Nonlinear Analysis of Structures

Geometrically Nonlinear Analysis of Plane Trusses

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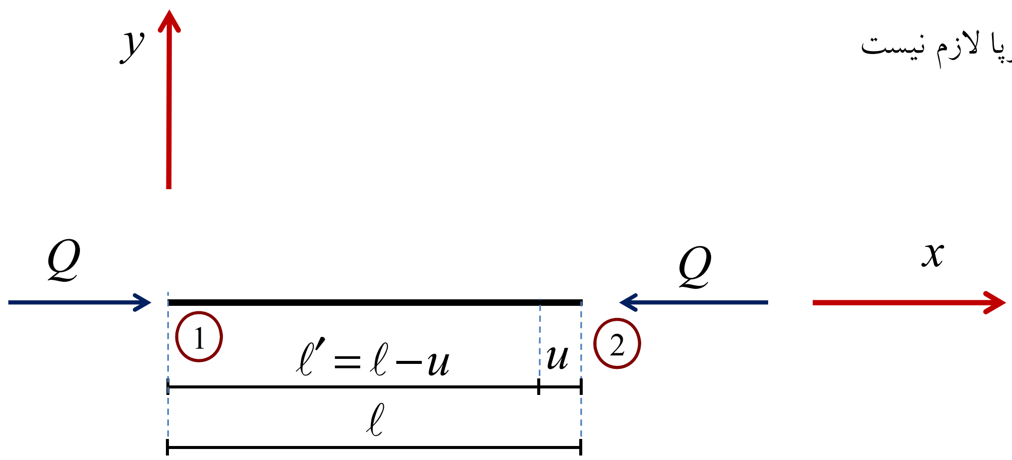
Geometrically Nonlinear Analysis of Plane Trusses

- **Member, Force-Deformation Relationship in local Coordinate**

الف – روابط نیرو تغییر شکل در دستگاه مختصات محلی

فرض انحنای کوچک برای خرپا لازم نیست

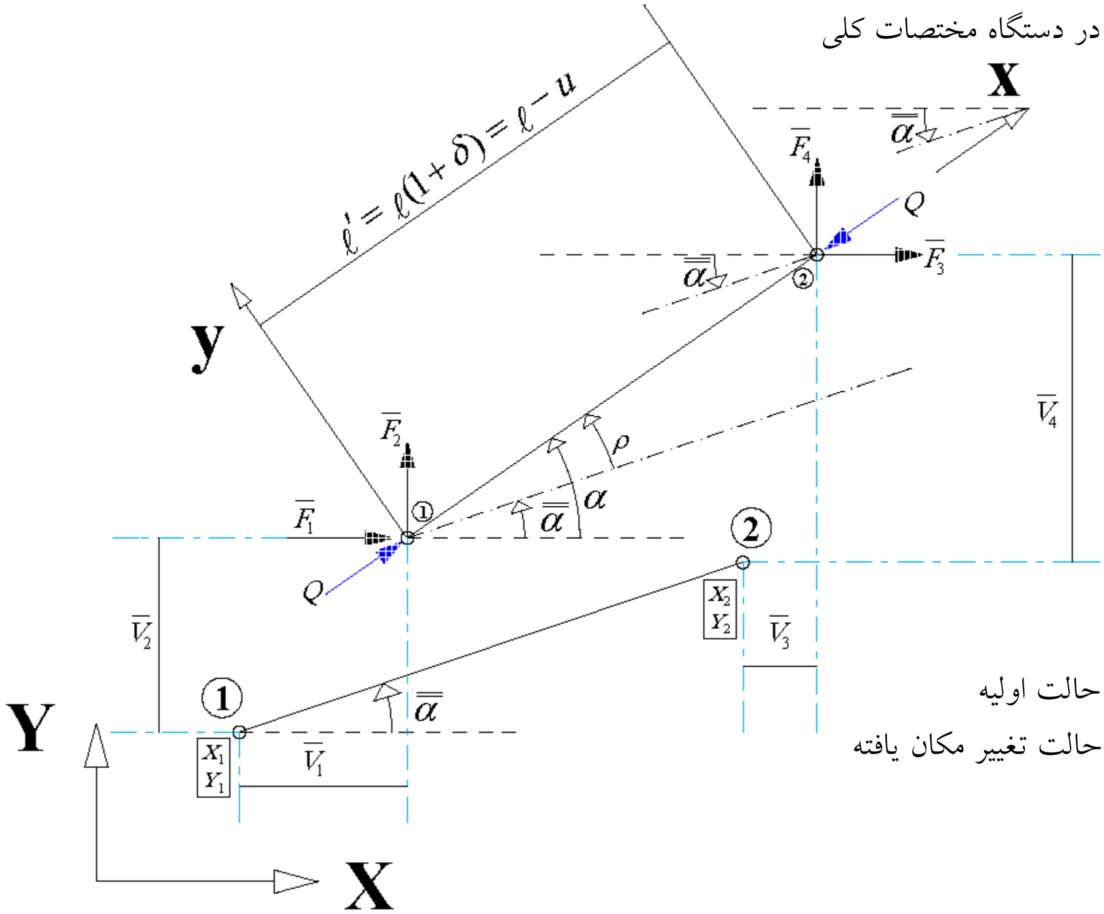


$$Q = \frac{AE}{\ell} u \qquad (1)$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Member, Force-Deformation Relationship in Global Coordinate

ب – روابط نیرو تغییر شکل در دستگاه مختصات کلی



$\bar{\bar{\alpha}}$ موقعیت محور عضو در حالت اولیه

α موقعیت محور عضو در حالت تغییر مکان یافته

ρ دوران محور عضو

ℓ' طول کوتاه شدگی

نیروها در دستگاه مختصات کلی از رابطه زیر به دست می‌آید.

