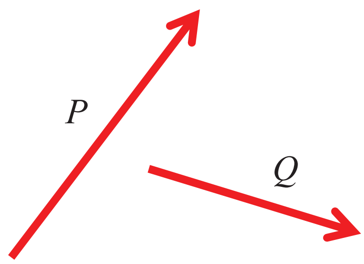
Statics of Particles

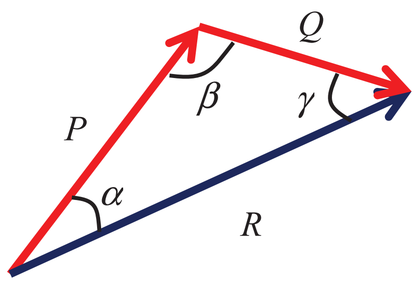
❑ Addition of Vectors

- ***Law of sines,***



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$





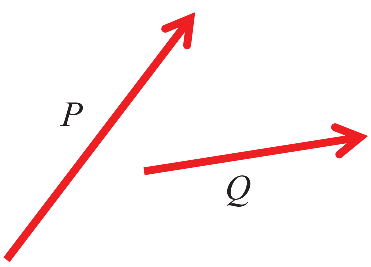
$$\frac{Q}{\sin\alpha} = \frac{R}{\sin\beta} = \frac{P}{\sin\gamma}$$

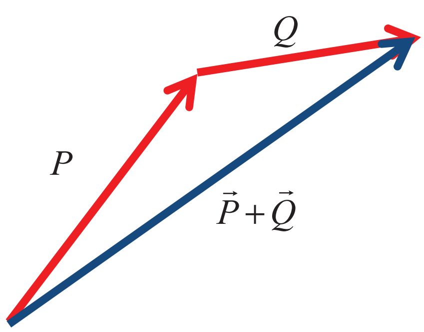
Statics of Particles

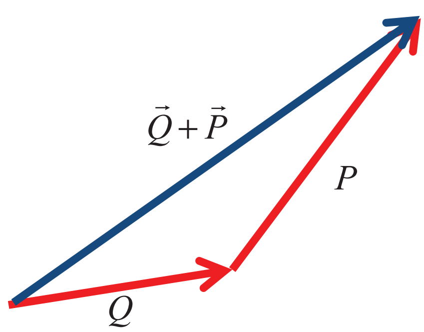
❑ Addition of Vectors

- Vector addition is ***commutative***,

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$







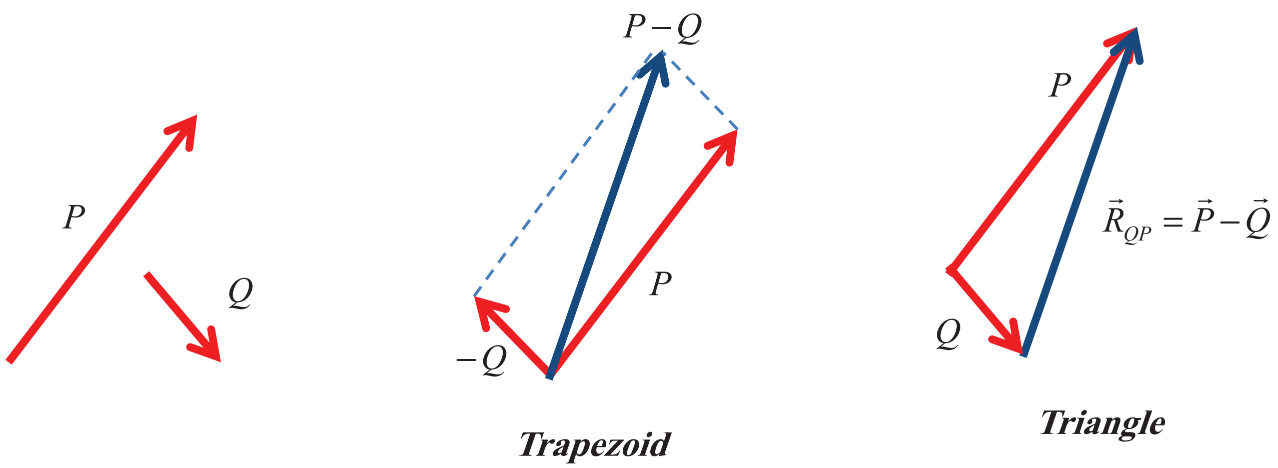
Statics of Particles

❑ Addition of Vectors

Vector ***subtraction***

The subtraction of a vector is defined as the addition of the corresponding negative vector.

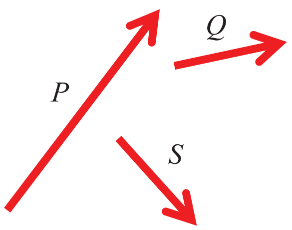
$$\vec{P} + (-\vec{Q}) = \vec{P} - \vec{Q}$$

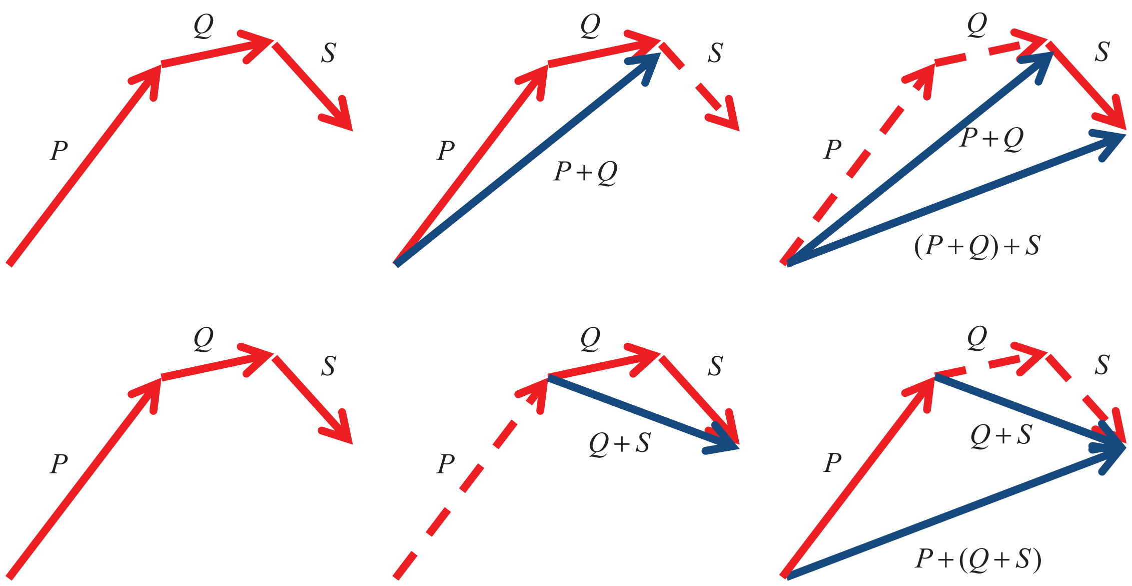


Statics of Particles

❑ Addition of Vectors

- ***Addition of three or more vectors*** through repeated application of the triangle rule

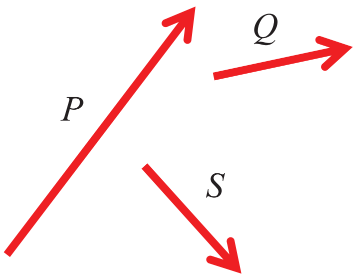


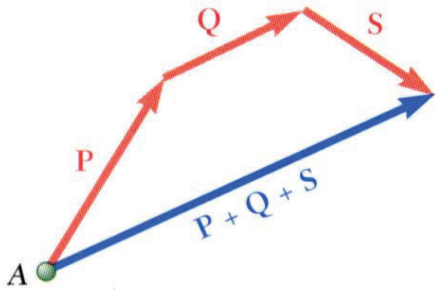


Statics of Particles

❑ Addition of Vectors

- ***The polygon rule for the addition*** of three or more vectors.





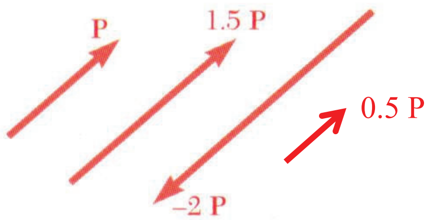
- Vector addition is ***associative***,

$$\vec{P}+\vec{Q}+\vec{S}=(\vec{P}+\vec{Q})+\vec{S}=\vec{P}+(\vec{Q}+\vec{S})$$

Statics of Particles

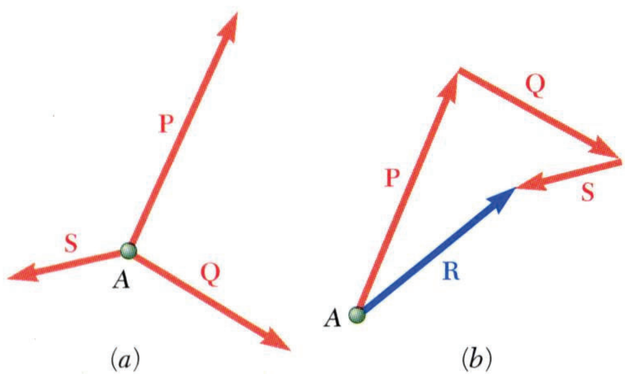
❑ Resultant of Several Concurrent Forces

- ***Multiplication of a vector by a scalar***



- ***Concurrent forces***: set of forces which all pass through the same point.

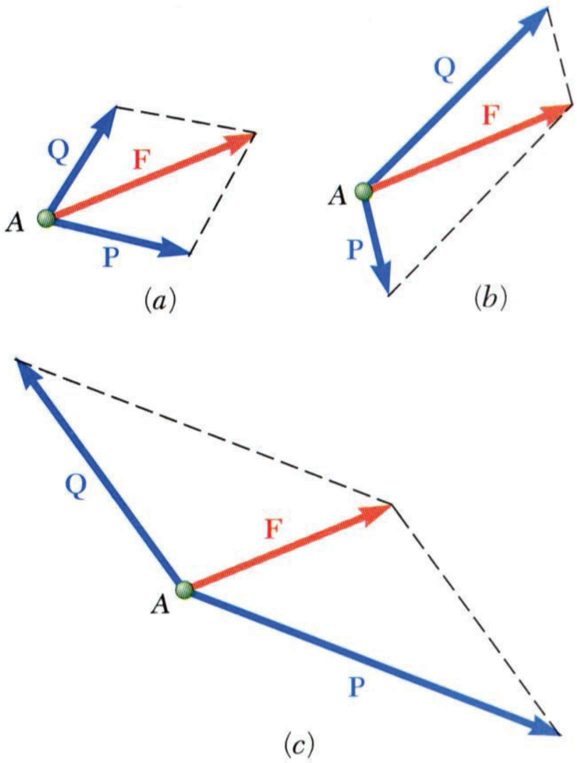
  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.



Statics of Particles

❑ Resolution of a Force into Components

- ***Vector force components***: a single force F acting on a particle may be replaced by two or more forces which, together, have the same effect on the particle. These forces are called the ***components*** of the Original force F , and the process of substituting them for F is called ***resolving*** the force F into components.

