# Mechanics of Materials

دانشگاه کردستان
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Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:
J.Wat Oler "Lectures notes on Mechanics od Materials"
Ibrahim A.Assakkaf "Lectures notes on Mechanics od Materials"

## Torsion

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***Torsion***

❑ **Torsion of Noncircular Members**

## Introduction

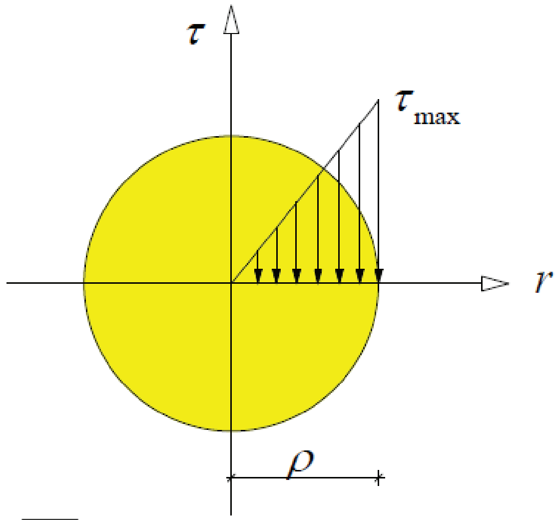
- The analysis of a noncircular torsion structural member is far ***more complicated*** than a circular shaft.
- The major difficulty basically lies in ***determining the shear-strain distribution***.
- In these noncircular members, the discussion presented previously for circular shafts is not applicable.

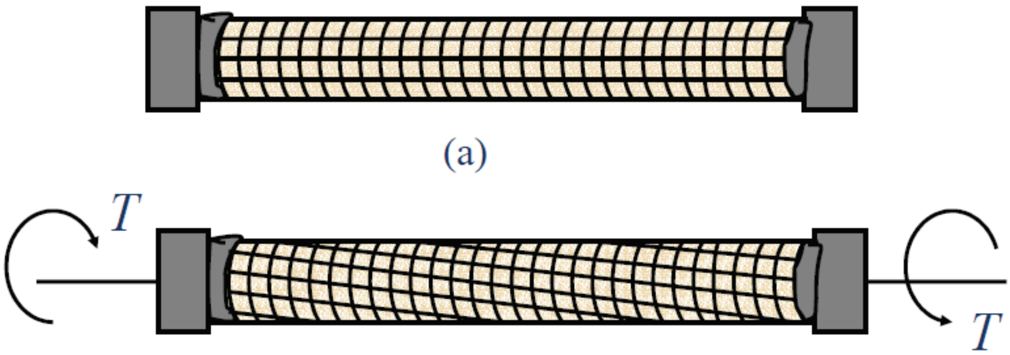
## *Torsion*

### ❑ Torsion of Noncircular Members

**Deformation of Circular Shaft Subjected to Torque T**

plane transverse sections remain plane and the shear strain varies linearly from the geometric center.




(a)

(b)

