Mechanics of Materials

Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:
J.Wat Oler "Lectures notes on Mechanics od Materials"
Ibrahim A.Assakkaf "Lectures notes on Mechanics od Materials"

Introduction to Plasticity

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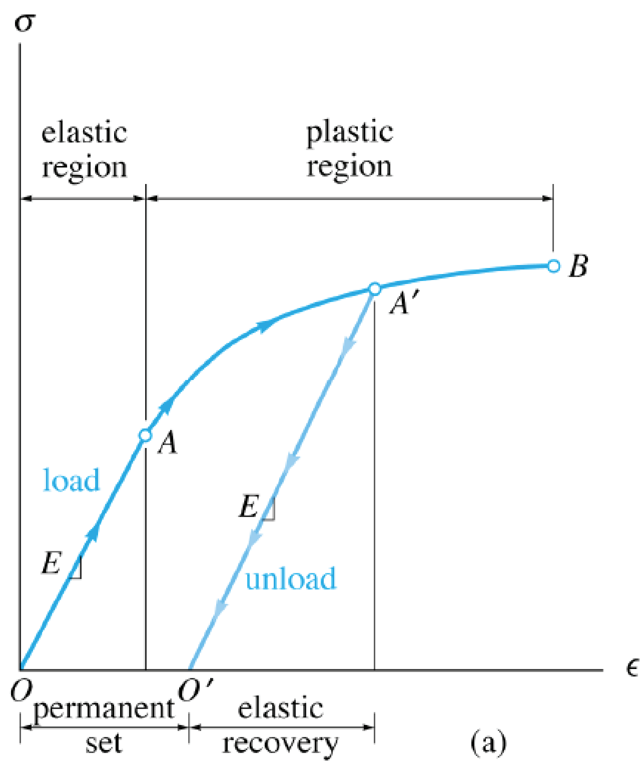
Introduction to Plasticity

❑ Inelastic (Plastic) Axial Deformation and Residual Stress

- Previous analyses based on assumption of linear stress-strain relationship, i.e., stresses below the yield stress
- Assumption is good for brittle material which rupture without yielding
- If the yield stress of ductile materials is exceeded, then plastic deformations occur



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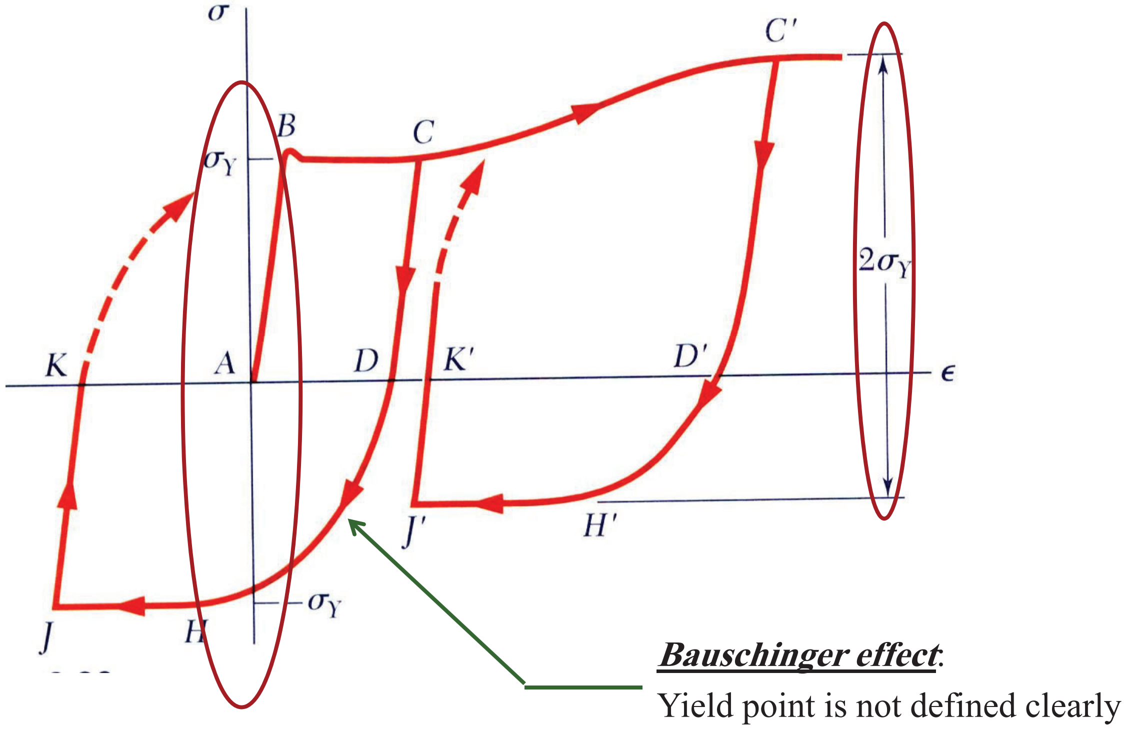
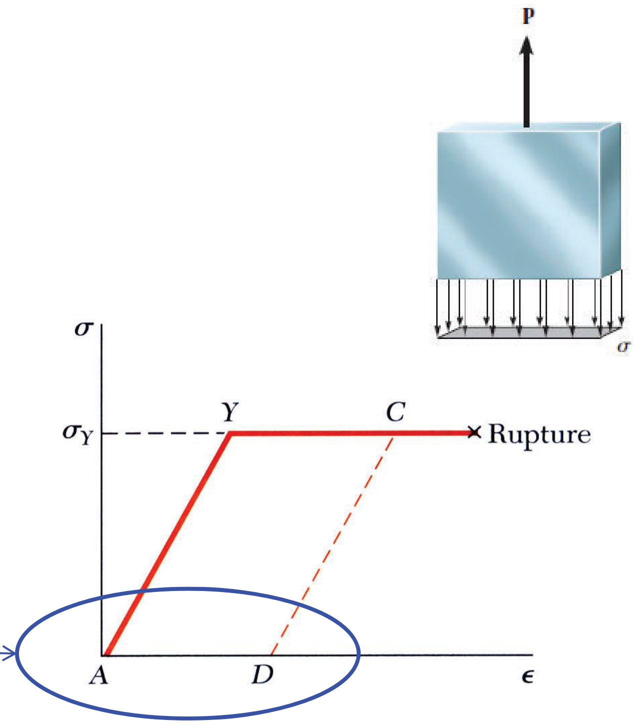
❑ Inelastic (Plastic) Axial Deformation and Residual Stress



Introduction to Plasticity

❑ Inelastic (Plastic) Axial Deformation and Residual Stress

- Analysis of plastic deformations is simplified by assuming an idealized ***Elastoplastic material***
- Deformations of an Elastoplastic material are divided into elastic and plastic ranges
- Permanent deformations result from loading beyond the yield stress



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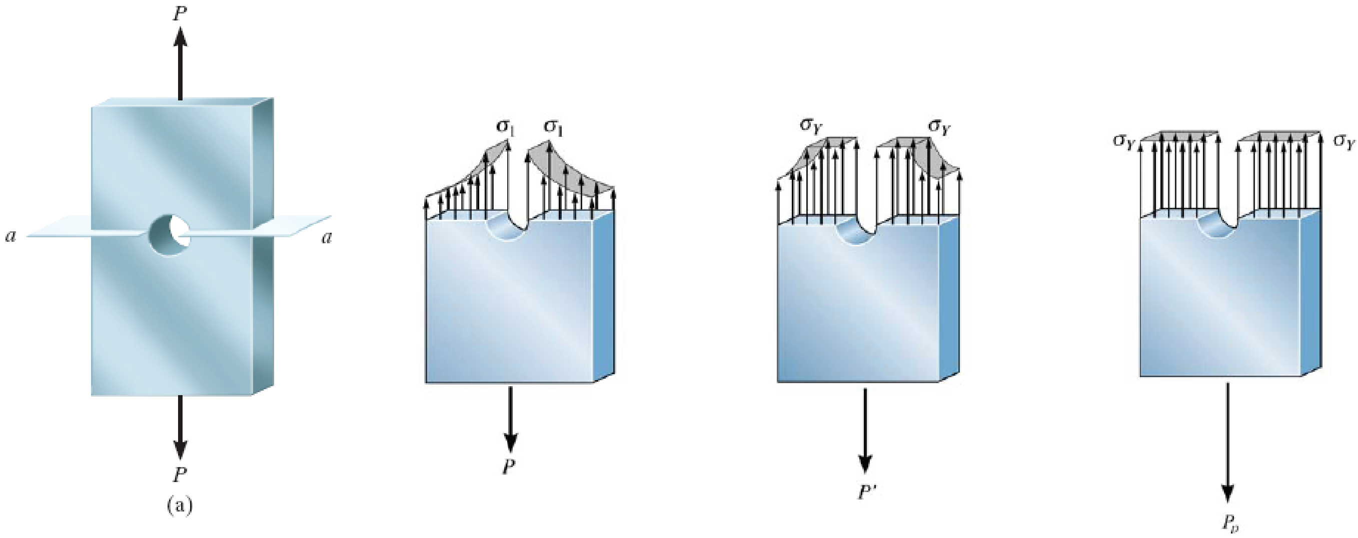
❑ Stress Concentration

Consider now the case of a bar with a hole through it (a stress concentration). The stress distribution across section a-a is not uniform due to the stress concentration. The material closest to the hole reaches the yield stress first. Once the entire section reaches the yield stress the section can sustain no greater load.



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❑ Stress Concentration

- Elastic deformation while maximum stress is less than yield stress

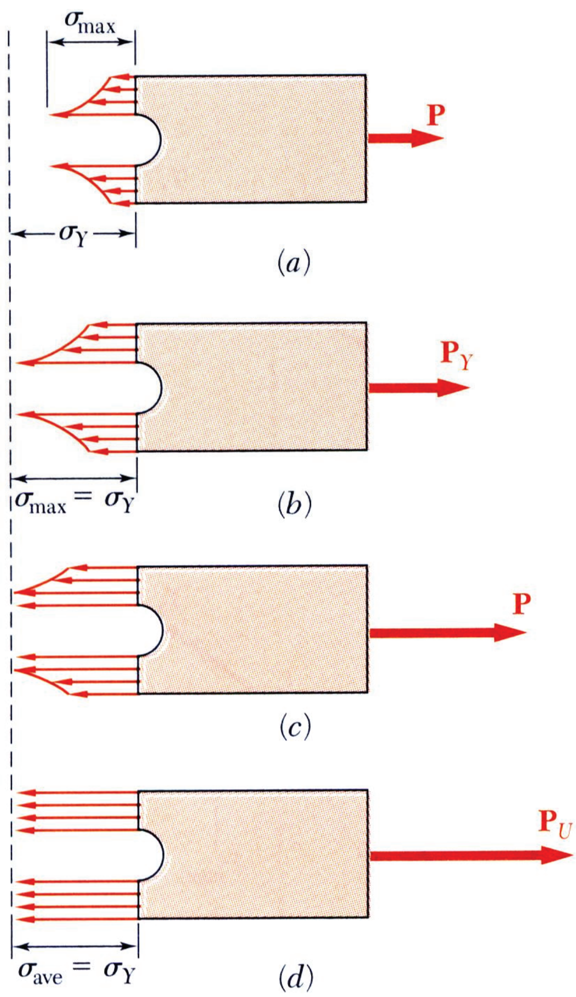
$$P = \sigma_{ave} A = \frac{\sigma_{max} A}{K}$$

- Maximum stress is equal to the yield stress at the maximum elastic loading

$$P_Y = \frac{\sigma_Y A}{K}$$

- At loadings above the maximum elastic load, a region of plastic deformations develop near the hole
- As the loading increases, the plastic region expands until the section is at a uniform stress equal to the yield stress

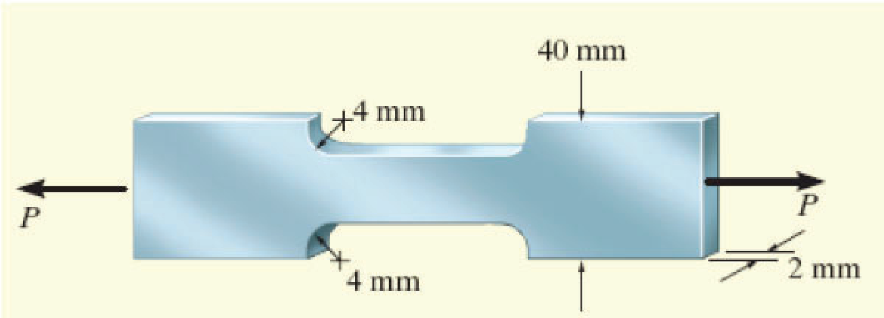
$$P_U = \sigma_Y A = K\, P_Y$$



❑ Plastic Deformations due to axial force

Example 01

The bar is made of steel that is assumed to be elastic perfectly plastic, with σ_Y = 250 MPa. Determine (a) the maximum value of the applied load P that can be applied without causing the steel to yield and (b) the maximum value of P that the bar can support. Sketch the stress distribution at the critical section for each case.



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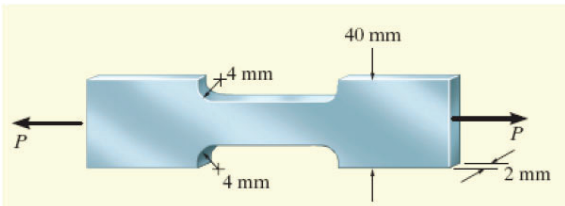
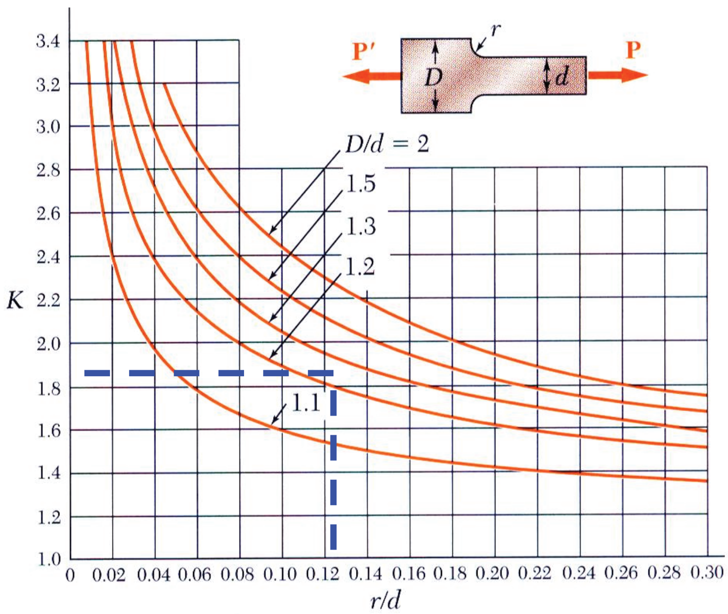
❑ Plastic Deformations due to axial force

Example 01





(b) Flat bars with fillets

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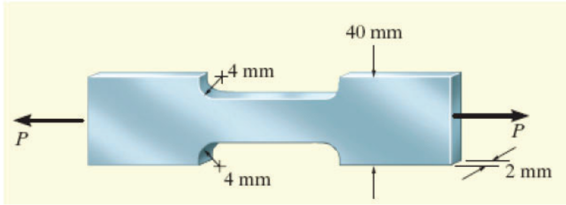
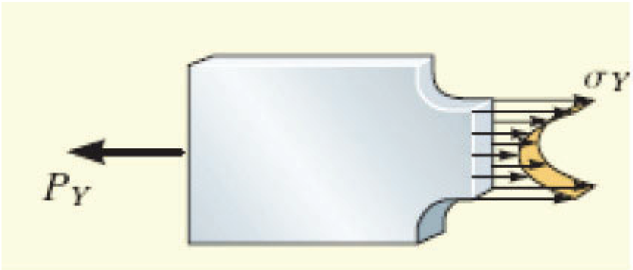
❑ **Plastic Deformations due to axial force**

Example 01





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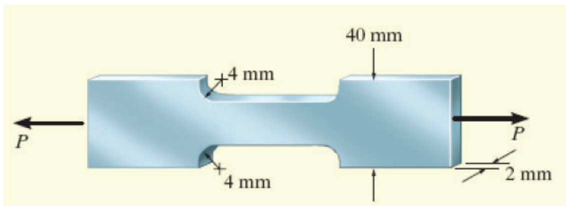
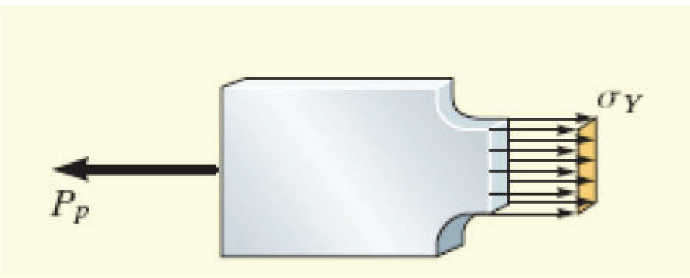
❑ **Plastic Deformations due to axial force**

Example 01



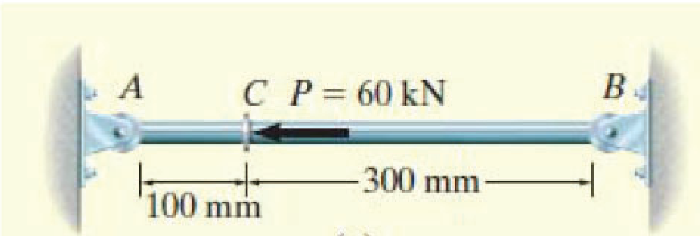


❑ Inelastic (Plastic) Axial Deformation and Residual Stress

- When a single structural element is loaded uniformly beyond its yield stress and then unloaded, it is permanently deformed but all stresses disappear. This is not the general result.
- ***Residual stresses*** will remain in a structure after loading and unloading if
  - only part of the structure undergoes plastic deformation
  - different parts of the structure undergo different plastic deformations
- Residual stresses also result from the ***uneven heating or cooling*** of structures or structural elements

Example 02

The rod shown below has a radius of 5 mm and is made of an elastic perfectly plastic material for which σ_Y = 420 Mpa and E = 70 GPa. If a force of P = 60 kN is applied to the rod and then removed, determine the residual stress in the rod.