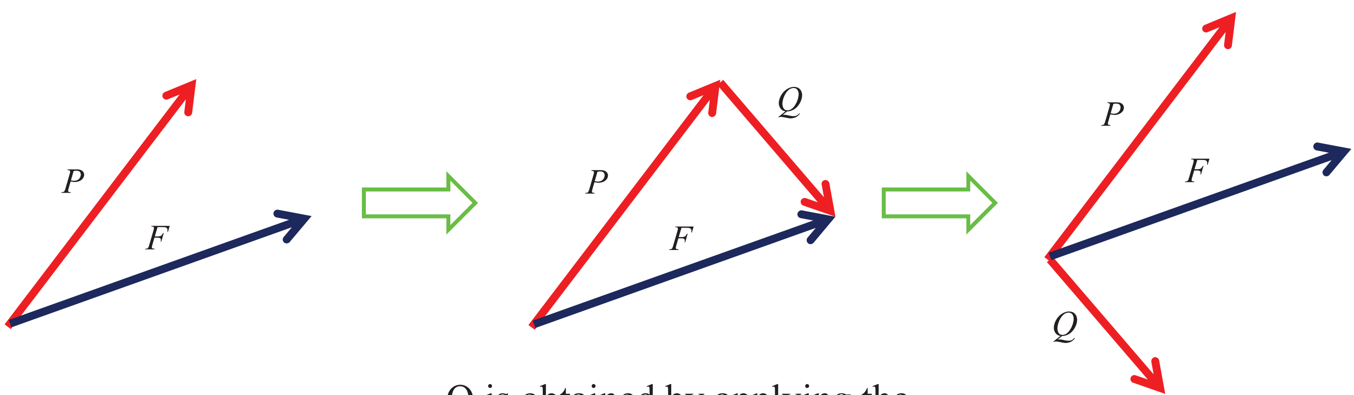


Statics of Particles

❑ Resolution of a Force into Components

The number of ways in which a given force F may be resolved into two components is **unlimited.** Two cases are of particular interest:

I. One of the Two Components, P, Is Known

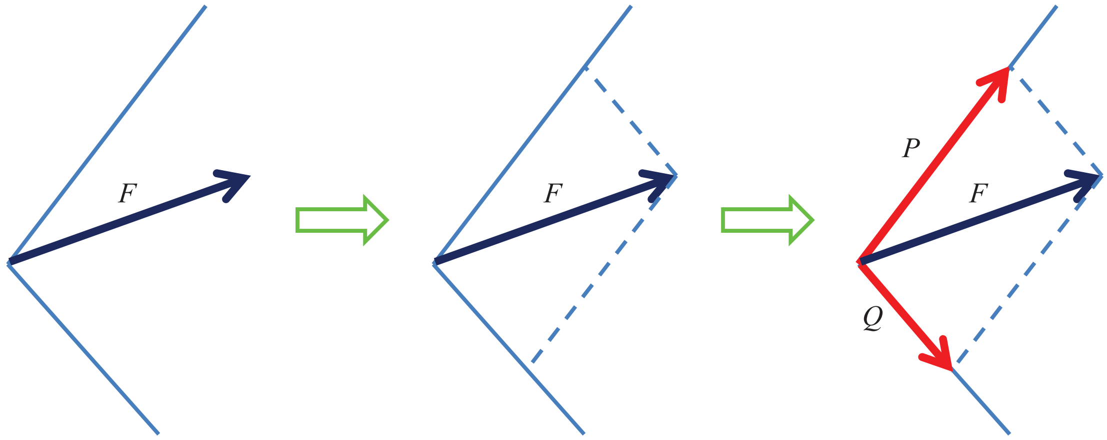


Q is obtained by applying the ***triangle rule*** and joining the tip of P to the tip of F

Statics of Particles

❑ Resolution of a Force into Components

II. The Line of Action of Each Component Is Known

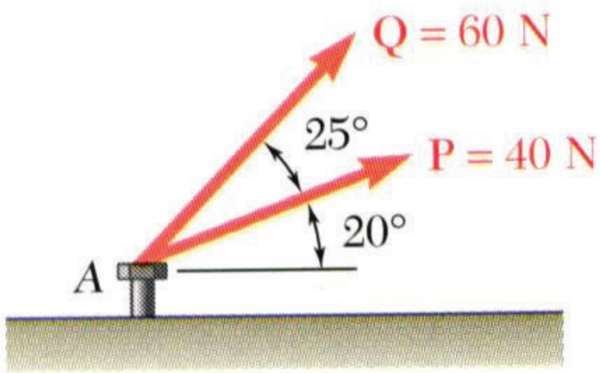


The magnitude and sense of the components are obtained by applying the ***parallelogram law*** and drawing lines, through the tip of F , parallel to the given lines of action

Statics of Particles

❑ Sample Problem 01

The two forces act on a bolt at *A*. Determine their resultant.



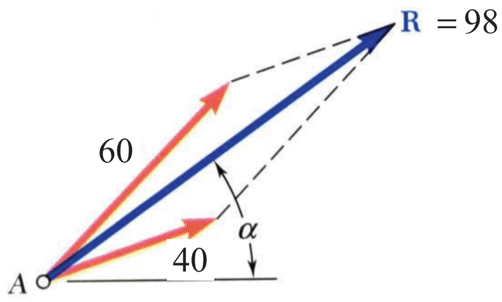
Statics of Particles

❑ Sample Problem 01

SOLUTION: Graphical solution

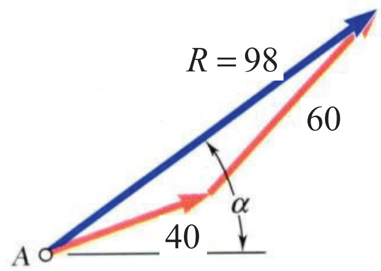
- A ***parallelogram*** with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

$\mathbf{R} = 98\text{ N} \quad \alpha = 35°$



- A ***triangle*** is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$\mathbf{R} = 98\text{ N} \quad \alpha = 35°$

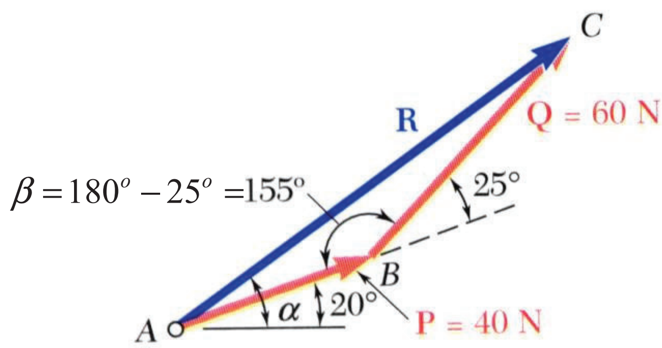


Statics of Particles

❑ Sample Problem 01

SOLUTION: Trigonometric solution

- ***The Law of Cosines,***



$\Rightarrow$ $R = 97.73\text{ (N)}$

- ***The Law of Sines,***

$\Rightarrow$ $A = 15.04°$

$\alpha = 35.04^{o}$

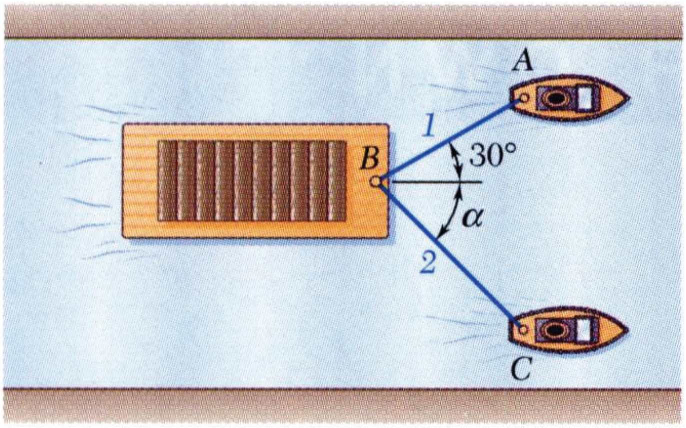
Statics of Particles

❑ Sample Problem 02

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

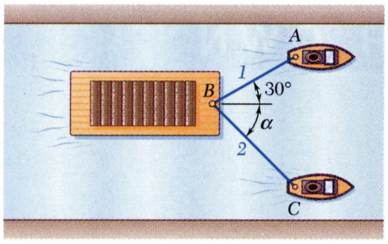
a) the tension in each of the ropes for α = 45°,

b) the value of α for which the tension in rope 2 is a minimum.



Statics of Particles

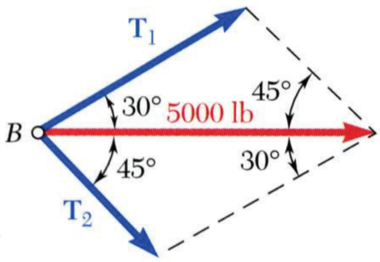
❑ Sample Problem 02



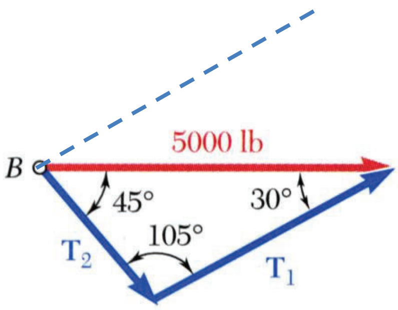
SOLUTION:

- ***Graphical solution*** - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.

$$T_1 = 3700 \text{ (lbf)} \quad T_2 = 2600 \text{ (lbf)}$$



- ***Trigonometric solution*** - Triangle Rule with Law of Sines

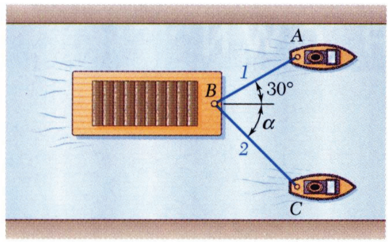


$$T_1 = 3660 \text{ (lbf)}$$

$$T_2 = 2590 \text{ (lbf)}$$

Statics of Particles

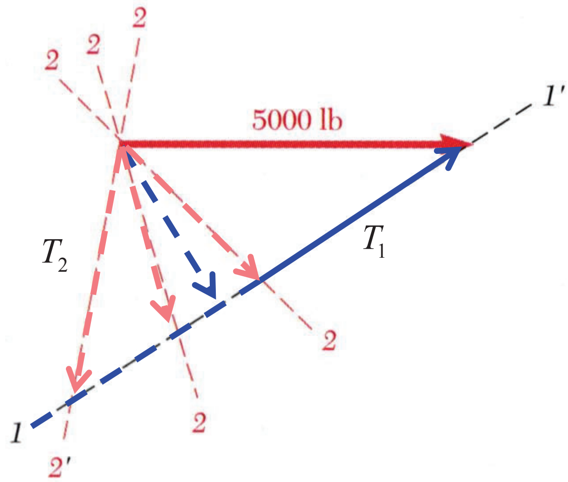
❑ Sample Problem 02



SOLUTION:

- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α .

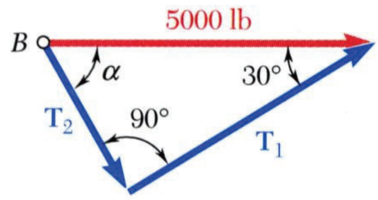
- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are ***perpendicular***.



$$T_2 = 2500\ (lbf\ )$$

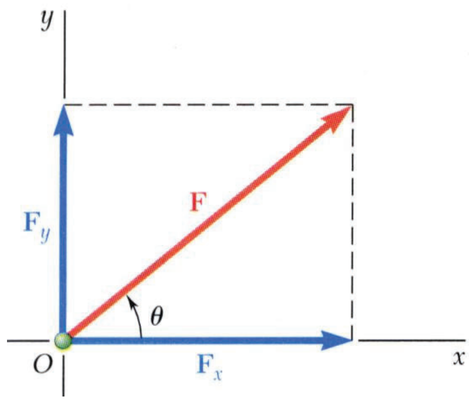
$$T_1 = 4330\ (lbf\ )$$

$$\alpha = 60°$$



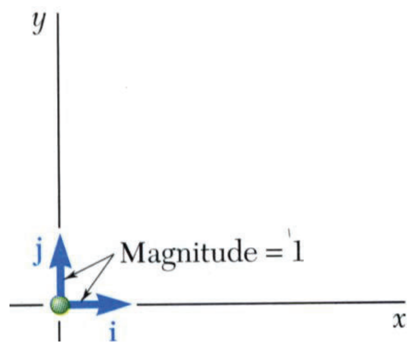
Statics of Particles

❑ Rectangular Components of a Force: Unit Vectors



- May resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as ***rectangular vector components*** and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$



- Define perpendicular ***unit vectors*** $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.