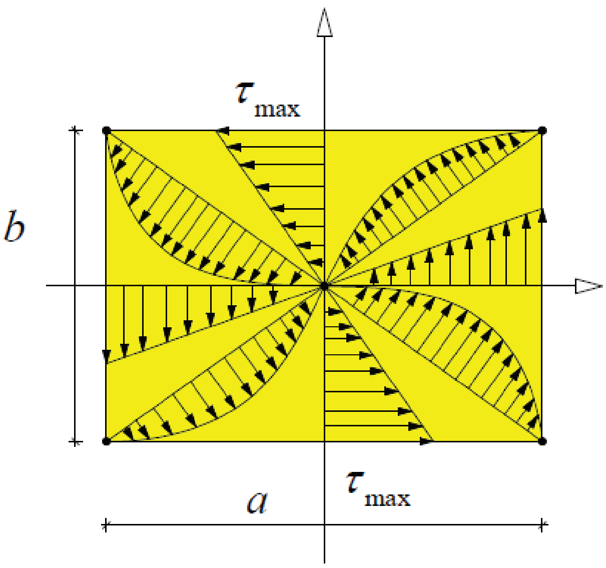


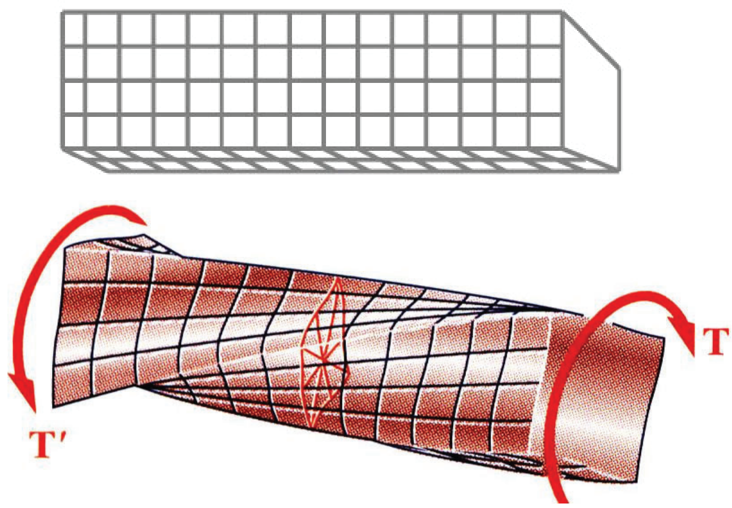
## *Torsion*

### ❑ Torsion of Noncircular Members

**Deformation of a Bar of Square Cross Section Subjected to Torque T**

Planar cross-sections of noncircular shafts do not remain planar and stress and strain distribution do not vary linearly

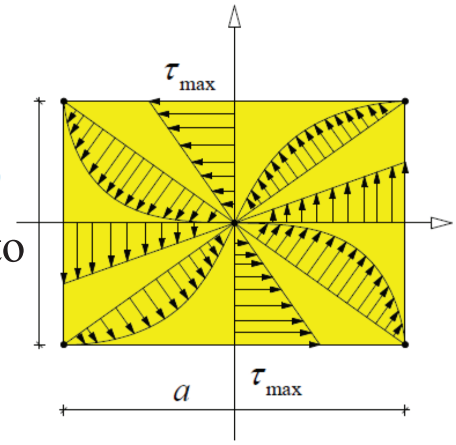


# Torsion

## ❑ Torsion of Noncircular Members

**Determination of Shearing Stress**




- Several rigorous methods have been derived to determine the shear strain distribution in noncircular torsion members.

- The solutions of many problems for solid noncircular torsion members can be found in several advanced books such as

  • Seely, F. B. and Smith, J. O., 1952. "***Advanced Mechanics of Materials***," 2nd edition, Wiley.

  • Timoshenko, S. P. and Goodier, J. N., 1970. "***Theory of Elasticity***," 3rd edition, McGraw-Hill, New York, section. 109.



# Torsion

## ❑ Torsion of Noncircular Members

For uniform rectangular cross-sections,

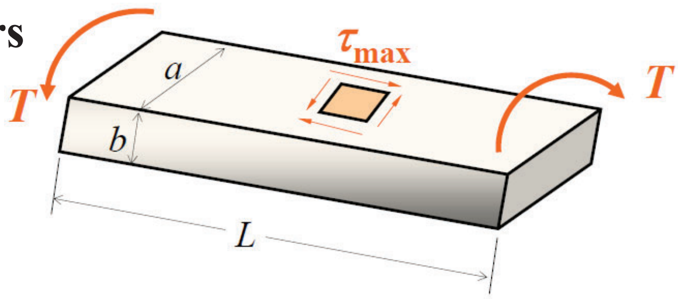



**TABLE 3.1. Coefficients for Rectangular Bars in Torsion**

| $a/b$ | $c_1$ | $c_2$ |
|---|---|---|
| 1.0 | 0.208 | 0.1406 |
| 1.2 | 0.219 | 0.1661 |
| 1.5 | 0.231 | 0.1958 |
| 2.0 | 0.246 | 0.229 |
| 2.5 | 0.258 | 0.249 |
| 3.0 | 0.267 | 0.263 |
| 4.0 | 0.282 | 0.281 |
| 5.0 | 0.291 | 0.291 |
| 10.0 | 0.312 | 0.312 |
| $\infty$ | 0.333 | 0.333 |