(a)

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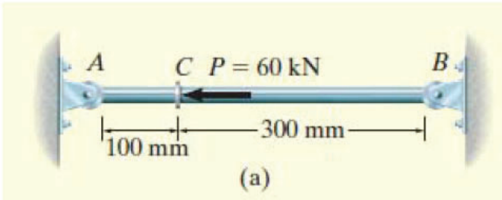
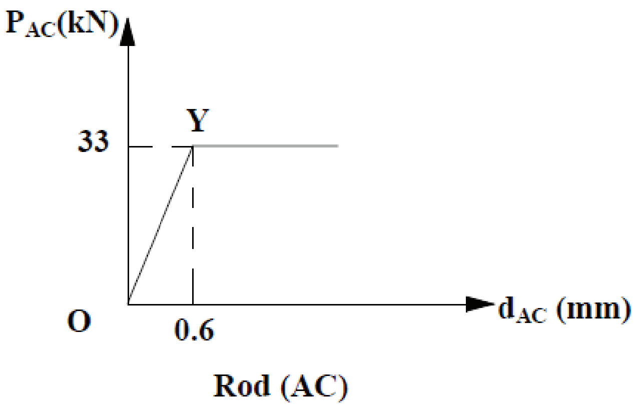
Example 02





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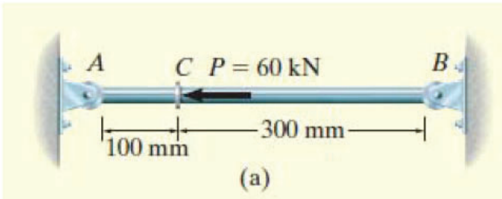
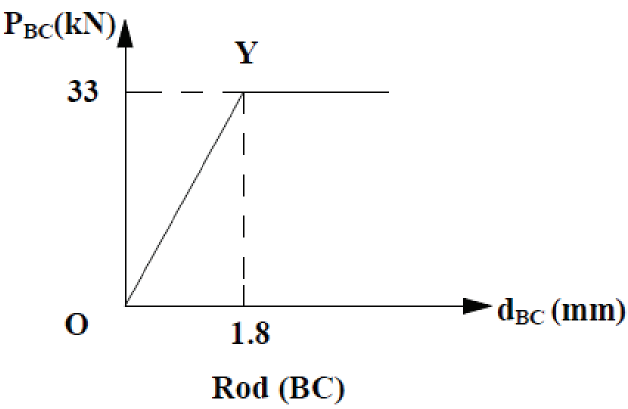
Example 02





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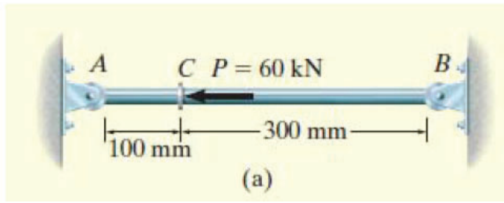
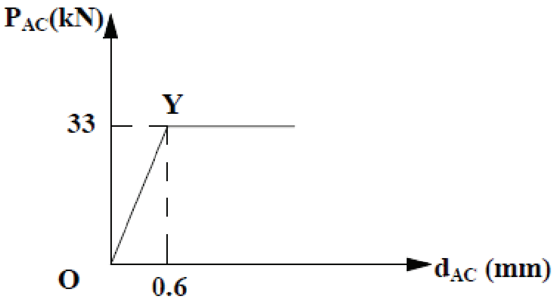
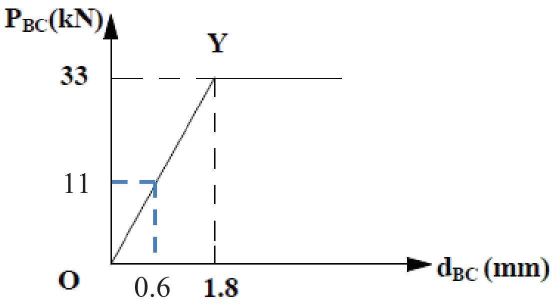
Example 02

$$P = P_{AC} + P_{BC}$$
$$\delta = \delta_{AC} = \delta_{BC}$$

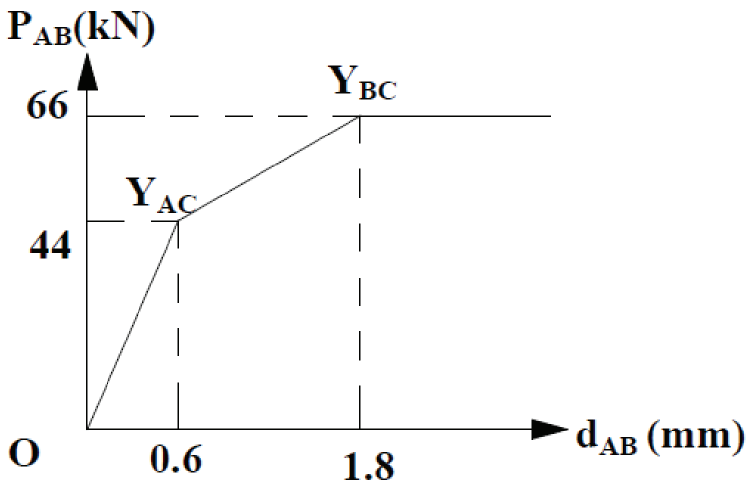




Rod (AC)



Rod (BC)



Rod (AB)

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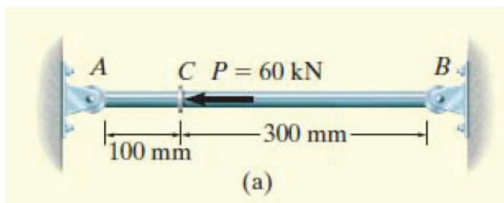
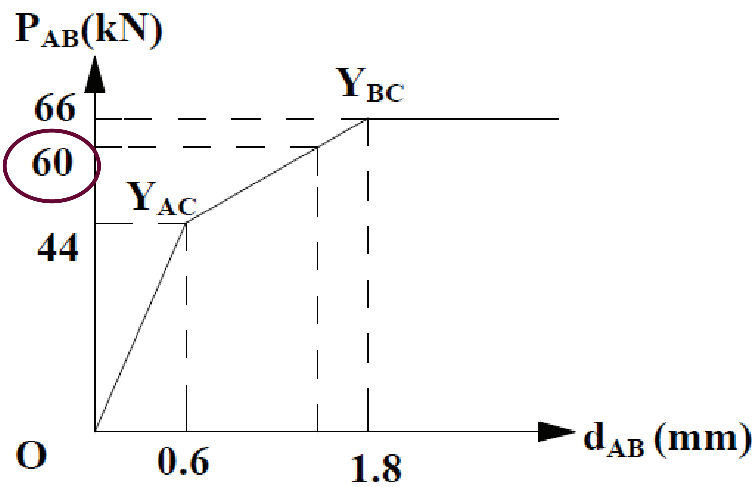
Example 02



- at a load of P= 60 kips, the portion AC has reached the plastic range while the portion BC is still in the elastic range



Rod (AB)

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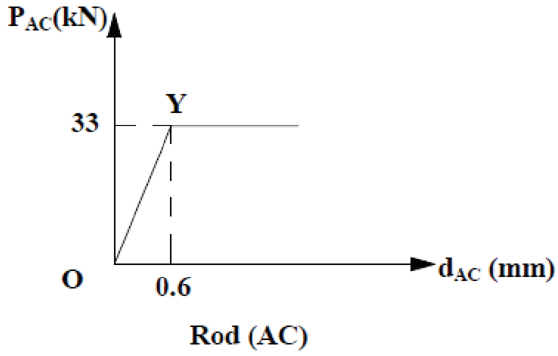
Example 02



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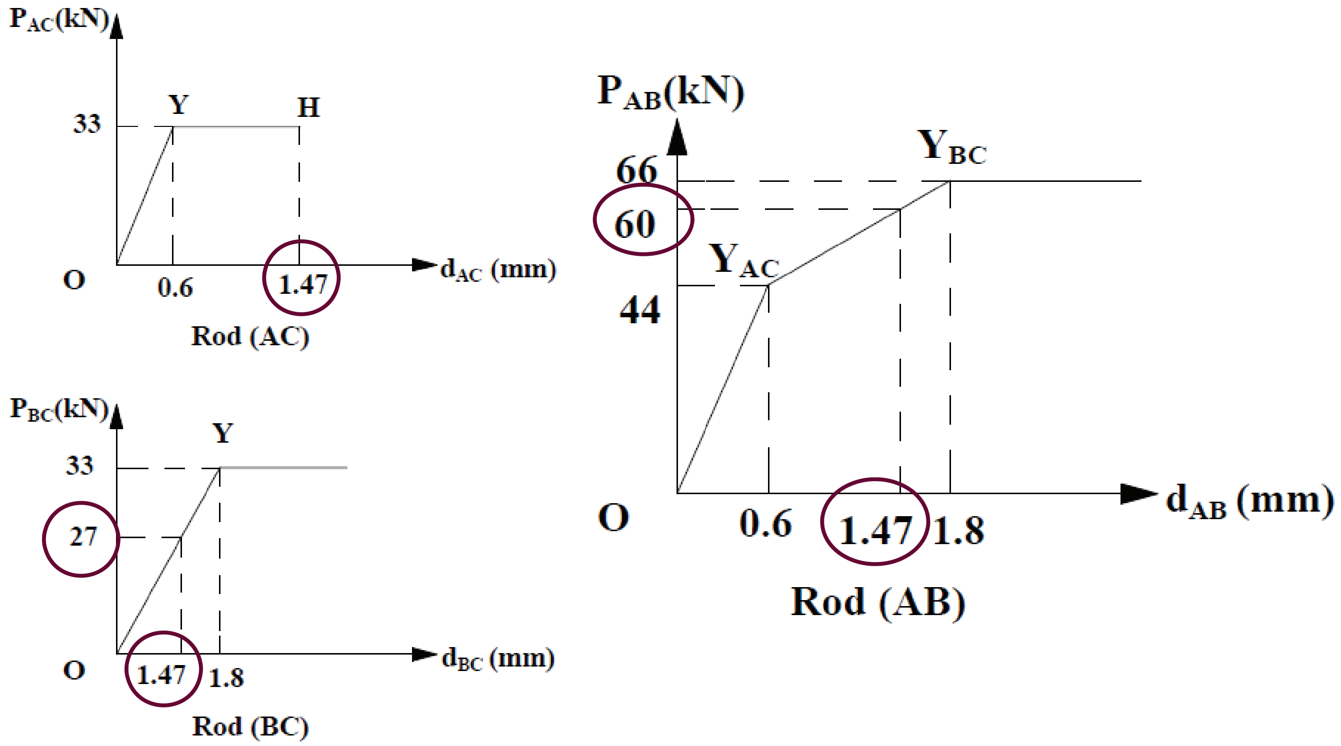
Example 02



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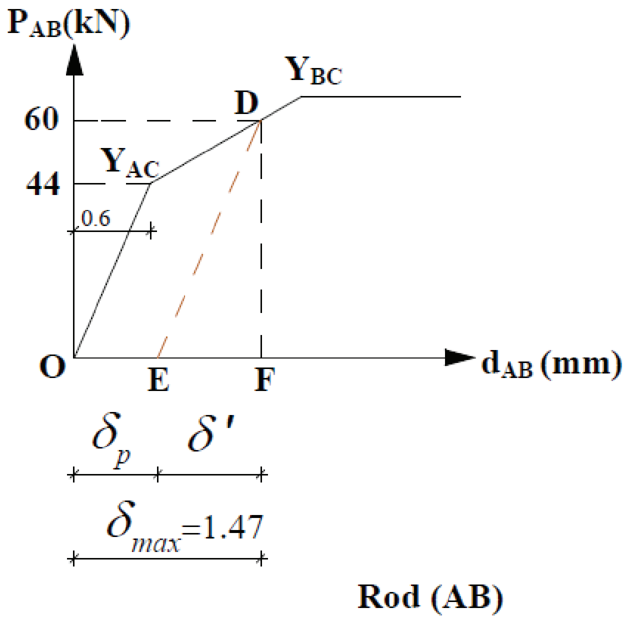
Example 02

- The unloads along a line parallel to O- Y_{AC}



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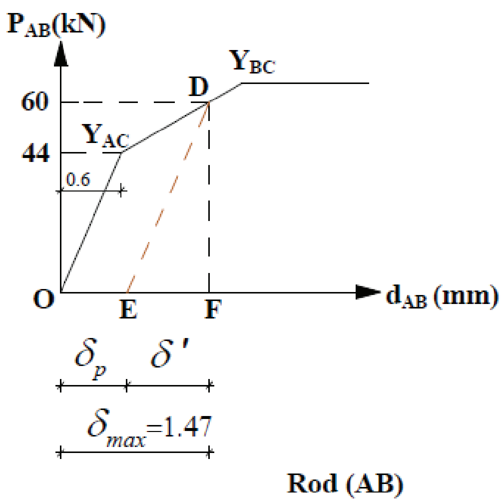
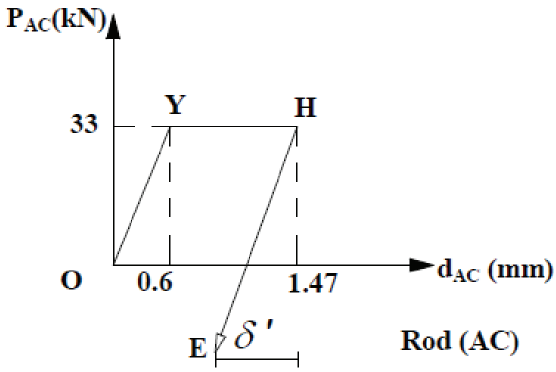
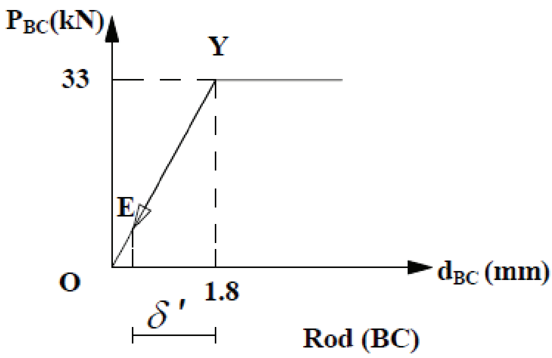
Example 02

- calculate the residual stresses in the bar.







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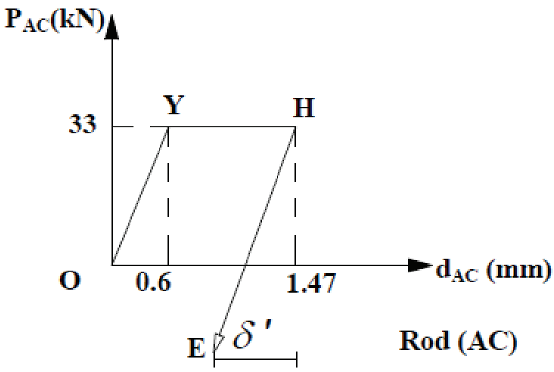
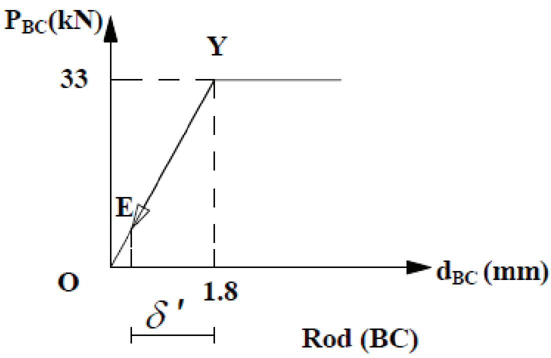
Example 02

calculate the reverse stresses in the bar caused by unloading and add them to the maximum stresses.





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Example 03

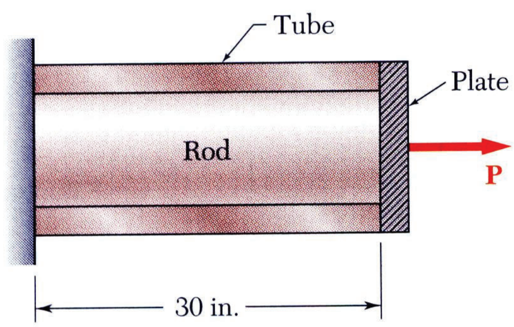
A cylindrical rod is placed inside a tube of the same length. The ends of the rod and tube are attached to a rigid support on one side and a rigid plate on the other. The load on the rod-tube assembly is increased from zero to 5.7 kips and decreased back to zero.

a) draw a load-deflection diagram for the rod-tube assembly

b) determine the maximum elongation

c) determine the permanent elongation

d) calculate the residual stresses in the rod and tube.