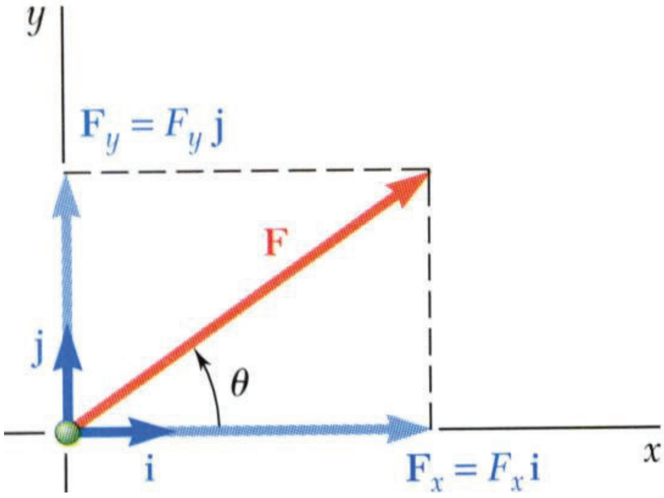
Statics of Particles

❑ Rectangular Components of a Force: Unit Vectors

- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

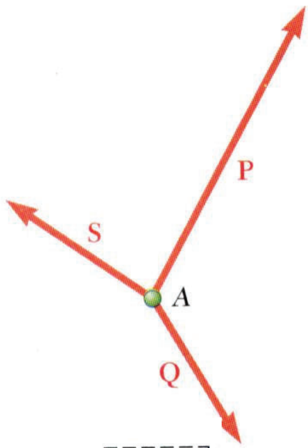
$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$



Statics of Particles

❑ Addition of Forces by Summing Componen



- Wish to find the resultant of 3 or more concurrent forces,

$$\vec{P} = P_x\vec{i} + P_y\vec{j} \qquad \vec{Q} = Q_x\vec{i} + Q_y\vec{j} \qquad \vec{S} = S_x\vec{i} + S_y\vec{j}$$

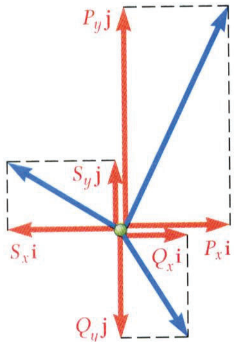
$$\vec{R} = \vec{P} + \vec{Q} + \vec{S} = R_x\vec{i} + R_y\vec{j}$$

- Resolve each force into rectangular components

$$\vec{R} = R_x\vec{i} + R_y\vec{j} = P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j}$$

$$\Rightarrow \vec{R} = (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j}$$

$$R_x = P_x + Q_x + S_x = \sum F_x \qquad R_y = P_y + Q_y + S_y = \sum F_y$$



Statics of Particles

❑ Addition of Forces by Summing Components

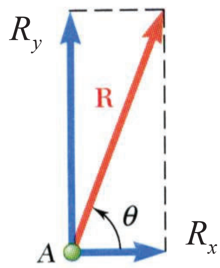
- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

$$\vec{R} = R_x\vec{i} + R_y\vec{j}$$

$$R_x = \sum_{i=1}^{n} Fx_i \qquad R_y = \sum_{i=1}^{n} Fy_i$$

- To find the resultant magnitude and direction,

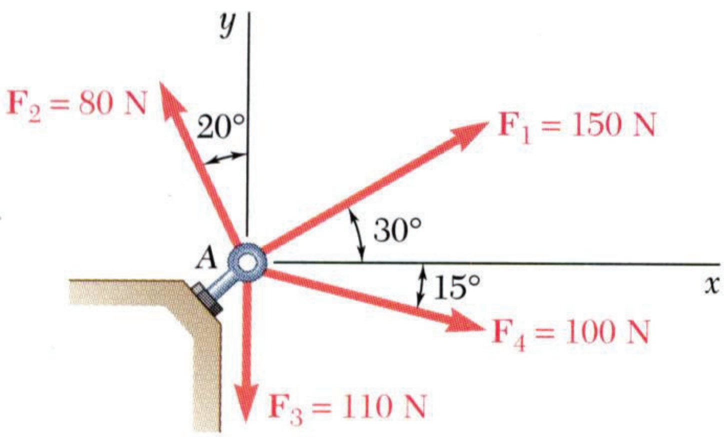
$$R = \sqrt{R_x^2 + R_y^2} \quad \& \quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Statics of Particles

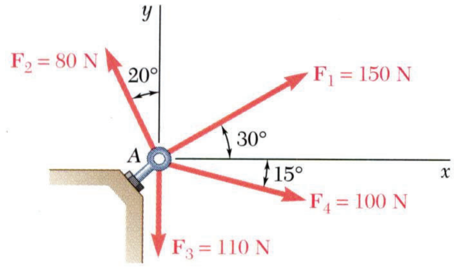
❑ Sample Problem 03

Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.



Statics of Particles

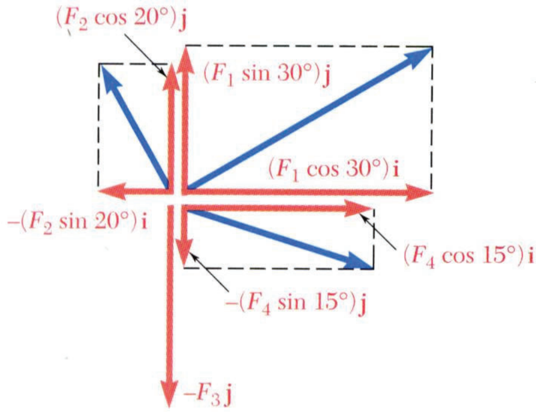
❑ Sample Problem 03



SOLUTION:

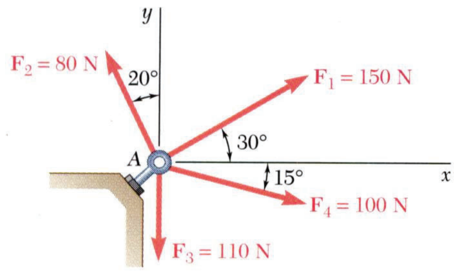
- Resolve each force into rectangular components.

$force$	mag	$x-comp$	$y-comp$
$\vec{F}_1$			
$\vec{F}_2$			
$\vec{F}_3$			
$\vec{F}_4$			



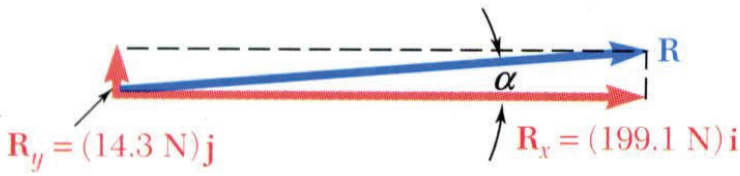
Statics of Particles

❑ Sample Problem 03



SOLUTION:

- Calculate the magnitude and direction.



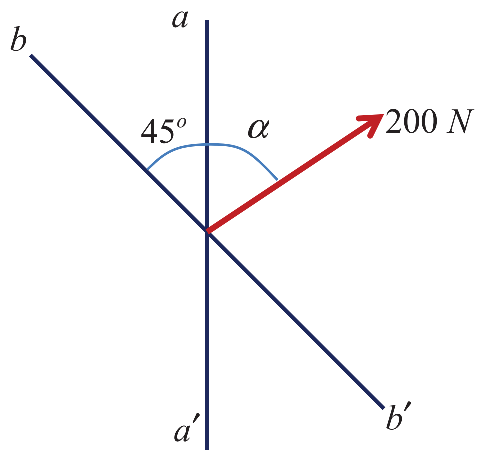
$$R = 199.6\,(N)$$

$$\alpha = 4.1^o$$

Statics of Particles

❑ Sample Problem 04

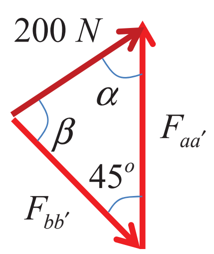
The 200-N force is to be resolved into components along lines a-a' and b-b'. (a) Determine the angle α using trigonometry knowing that the component along a-a ' is to be 150 N. (b) What is the corresponding value of the component along b-b'?

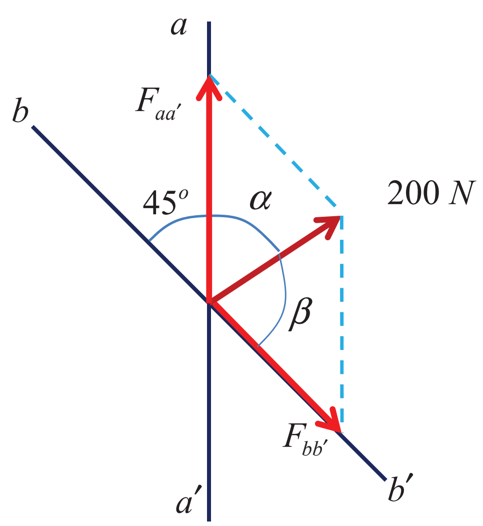


Statics of Particles

❑ Sample Problem 04

SOLUTION:





$\beta = 32.03^o$

$\alpha = 102.97^o \approx 103^o$

$F_{bb'} = 275.63\ N$

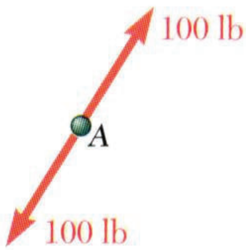
Statics of Particles

❑ Equilibrium of a Particle

When the resultant of all forces acting on a particle is zero, the particle is in ***equilibrium***.

- ***Newton's First Law***: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.