The maximum shearing stress occurs along the center line of the wider face of the bar and is equal to

$$\boxed{\tau_{max} = \frac{T}{c_1 ab^2}}$$

The angle of twist

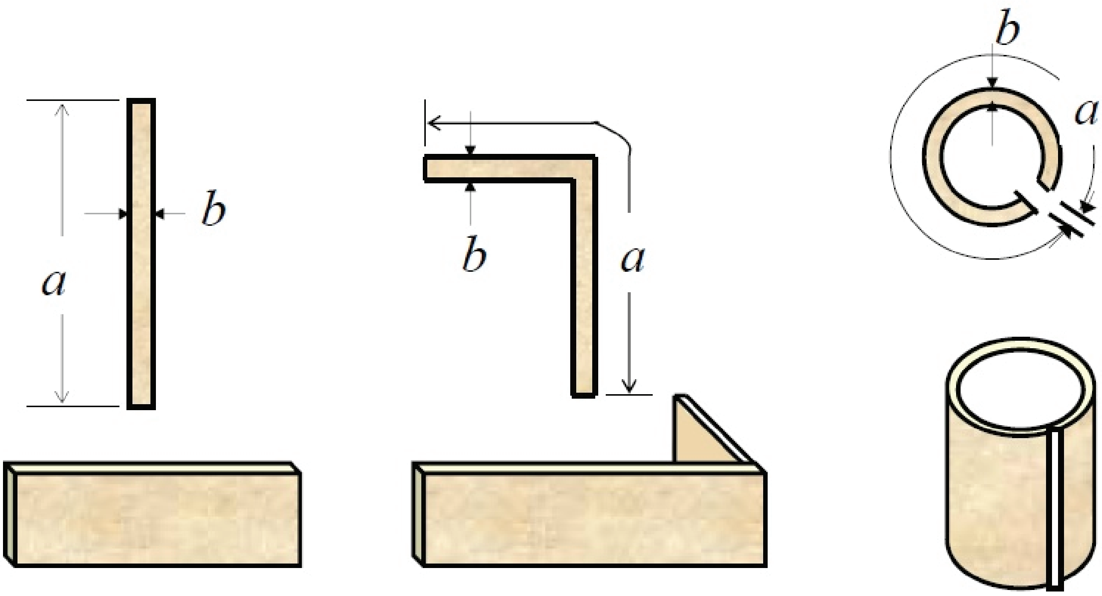
$$\boxed{\phi = \frac{TL}{c_2 ab^3 G}}$$

$$\boxed{If\ \frac{a}{b} \geq 5 \Rightarrow C_1 = C_2 = \frac{1}{3}\left(1 - \frac{0.630}{(a/b)}\right)}$$

# Torsion

## ❑ Torsion of Noncircular Members

### Thin-Walled Member of Uniform Thickness and Arbitrary Shape



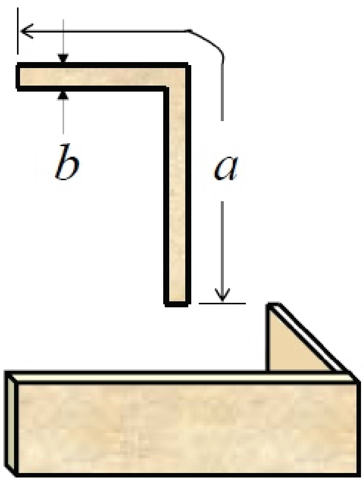



# Torsion

## ❑ Torsion of Noncircular Members

### Thin-Walled Member of Uniform Thickness and Arbitrary Shape

Based on the ***membrane analogy*** (Timoshenko and Goodier 1970), for a thin walled member of uniform thickness and arbitrary shape, ***the maximum shearing stress is the same as for a rectangular bar with a very large value of a/b***




$$\frac{a}{b} >> 5 \quad \Rightarrow \quad C_1 = C_2 = \frac{1}{3}$$

$$\Rightarrow \quad \boxed{\tau_{\max} = \frac{3Tb_{\max}}{\sum_{i=1}^{n} a_i b_i^3}} \qquad \boxed{\phi = \frac{3TL}{G\sum_{i=1}^{n} a_i b_i^3}}$$

## Torsion

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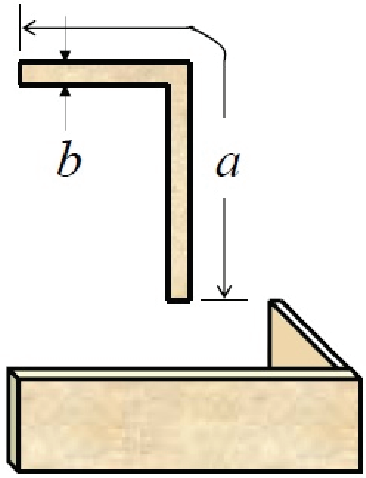