$A_r = 0.075\,\text{in.}^2$ $\quad A_t = 0.100\,\text{in.}^2$

$E_r = 30\times10^6\,\text{psi}$ $\quad E_t = 15\times10^6\,\text{psi}$

$\sigma_{Y,r} = 36\,\text{ksi}$ $\quad \sigma_{Y,t} = 45\,\text{ksi}$

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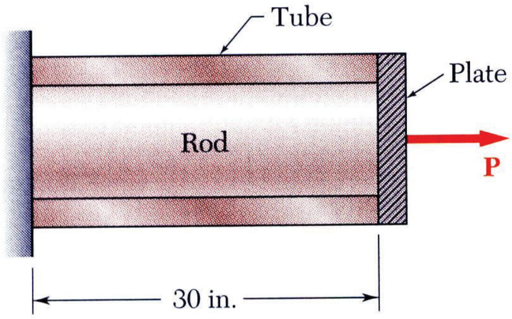
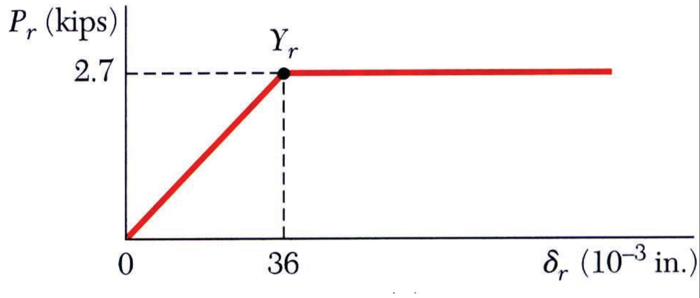
Example 03

a) draw a load-deflection diagram for the rod-tube assembly





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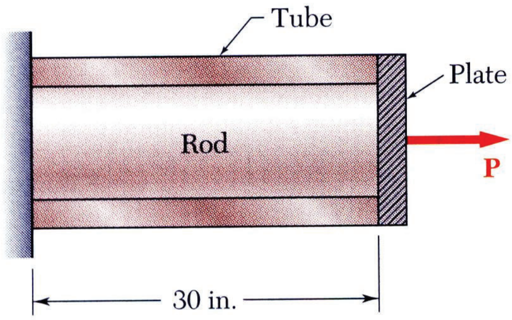
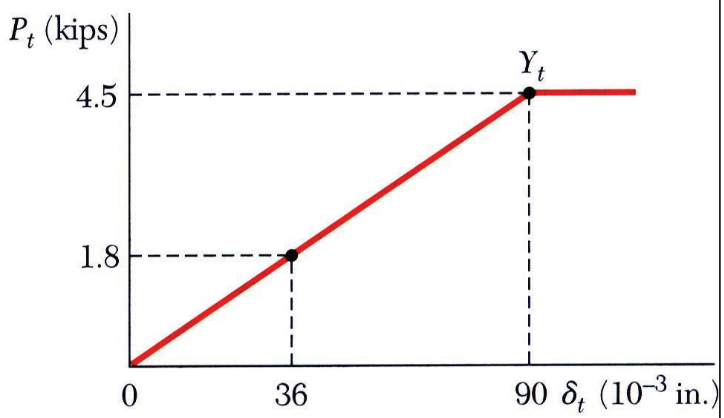
Example 03

a) draw a load-deflection diagram for the rod-tube assembly





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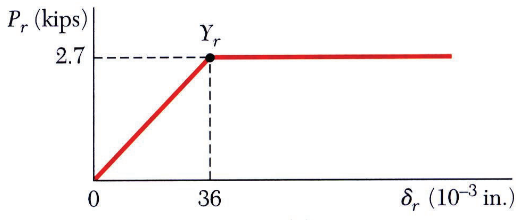
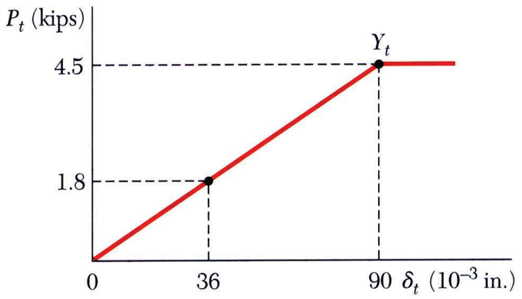
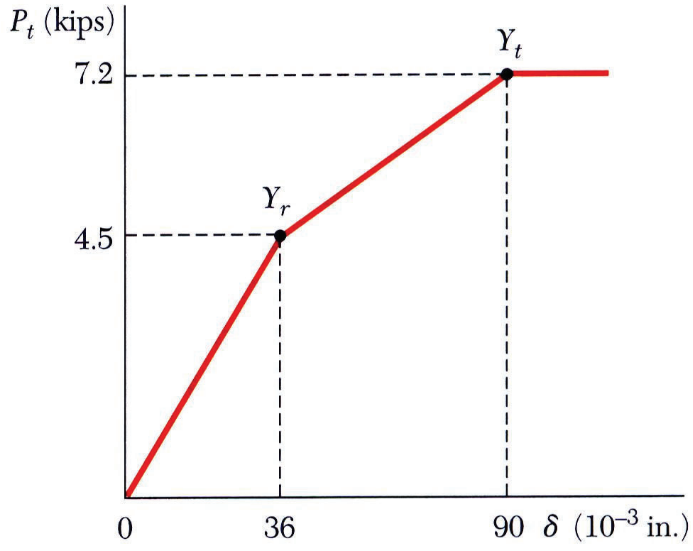
Example 03

a) draw a load-deflection diagram for the rod-tube assembly

$$P = P_r + P_t$$
$$\delta = \delta_r = \delta_t$$







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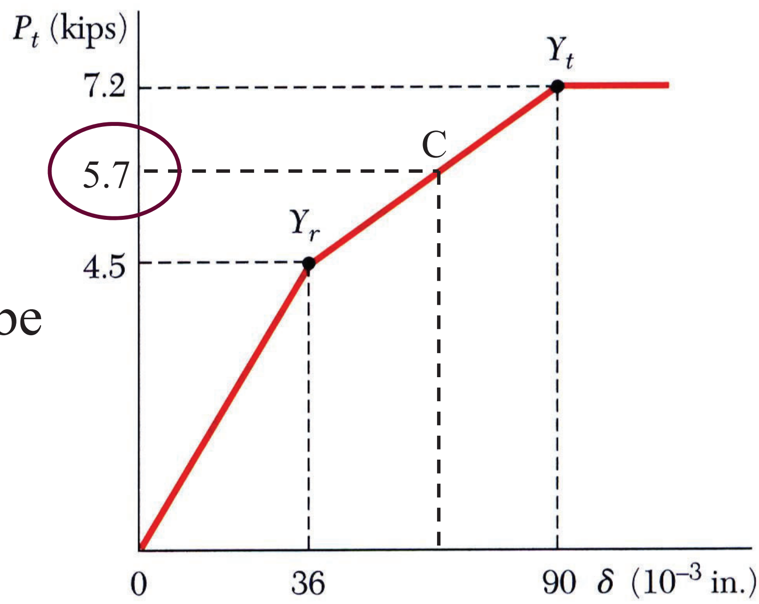
Example 03

- at a load of P= 5.7 kips, the rod has reached the plastic range while the tube is still in the elastic range



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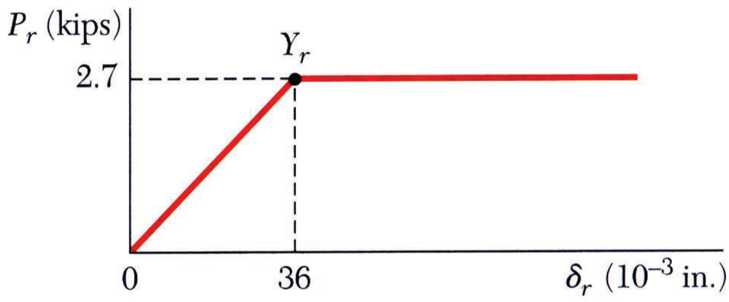
Example 03



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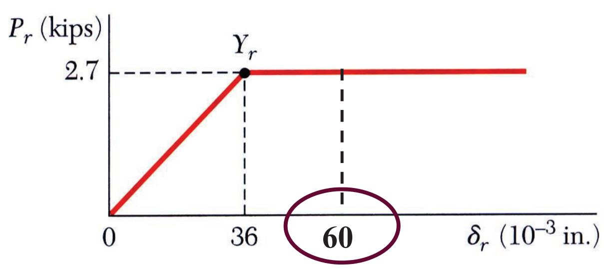
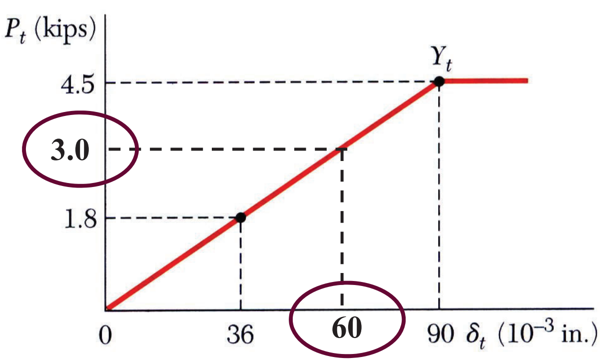
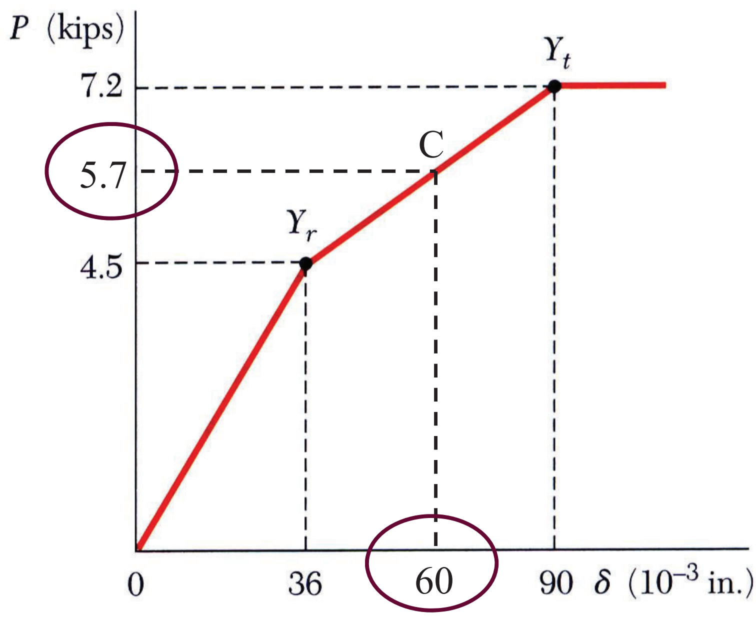
Example 03







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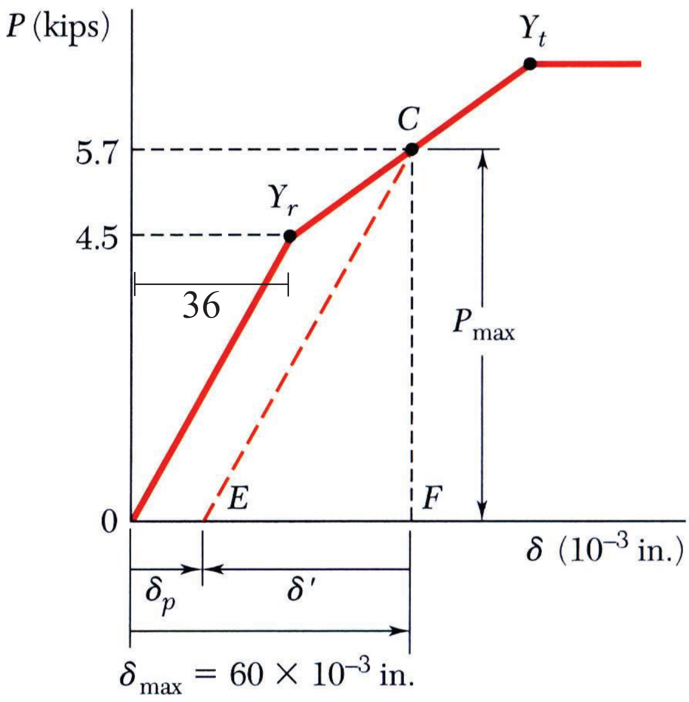
Example 03

- the rod-tube assembly unloads along a line parallel to 0- Y_r



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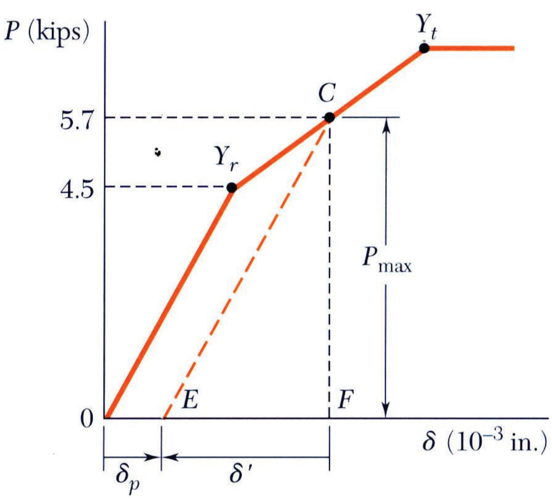
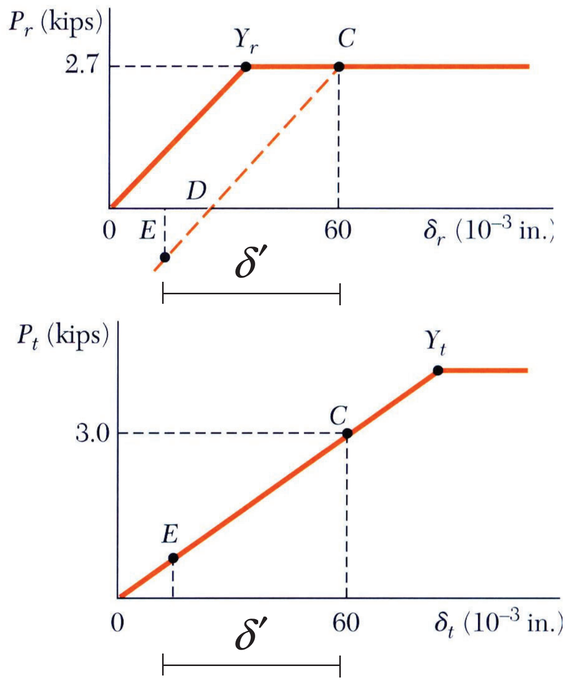
Example 03

- calculate the residual stresses in the rod and tube.





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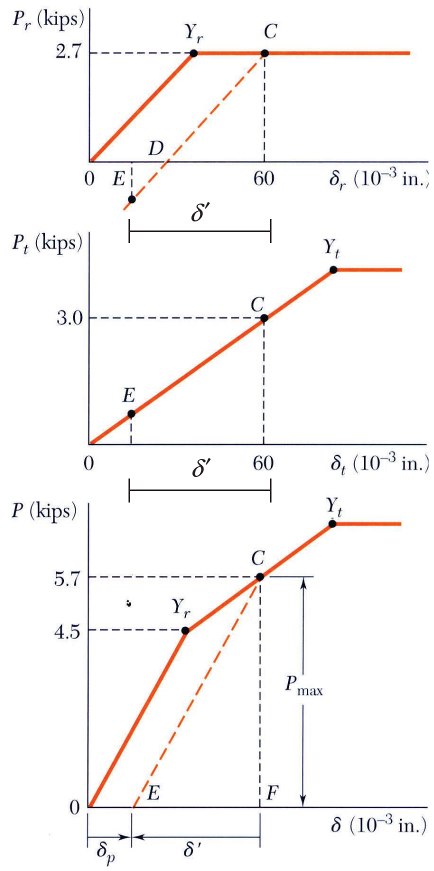
Example 03

calculate the reverse stresses in the rod and tube caused by unloading and add them to the maximum stresses.



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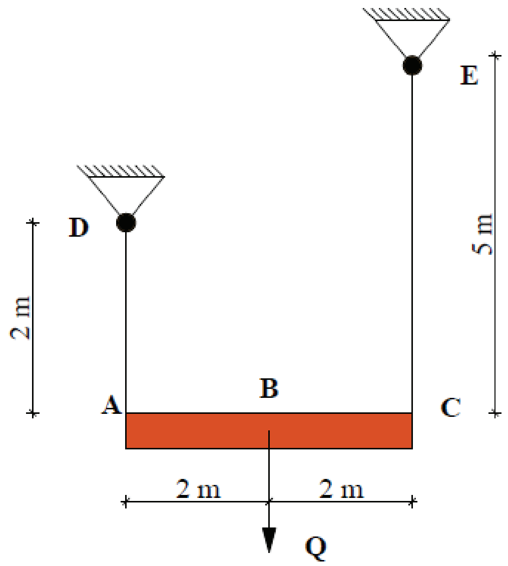
Example 04

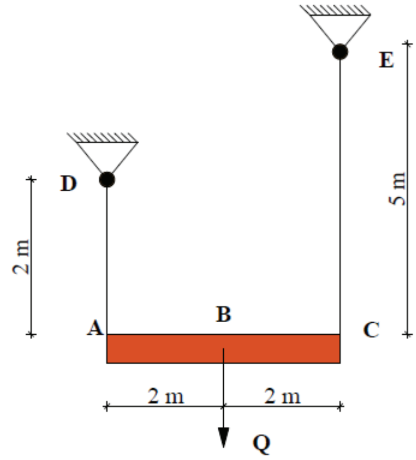
The rigid beam ABC is suspended from two steel rods as shown and is initially horizontal. The midpoint B of the beam is deflected 10 mm downward by the slow application of the force Q, after which the force is slowly removed. Knowing that the steel used for the rods is Elastoplastic with E = 200 Gpa and σy=300 MPa, determine

(a) the required maximum value of Q and the corresponding position of the. beam,
(b) the final position of the beam.



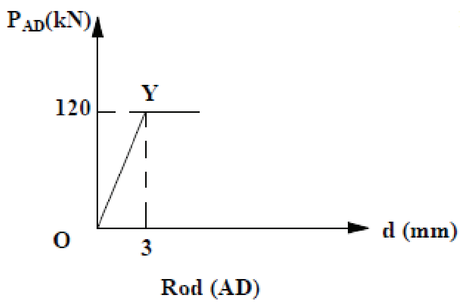
$$A_{AD} = 400\,mm^2$$
$$A_{CE} = 500\,mm^2$$

Example 04

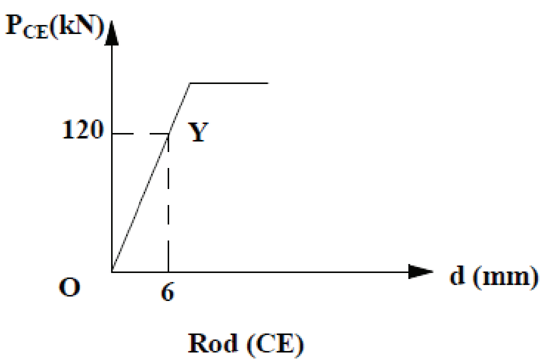


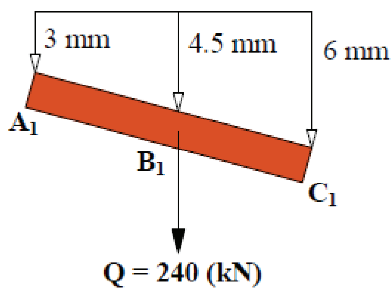
Since Q is applied at the midpoint of the beam, we have

The maximum value of Q and the maximum elastic deflection of point A occur when $\sigma_{AD} = \sigma_Y$ in rod AD.