- ***Particle acted upon by two forces***:
  - **equal magnitude**
  - **same line of action**
  - **opposite sense**



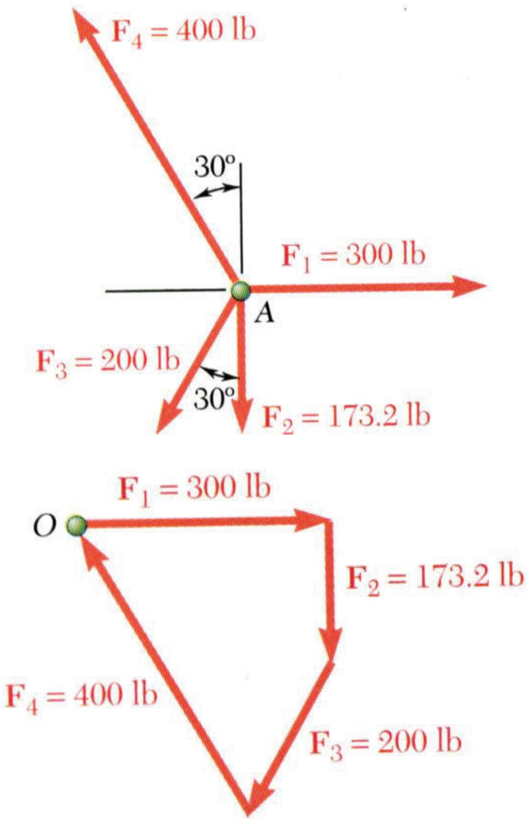
Statics of Particles

❑ Equilibrium of a Particle

- ***Particle acted upon by three or more forces:***
  - **graphical solution yields a closed polygon**
  - **algebraic solution**

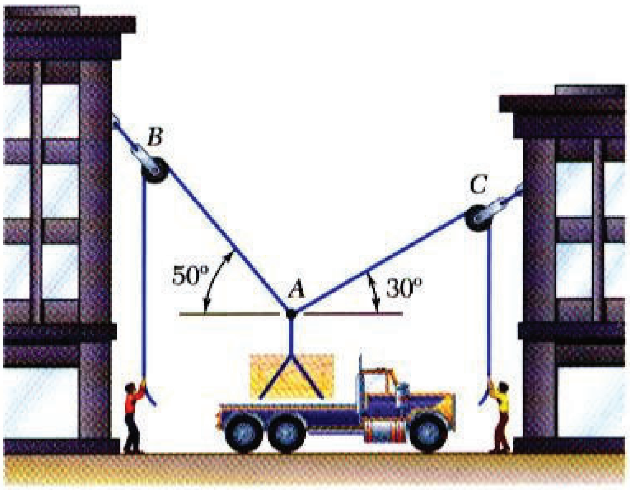
$$\vec{R} = \sum \vec{F} = 0 \quad \Rightarrow$$

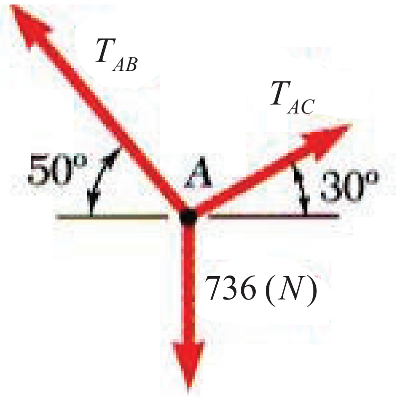
$$\sum F_x = 0 \quad \& \quad \sum F_y = 0$$



Statics of Particles

❑ Free-Body Diagrams





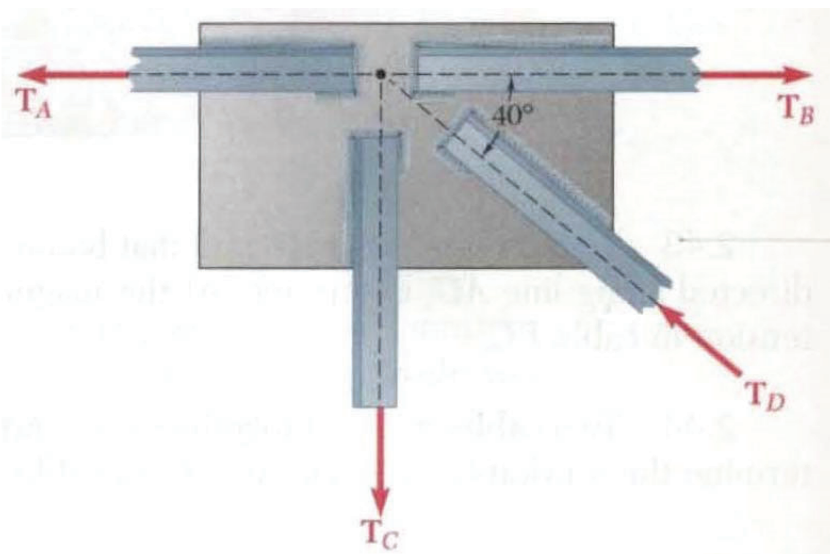
Space Diagram: A sketch showing the physical conditions of the problem.

Free-Body Diagram: A sketch showing only the forces on the selected particle.

Statics of Particles

❑ Sample Problem 05

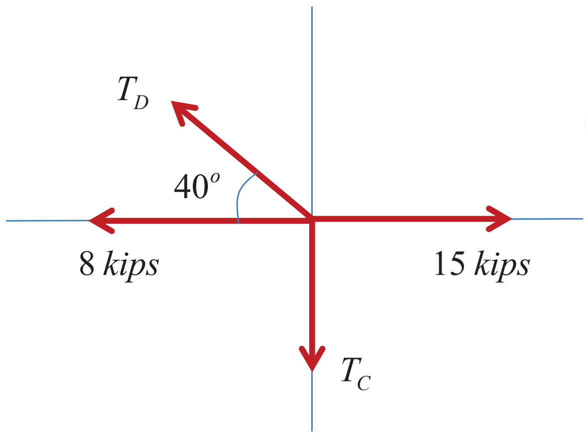
Two forces of magnitude $T_A = 8\ kips$ and $T_B = 15\ kips$ are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces T_C and T_D .

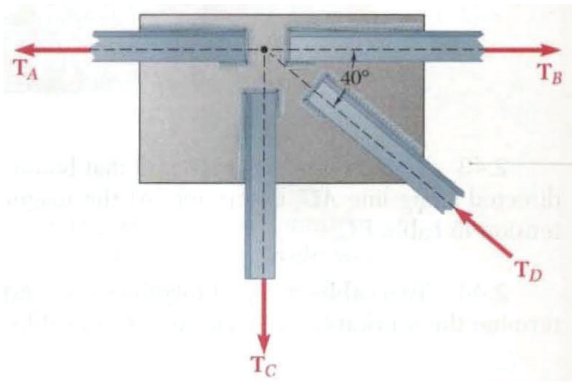


Statics of Particles

❑ Sample Problem 05

SOLUTION:





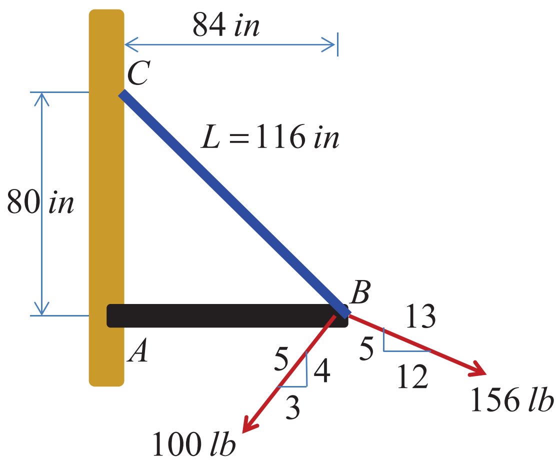
$$T_D = 9.14\ kips$$

$$T_C = 5.87\ kips$$

Statics of Particles

❑ Sample Problem 06

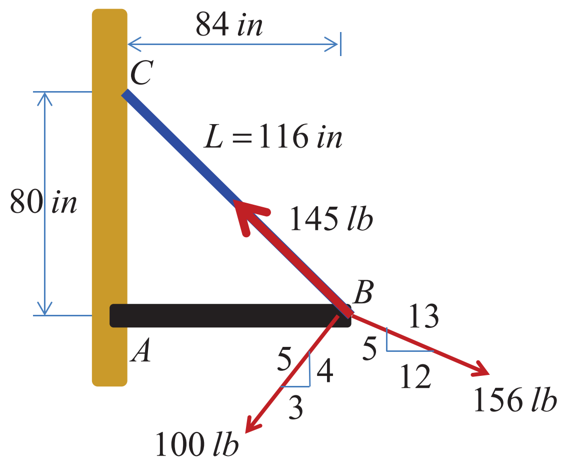
Knowing that the tension in cable BC is 145 lb, determine tile resultant of the three forces exerted at point B of beam AB.



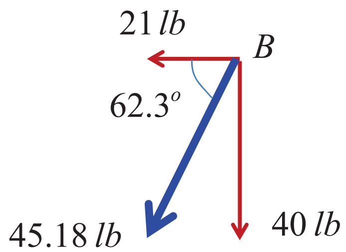
Statics of Particles

❑ Sample Problem 06

SOLUTION:



$\Rightarrow$ $F_x = -21\,lb$



$\Rightarrow$ $F_y = -40\,lb$

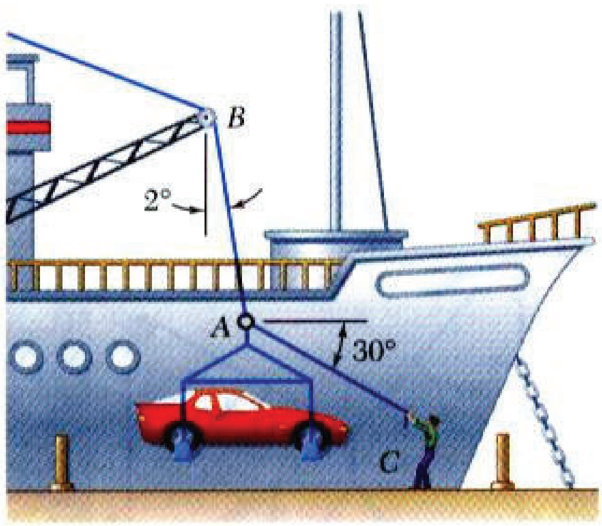
$F = 45.18\,lb$

$\theta = 62.3^o$

Statics of Particles

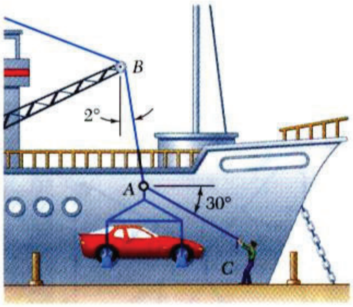
❑ Sample Problem 07

In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?



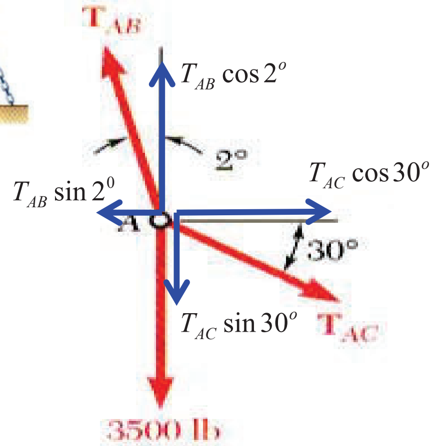
Statics of Particles

❑ Sample Problem 07



SOLUTION:

- Construct a free-body diagram for the particle at *A*.
- Apply the conditions for equilibrium.