$$\{\bar{F}\}_{4\times1} = [\bar{B}]_{4\times1} Q \quad (2)$$

که در آن

$\{\bar{F}\}$ نیروهای داخلی در دستگاه کلی

$$\{\bar{F}\} = \begin{Bmatrix} \bar{F}_1 \\ \bar{F}_2 \\ \bar{F}_3 \\ \bar{F}_4 \end{Bmatrix}$$

$[\bar{B}]$ ماتریس انتقال (Transformation Matrix)

$$[\bar{B}] = \begin{bmatrix} m \\ n \\ -m \\ -n \end{bmatrix} \quad , \quad \begin{matrix} m = \cos\alpha \\ n = \sin\alpha \end{matrix} \quad (3)$$

با توجه به شکل

$$u = \ell - \ell'$$

$$\ell' = \left[\left((X_2+\bar{V}_3)-(X_1+\bar{V}_1)\right)^2 + \left((Y_2+\bar{V}_4)-(Y_1+\bar{V}_2)\right)^2\right]^{\frac{1}{2}} \quad (4)$$

$$m = \frac{(X_2+\bar{V}_3)-(X_1+\bar{V}_1)}{\ell'} \quad , \quad n = \frac{(Y_2+\bar{V}_4)-(Y_1+\bar{V}_2)}{\ell'}$$

Geometrically Nonlinear Analysis of Plane Trusses

Member, Force-Deformation Relationship in Global Coordinate

$$\{\Delta \bar{F}\} = [T]\{\Delta \bar{V}\} \qquad (5)$$

$$[T] = \left[\frac{\partial \bar{F}_i^{(i)}}{\partial \bar{V}_j}\right]$$ M.T.S.M in Global Coordinate

که در آن

$\dfrac{\partial \bar{F}_i^{(i)}}{\partial \bar{V}_j}$ مشتق $\bar{F}_i$ نسبت به هر یک از درجات آزادی

$$\{\bar{V}\}_{4\times1} = \begin{Bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \end{Bmatrix}$$

$\{\bar{V}\}$ تغییر مکان گره‌ای در دستگاه کلی

شبیه $[t]$ در قاب‌ها است ($\frac{AE}{\ell}$)

$$[T] = \frac{AE}{\ell}[\bar{B}][\bar{B}]^T + Q\,[g] \qquad (6)$$

M.T.S.M in Global Coordinate

Geometrically Nonlinear Analysis of Plane Trusses

Member, Force-Deformation Relationship in Global Coordinate

$$[g] = \frac{1}{\ell'}\begin{bmatrix} -n^2 & mn & n^2 & -mn \\ mn & -m^2 & -mn & m^2 \\ n^2 & -mn & -n^2 & mn \\ -mn & m^2 & mn & -m^2 \end{bmatrix} \qquad (7)$$

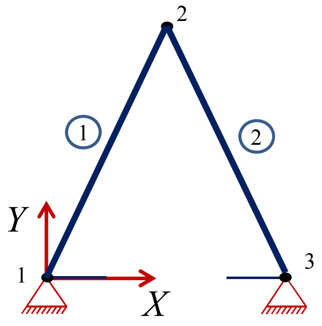
$$[\bar{B}][\bar{B}]^T = \begin{bmatrix} m^2 & mn & -m^2 & -mn \\ mn & n^2 & -mn & -n^2 \\ -m^2 & -mn & m^2 & mn \\ -mn & -n^2 & mn & n^2 \end{bmatrix} \qquad (8)$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Member, Force-Deformation Relationship in Global Coordinate

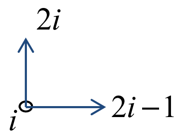
مراحل

$$\{\Delta P\}=[\tau]\{\Delta x\}$$



۱) مختصات کلی تعیین می‌گردد (X,Y) Establish General Coordinate

۲) شماره‌گذاری اعضا



Node i

۳) T برای هر عضو به کمک رابطه (6) محاسبه می‌شود.

۴) $[\tau]$ به کمک Assemble کردن تمام T ها به دست می‌آید.

$$[T^{(1)}]=\begin{bmatrix} 1 & 2 & 3 & 4 & \\ & & & & 1 \\ & & & & 2 \\ & & & & 3 \\ & & & & 4 \end{bmatrix} \qquad [T^{(2)}]=\begin{bmatrix} 3 & 4 & 5 & 6 & \\ & & & & 3 \\ & & & & 4 \\ & & & & 5 \\ & & & & 6 \end{bmatrix}$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Member, Force-Deformation Relationship in Global Coordinate

$$[\tau]=\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & \\ T_{11}^{(1)} & T_{12}^{(1)} & T_{13}^{(1)} & T_{14}^{(1)} & 0 & 0 & 1 \\ T_{21}^{(1)} & T_{22}^{(1)} & T_{23}^{(1)} & T_{24}^{(1)} & 0 & 0 & 2 \\ T_{31}^{(1)} & T_{32}^{(1)} & T_{33}^{(1)}+T_{33}^{(2)} & T_{34}^{(1)}+T_{34}^{(2)} & T_{35}^{(2)} & T_{36}^{(2)} & 3 \\ T_{41}^{(1)} & T_{42}^{(1)} & T_{43}^{(1)}+T_{43}^{(2)} & T_{44}^{(1)}+T_{44}^{(2)} & T_{45}^{(2)} & T_{45}^{(2)} & 4 \\ 0 & 0 & T_{53}^{(2)} & T_{54}^{(2)} & T_{55}^{(2)} & T_{55}^{(2)} & 5 \\ 0 & 0 & T_{63}^{(2)} & T_{64}^{(2)} & T_{65}^{(2)} & T_{65}^{(2)} & 6 \end{bmatrix}$$