Example 04





Since we must have $\delta_{B_1} = 10mm$ **, we conclude that plastic deformation will occur.**

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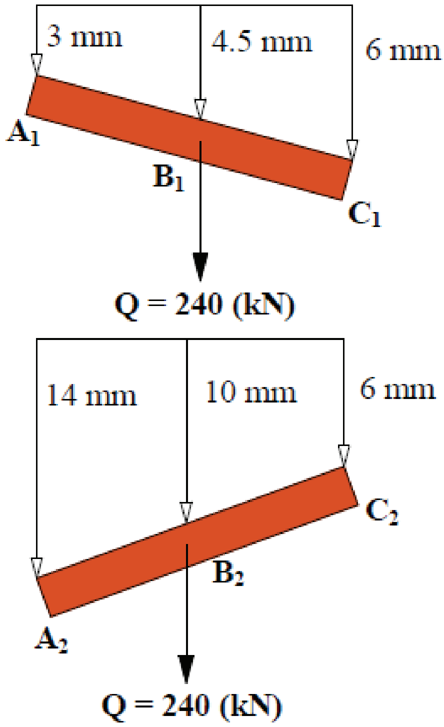
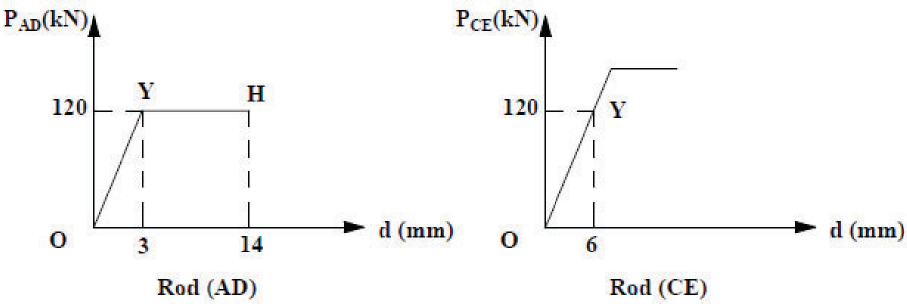
Example 04

Plastic Deformation. For $Q = 240$ kN, plastic deformation occurs in rod *AD*, where $\sigma_{AD} = \sigma_Y$, Since the stress in rod *CE* is within the elastic range, δ_C **remains equal to 6 mm**. The deflection δ_A for which $\delta_B = 10mm$ is obtained by writing





Load-Deflection diagrams

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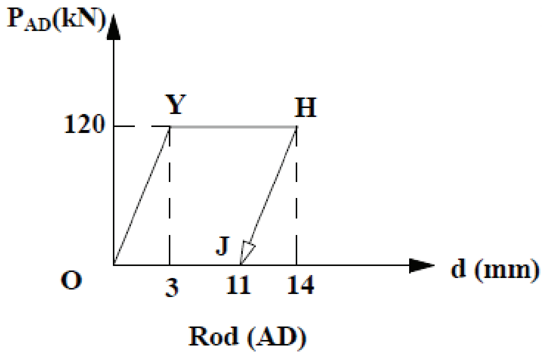
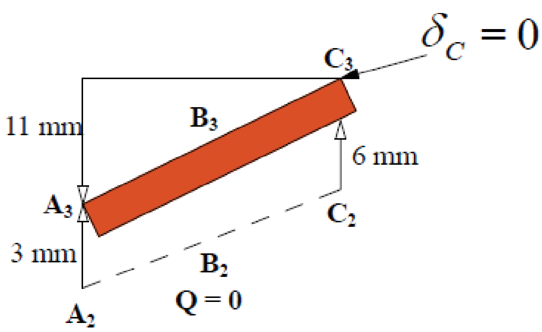
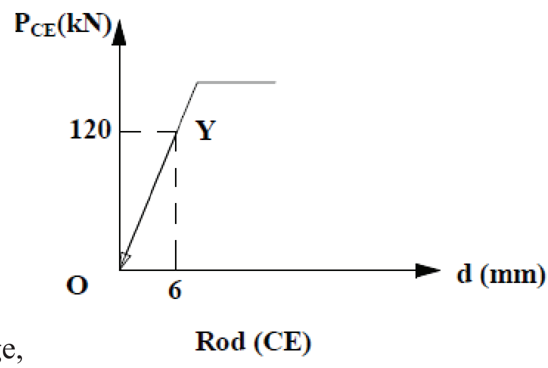
Example 04

Unloading. As force Q is slowly removed, the force *PAD* decreases along line *HI* parallel to the initial portion of the load-deflection diagram of rod *AD*. The final deflection of point A is







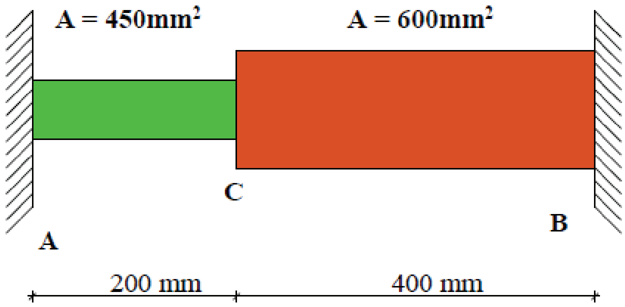
Load-Deflection diagrams

Since the stress in rod CE remained within the elastic range, we note that the final deflection of point C is zero.

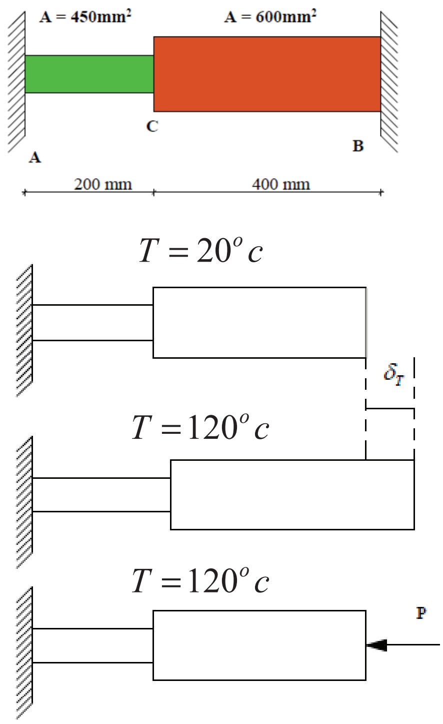
Example 05

The steel rod ABC is attached to rigid supports and is unstressed at a temperature of $20^o c$. The steel is assumed Elastoplastic, with $\sigma_Y = 250$ MPa and $E = 200$ GPa. The temperature of both portions of the rod is then raised to $120^o c$. Knowing that $\alpha = 11.7 \times 10^{-6} / ^oC$, **determine *(a)* the stress in portion *AC*, *(b)* the deflection of point C.**

assuming that the temperature of the rod is raised to 120°C and then returned to 20°C, **determine *(C)* the stress in portion *AC*, *(D)* the deflection of point C.**

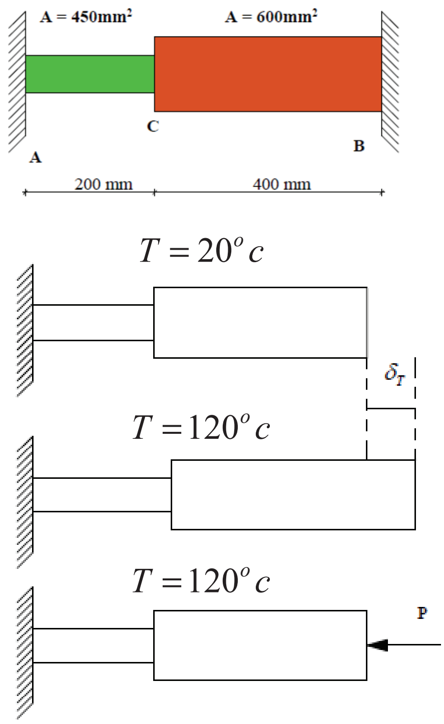


Example 05



Introduction to Plasticity

Example 05



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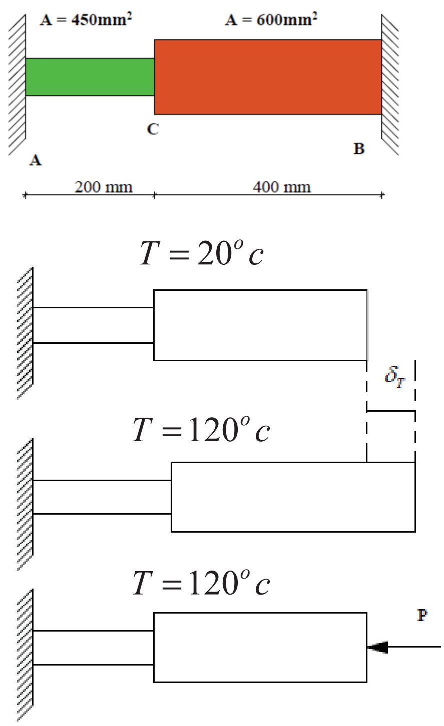
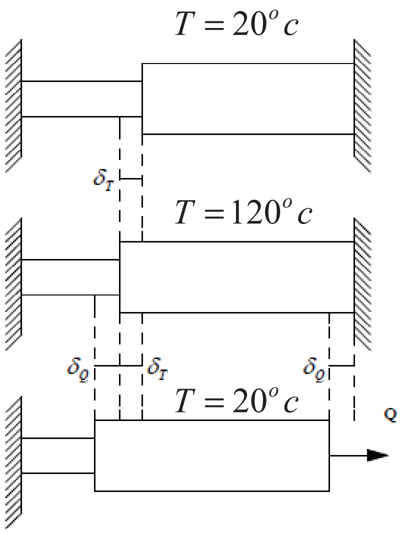
Example 05



Introduction to Plasticity

Example 05

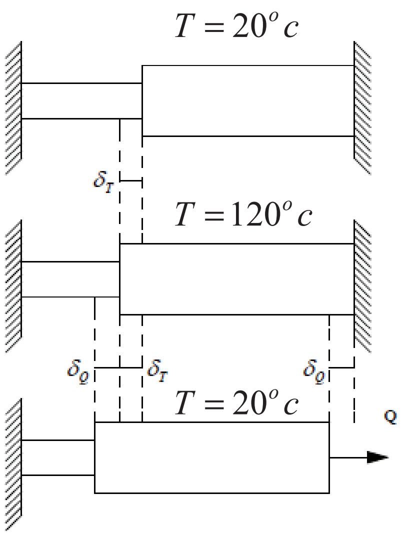
The temperature of the rod returned to 20°C



Introduction to Plasticity

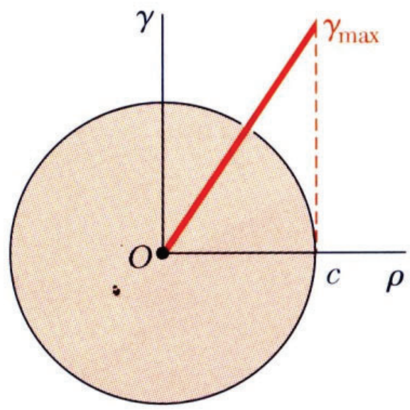
Example 05

The temperature of the rod returned to 20°C



Introduction to Plasticity

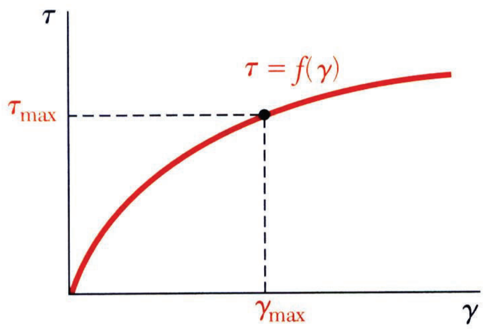
❑ Plastic Deformations due to torsion



Shearing strain varies linearly regardless of material properties.

$$\gamma = \frac{\rho}{c}\gamma_{max}$$

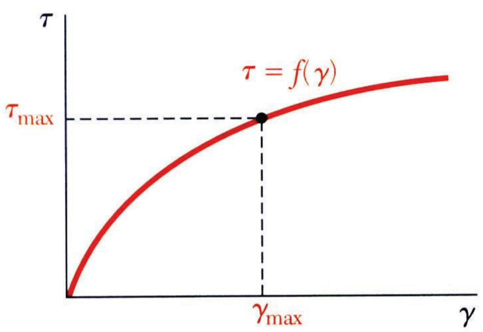
- If the yield strength is exceeded or the material has a nonlinear shearing-stress-strain curve, this expression does not hold.



Introduction to Plasticity

❑ Plastic Deformations due to torsion

•Application of shearing-stress-strain curve allows determination of stress distribution.



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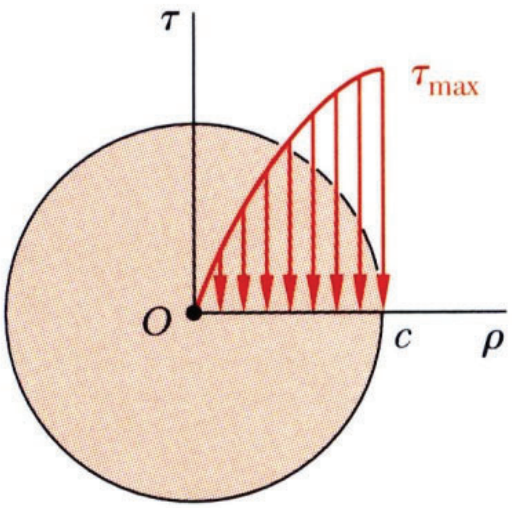


$$\gamma = \frac{\rho}{c}\gamma_{max}$$

Introduction to Plasticity

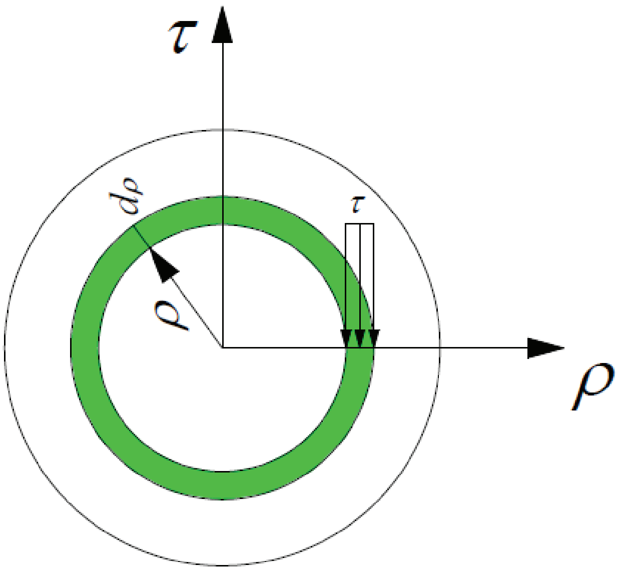
❑ Plastic Deformations due to torsion

- The integral of the moments from the internal stress distribution is equal to the torque on the shaft at the section,

$$T = \int_0^c \rho \tau dA \quad \Rightarrow$$

$$T = \int_0^c \rho \tau (2\pi \rho \, d\rho)$$

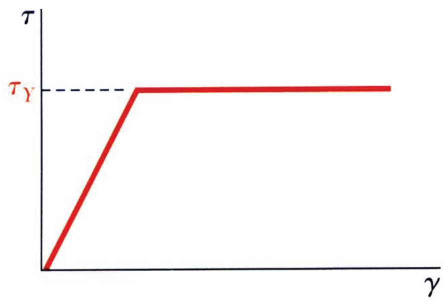
$$\Rightarrow \boxed{T = 2\pi \int_0^c \rho^2 \tau \, d\rho}$$

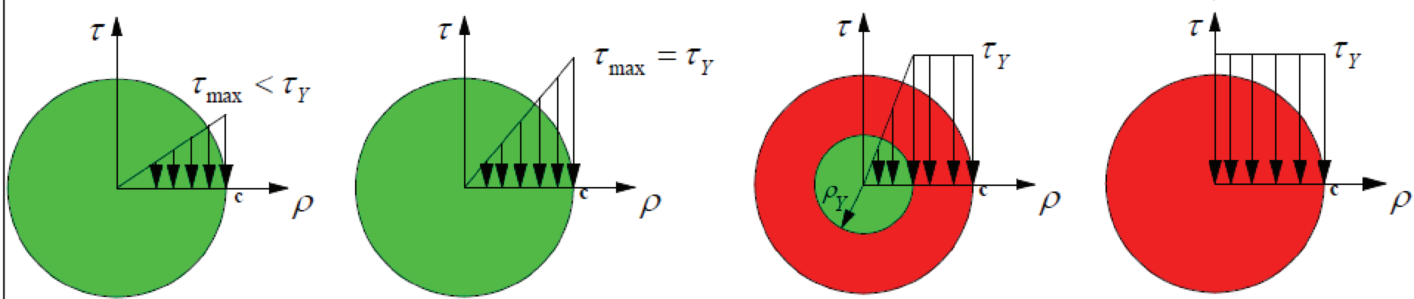


Introduction to Plasticity

❑ CIRCULAR SHAFTS MADE OF AN ELASTOPlASTIC MATERIAL

- Considering the idealized case of a solid circular shaft made of an Elastoplastic material.