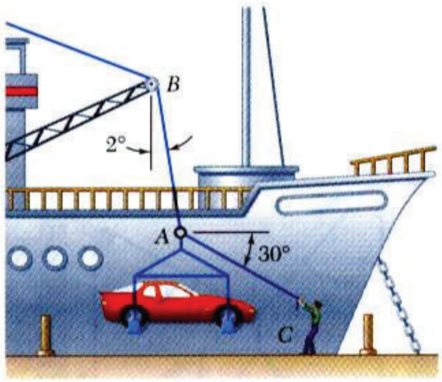


$$T_{AC} = T_{AB}\frac{\sin 2^0}{\cos 30^o} \qquad (I)$$

$$T_{AB} = 3574.2\ (lb)$$

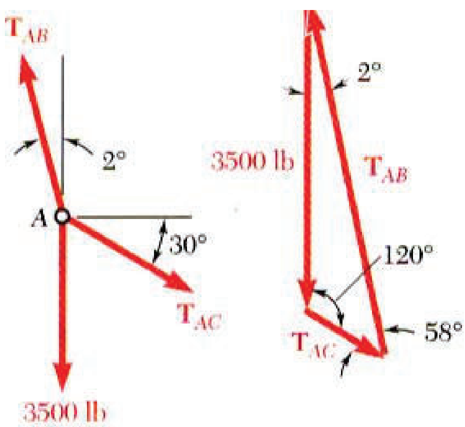
$$T_{AC} = 144\ (lb)$$

❑ Sample Problem 07



SOLUTION:

- Alternative solution : ***Law of sines***



$$T_{AB} = 3574.2\ (lb)$$

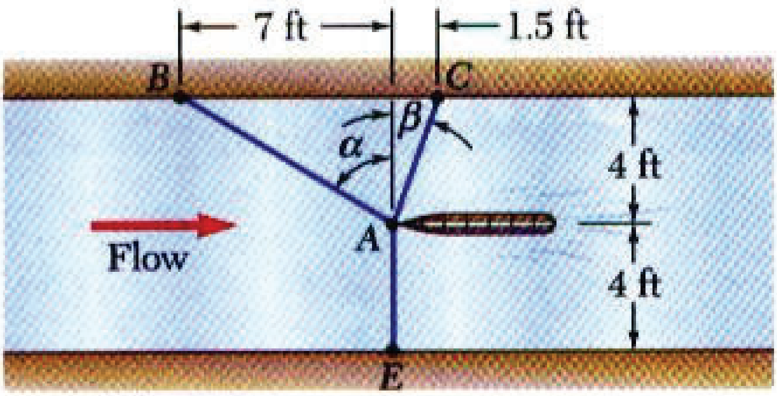
$$T_{AC} = 144\ (lb)$$

Statics of Particles

❑ Sample Problem 08

It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 40 lb in cable *AB* and 60 lb in cable *AE*.

Determine the drag force exerted on the hull and the tension in cable *AC*.

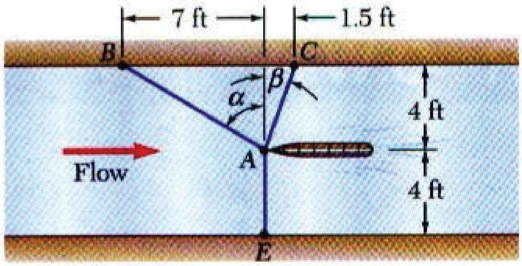


Statics of Particles

❑ Sample Problem 08

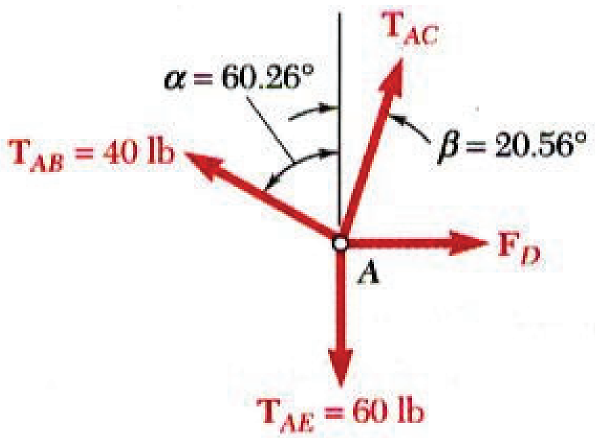
SOLUTION:

- Choosing the hull as the free body, draw a free-body diagram.



$\alpha = 60.26°$

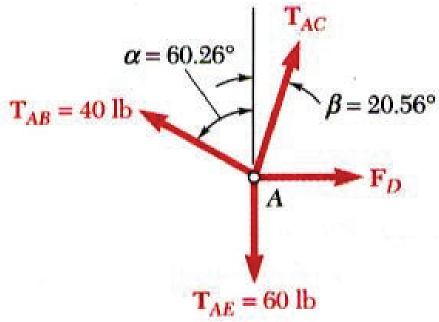
$\beta = 20.56°$



- Express the condition for ***equilibrium*** for the hull by writing that the sum of all forces must be zero.

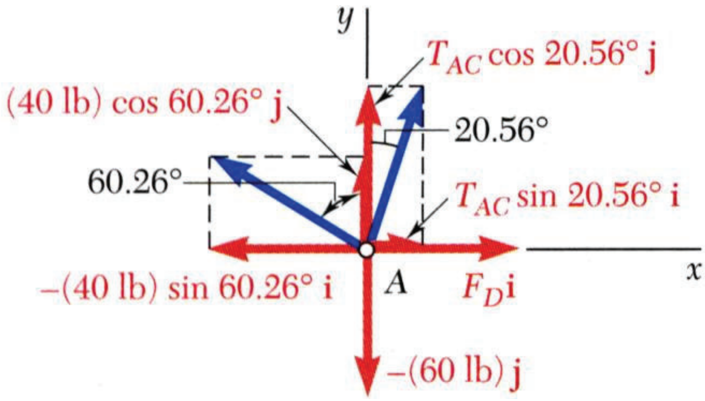
Statics of Particles

❑ Sample Problem 08



SOLUTION:

- Resolve the vector equilibrium equation into two comp equations. Solve for the two unknown cable tensions.



$$\vec{T}_{AB} = -(34.73)\vec{i} + (19.84)\vec{j}$$

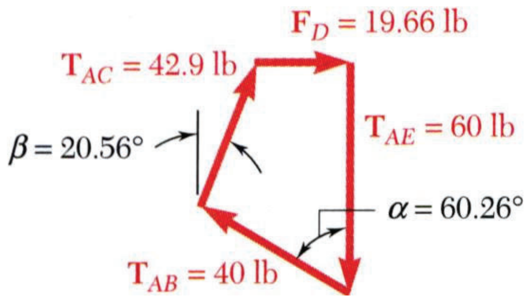
$$\vec{T}_{AC} = (0.3512T_{AC})\vec{i} + (0.9363T_{AC})\vec{j}$$

$$\vec{T}_{AE} = -(60)\vec{j} \quad \& \quad \vec{F}_D = F_D\,\vec{i}$$

Statics of Particles

❑ Sample Problem 08

SOLUTION:



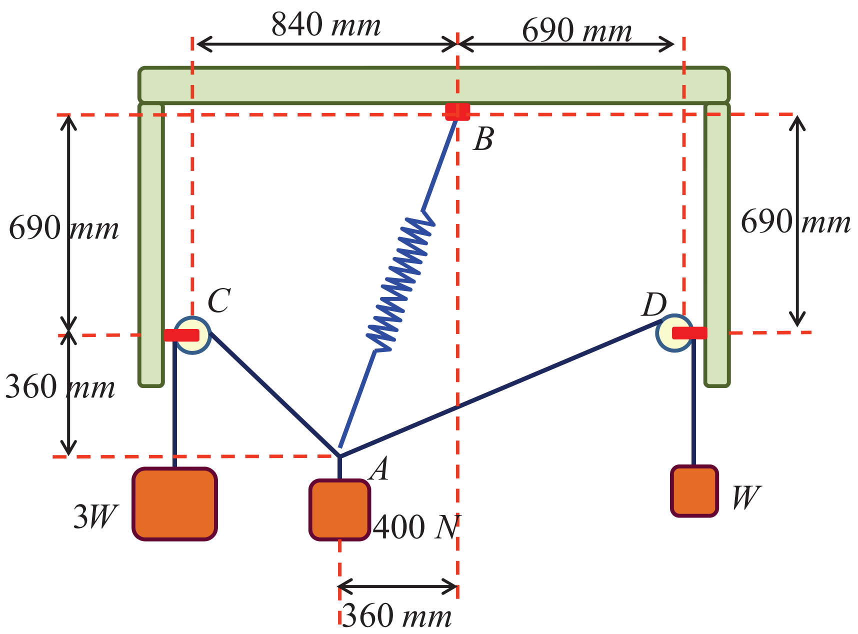
This equation is satisfied only if each component of the resultant is equal to zero

$$(I)\ \&\ (II) \quad \Rightarrow \begin{cases} T_{AC} = 42.9\ \text{(lb)} \\ F_D = 19.66\ \text{(lb)} \end{cases}$$

Statics of Particles

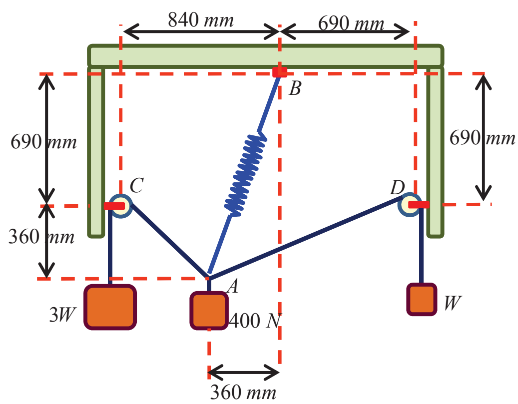
❑ Sample Problem 09

A load of weight 400 N is suspended from a spring and two cords that are attached to blocks of weights 3W and W as shown. Knowing that the constant of the spring is 800 N/m, determine (a) the value of W, (b) the unstretched length of the spring.



Statics of Particles

❑ Sample Problem 09



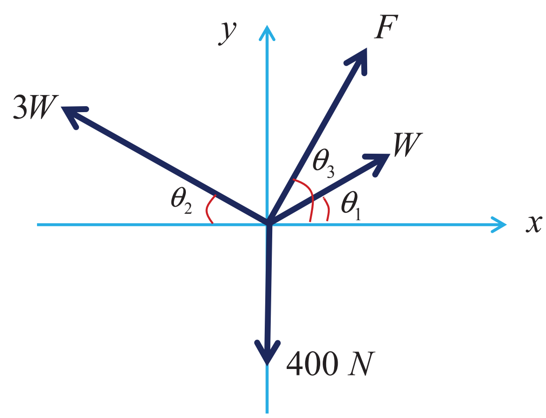
SOLUTION:

draw a free-body diagram.

$\theta_1 = 18.92^o$

$\theta_2 = 36.87^o$

$\theta_3 = 71.08^o$

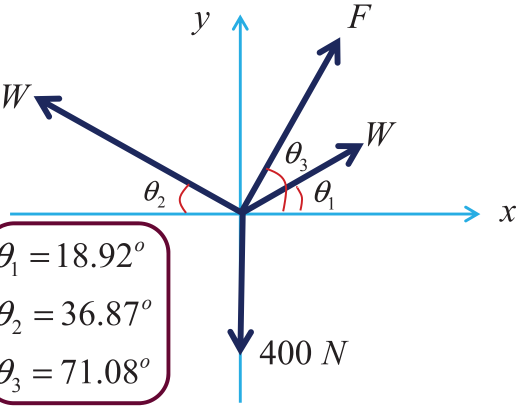


Statics of Particles

❑ Sample Problem 09

SOLUTION:

Apply the conditions for equilibrium.



$$\theta_1 = 18.92^o$$
$$\theta_2 = 36.87^o$$
$$\theta_3 = 71.08^o$$

$$\Rightarrow \quad 0.32F - 1.45W = 0 \qquad (I)$$

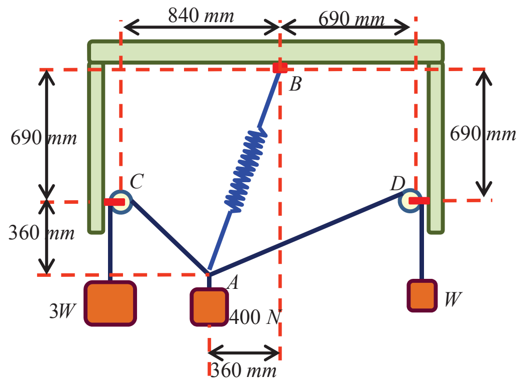
$$\Rightarrow \quad 0.95F + 2.12W = 400 \qquad (II)$$

$$(I)\ \&\ (II) \quad \Rightarrow \quad \begin{cases} W = 62.31\,(N) \\ F = 282.24\,(N) \end{cases}$$

Statics of Particles

❑ Sample Problem 09

SOLUTION:



$$\Delta x = 352.8\,(mm)$$

$$L_1 = 757.20\,(mm)$$

Statics of Particles

❑ Unit Vector

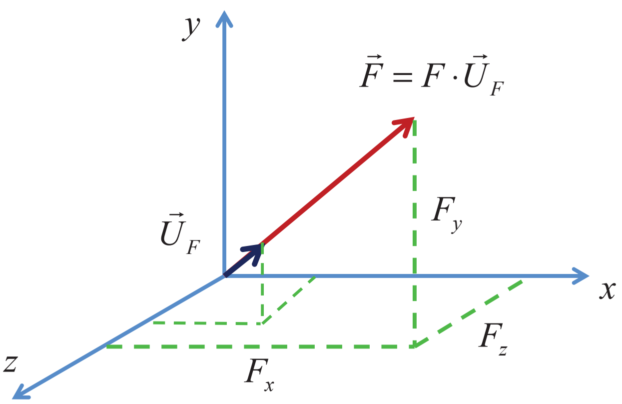
A unit vector is a vector who has the unit length.