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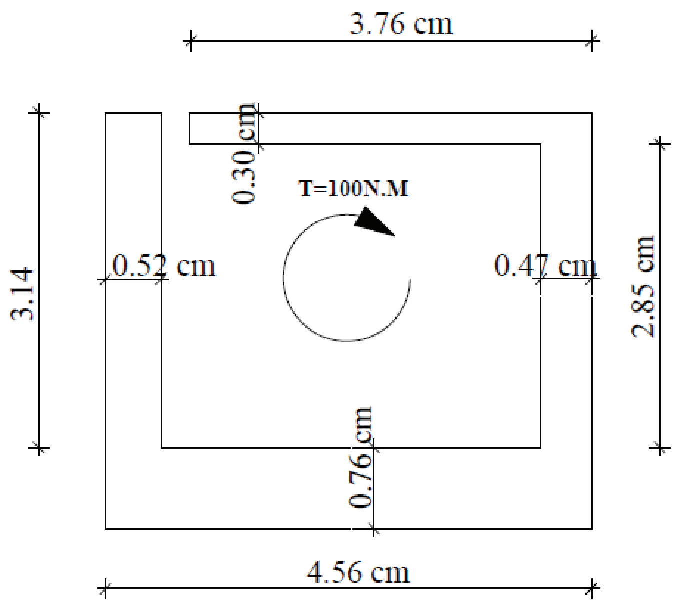
## Torsion

### ❑ Torsion of Noncircular Members

Example 01

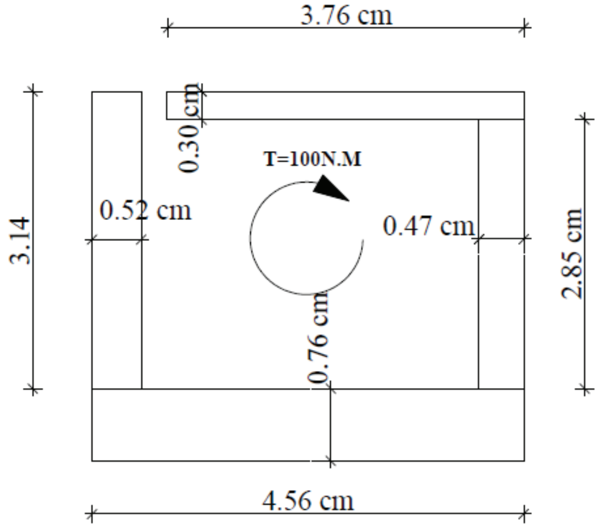
Determine the maximum shear stress due to exerted torque.