$$\tau_{max} = \frac{Tc}{J}$$

$$\tau_Y = \frac{T_Y c}{J}$$

$$T_Y = \frac{J}{c}\tau_Y = \frac{1}{2}\pi c^3 \tau_Y$$

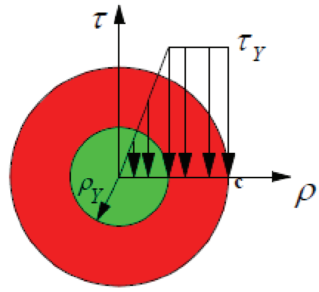
$$\tau = \frac{\tau_Y}{\rho_Y}\rho$$

$$T_p = \frac{4}{3}T_Y$$

Plastic torque **of the shaft**

Introduction to Plasticity

❑ Plastic Deformations due to torsion



$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho$$

$$\tau = \frac{\tau_Y}{\rho_Y} \rho$$

$$\Rightarrow \quad T = 2\pi \int_0^{\rho_Y} \rho^2 \left(\frac{\tau_Y}{\rho_Y} \rho \right) d\rho + 2\pi \int_{\rho_Y}^{c} \rho^2 \tau_Y \, d\rho$$

$$\Rightarrow \quad T = \frac{1}{2} \pi {\rho_Y}^3 \tau_Y + \frac{2}{3} \pi c^3 \tau_Y - \frac{2}{3} \pi {\rho_Y}^3 \tau_Y$$

$$\Rightarrow \quad T = \frac{2}{3} \pi c^3 \tau_Y \left(1 - \frac{1}{4} \frac{{\rho_Y}^3}{c^3} \right)$$

$$\tau = \frac{\tau_Y}{\rho_Y} \rho$$

$$\Rightarrow \quad T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{{\rho_Y}^3}{c^3} \right)$$

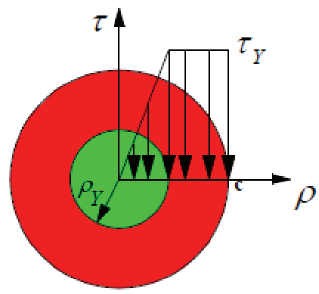
If $\rho_Y \to 0$

$$\Rightarrow \quad T_p = \frac{4}{3} T_Y$$

Plastic torque **of the shaft**

Introduction to Plasticity

❑ Plastic Deformations due to torsion



Stress distribution is linear then hooks law is justified

$$\rho_Y = \frac{L \gamma_Y}{\phi}$$

$$\tau = \frac{\tau_Y}{\rho_Y} \rho$$

The angle of twist at onset of yield

$$\rho_Y = c \qquad \phi = \phi_Y$$

$$\Rightarrow \quad c = \frac{L \gamma_Y}{\phi_Y} \quad \Rightarrow \quad \frac{\rho_Y}{c} = \frac{\phi_Y}{\phi}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{{\rho_Y}^3}{c^3} \right)$$

$$\Rightarrow \quad T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{{\phi_Y}^3}{\phi^3} \right)$$

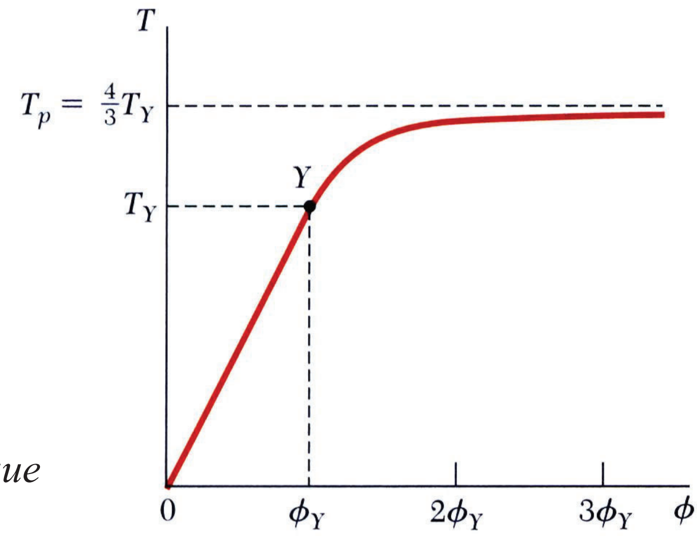
Introduction to Plasticity

- **Plastic Deformations due to torsion**

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\phi_Y^{\ 3}}{\phi^3} \right)$$

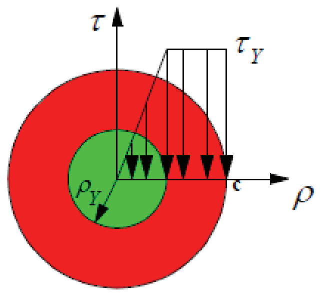
- As $\phi \to \infty$, the torque approaches

a limiting value, $T_P = \frac{4}{3} T_Y = plastic\ torque$

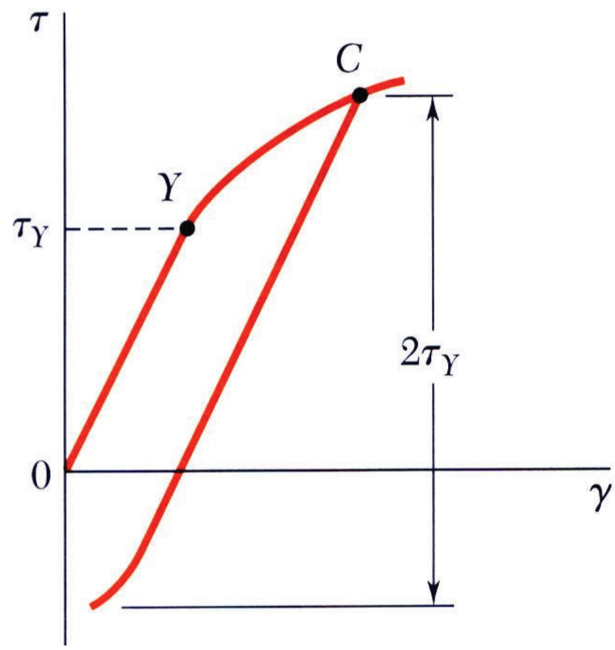


Introduction to Plasticity

- **Plastic Deformations due to torsion**



- Plastic region develops in a shaft when subjected to a large enough torque
- When the torque is removed, the reduction of stress and strain at each point takes place along a straight line to a generally non-zero residual stress



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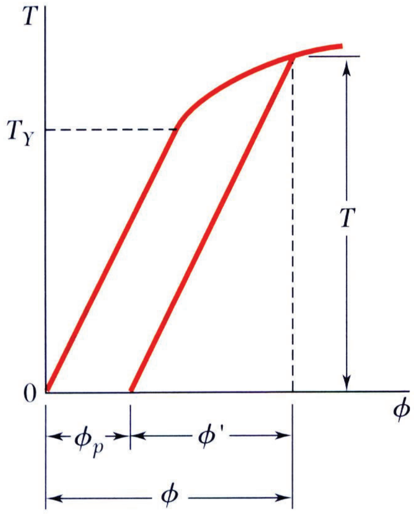
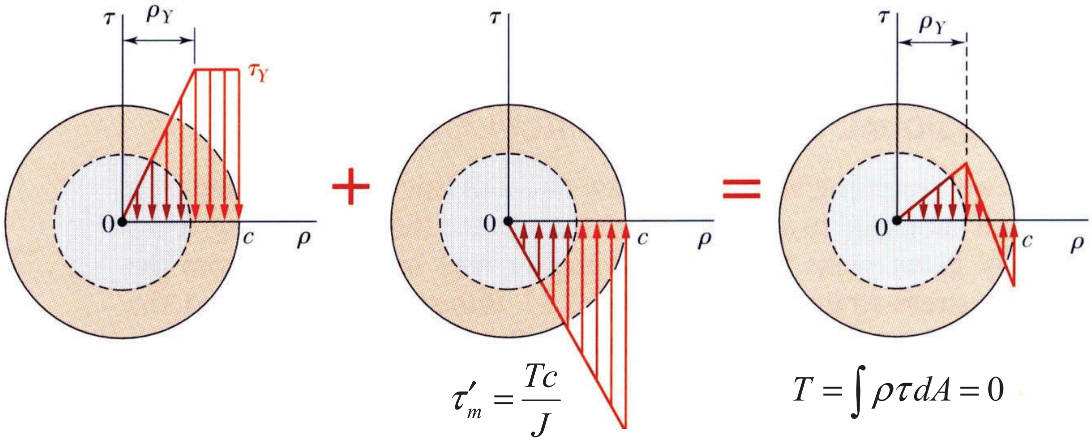
❑ Plastic Deformations due to torsion



- On a T-ϕ curve, the shaft unloads along a straight line to an angle greater than zero

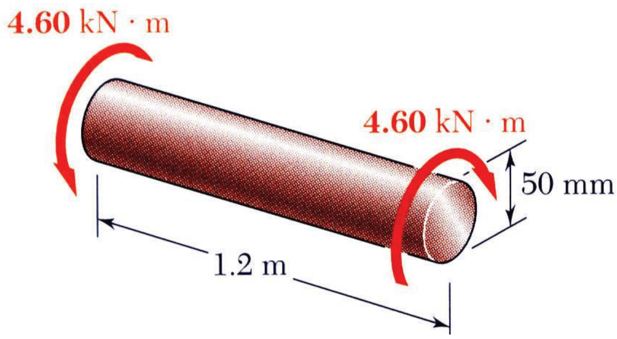
- Residual stresses found from principle of superposition



Introduction to Plasticity

Example 06

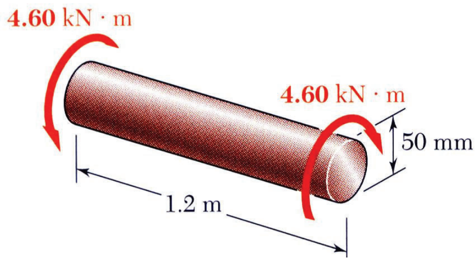
A solid circular shaft is subjected to a torque $T = 4.6\,\text{kN}\cdot\text{m}$ at each end. Assuming that the shaft is made of an elastoplastic material with $\tau_Y = 150\,\text{MPa}$ and $G = 77\,\text{GPa}$ determine (a) the radius of the elastic core, (b) the angle of twist of the shaft.



When the torque is removed, determine (c) the permanent twist, (d) the distribution of residual stresses.

Introduction to Plasticity

Example 06



The radius of the elastic core

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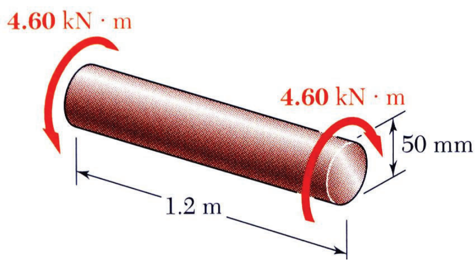
Example 06

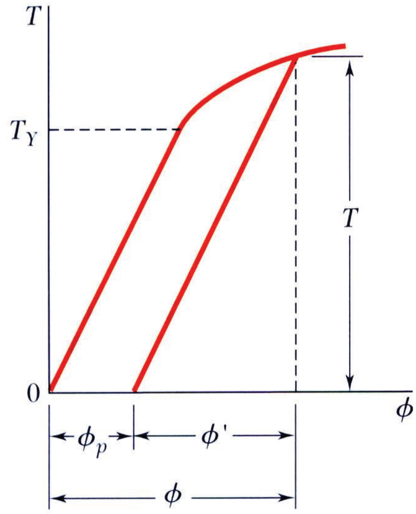


- The angle of twist

Introduction to Plasticity

Example 06

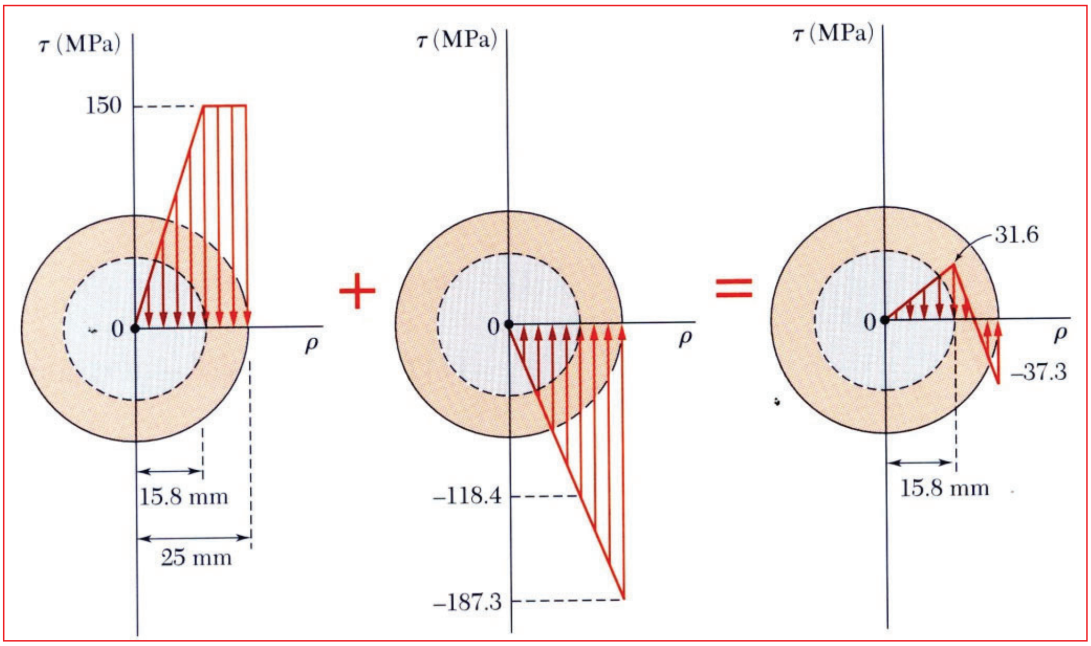
- The permanent twist is the difference between the angles of twist and untwist



Introduction to Plasticity

Example 06

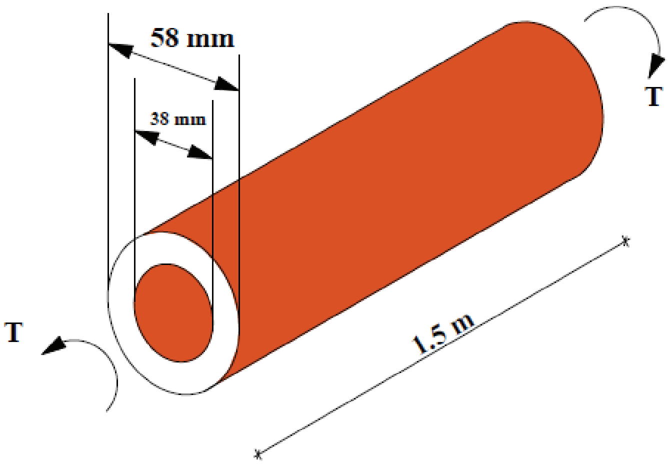
- Find the residual stress distribution by a superposition of the stress due to twisting and untwisting the shaft



Introduction to Plasticity

Example 07

Shaft *AB* is made of a mild steel which is assumed to be elastoplastic with G = 77 GPa and τ_Y = 145 MPa. A torque T is applied and gradually increased in magnitude.
Determine the magnitude of T and the corresponding angle of twist *(a)* when yield first occurs, *(b)* when the deformation has become fully plastic. *(C)* the residual stresses and the permanent angle of twist after the torque Tp = 5.32 kN. m has been removed.