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$F = \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}}$$

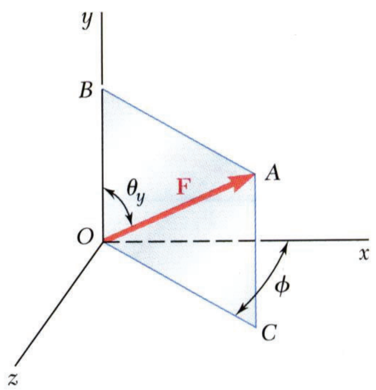
$$\vec{U}_F = \frac{\vec{F}}{F} \quad \Rightarrow \quad \boxed{\vec{U}_F = \frac{F_x}{F}\vec{i} + \frac{F_y}{F}\vec{j} + \frac{F_z}{F}\vec{k}}$$

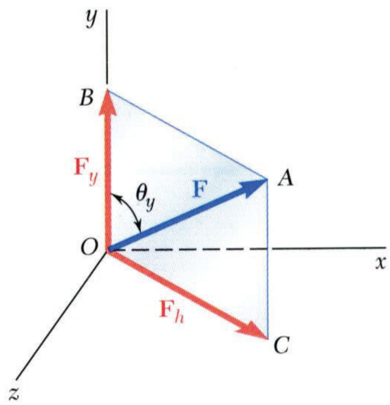
$$\boxed{U_F = \sqrt{\left(\frac{F_x}{F}\right)^2 + \left(\frac{F_y}{F}\right)^2 + \left(\frac{F_z}{F}\right)^2} = 1}$$

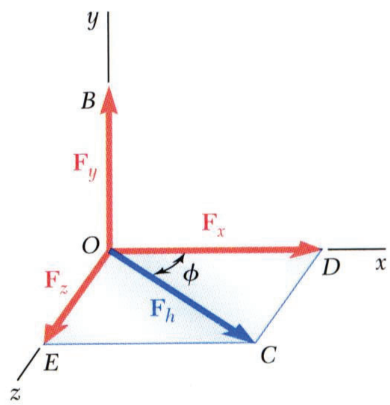


Statics of Particles

❑ Rectangular Components in Space







- The vector $\vec{F}$ is contained in the plane *OBAC*.
- Resolve $\vec{F}$ into horizontal and vertical components.

$$\boxed{F_y = F\cos\theta_y}$$

$$F_h = F\sin\theta_y$$

- Resolve F_h into rectangular components

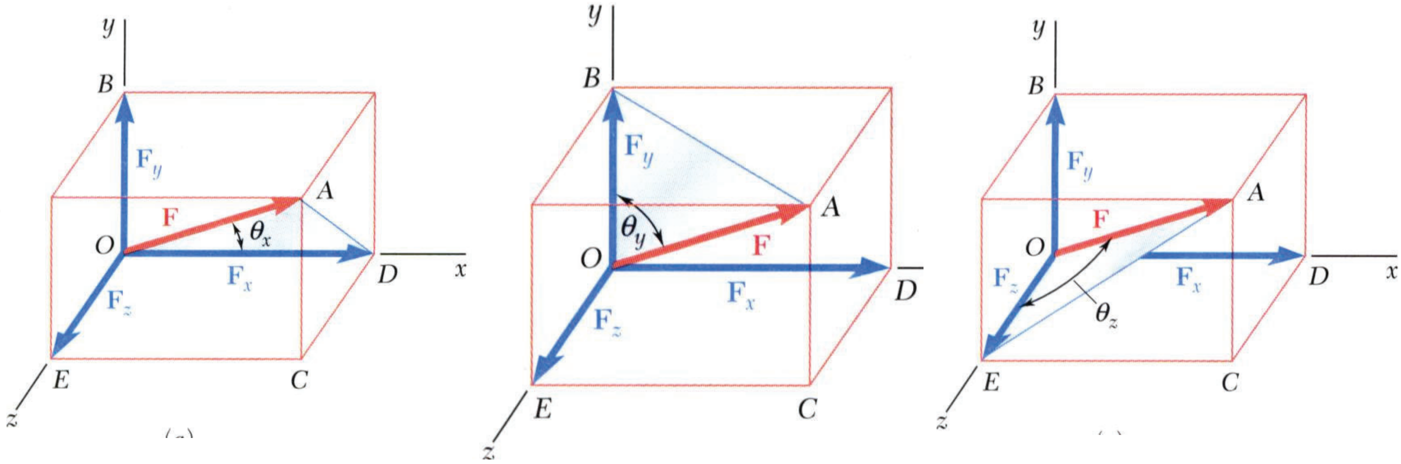
$$F_x = F_h\cos\phi \quad \Rightarrow \quad F_x = F\sin\theta_y\cos\phi$$

$$F_z = F_h\sin\phi \quad \Rightarrow \quad F_z = F\sin\theta_y\sin\phi$$

Statics of Particles

❑ Rectangular Components in Space

- With the angles between $\vec{F}$ and the axes,



$F_x = F\cos\theta_x$ $\qquad$ $F_y = F\cos\theta_y$ $\qquad$ $F_z = F\cos\theta_z$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} \quad \Rightarrow \quad \boxed{\vec{F} = F(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k})}$$

Statics of Particles

❑ Rectangular Components in Space

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for $\vec{F}$

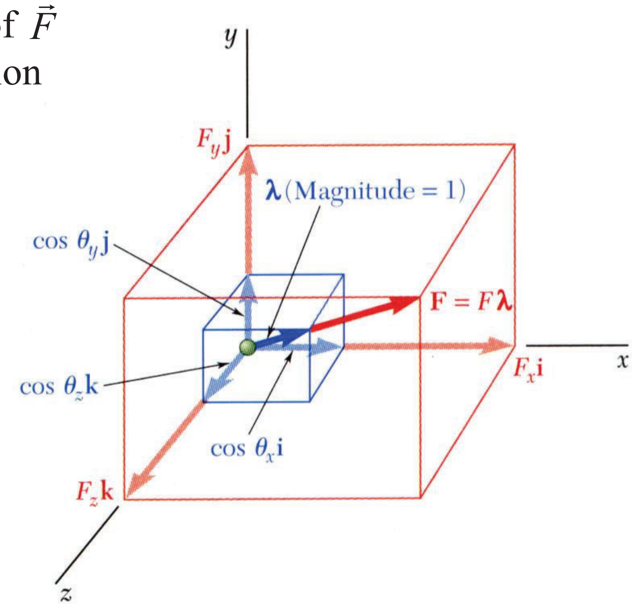
$$\boxed{\vec{F} = F\vec{\lambda}}$$

$$\boxed{\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}}$$

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right)$$

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$$

$$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right)$$



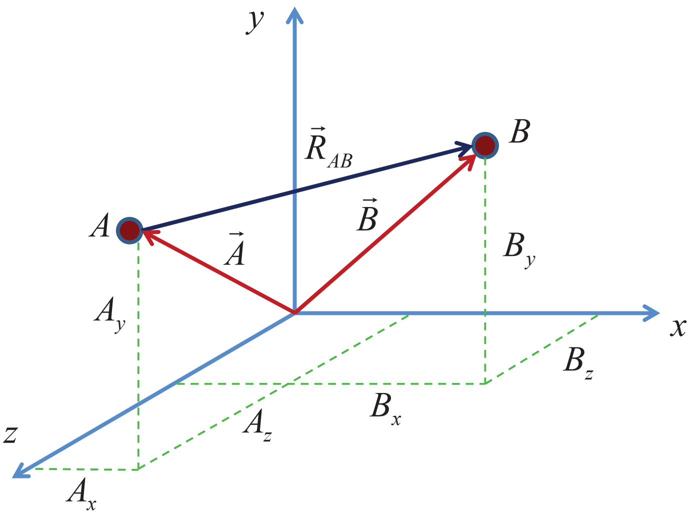
$$\lambda = 1 \quad \Rightarrow \quad \boxed{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1}$$

Statics of Particles

❑ Rectangular Components in Space

Direction of the vector is defined by the location of two points,

$A(A_x, A_y, A_z)$ and $B(B_x, B_y, B_z)$



$$\vec{A} = A_x\,\vec{i} + A_y\,\vec{j} + A_z\,\vec{k}$$

$$\vec{B} = B_x\,\vec{i} + B_y\,\vec{j} + B_z\,\vec{k}$$

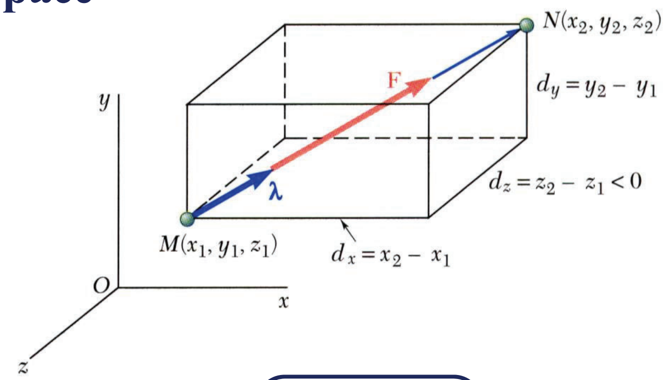
$$\vec{R}_{AB} = \vec{B} - \vec{A} \quad \Rightarrow \quad \boxed{\vec{R}_{AB} = (B_x - A_x)\,\vec{i} + (B_y - A_y)\,\vec{j} + (B_z - A_z)\,\vec{k}}$$

Statics of Particles

❑ Rectangular Components in Space

Direction of the force is defined by the location of two points,

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



vector joining M and N: $\vec{d} = d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$

$$\boxed{d = \sqrt{d_x^{\,2} + d_y^{\,2} + d_z^{\,2}}}$$

$$\boxed{\begin{aligned} d_x &= (x_2 - x_1) \\ d_y &= (y_2 - y_1) \\ d_z &= (z_2 - z_1) \end{aligned}}$$

$$\boxed{\vec{\lambda} = \frac{1}{d}(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}) \quad \Rightarrow \quad \vec{F} = F\vec{\lambda}}$$

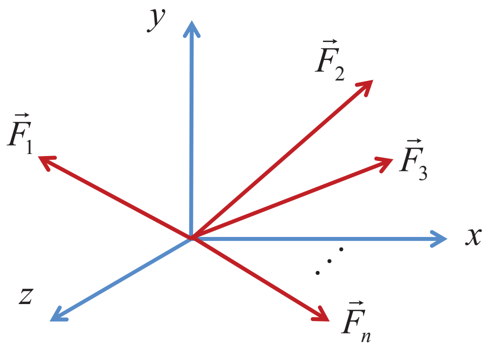
$$\boxed{F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}}$$