$$\{\Delta P\}_{6\times1}=[\tau]_{6\times6}\{\Delta x\}_{6\times1} \Rightarrow \begin{Bmatrix} \{\Delta P\}_{2\times1}\Big|_{3,4} \\ \{\Delta R\}_{4\times1}\Big|_{1,2,5,6} \end{Bmatrix}_{6\times1} = \begin{bmatrix} \overset{3\ 4}{[\tau_{PP}]_{2\times2}} & \overset{1\ 2\ 5\ 6}{[\tau_{PR}]_{2\times4}} \\ [\tau_{RP}]_{4\times2} & [\tau_{RR}]_{4\times4} \end{bmatrix}_{6\times6} \begin{Bmatrix} \{\Delta x\}_{2\times1}\Big|_{3,4} \\ \{\Delta x_{Support}\}_{4\times1}\Big|_{1,2,5,6} \end{Bmatrix}_{6\times1}$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Member, Force-Deformation Relationship in Global Coordinate

$$\begin{Bmatrix} \{\Delta P\}_{2\times1} \begin{matrix} 3 \\ 4 \end{matrix} \\ \{\Delta R\}_{4\times1} \begin{matrix} 1 \\ 2 \\ 5 \\ 6 \end{matrix} \end{Bmatrix}_{6\times1} = \begin{bmatrix} \overset{3 \quad 4}{[\tau_{PP}]_{2\times2}} & \overset{1 \quad 2 \quad 5 \quad 6}{[\tau_{PR}]_{2\times4}} \\ [\tau_{RP}]_{4\times2} & [\tau_{RR}]_{4\times4} \end{bmatrix}_{6\times6} \begin{Bmatrix} \{\Delta x\}_{2\times1} \begin{matrix} 3 \\ 4 \end{matrix} \\ \{\Delta x_{Support}\}_{4\times1} \begin{matrix} 1 \\ 2 \\ 5 \\ 6 \end{matrix} \end{Bmatrix}_{6\times1}$$

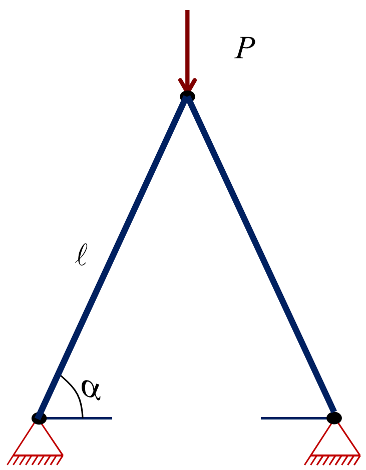
$$\Rightarrow \quad \{\Delta P\}_{2\times1} = [\tau_{PP}]_{2\times2}\{\Delta x\}_{2\times1} + [\tau_{PR}]_{2\times4}\{\Delta x_{Support}\}_{4\times1} \quad \Rightarrow \quad \{\Delta x\}_{2\times1} \ ✓$$

$$\Rightarrow \quad \{\Delta R\}_{4\times1} = [\tau_{RP}]_{4\times2}\{\Delta x\}_{2\times1} + [\tau_{RR}]_{4\times4}\{\Delta x_{Support}\}_{4\times1} \quad \Rightarrow \quad \{\Delta R\}_{4\times1} \ ✓$$

Geometrically Nonlinear Analysis of Plane Trusses

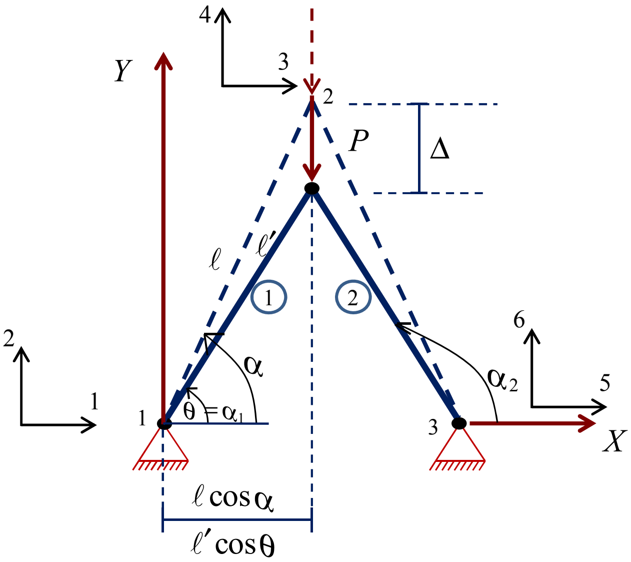
❑ Example

مثال: در خرپای نشان داده شده در شکل زیر مطلوب است تعیین الف) بار بحرانی
و ب) جابجایی نقطه اثر بار P=1500 kips .



$$\alpha = 30^0$$
$$\ell = 100\ (in.)$$
$$E = 30000\ (ksi)$$
$$A = 1\ (in.^2)$$

❑ Example



Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

$$[T_1]=\begin{bmatrix} 1 & 2 & 3 & 4 & \\ \left(\frac{AE}{\ell}\right)_1 m_1^2-\frac{Q_1}{\ell'_1}n_1^2 & \left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1 & -\left(\frac{AE}{\ell}\right)_1 m_1^2+\frac{Q_1}{\ell'_1}n_1^2 & -\left(\frac{AE}{\ell}\right)_1 m_1n_1-\frac{Q_1}{\ell'_1}m_1n_1 & 1 \\ \left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1 & \left(\frac{AE}{\ell}\right)_1 n_1^2-\frac{Q_1}{\ell'_1}m_1^2 & -\left(\frac{AE}{\ell}\right)_1 m_1n_1-\frac{Q_1}{\ell'_1}m_1n_1 & -\left(\frac{AE}{\ell}\right)_1 n_1^2+\frac{Q_1}{\ell'_1}m_1^2 & 2 \\ -\left(\frac{AE}{\ell}\right)_1 m_1^2+\frac{Q_1}{\ell'_1}n_1^2 & -\left(\frac{AE}{\ell}\right)_1 m_1n_1-\frac{Q_1}{\ell'_1}m_1n_1 & \left(\frac{AE}{\ell}\right)_1 m_1^2-\frac{Q_1}{\ell'_1}n_1^2 & \left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1 & 3 \\ -\left(\frac{AE}{\ell}\right)_1 m_1n_1-\frac{Q_1}{\ell'_1}m_1n_1 & -\left(\frac{AE}{\ell}\right)_1 n_1^2+\frac{Q_1}{\ell'_1}m_1^2 & \left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1 & \left(\frac{AE}{\ell}\right)_1 n_1^2-\frac{Q_1}{\ell'_1}m_1^2 & 4 \end{bmatrix}$$