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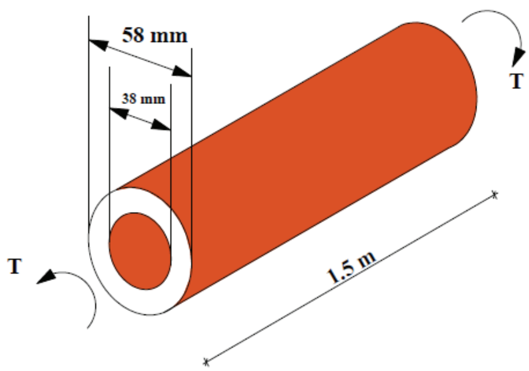
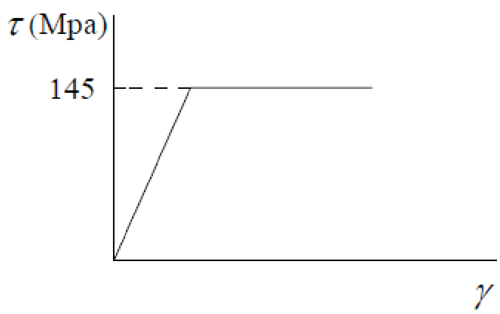
Example 07



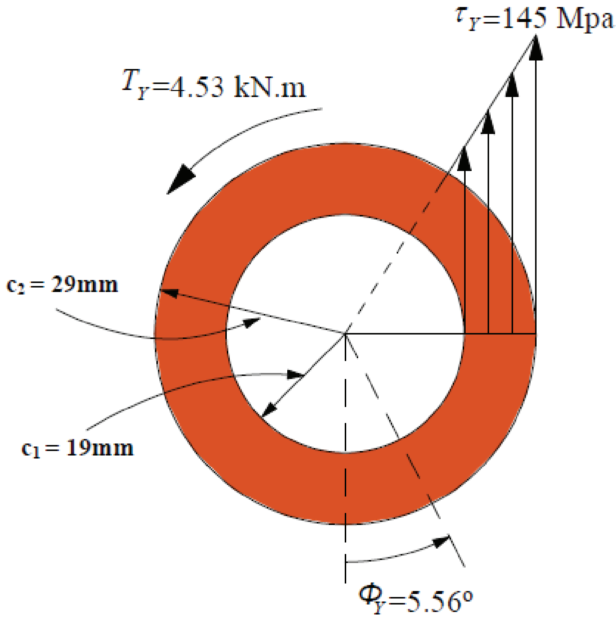
The geometric properties of the cross section are



Introduction to Plasticity

Example 07

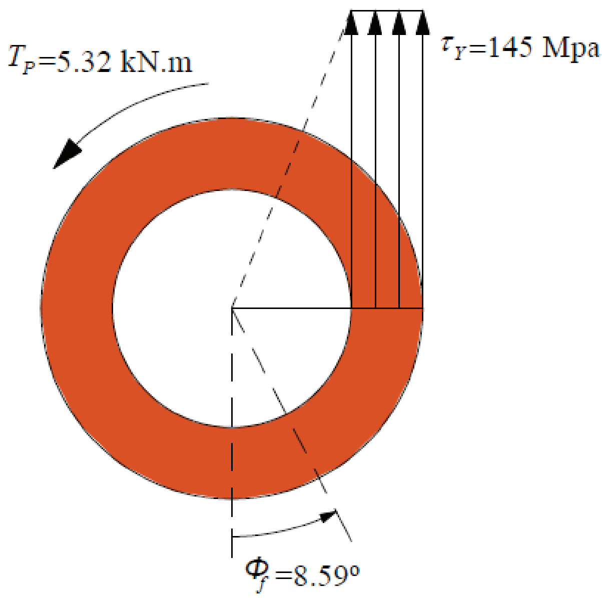
Onset of Yield



Introduction to Plasticity

Example 07

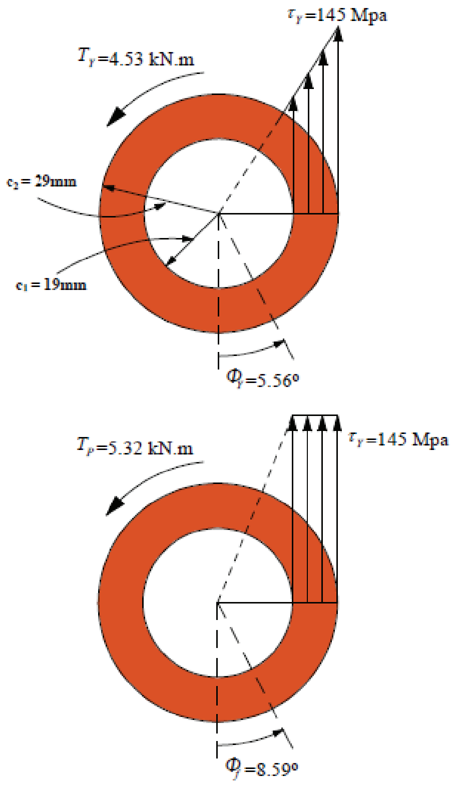
Fully Plastic Deformation



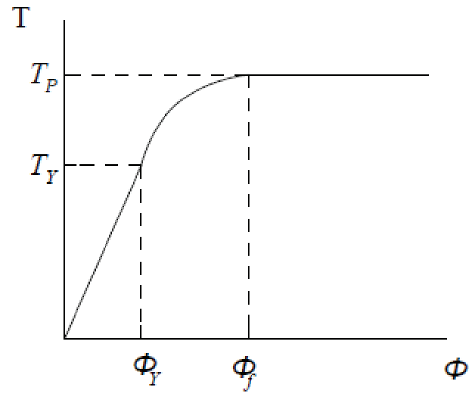
When yield first occurs on the inner surface, the deformation is fully plastic

Introduction to Plasticity

Example 07



For larger angles of twist, the torque remains constant; the T-ϕ diagram of the shaft is as shown.



Introduction to Plasticity

Example 07

Elastic Unloading. We unload the shaft by applying a 5.32 kN. m torque in the sense shown in Fig. 2. During this unloading, ***the behavior of the material is linear***.

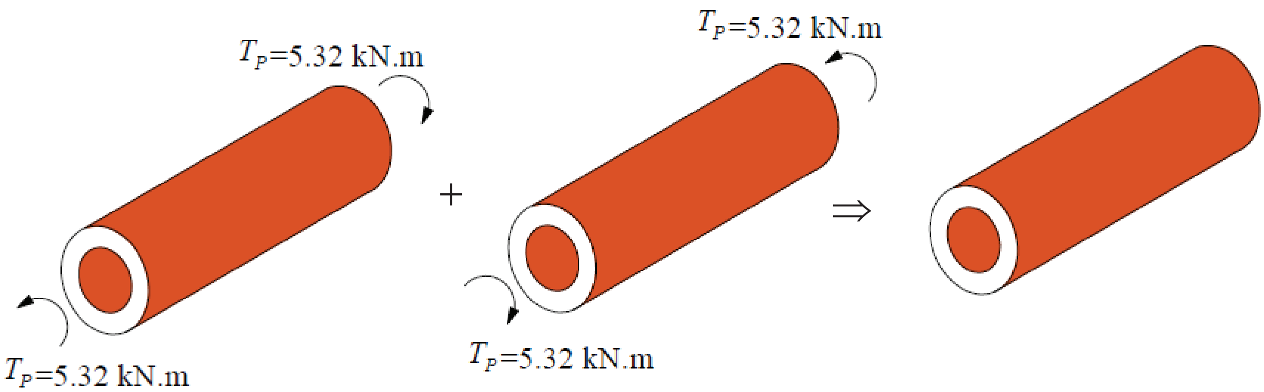


Fig. 1 Fig. 2 Fig. 3

Introduction to Plasticity

Example 07

Elastic Unloading. We unload the shaft by applying a 5.32 kN. m torque in the sense shown in Fig. 2. During this unloading, ***the behavior of the material is linear***.

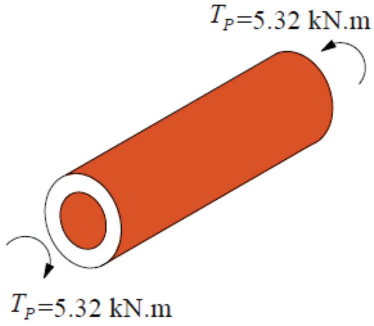


Fig. 2

Introduction to Plasticity

Example 07

Residual Stresses and Permanent Twist.
The results of the loading (Fig. 1) and the unloading (Fig. 2) are superposed (Fig. 3) to obtain the residual stresses and the permanent angle of twist ϕp

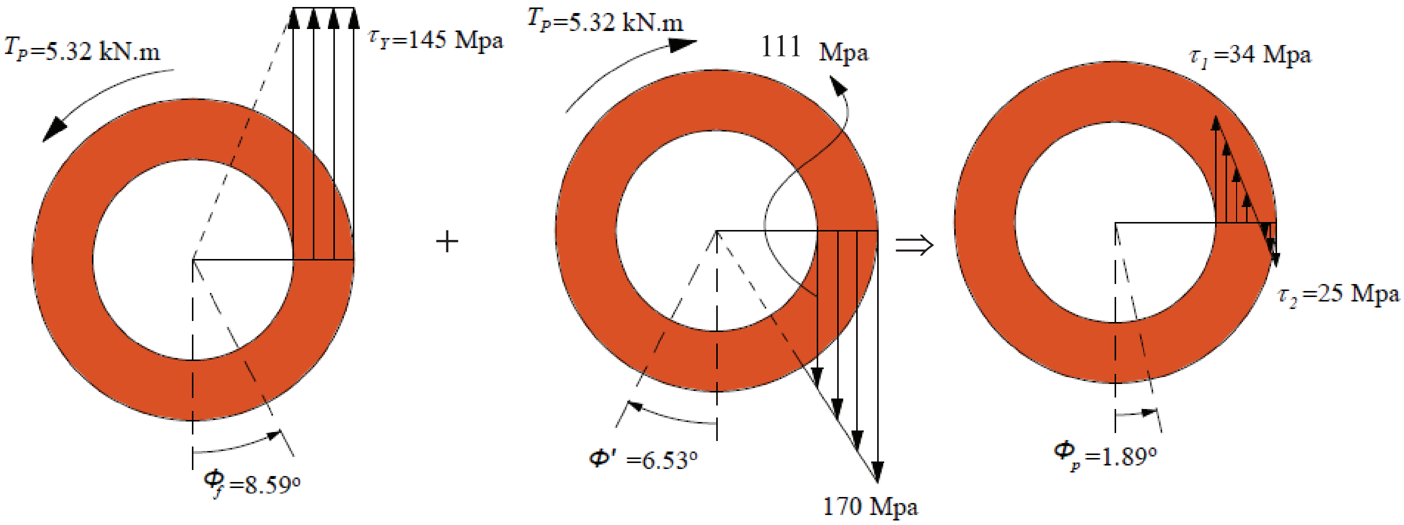


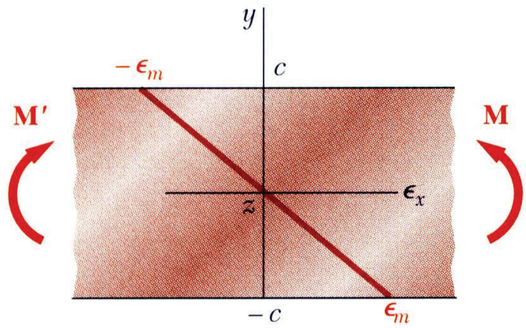
Fig. 1 Fig. 2 Fig. 3

❑ Plastic Deformations due to pure bending

- For any member subjected to pure bending

strain varies linearly across the section

$$\varepsilon_x = -\frac{y}{c}\varepsilon_m$$

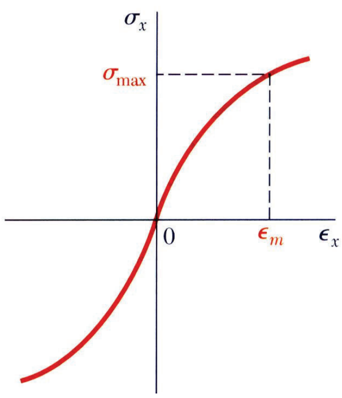


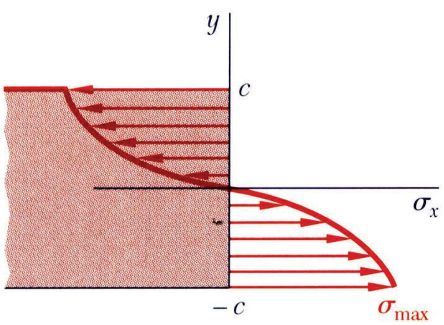
- If the member is made of a ***linearly elastic material,*** the neutral axis passes through the section centroid and

$$\sigma_x = -\frac{My}{I}$$

Introduction to Plasticity

❑ Plastic Deformations due to pure bending





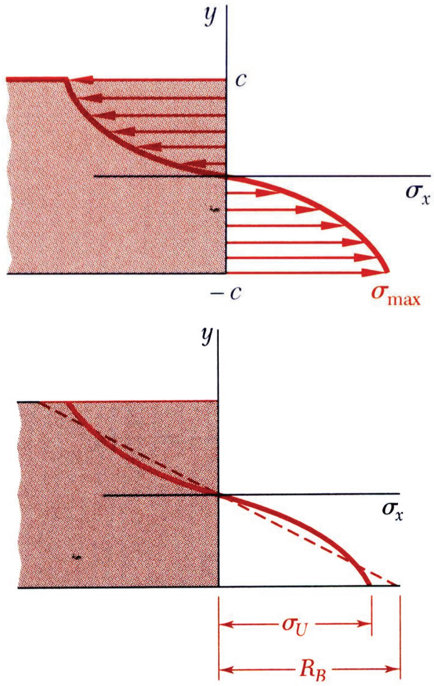
- For a material with a nonlinear stress-strain curve, the neutral axis location is found by satisfying

$$F_x = \int \sigma_x \, dA = 0 \qquad M = \int -y\sigma_x \, dA$$

- For a member with ***vertical and horizontal planes*** of symmetry and a material with the ***same tensile and compressive stress-strain relationship***, the neutral axis is located at ***the section centroid*** and the stress-strain relationship may be used to map the strain distribution from the stress distribution.

Introduction to Plasticity

❑ Plastic Deformations due to pure bending



- When the maximum stress is equal to the ultimate strength of the material, failure occurs and the corresponding moment M_U is referred to as the ***ultimate bending moment.***
- The ***modulus of rupture in bending,*** R_B, is found from an experimentally determined value of M_U and a fictitious linear stress distribution.

$$R_B = \frac{M_U c}{I}$$

- R_B may be used to determine M_U of any member made of the same material and with the same cross sectional shape but different dimensions.

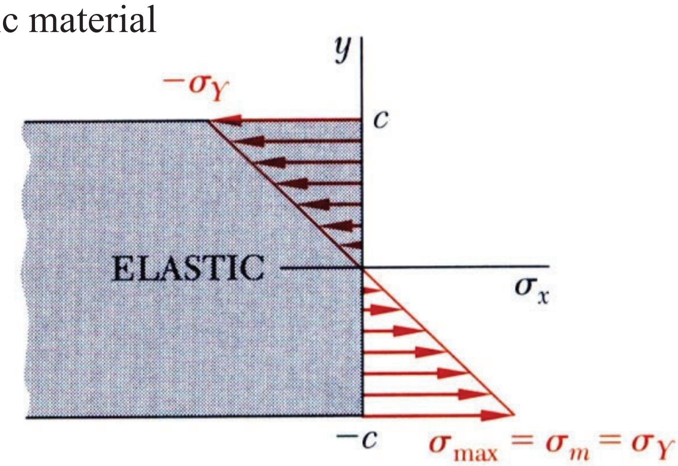
Introduction to Plasticity

❑ Plastic Deformations due to pure bending

- Rectangular beam made of an Elastoplastic material

$$\sigma_x \le \sigma_Y$$

$$\sigma_m = \frac{Mc}{I} = \sigma_Y$$



(b) $M = M_Y$

$$M_Y = \frac{I}{c}\sigma_Y = \frac{\frac{1}{12}b(2c)^3}{c}\sigma_Y$$

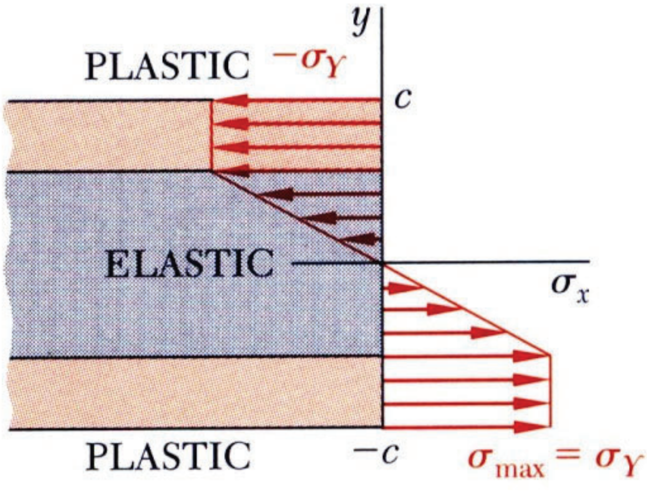
$$\Rightarrow M_Y = \frac{2}{3}bc^3\sigma_Y$$

Maximum Elastic Moment

Introduction to Plasticity

❑ Plastic Deformations due to pure bending

- If the moment is increased beyond the maximum elastic moment, ***plastic zones develop around an elastic core***.



(c) $M > M_Y$

$$y_Y = \text{elastic core half - thickness}$$

$$0 \le y \le y_Y \quad \Rightarrow \quad \sigma_x = -\frac{y}{y_Y}\sigma_Y$$

$$y_Y \le y \le c \quad \Rightarrow \quad \sigma_x = -\sigma_Y$$

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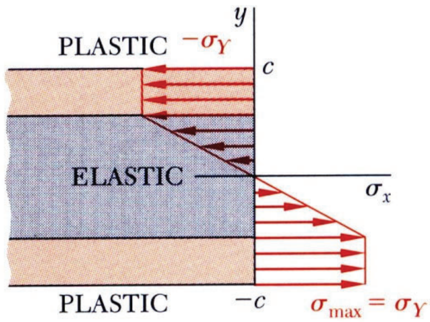
❑ Plastic Deformations due to pure bending



(c) $M > M_Y$

$$M = -\int_{-c}^{c} y\sigma_x dA = -\int_{-c}^{c} y\sigma_x (b\,dy)$$

$$\Rightarrow \quad M = -2b\int_0^c y\sigma_x\,dy$$

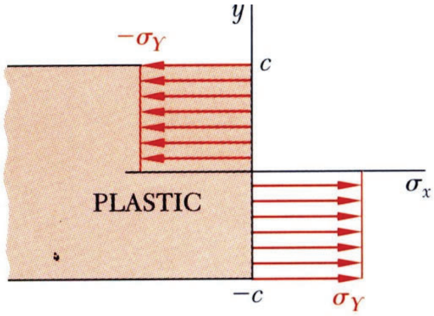
$$M = -2b\int_0^{y_Y} y\left(-\frac{y}{y_Y}\sigma_Y\right)dy - 2b\int_{y_Y}^{c} y(-\sigma_Y)dy$$

$$M = bc^2\sigma_Y\left(1 - \frac{1}{3}\frac{y_Y^{\,2}}{c^2}\right) \quad \Rightarrow \quad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^{\,2}}{c^2}\right)$$

$$y_Y = \varepsilon_Y \rho$$
$$c = \varepsilon_Y \rho_Y$$
$$\Rightarrow \quad \frac{y_Y}{c} = \frac{\rho}{\rho_Y} \quad \Rightarrow \quad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^{\,2}}\right)$$

Introduction to Plasticity

- **Plastic Deformations due to pure bending**



(d) $M = M_p$

- In the limit as the moment is increased further, the elastic core thickness goes to zero, corresponding to ***a fully plastic deformation.***

$$M_p = \frac{3}{2} M_Y = \text{plastic moment}$$

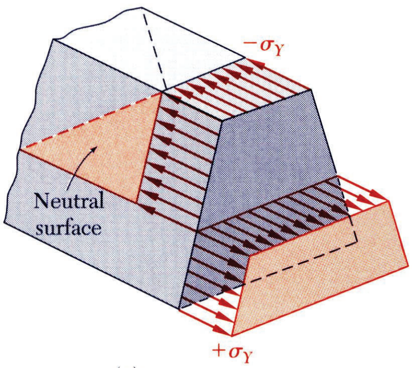
$$Z = \frac{M_p}{\sigma_Y} \qquad S = \frac{M_Y}{\sigma_Y}$$

Z : plastic modulus
"It can also be defined as the first moment of area about the neutral axis when the areas above and below the neutral axis are equal."

$$\frac{M_p}{M_Y} = \frac{Z}{S} = k$$ shape factor (depends only on cross section shape)

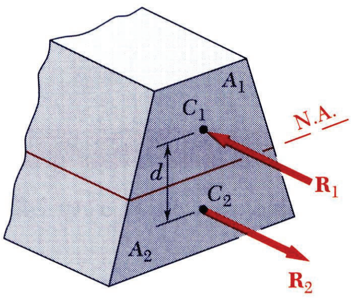
Introduction to Plasticity

- **Plastic Deformations due to pure bending**



(a)

=



(b)

- Fully plastic deformation of a beam with ***only a vertical plane of symmetry.***
- The neutral axis cannot be assumed to pass through the section centroid.
- Resultants R_1 and R_2 of the elementary compressive and tensile forces form a couple.