## Torsion

### ❑ Torsion of Noncircular Members

Example 01

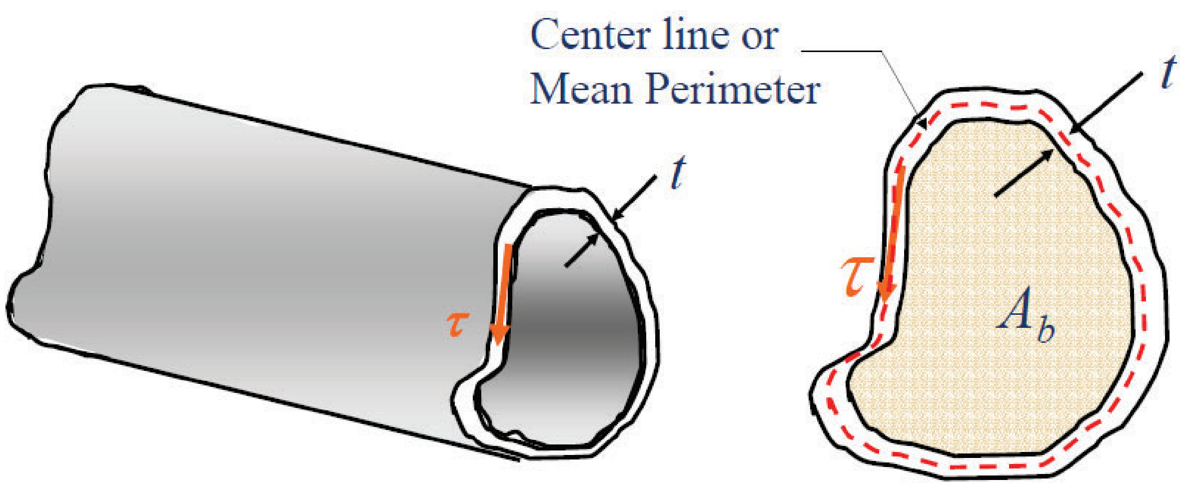





## Torsion

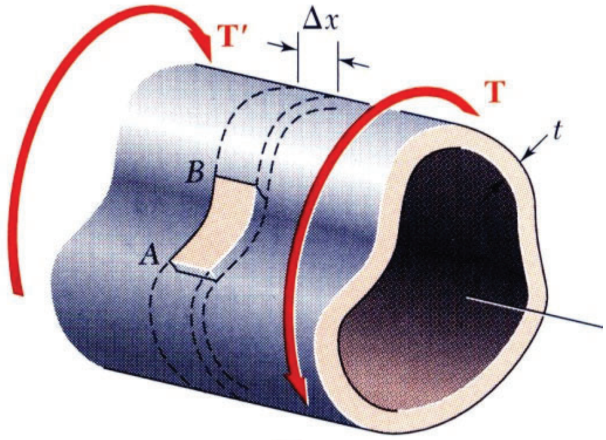
### ❑ Torsion of Noncircular Members

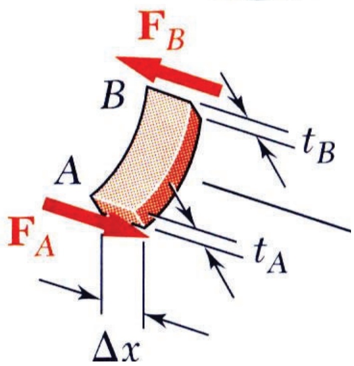
**Thin-Walled Hollow Shafts**



## ❑ Torsion of Noncircular Members

### Thin-Walled Hollow Shafts




- Summing forces in the x-direction on *AB*,

$$\sum F_x = 0 = \tau_A(t_A\Delta x) - \tau_B(t_B\Delta x)$$

$$\tau_A t_A = \tau_B t_B = \tau t$$

$$\Rightarrow \quad \boxed{q = \tau t = \text{shear flow} = \text{cte}}$$

shear stress varies inversely with thickness

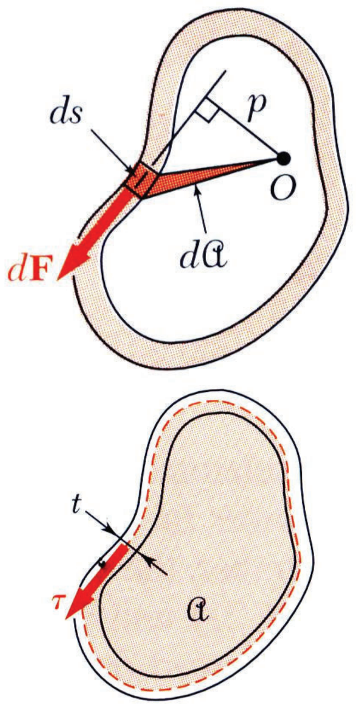
$$\boxed{\tau = \frac{q}{t} \neq \text{cte}}$$