$$[T_2]=\left(\frac{AE}{\ell}\right)_2[\overline{B}_2][\overline{B}_2]^T+Q_2\,[g_2]=\left(\frac{AE}{\ell}\right)_2\begin{bmatrix} m_2 \\ n_2 \\ -m_2 \\ -n_2 \end{bmatrix}\begin{bmatrix} m_2 \\ n_2 \\ -m_2 \\ -n_2 \end{bmatrix}^T+\frac{Q_2}{\ell'_2}\begin{bmatrix} -n_2^2 & m_2n_2 & n_2^2 & -m_2n_2 \\ m_2n_2 & -m_2^2 & -m_2n_2 & m_2^2 \\ n_2^2 & -m_2n_2 & -n_2^2 & m_2n_2 \\ -m_2n_2 & m_2^2 & m_2n_2 & -m_2^2 \end{bmatrix}$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

$$[T_2]=\left[\begin{array}{ccccc}
3 & 4 & 5 & 6 & \\
\left(\frac{AE}{\ell}\right)_2 m_2^{\,2}-\frac{Q_2}{\ell'_2}n_2^{\,2} & \left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & -\left(\frac{AE}{\ell}\right)_2 m_2^{\,2}+\frac{Q_2}{\ell'_2}n_2^{\,2} & -\left(\frac{AE}{\ell}\right)_2 m_2n_2-\frac{Q_2}{\ell'_2}m_2n_2 & 3\\
\left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & \left(\frac{AE}{\ell}\right)_2 n_2^{\,2}-\frac{Q_2}{\ell'_2}m_2^{\,2} & -\left(\frac{AE}{\ell}\right)_2 m_2n_2-\frac{Q_2}{\ell'_2}m_2n_2 & -\left(\frac{AE}{\ell}\right)_2 n_2^{\,2}+\frac{Q_2}{\ell'_2}m_2^{\,2} & 4\\
-\left(\frac{AE}{\ell}\right)_2 m_2^{\,2}+\frac{Q_2}{\ell'_2}n_2^{\,2} & -\left(\frac{AE}{\ell}\right)_2 m_2n_2-\frac{Q_2}{\ell'_2}m_2n_2 & \left(\frac{AE}{\ell}\right)_2 m_2^{\,2}-\frac{Q_2}{\ell'_2}n_2^{\,2} & \left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & 5\\
-\left(\frac{AE}{\ell}\right)_2 m_2n_2-\frac{Q_2}{\ell'_2}m_2n_2 & -\left(\frac{AE}{\ell}\right)_2 n_2^{\,2}+\frac{Q_2}{\ell'_2}m_2^{\,2} & \left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & \left(\frac{AE}{\ell}\right)_2 n_2^{\,2}-\frac{Q_2}{\ell'_2}m_1^{\,2} & 6
\end{array}\right]$$

$$[\tau]=\left[\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & \\
T_{11}^{\ (1)} & T_{12}^{\ (1)} & T_{13}^{\ (1)} & T_{14}^{\ (1)} & 0 & 0 & 1\\
T_{21}^{\ (1)} & T_{22}^{\ (1)} & T_{23}^{\ (1)} & T_{24}^{\ (1)} & 0 & 0 & 2\\
T_{31}^{\ (1)} & T_{32}^{\ (1)} & T_{33}^{\ (1)}+T_{33}^{\ (2)} & T_{34}^{\ (1)}+T_{34}^{\ (2)} & T_{35}^{\ (2)} & T_{36}^{\ (2)} & 3\\
T_{41}^{\ (1)} & T_{42}^{\ (1)} & T_{43}^{\ (1)}+T_{43}^{\ (2)} & T_{44}^{\ (1)}+T_{44}^{\ (2)} & T_{45}^{\ (2)} & T_{45}^{\ (2)} & 4\\
0 & 0 & T_{53}^{\ (2)} & T_{54}^{\ (2)} & T_{55}^{\ (2)} & T_{55}^{\ (2)} & 5\\
0 & 0 & T_{63}^{\ (2)} & T_{64}^{\ (2)} & T_{65}^{\ (2)} & T_{65}^{\ (2)} & 6
\end{array}\right]$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

$$[\tau_{PP}]_{2\times 2}=\left[\begin{array}{ccc}
3 & 4 & \\
\left(\frac{AE}{\ell}\right)_1 m_1^{\,2}-\frac{Q_1}{\ell'_1}n_1^{\,2}+\left(\frac{AE}{\ell}\right)_2 m_2^{\,2}-\frac{Q_2}{\ell'_2}n_2^{\,2} & \left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1+\left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & 3\\
\left(\frac{AE}{\ell}\right)_1 m_1n_1+\frac{Q_1}{\ell'_1}m_1n_1+\left(\frac{AE}{\ell}\right)_2 m_2n_2+\frac{Q_2}{\ell'_2}m_2n_2 & \left(\frac{AE}{\ell}\right)_1 n_1^{\,2}-\frac{Q_1}{\ell'_1}m_1^{\,2}+\left(\frac{AE}{\ell}\right)_2 n_2^{\,2}-\frac{Q_2}{\ell'_2}m_2^{\,2} & 4
\end{array}\right]$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

$$\left(\frac{AE}{\ell}\right)_1 = \left(\frac{AE}{\ell}\right)_2 = \frac{AE}{\ell} \quad \& \quad \frac{Q_1}{\ell'_1} = \frac{Q_2}{\ell'_2} = \frac{Q}{\ell'} \Rightarrow$$