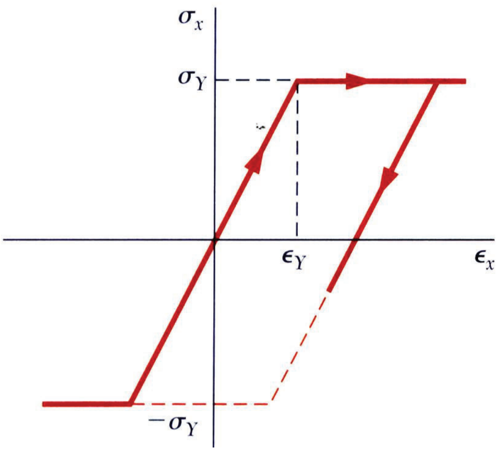
$$R_1 = R_2 \quad \Rightarrow \quad A_1\sigma_Y = A_2\sigma_Y \quad \Rightarrow \quad A_1 = A_2$$

The neutral axis divides the section into equal areas.

- The plastic moment for the member, $$M_p = \left(\frac{1}{2} A\sigma_Y\right) d$$

Introduction to Plasticity

❑ Plastic Deformations due to pure bending



- Plastic zones develop in a member made of an Elastoplastic material if the bending moment is large enough.
- Since the linear relation between normal stress and strain applies at all points during the unloading phase, it may be handled by assuming the member to be fully elastic.
- Residual stresses are obtained by applying the principle of superposition to combine the stresses due to loading with a moment *M* (elastoplastic deformation) and unloading with a moment -*M* (elastic deformation).
  - The final value of stress at a point will not, in general, be zero.

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Example 08

A member of uniform rectangular cross section is subjected to a bending moment M = 36.8 kN-m. The member is made of an Elastoplastic material with a yield strength of 240 MPa and a modulus of elasticity of 200 GPa.

Determine

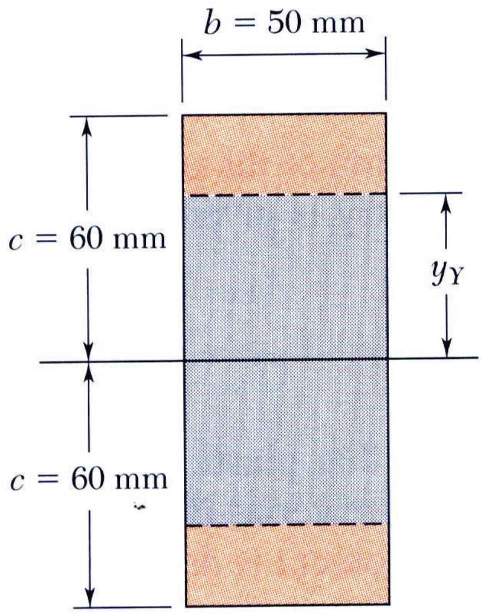
(a) the thickness of the elastic core,

(b) the radius of curvature of the neutral surface.

After the loading has been reduced back to zero, determine

(c) the distribution of residual stresses,

(d) radius of curvature.



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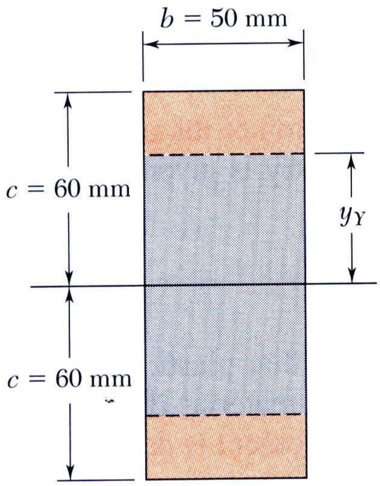
Example 08



- Maximum elastic moment:

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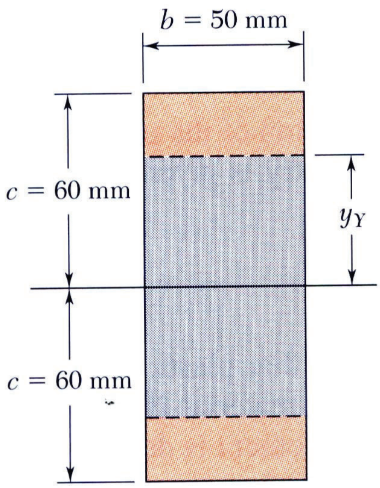
Example 08



- Thickness of elastic core:

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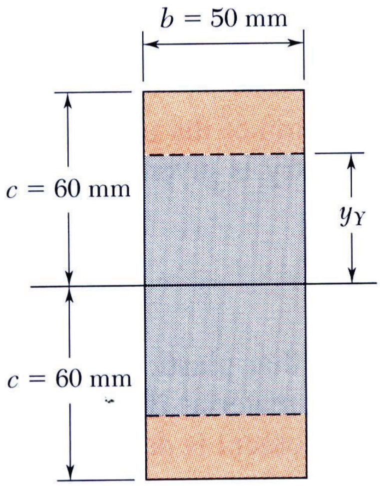
Example 08

- Radius of curvature:



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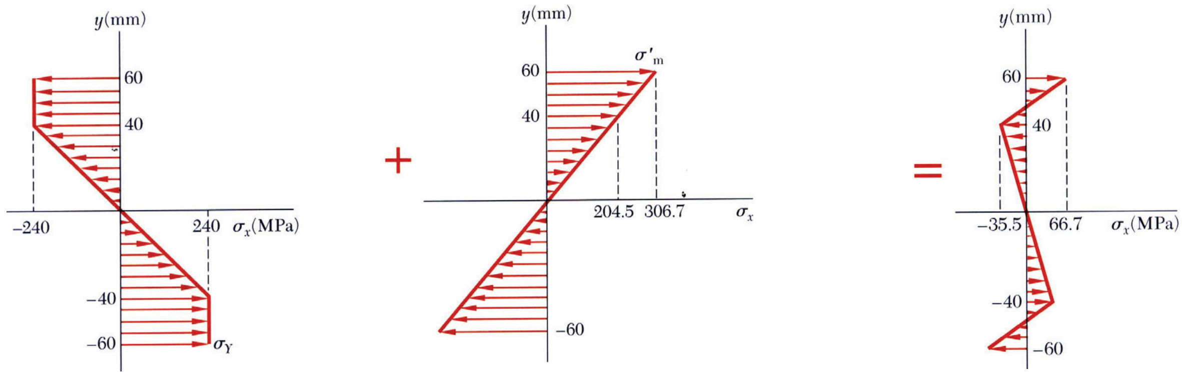
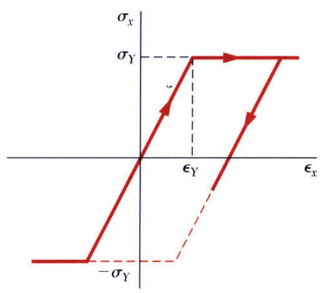
Example 08



- M = 36.8 kN-m
- M = -36.8 kN-m
- M = 0



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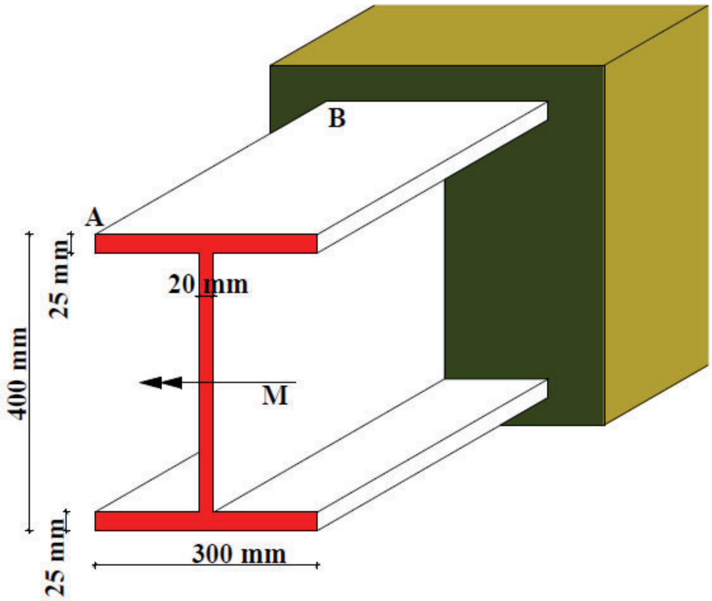
Example 09

Beam *AB* has been fabricated from a high-strength low-alloy steel that is assumed to be Elastoplastic with $E = 200$ GPa and $\sigma y = 350$ MPa. Neglecting the effect of fillets, determine the bending moment *M* and the corresponding radius of curvature ***(a)*** when yield first occurs, ***(b)*** when the flanges have just become fully plastic. ***(c)*** determine the residual stresses and the permanent radius of curvature after couple M in part (b) has been removed.



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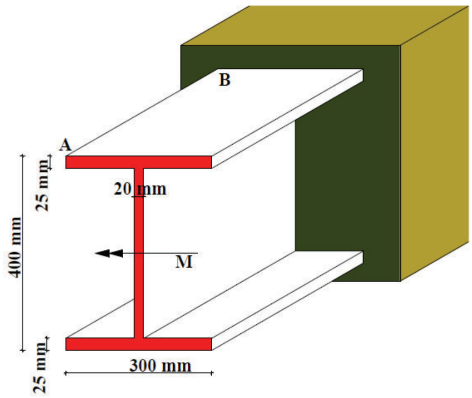
Example 09



a. Onset of Yield

The centroidal moment of inertia of the section is

Bending Moment.

Radius of curvature.

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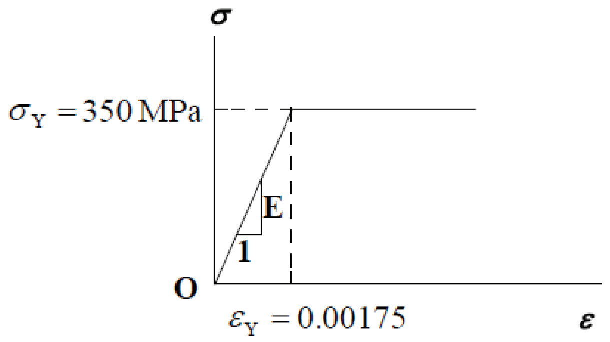
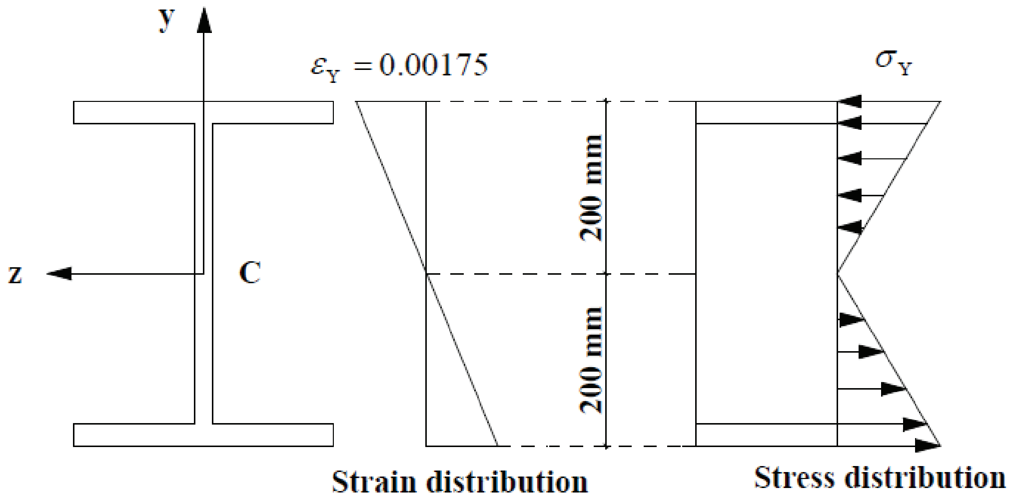
Example 09

a. Onset of Yield





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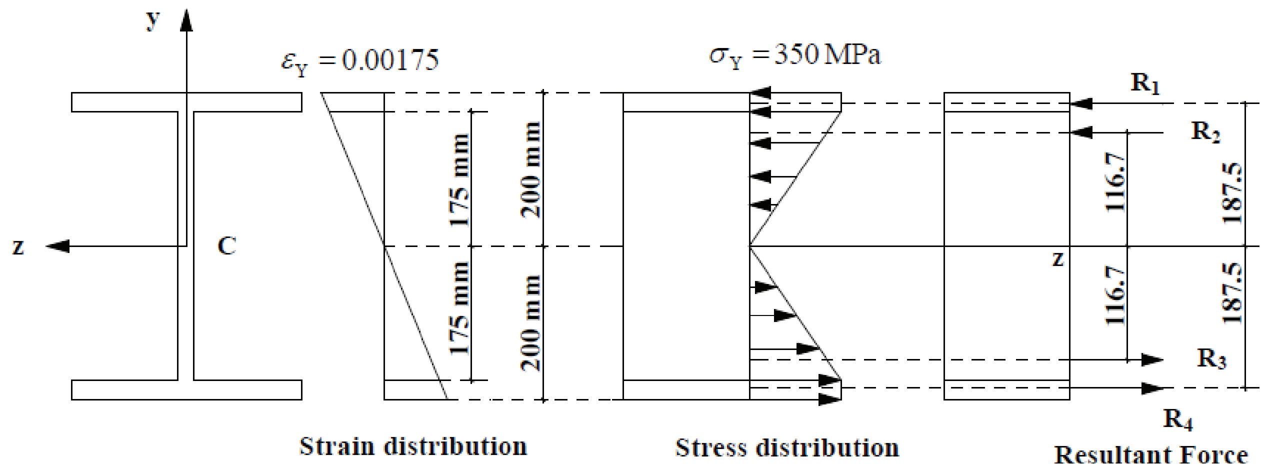
Example 09

b. Flanges Fully Plastic

When the flanges have just become fully plastic, the strains and stresses in the section are as shown in the figure below. We replace the elementary compressive forces exerted on the top flange and on the top half of the web by their resultants Rl and R2, and similarly replace the tensile forces by R3, and R4



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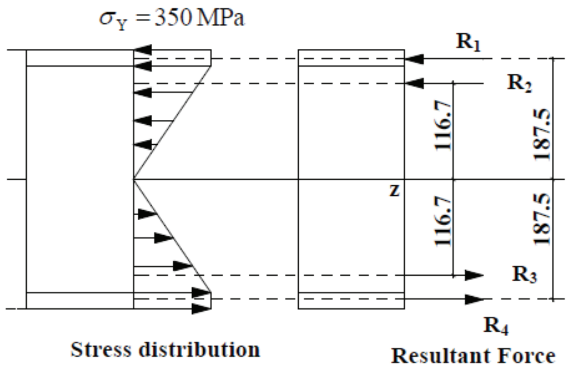
Example 09

b. Flanges Fully Plastic



Bending Moment.

Radius of curvature.