## ❑ Torsion of Noncircular Members

### Thin-Walled Hollow Shafts




- Compute the shaft torque from the integral of the moments due to shear stress

$$dM_0 = dF \cdot p = \tau\, t\, ds \cdot p = q\, ds \cdot p = q \cdot 2\,dA$$

$$T = \oint dM_0 = \oint 2q\,dA = 2qA_m$$

$$\Rightarrow \quad \boxed{\tau = \frac{T}{2tA_m}}$$

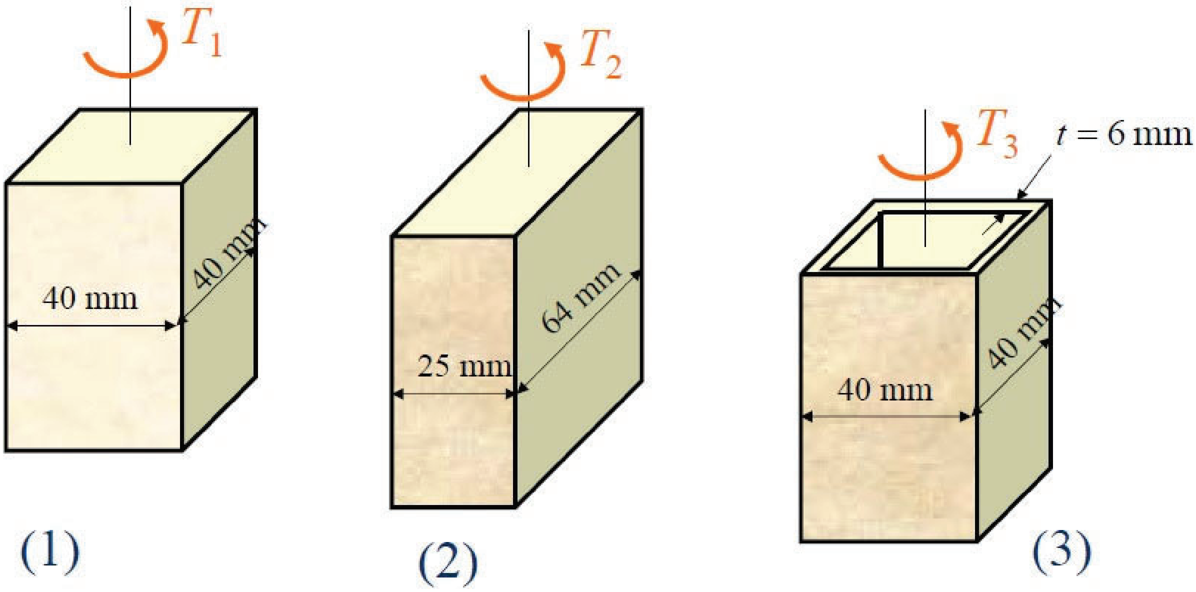
- Angle of twist

$$\boxed{\phi = \frac{TL}{4A_m{}^2 G}\oint \frac{ds}{t}}$$

## ❑ Torsion of Noncircular Members

### Example 02

Using τall = 40 MPa, determine the largest torque which may be applied to each of the brass bars and to the brass tube shown. Note that the two solid bars have the same cross-sectional area, and that the square bar and square tube have the same outside dimensions.