$$[\tau_{PP}]_{2\times2} = \frac{AE}{\ell}\begin{bmatrix} 3 & 4 & \\ m_1^2 + m_2^2 & m_1 n_1 + m_2 n_2 & 3 \\ m_1 n_1 + m_2 n_2 & n_1^2 + n_2^2 & 4 \end{bmatrix} + \frac{Q}{\ell'}\begin{bmatrix} 3 & 4 & \\ -n_1^2 - n_2^2 & m_1 n_1 + m_2 n_2 & 3 \\ m_1 n_1 + m_2 n_2 & -m_1^2 - m_2^2 & 4 \end{bmatrix}$$

$$\begin{matrix} m = m_1 = -m_2 = \cos\theta \\ n = n_1 = n_2 = \sin\theta \end{matrix} \Rightarrow [\tau_{PP}] = \frac{AE}{\ell}\begin{bmatrix} 2m^2 & 0 \\ 0 & 2n^2 \end{bmatrix} + \frac{Q}{\ell'}\begin{bmatrix} -2n^2 & 0 \\ 0 & -2m^2 \end{bmatrix}$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

$$\Rightarrow [\tau_{PP}] = \frac{2AE}{\ell\cos\alpha}\begin{bmatrix} \cos\alpha - \sin^2\theta\cos\theta & 0 \\ 0 & \cos\alpha - \cos^3\theta \end{bmatrix}$$

Geometrically Nonlinear Analysis of Plane Trusses

Example

خرپا چه زمانی ناپایدار می‌شود؟

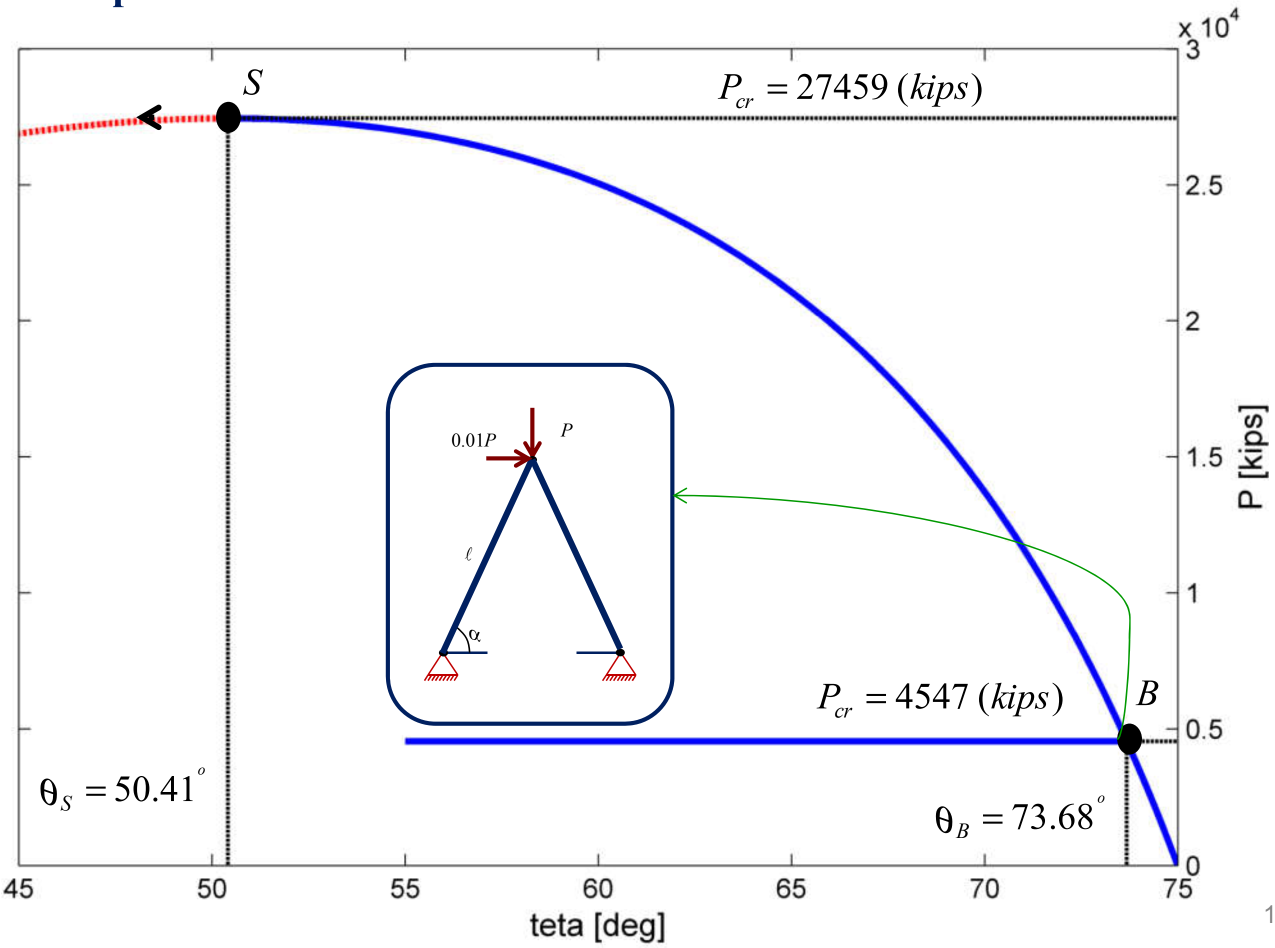
Geometrically Nonlinear Analysis of Plane Trusses

Example

در چه حالتی مودهای ناپایداری Bifurcation و Snap through همزمان اتفاق می‌افتد.