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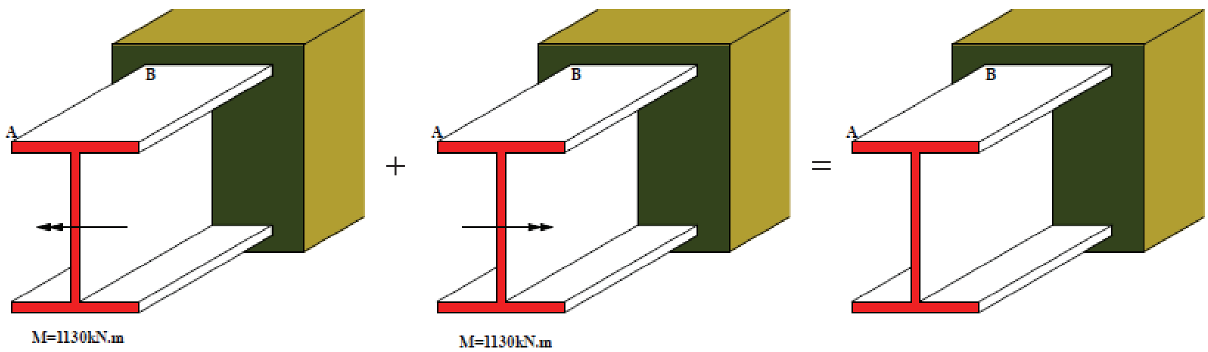
Example 09

c. Residual stress

Elastic Unloading

The beam is unloaded by the application of a coupe of moment M = -1130 kN . m (which is equal and opposite to the couple originally applied).

c. Residual stress



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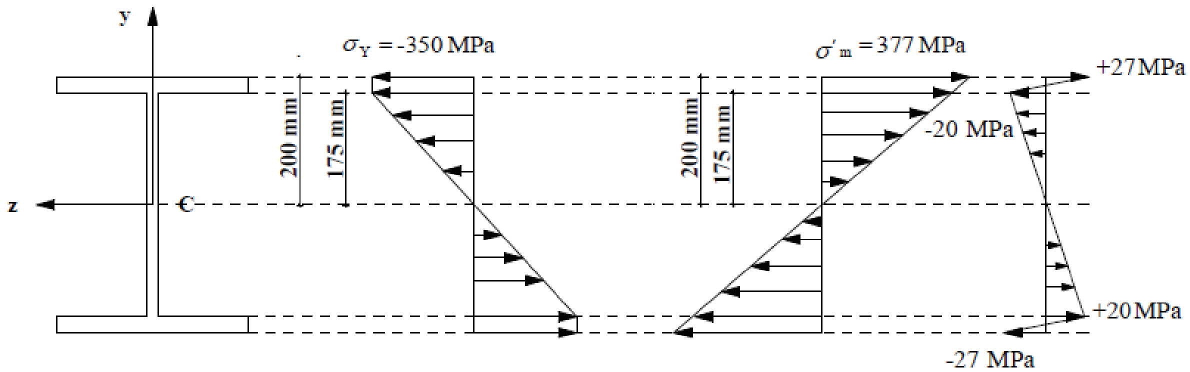
Example 09

c. Residual stress



Permanent Radius of Curvature. At y = 175 mm the residual stress is σy= -20 MPa. Since no plastic deformation occurred at this point, Hooke's law can be used

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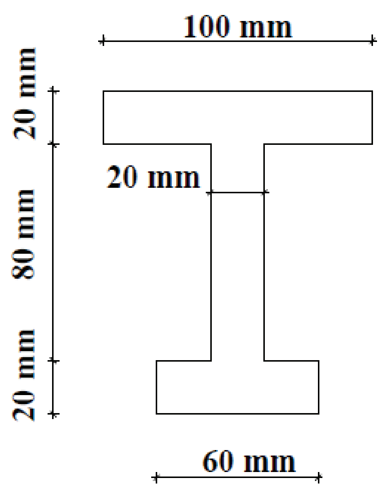
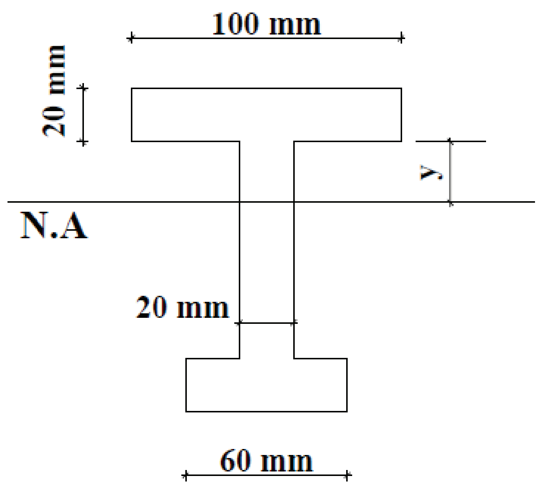
Example 10

Determine the plastic moment Mp of a beam with the cross section shown when the beam is bent about a horizontal axis. Assume that the material is Elastoplastic with a yield strength of 240 MPa.



Example 10

Neutral Axis. When the deformation is fully plastic, the neutral axis divides the cross section into two portions of equal areas. Since the total area is



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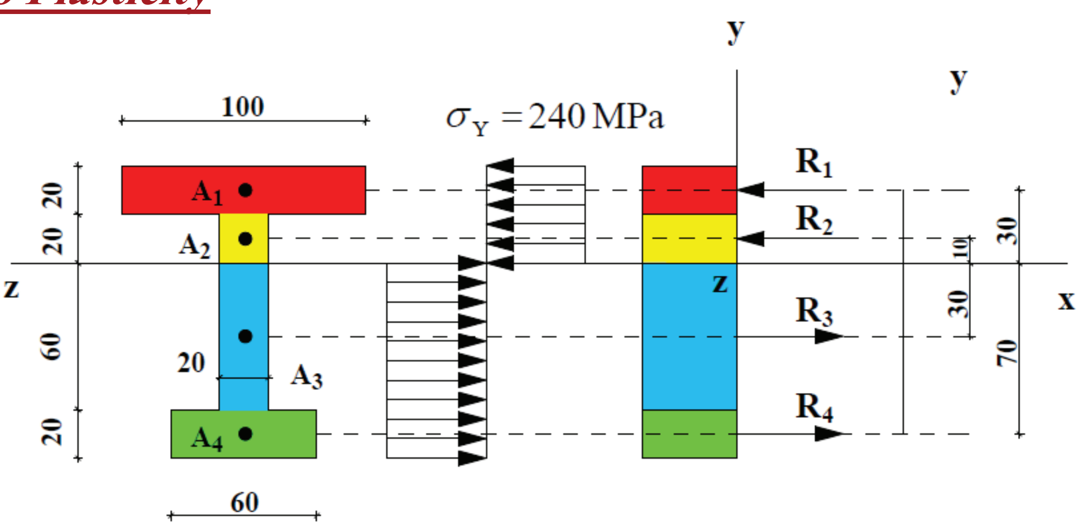
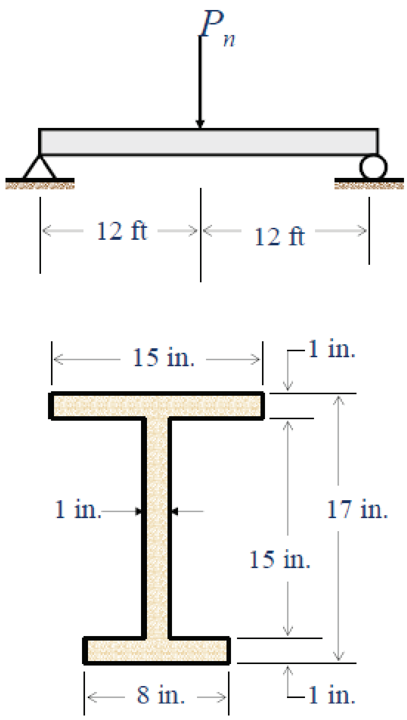
Example 10

Plastic Moment



Example 11



Determine the yield moment My, the plastic Mp, and the plastic modulus Z for the simply supported beam having the cross section shown in Figure. Also calculate the shape factor and plastic load Pn acting transversely through the midspan of the beam. Assume that σY = 50 ksi.

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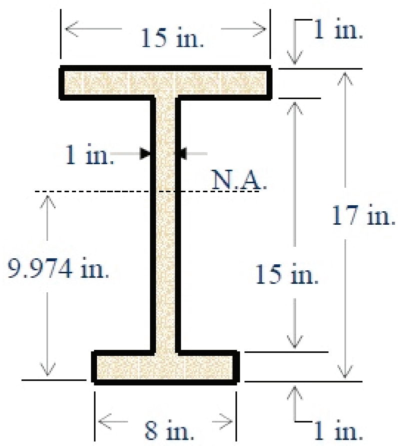
Example 11

Elastic Calculations:



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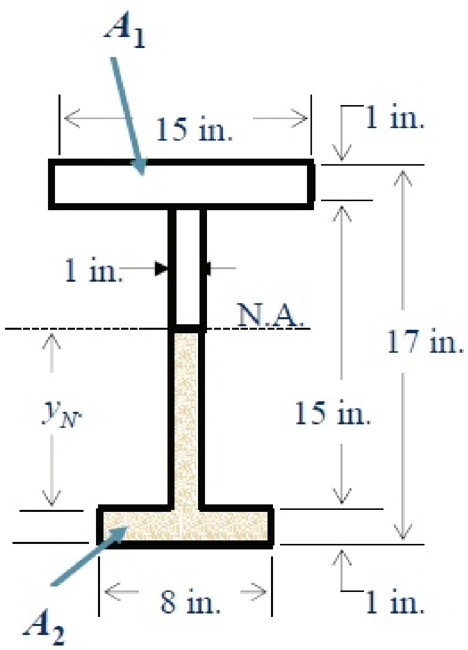
Example 11

Plastic Calculations:



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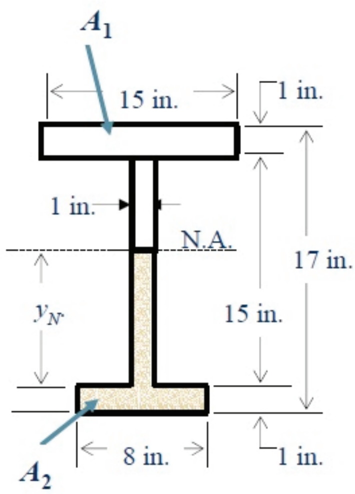
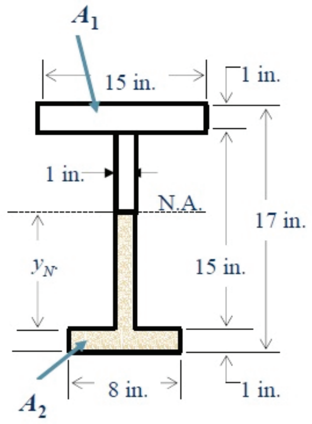
Example 11

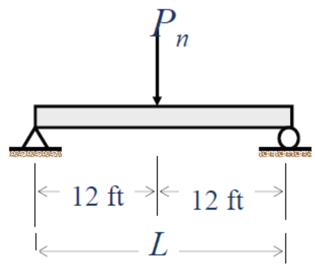
Plastic Calculations:



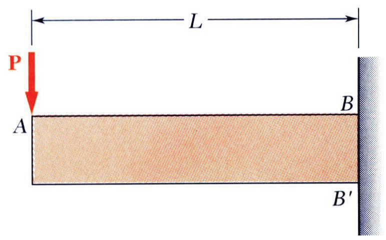
Example 11

Plastic Calculations:



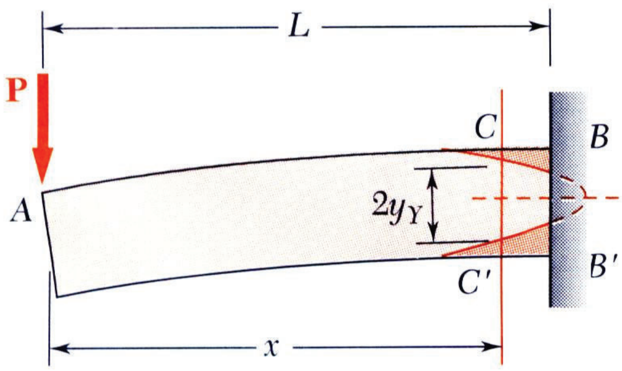


❑ Plastic Deformations due to shearing



- Recall: $M_Y = \frac{I}{c}\sigma_Y =$ maximum elastic moment
- For $M = PL < M_Y$, the normal stress does not exceed the yield stress anywhere along the beam.

- For $PL > M_Y$, yield is initiated at B and B'. For an Elastoplastic material, the half-thickness of the elastic core is found from



$$Px = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right) \Rightarrow y_Y = c\sqrt{3\left(1 - \frac{2Px}{3M_Y}\right)}$$

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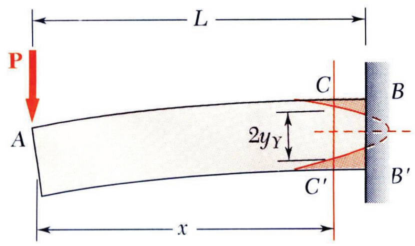
❑ Plastic Deformations due to shearing

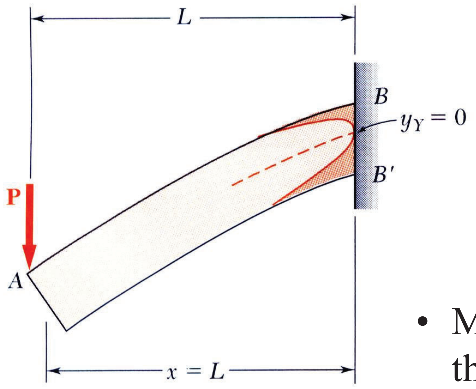


- The section becomes fully plastic at the wall when

$$at \quad x = L \quad \Rightarrow M = PL$$

$$If \quad PL = \frac{3}{2}M_Y = M_p \quad \Rightarrow \quad y_Y = c\sqrt{3\left(1-\frac{2}{3}\times\frac{3}{2}\frac{M_Y}{M_Y}\right)} = 0$$

$$\rho = \frac{y_Y}{\varepsilon_Y} = \frac{0}{\varepsilon_Y} \Rightarrow \rho = 0 \Rightarrow \frac{1}{\rho} = \infty \Rightarrow M \to \infty$$



indicating the presence of a ***sharp bend*** in the beam at its fixed end, We say that a ***plastic hinge*** has developed at that point.

- Maximum load which the beam can support is

$$P_{\max} = \frac{M_p}{L}$$

Introduction to Plasticity

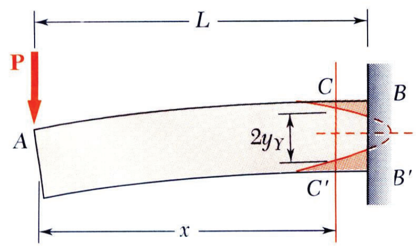
❑ Plastic Deformations due to shearing

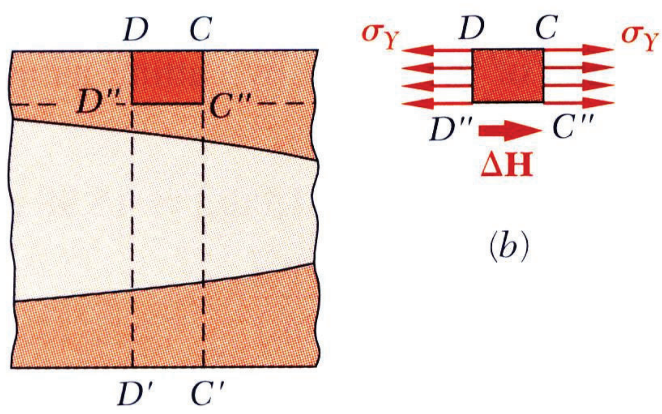
- Preceding discussion was based on normal stresses only

- Consider horizontal shear force on an element within the plastic zone,

$$\Delta H = -(\sigma_C - \sigma_D)dA = -(\sigma_Y - \sigma_Y)dA = 0$$

Therefore, ***the shear stress is zero in the plastic zone.***





(b)

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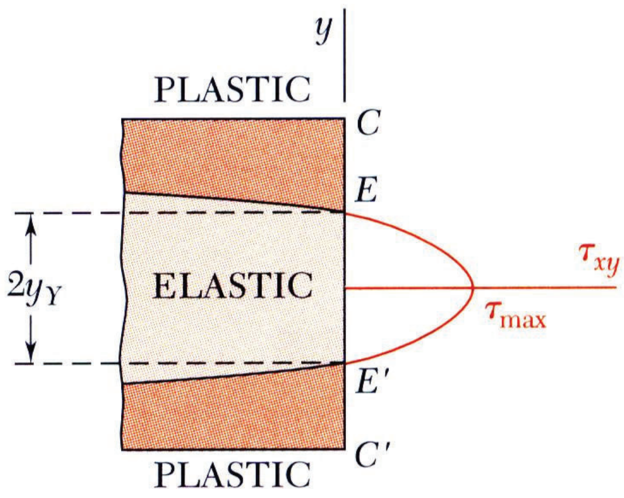
❑ Plastic Deformations due to shearing

- ***Shear load is carried by the elastic core,***

It can be shown that the distribution of the shearing stresses over EE' is the same as in an elastic rectangular beam of the same width b as beam AB, and of depth equal to the thickness $2y_Y$ of the elastic zone. Denoting by A' the area $2by_Y$ of the elastic portion of the cross section. we have



$$\tau_{xy} = \frac{3}{2}\frac{P}{A'}\left(1 - \frac{y^2}{y_Y^2}\right) \quad \Rightarrow \quad \tau_{max} = \frac{3}{2}\frac{P}{A'}$$

where $A' = 2by_Y$

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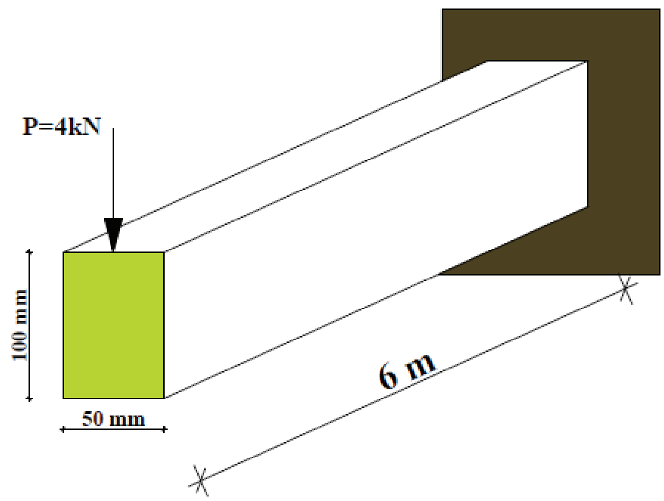
Example 12

Determine shear stress distribution at fixed point . Assume that σ_Y = 240 Mpa.



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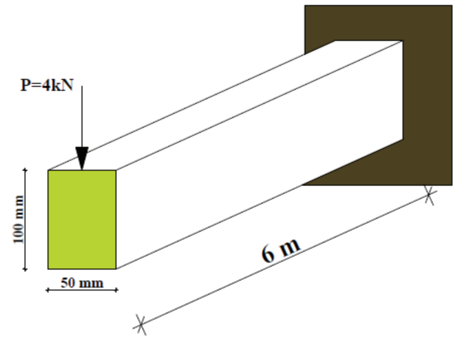
Example 12



- Maximum elastic moment:

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Example 12