## ❑ Torsion of Noncircular Members

### Example 02

1. Bar with Square Cross Section:

## ❑ Torsion of Noncircular Members

### Example 02

2. Bar with Rectangular Cross Section:

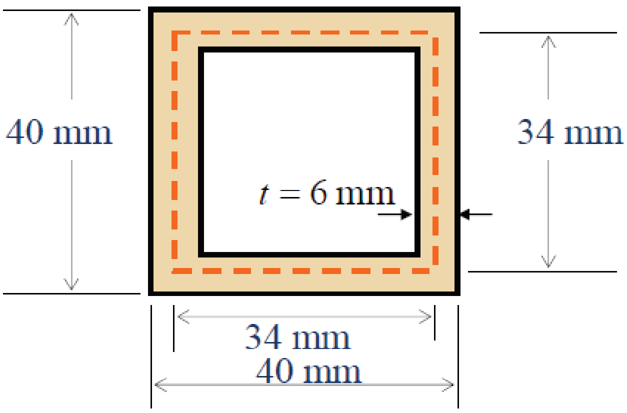




## ❑ Torsion of Noncircular Members

### Example 02

3. Square Tube:

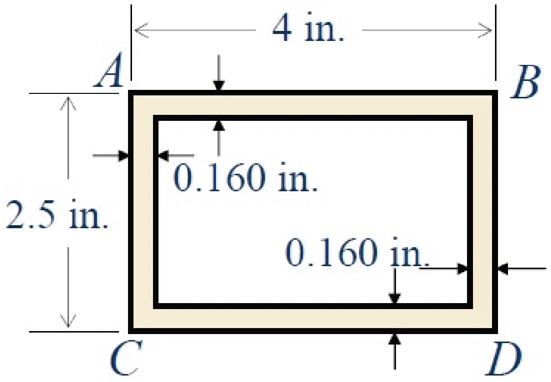


## *Torsion*

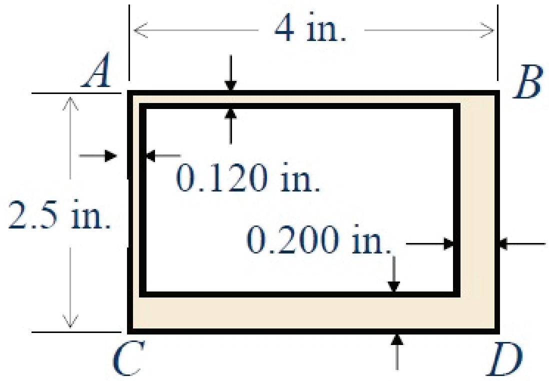
### ❑ Torsion of Noncircular Members

Example 03

Structural aluminum tubing of 2.5 × 4-in. rectangular cross section was fabricated by extrusion. Determine the shearing stress in each of the four walls of a portion of such tubing when it is subjected to a torque of 24 kip·in., assuming (a) a uniform 0.160- in. wall thickness, (b) that, as a result of defective fabrication, walls AB and AC are 0.120-in thick, and walls BD and CD are 0.200-in thick.




(a)




(b)

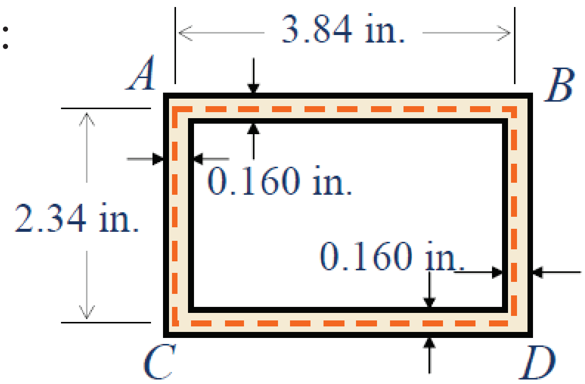


## *Torsion*

### ❑ Torsion of Noncircular Members

Example 03

(a) Tubing of Uniform Wall Thickness:



## ❑ Torsion of Noncircular Members

Example 03

(b) Tubing with Variable Wall Thickness:

Observing that the area Am bounded by the center line is the same as in Part a





## ❑ Torsion of Noncircular Members

**R: Torsional resistance**

$$\boxed{\tau_{\max} = \frac{T}{R}}$$

Rectangular section: $\tau_{\max} = \dfrac{T}{c_1 a b^2} \Rightarrow \boxed{R = c_1 a b^2}$

Opened thin-Walled Member: $\tau_{\max} = \dfrac{3 T b_{\max}}{\sum_{i=1}^{n} a_i b_i^3} \Rightarrow \boxed{R = \dfrac{\sum_{i=1}^{n} a_i b_i^3}{3 b_{\max}}}$

closed thin-Walled Member: $\tau_{\max} = \dfrac{T}{2 t_{\min} A_m} \Rightarrow \boxed{R = 2 t_{\min} A_m}$

## *Torsion*

### ❑ Torsion of Noncircular Members

**K: Torsional Rigidity**

$$\boxed{\phi = \frac{T}{K}}$$

Rectangular section: $\phi = \dfrac{TL}{c_2 ab^3 G} \Rightarrow \boxed{K = \dfrac{c_2 ab^3 G}{L}}$

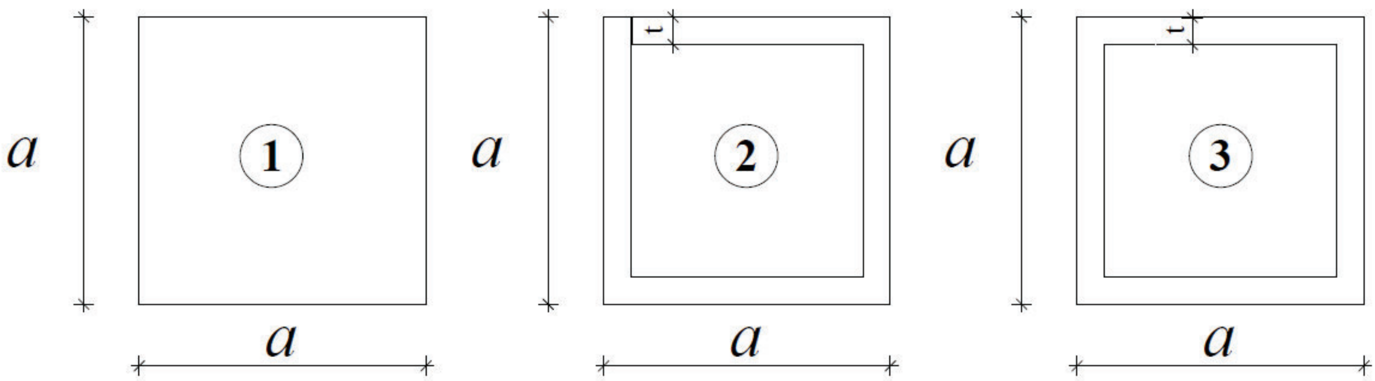
Opened thin-Walled Member: $\phi = \dfrac{3TL}{G\sum_{i=1}^{n} a_i b_i^3} \Rightarrow \boxed{K = \dfrac{G\sum_{i=1}^{n} a_i b_i^3}{3L}}$