Statics of Particles

Addition of n Forces by Summing Components in Space

$$\vec{F}_1 = Fx_1\,\vec{i} + Fy_1\,\vec{j} + Fz_1\,\vec{k}$$
$$\vec{F}_2 = Fx_2\,\vec{i} + Fy_2\,\vec{j} + Fz_2\,\vec{k}$$
$$\vdots$$
$$\vec{F}_n = Fx_n\,\vec{i} + Fy_n\,\vec{j} + Fz_n\,\vec{k}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$



$$\Rightarrow\quad \vec{F} = (Fx_1 + Fx_2 + \cdots + Fx_n)\,\vec{i} + (Fy_1 + Fy_2 + \cdots + Fy_n)\,\vec{j} + (Fz_1 + Fz_2 + \cdots + Fz_n)\,\vec{k}$$

$$\Rightarrow\quad \vec{F} = \left(\sum_{r=1}^{n} Fx_r\right)\vec{i} + \left(\sum_{r=1}^{n} Fy_r\right)\vec{j} + \left(\sum_{r=1}^{n} Fz_r\right)\vec{k}$$

$$\Rightarrow\quad F = \sqrt{\left(\sum_{r=1}^{n} Fx_r\right)^2 + \left(\sum_{r=1}^{n} Fy_r\right)^2 + \left(\sum_{r=1}^{n} Fz_r\right)^2}$$

$$\theta_x = \cos^{-1}\left(\sum_{r=1}^{n} Fx_r \Big/ F\right)$$
$$\theta_y = \cos^{-1}\left(\sum_{r=1}^{n} Fy_r \Big/ F\right)$$
$$\theta_z = \cos^{-1}\left(\sum_{r=1}^{n} Fz_r \Big/ F\right)$$

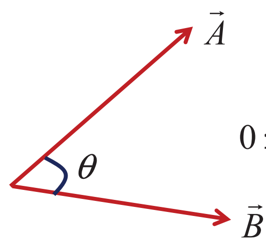
Statics of Particles

Note on Vectors

- ***Dot or inner product***

The dot product is a method for multiplying two vectors. Because the product of the multiplication is a scalar, the dot product is sometimes referred to as the scalar product.

$$\vec{A}\cdot\vec{B} = A\cdot B\cos\theta \quad\Rightarrow\quad \theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{A\cdot B}\right)$$



$$0 \le \theta \le 180^o$$

$$if\quad \vec{A}\cdot\vec{B} = 0 \quad\Rightarrow\quad \theta = 90^o \quad\Rightarrow\quad \vec{A}\perp\vec{B}$$

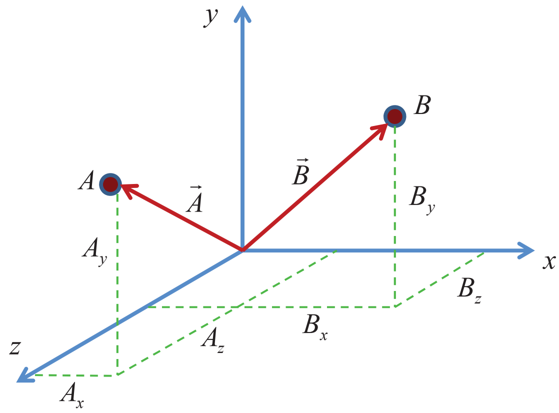
$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$
$$a(\vec{A}\cdot\vec{B}) = (a\vec{A})\cdot\vec{B} = \vec{A}\cdot(a\vec{B}) = (\vec{A}\cdot\vec{B})a$$
$$\vec{A}\cdot(\vec{B}+\vec{D}) = (\vec{A}\cdot\vec{B}) + (\vec{A}\cdot\vec{D})$$

$$\vec{i}\cdot\vec{j} = \vec{i}\cdot\vec{k} = \vec{k}\cdot\vec{j} = (1)\,(1)\cos 90^0 = 0$$
$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = (1)\,(1)\cos 0^0 = 1$$

Statics of Particles

❑ Note on Vectors

The dot product to vectors in space



$$\vec{A} = A_x\,\vec{i} + A_y\,\vec{j} + A_z\,\vec{k}$$
$$\vec{B} = B_x\,\vec{i} + B_y\,\vec{j} + B_z\,\vec{k}$$

$$\vec{A}\cdot\vec{B} = (A_x\,\vec{i} + A_y\,\vec{j} + A_z\,\vec{k})\cdot(B_x\,\vec{i} + B_y\,\vec{j} + B_z\,\vec{k})$$
$$= A_xB_x(\vec{i}\cdot\vec{i}) + A_xB_y(\vec{i}\cdot\vec{j}) + A_xB_z(\vec{i}\cdot\vec{k})$$
$$+ A_yB_x(\vec{j}\cdot\vec{i}) + A_yB_y(\vec{j}\cdot\vec{j}) + A_yB_z(\vec{j}\cdot\vec{k})$$
$$+ A_zB_x(\vec{k}\cdot\vec{i}) + A_zB_y(\vec{k}\cdot\vec{j}) + A_zB_z(\vec{k}\cdot\vec{k})$$

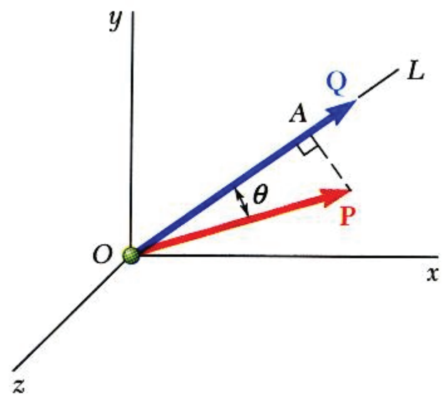
$$\Rightarrow \quad \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Statics of Particles

❑ Note on Vectors

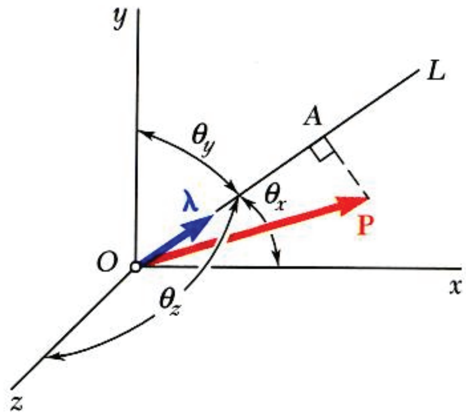
- ***Projection of a vector on a given axis:***

$$P_{OL} = P\cos\theta = \text{projection of } P \text{ along } OL$$
$$\vec{P}\bullet\vec{Q} = PQ\cos\theta$$
$$\frac{\vec{P}\bullet\vec{Q}}{Q} = P\cos\theta = P_{OL}$$



- ***For an axis defined by a unit vector:***

$$P_{OL} = \vec{P}\bullet\vec{\lambda}$$
$$= P_x\cos\theta_x + P_y\cos\theta_y + P_z\cos\theta_z$$



Statics of Particles

❑ Note on Vectors

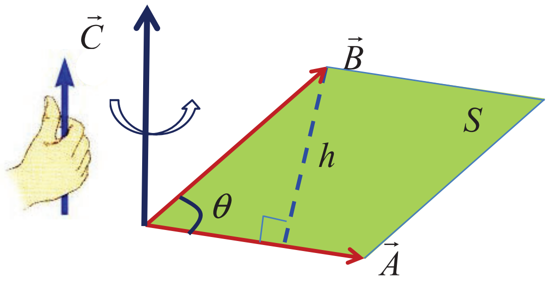
- ***The Cross Product (Vector Product)***

The cross product $\vec{A}\times\vec{B}$ between two vectors $\vec{A}$ and $\vec{B}$ is a new vector perpendicular to the plane defined by the original two vectors.

$$\vec{C}=\vec{A}\times\vec{B} \quad , \quad \vec{C}\perp(\vec{A}\,\&\,\vec{B})$$

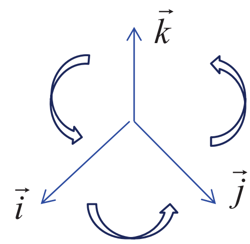
$$\boxed{C=A\cdot B\sin\theta}$$

$$S=h\cdot A=(B\sin\theta)\cdot A \quad \Rightarrow \quad \boxed{C=S}$$



$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$
$$(a\vec{A})\times\vec{B}=a(\vec{A}\times\vec{B})$$
$$(\vec{A}+\vec{B})\times\vec{C}=(\vec{A}\times\vec{C})+(\vec{B}\times\vec{C})$$
$$(\vec{A}\times\vec{B})\times\vec{C}\neq\vec{A}\times(\vec{B}\times\vec{C})$$

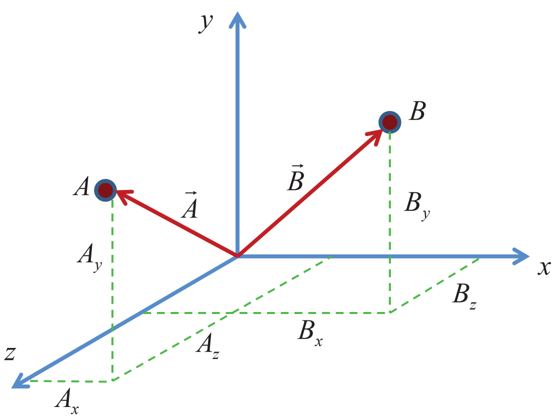
$$\vec{i}\times\vec{j}=\vec{k} \quad \vec{j}\times\vec{k}=\vec{i} \quad \vec{k}\times\vec{i}=\vec{j}$$
$$\vec{i}\times\vec{i}=\vec{j}\times\vec{j}=\vec{k}\times\vec{k}=0$$



Statics of Particles

❑ Note on Vectors

The Cross product to vectors in space



$$\vec{A}=A_x\,\vec{i}+A_y\,\vec{j}+A_z\,\vec{k}$$
$$\vec{B}=B_x\,\vec{i}+B_y\,\vec{j}+B_z\,\vec{k}$$

$$\vec{A}\times\vec{B}=(A_x\,\vec{i}+A_y\,\vec{j}+A_z\,\vec{k})\times(B_x\,\vec{i}+B_y\,\vec{j}+B_z\,\vec{k})=A_xB_x(\vec{i}\times\vec{i})+A_xB_y(\vec{i}\times\vec{j})+A_xB_z(\vec{i}\times\vec{k})$$
$$+A_yB_x(\vec{j}\times\vec{i})+A_yB_y(\vec{j}\times\vec{j})+A_yB_z(\vec{j}\times\vec{k})+A_zB_x(\vec{k}\times\vec{i})+A_zB_y(\vec{k}\times\vec{j})+A_zB_z(\vec{k}\times\vec{k})$$

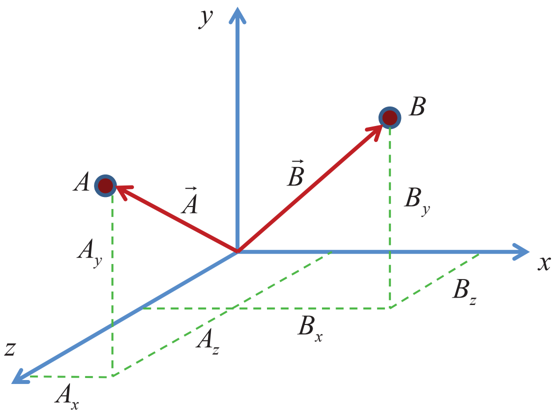
$$\Rightarrow \quad \vec{A}\times\vec{B}=A_xB_y(\vec{k})+A_xB_z(-\vec{j})+A_yB_x(-\vec{k})+A_yB_z(\vec{i})+A_zB_x(\vec{j})+A_zB_y(-\vec{i})$$

$$\Rightarrow \quad \boxed{\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\vec{i}-(A_xB_z-A_zB_x)\vec{j}+(A_xB_y-A_yB_x)\vec{k}}$$