Geometrically Nonlinear Analysis of Plane Trusses

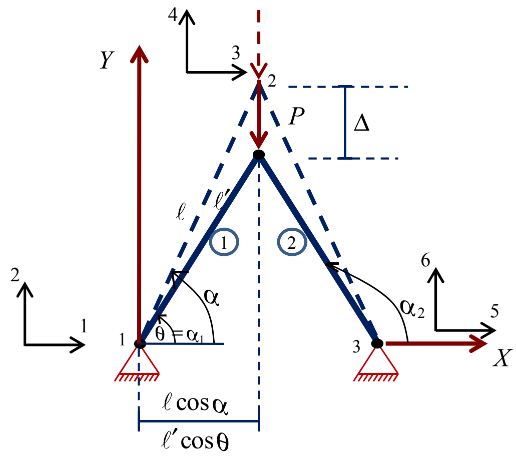
- **Example**



Geometrically Nonlinear Analysis of Plane Trusses

- **Example**

$$\Rightarrow \ell_1' = \ell\left[1-2\frac{\Delta}{\ell}\sin\alpha+\left(\frac{\Delta}{\ell}\right)^2\right]^{\frac{1}{2}}$$



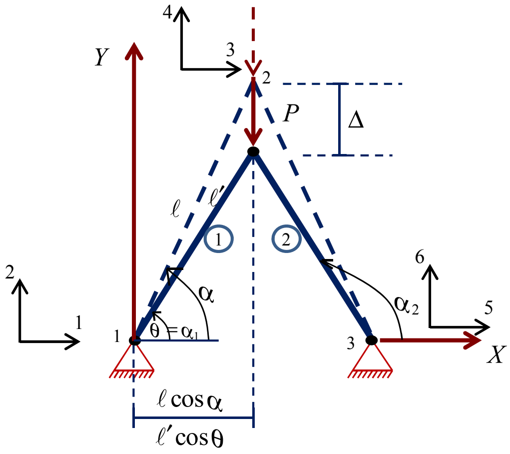
$$\Rightarrow \ell_2' = \ell_1' = \ell' = \ell\left[1-2\frac{\Delta}{\ell}\sin\alpha+\left(\frac{\Delta}{\ell}\right)^2\right]^{\frac{1}{2}}$$

Geometrically Nonlinear Analysis of Plane Trusses

- ❑ **Example**

$$m_1 = \frac{\ell \cos\alpha}{\ell'}$$

$$n_1 = \frac{\ell \sin\alpha - \Delta}{\ell'}$$



$$m_2 = -\frac{\ell \cos\alpha}{\ell'}$$

$$n_2 = \frac{\ell \sin\alpha - \Delta}{\ell'}$$

$$\Rightarrow \; [k_{PP}] = \frac{AE}{\ell}\begin{bmatrix} 2m^2 & 0 \\ 0 & 2n^2 \end{bmatrix} + \frac{Q}{\ell'}\begin{bmatrix} -2n^2 & 0 \\ 0 & -2m^2 \end{bmatrix}$$

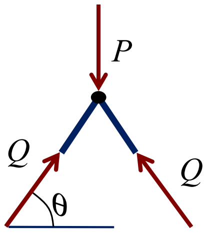
Geometrically Nonlinear Analysis of Plane Trusses

- ❑ **Example**

Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

در هر مرحله باید تعادل گره‌ای بررسی شود برای این منظور باید تعادل را در دو راستای 3 و 4 کنترل کرد. اما چون در راستای 3 نیرویی وارد نمی‌شود نیازی به کنترل تعادل در این راستا نیست.



Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

یادآوری

$$[\tau_{PP}]=\frac{AE}{\ell}\begin{bmatrix}2m^2 & 0\\ 0 & 2n^2\end{bmatrix}+\frac{Q}{\ell'}\begin{bmatrix}-2n^2 & 0\\ 0 & -2m^2\end{bmatrix}$$

$$\Rightarrow\quad [\tau_{PP}]=\frac{AE}{\ell}\begin{bmatrix}2\left(\frac{\ell\cos\alpha}{\ell\beta}\right)^2 & 0\\ 0 & 2\left(\frac{\ell\sin\alpha-\Delta}{\ell\beta}\right)^2\end{bmatrix}+\frac{AE}{\ell}\left(\frac{1-\beta}{\beta}\right)\begin{bmatrix}-2\left(\frac{\ell\sin\alpha-\Delta}{\ell\beta}\right)^2 & 0\\ 0 & -2\left(\frac{\ell\cos\alpha}{\ell\beta}\right)^2\end{bmatrix}$$

$$\Rightarrow\quad [\tau_{PP}]=\frac{2AE}{\ell}\begin{bmatrix}\left(\frac{\ell\cos\alpha}{\ell\beta}\right)^2-\left(\frac{1-\beta}{\beta}\right)\left(\frac{\sin\alpha-\frac{\Delta}{\ell}}{\beta}\right)^2 & 0\\ 0 & \left(\frac{\sin\alpha-\frac{\Delta}{\ell}}{\beta}\right)^2-\left(\frac{1-\beta}{\beta}\right)\left(\frac{\ell\cos\alpha}{\ell\beta}\right)^2\end{bmatrix}$$

❑ Example

$$\Rightarrow \quad [\tau_{PP}] = \frac{2AE}{\ell\beta^2}\begin{bmatrix} \cos^2\alpha - \left(\frac{1}{\beta}-1\right)\left(\sin\alpha - \frac{\Delta}{\ell}\right)^2 & 0 \\ 0 & \left(\sin\alpha - \frac{\Delta}{\ell}\right)^2 - \left(\frac{1}{\beta}-1\right)\cos^2\alpha \end{bmatrix}$$

$$\Rightarrow \quad [\tau_{PP}] = \frac{2AE}{\ell\beta^2}\begin{bmatrix} \cos^2\alpha - \frac{\left(\sin\alpha - \frac{\Delta}{\ell}\right)^2}{\beta} + \left(\sin\alpha - \frac{\Delta}{\ell}\right)^2 & 0 \\ 0 & \left(\sin\alpha - \frac{\Delta}{\ell}\right)^2 - \frac{\cos^2\alpha}{\beta} + \cos^2\alpha \end{bmatrix}$$