closed thin-Walled Member: $\phi = \dfrac{TL}{4A_m^2 G}\oint \dfrac{ds}{t} \Rightarrow \boxed{K = \dfrac{4A_m^2 G}{L\oint \dfrac{ds}{t}}}$



## *Torsion*

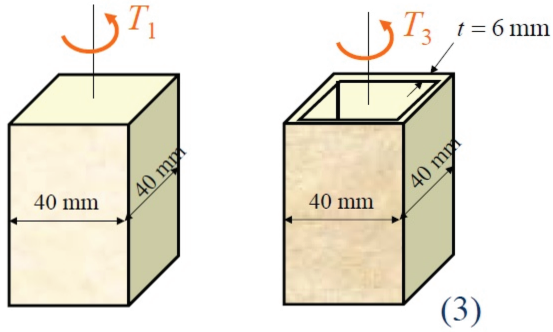
### ❑ Torsion of Noncircular Members




$R_1 = 0.208a^3$ $\quad$ $R_2 = \dfrac{4}{3}(a-t)t^2$ $\quad$ $R_3 = 2(a-t)^2 t$

## ***Torsion***

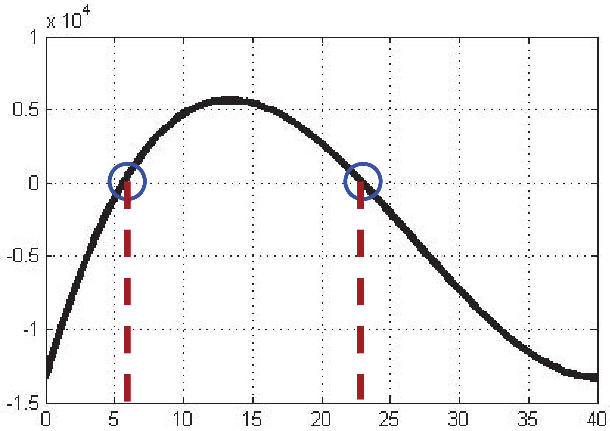
### ❑ Torsion of Noncircular Members




$$\frac{R_3}{R_1} > 1 \quad \Rightarrow \quad \frac{2(a-t)^2 t}{0.208a^3} > 1$$

$$\Rightarrow \quad \boxed{2(a-t)^2 t - 0.208a^3 > 0}$$

$$If = 40mm \quad \Rightarrow \quad 2(40-t)^2 t - 0.208(40)^3 > 0$$






## ***Torsion***

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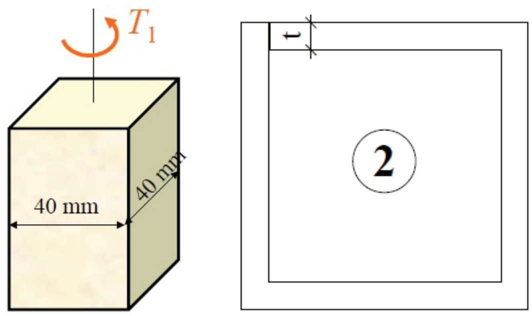


$$\frac{R_2}{R_1} > 1 \quad \Rightarrow \quad \frac{\frac{4}{3}(a-t)t^2}{0.208a^3} > 1$$

$$\Rightarrow \quad \boxed{\frac{4}{3}(a-t)t^2 - 0.208a^3 > 0}$$

$$If = 40mm \quad \Rightarrow \quad \frac{4}{3}(40-t)t^2 - 0.208(40)^3 > 0$$