Statics of Particles

❑ Note on Vectors

The Cross product to vectors in space



$$\vec{A} = A_x\,\vec{i} + A_y\,\vec{j} + A_z\,\vec{k}$$
$$\vec{B} = B_x\,\vec{i} + B_y\,\vec{j} + B_z\,\vec{k}$$

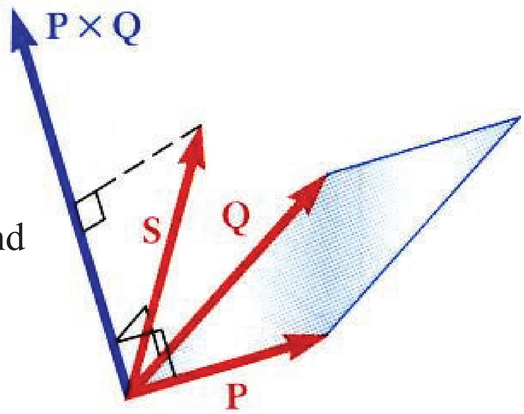
$$\Rightarrow \vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\vec{i} - (A_xB_z - A_zB_x)\vec{j} + (A_xB_y - A_yB_x)\vec{k}$$

Statics of Particles

❑ Note on Vectors

- Mixed triple product of three vectors,

$$\vec{S}\bullet(\vec{P}\times\vec{Q}) = \text{scalar result}$$



- The six mixed triple products formed from ***S***, ***P***, and ***Q*** have equal magnitudes but not the same sign,

$$\vec{S}\bullet(\vec{P}\times\vec{Q}) = \vec{P}\bullet(\vec{Q}\times\vec{S}) = \vec{Q}\bullet(\vec{S}\times\vec{P})$$
$$= -\vec{S}\bullet(\vec{Q}\times P) = -\vec{P}\bullet(\vec{S}\times\vec{Q}) = -\vec{Q}\bullet(\vec{P}\times\vec{S})$$

- Evaluating the mixed triple product,

$$\vec{S}\bullet(\vec{P}\times\vec{Q}) = S_x(P_yQ_z - P_zQ_y) + S_y(P_zQ_x - P_xQ_z) + S_z(P_xQ_y - P_yQ_x) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

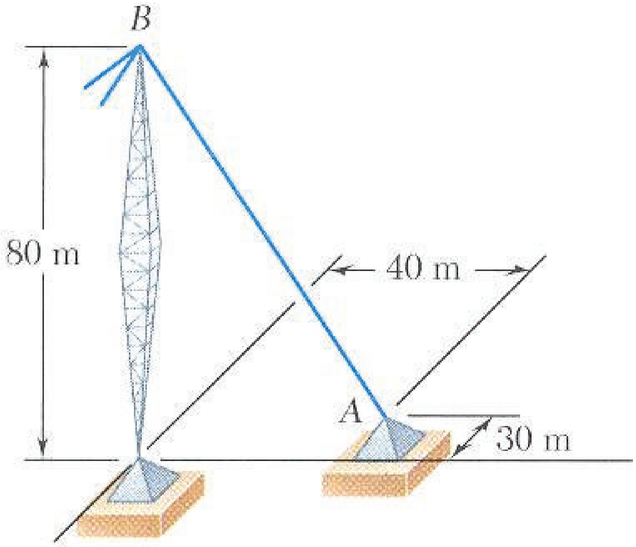
Statics of Particles

❑ Sample Problem 10

The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at A,

b) the angles θ_x, θ_y, θ_z defining the direction of the force

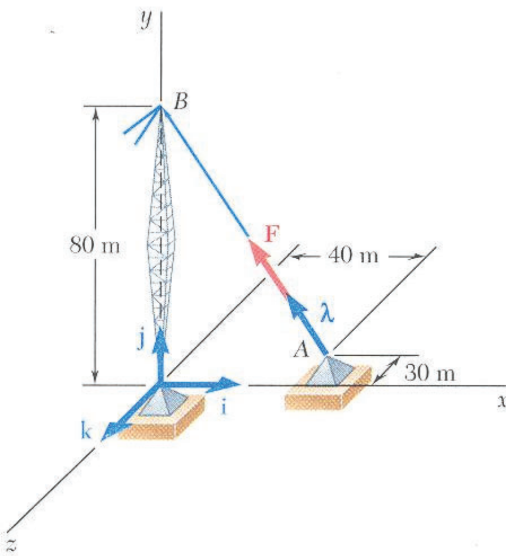


Statics of Particles

❑ Sample Problem 10

SOLUTION:

- Determine the unit vector pointing from A towards B.



$$\vec{AB} = (-40)\vec{i} + (80)\vec{j} + (30)\vec{k}$$

$$AB = 94.3 \text{ m}$$

- Determine the components of the force.

$$\vec{\lambda} = -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

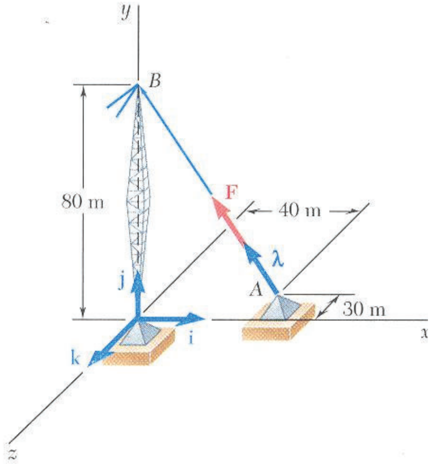
$$\vec{F} = (-1060 \text{ N})\vec{i} + (2120 \text{ N})\vec{j} + (795 \text{ N})\vec{k}$$

Statics of Particles

❑ Sample Problem 10

SOLUTION:

- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

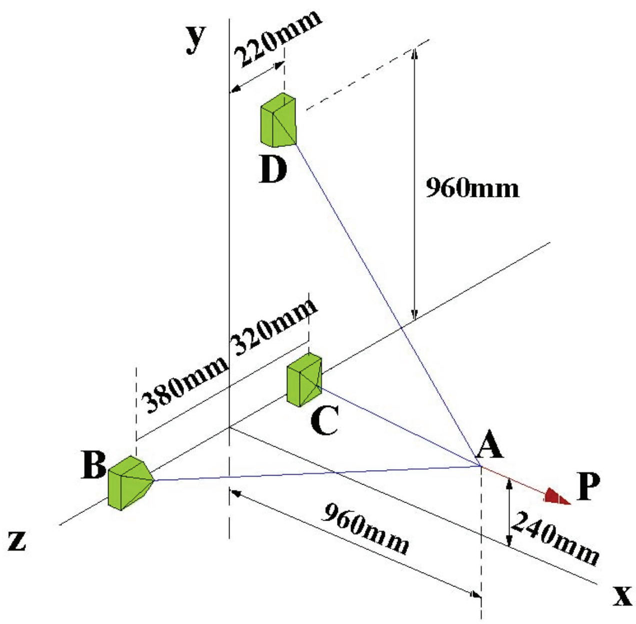


$$\Rightarrow \quad \theta_x = 115.1^\circ \quad \theta_y = 32.0^\circ \quad \theta_z = 71.5^\circ$$

Statics of Particles

❑ Sample Problem 11

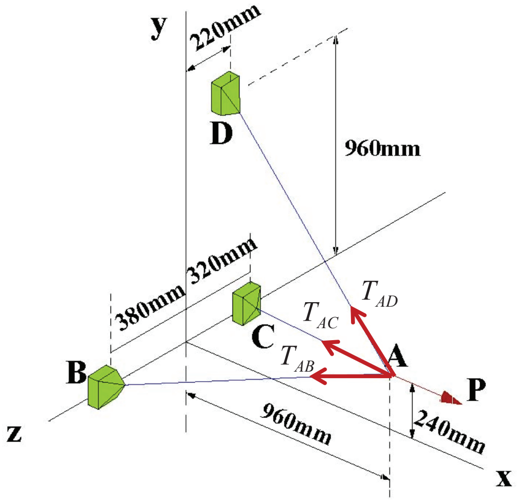
Determine the magnitude of P if there is 305 N tension force in Cable AD.



Statics of Particles

❑ Sample Problem 11

SOLUTION:



$AB = 1060\ mm$

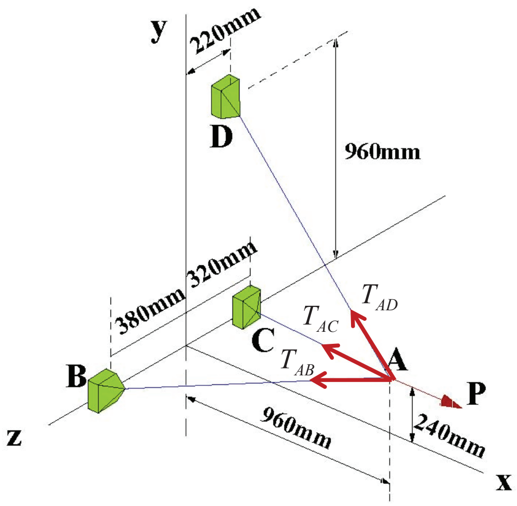
$AC = 1040\ mm$

$AD = 1220\ mm$

Statics of Particles

❑ Sample Problem 11

SOLUTION:



$$\vec{\lambda}_{AC} = \frac{\overrightarrow{AC}}{AC} = -\left(\frac{12}{13}\right)\vec{i} - \left(\frac{3}{13}\right)\vec{j} - \left(\frac{4}{13}\right)\vec{k}$$

$$\vec{\lambda}_{AD} = \frac{\overrightarrow{AD}}{AD} = -\left(\frac{48}{61}\right)\vec{i} + \left(\frac{36}{61}\right)\vec{j} - \left(\frac{11}{61}\right)\vec{k}$$

$$\vec{T}_{AC} = \vec{\lambda}_{AC} \cdot T_{AC} = \left(-\frac{12}{13}\vec{i} - \frac{3}{13}\vec{j} - \frac{4}{13}\vec{k}\right) \cdot T_{AC}$$

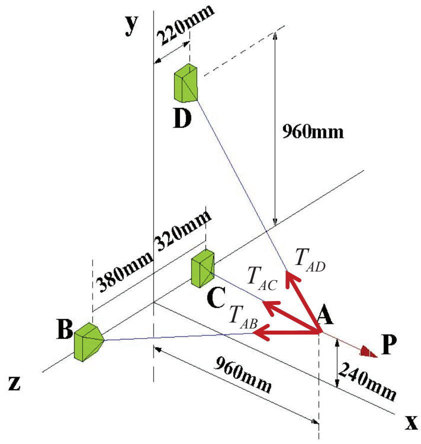
$$\vec{T}_{AD} = \vec{\lambda}_{AD} \cdot (305) = -240\vec{i} + 180\vec{j} - 55\vec{k}$$

$$\vec{P} = P\vec{i}$$

❑ Sample Problem 11

SOLUTION:

- Apply the conditions for equilibrium.



$$(I),(II),(III) \Rightarrow \begin{cases} T_{AB} = 446.71\,(N) \\ T_{AC} = 341.71\,(N) \\ \boxed{P = 4960\,(N)} \end{cases}$$