❑ Example

$$\Rightarrow \quad [\tau_{PP}] = \frac{2AE}{\ell\beta^2}\begin{bmatrix} \cos^2\alpha - \frac{\beta^2 - \cos^2\alpha}{\beta} + \beta^2 - \cos^2\alpha & 0 \\ 0 & \beta^2 - \cos^2\alpha - \frac{\cos^2\alpha}{\beta} + \cos^2\alpha \end{bmatrix}$$

$$\Rightarrow \quad [\tau_{PP}] = \frac{2AE}{\ell}\begin{bmatrix} 1 - \frac{\beta^2 - \cos^2\alpha}{\beta^3} & 0 \\ 0 & 1 - \frac{\cos^2\alpha}{\beta^3} \end{bmatrix} \quad (II)$$

$$\beta = \left[1 - 2\frac{\Delta}{\ell}\sin\alpha + \left(\frac{\Delta}{\ell}\right)^2\right]^{\frac{1}{2}}$$

Geometrically Nonlinear Analysis of Plane Trusses

- **Example**

خرپا چه زمانی ناپایدار می‌شود؟

Linear Incremental Method with Iteration با استفاده از روش

Updated Newton Raphson Iteration (Updated N.R.It)

Geometrically Nonlinear Analysis of Plane Trusses

- **Example**

Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

②

$$\{\mathbf{x}^{(2)}\}=\{\mathbf{x}^{(1)}\}+\{\Delta\mathbf{x}^{(1)}\}=\begin{Bmatrix}0\\10\end{Bmatrix}+\begin{Bmatrix}0\\4.18\end{Bmatrix}\Rightarrow \boxed{\{\mathbf{x}^{(2)}\}=\begin{Bmatrix}0\\14.18\end{Bmatrix}(in)} \overset{(I)}{\Rightarrow} \boxed{\{\mathbf{P}_{\mathbf{x}^{(2)}}^{(2)}\}=\begin{Bmatrix}0\\1440.6\end{Bmatrix}(kips)}$$

$$\{\Delta\mathbf{Q}^{(2)}\}=\begin{Bmatrix}0\\1500\end{Bmatrix}-\{\mathbf{P}_{\mathbf{x}^{(2)}}^{(2)}\}=\begin{Bmatrix}0\\1500\end{Bmatrix}-\begin{Bmatrix}0\\1440.6\end{Bmatrix}\Rightarrow \boxed{\{\Delta\mathbf{Q}^{(2)}\}=\begin{Bmatrix}0\\59.384\end{Bmatrix}(kips)}$$

$$(II)\Rightarrow \boxed{[\boldsymbol{\tau}_{\mathbf{x}^{(2)}}^{(2)}]=\begin{bmatrix}506.47 & 0\\0 & 53.308\end{bmatrix}(kips/in)}$$

$$\overset{(Equation\ 2\text{–}22)}{\Rightarrow}\{\Delta\mathbf{x}^{(2)}\}=[\boldsymbol{\tau}^{(2)}]^{-1}\{\Delta\mathbf{Q}^{(2)}\}\Rightarrow \boxed{\{\Delta\mathbf{x}^{(2)}\}=\begin{Bmatrix}0\\1.114\end{Bmatrix}(in.)}$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

③

$$\{\mathbf{x}^{(3)}\}=\{\mathbf{x}^{(2)}\}+\{\Delta\mathbf{x}^{(2)}\}=\begin{Bmatrix}0\\14.18\end{Bmatrix}+\begin{Bmatrix}0\\1.114\end{Bmatrix}\Rightarrow \boxed{\{\mathbf{x}^{(3)}\}=\begin{Bmatrix}0\\15.294\end{Bmatrix}(in)} \overset{(I)}{\Rightarrow} \boxed{\{\mathbf{P}_{\mathbf{x}^{(3)}}^{(3)}\}=\begin{Bmatrix}0\\1495.9\end{Bmatrix}(kips)}$$

$$\{\Delta\mathbf{Q}^{(3)}\}=\begin{Bmatrix}0\\1500\end{Bmatrix}-\{P_{\{\mathbf{x}^{(3)}\}}^{(3)}\}=\begin{Bmatrix}0\\1500\end{Bmatrix}-\begin{Bmatrix}0\\1495.9\end{Bmatrix}\Rightarrow \boxed{\{\Delta\mathbf{Q}^{(3)}\}=\begin{Bmatrix}0\\4.1372\end{Bmatrix}(kips)}$$

$$(II)\Rightarrow \boxed{[\boldsymbol{\tau}_{\mathbf{x}^{(3)}}^{(3)}]=\begin{bmatrix}511.01 & 0\\0 & 45.89\end{bmatrix}(kips/in)}$$

$$\overset{(Equation\ 2\text{–}22)}{\Rightarrow}\{\Delta\mathbf{x}^{(3)}\}=[\boldsymbol{\tau}^{(3)}]^{-1}\{\Delta\mathbf{Q}^{(3)}\}\Rightarrow \boxed{\{\Delta\mathbf{x}^{(3)}\}=\begin{Bmatrix}0\\0.090156\end{Bmatrix}(in.)}$$

Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

④

$$\{\mathbf{x}^{(4)}\}=\{\mathbf{x}^{(3)}\}+\{\Delta\mathbf{x}^{(3)}\}=\begin{Bmatrix}0\\15.294\end{Bmatrix}+\begin{Bmatrix}0\\0.0902\end{Bmatrix}\Rightarrow\boxed{\{\mathbf{x}^{(4)}\}=\begin{Bmatrix}0\\15.38416\end{Bmatrix}(in)}\overset{(I)}{\Rightarrow}\boxed{\{\mathbf{P}^{(4)}_{\mathbf{x}^{(4)}}\}=\begin{Bmatrix}0\\1499.97\end{Bmatrix}(kips)}$$

$$\{\Delta\mathbf{Q}^{(4)}\}=\begin{Bmatrix}0\\1500\end{Bmatrix}-\{P^{(4)}_{\{\mathbf{x}^{(4)}\}}\}=\begin{Bmatrix}0\\1500\end{Bmatrix}-\begin{Bmatrix}0\\1499.97\end{Bmatrix}\Rightarrow\boxed{\{\Delta\mathbf{Q}^{(4)}\}=\begin{Bmatrix}0\\0.0269\end{Bmatrix}(kips)}$$

$$(II)\Rightarrow\boxed{[\boldsymbol{\tau}^{(4)}_{\mathbf{x}^{(4)}}]=\begin{bmatrix}511.38 & 0\\0 & 45.287\end{bmatrix}(kips/in)}$$

$$\overset{(Equation\ 2-22)}{\Rightarrow}\{\Delta\mathbf{x}^{(4)}\}=[\boldsymbol{\tau}^{(4)}]^{-1}\{\Delta\mathbf{Q}^{(4)}\}\Rightarrow\boxed{\{\Delta\mathbf{x}^{(4)}\}=\begin{Bmatrix}0\\0.0006\end{Bmatrix}(in.)}$$

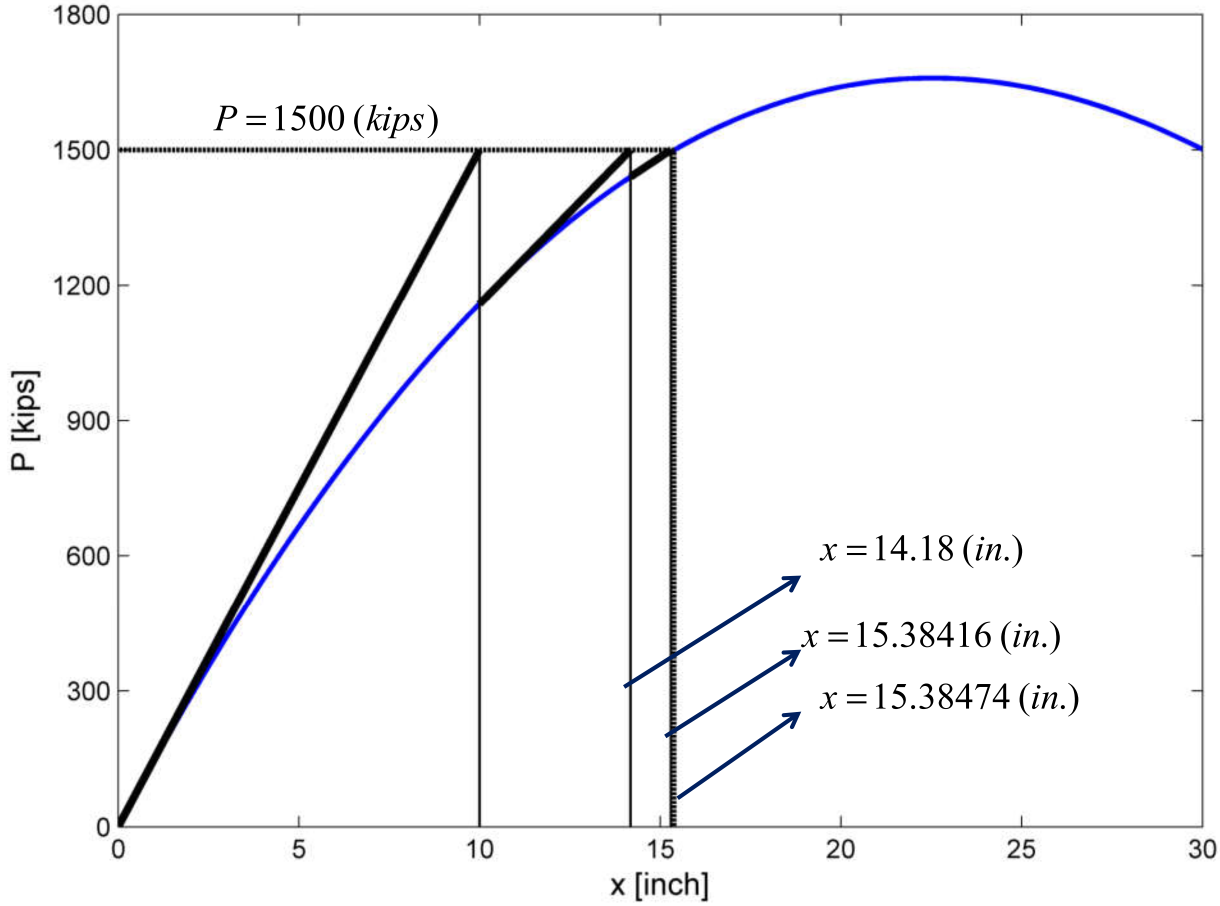
Geometrically Nonlinear Analysis of Plane Trusses

❑ Example

Iteration	$P^{(i)}$ $(kips)$	$x^{(i)}$ $(in.)$	$\tau^{(i)}$ $(kips/in.)$	$\Delta Q^{(i)}$ $(kips)$	$\Delta x^{(i)}$ $(in.)$	$x^{(i+1)}$ $(in.)$
0	0	0	150.000	1500	10	10
1	1158.836	10	81.617	341.1639	4.1800	14.18004
2	1440.618	14.18004	53.308	59.38181	1.1139	15.29399
3	1495.862	15.29399	45.890	4.137854	0.0902	15.38416
4	1499.973	15.38416	45.292	0.026937	0.0006	15.38475
5	**1500**	15.38475	45.288	1.17E-06	0.0000	**15.38475**

Geometrically Nonlinear Analysis of Plane Trusses

- **Example**



Updated Newton Raphson Iteration (Updated N.R.It)