Mechanics of Materials

دانشگاه کردستان
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Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:
Engineering Mechanics of Solids. Popov, Igor Paul.
J.Wat Oler "Lectures notes on Mechanics od Materials"
Ibrahim A.Assakkaf "Lectures notes on Mechanics od Materials"

Thin & Thick Walled Vessels

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Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels

- The thin-walled pressure vessels provide an important application of plane-stress analysis.

Two types of thin-walled vessels are investigated:

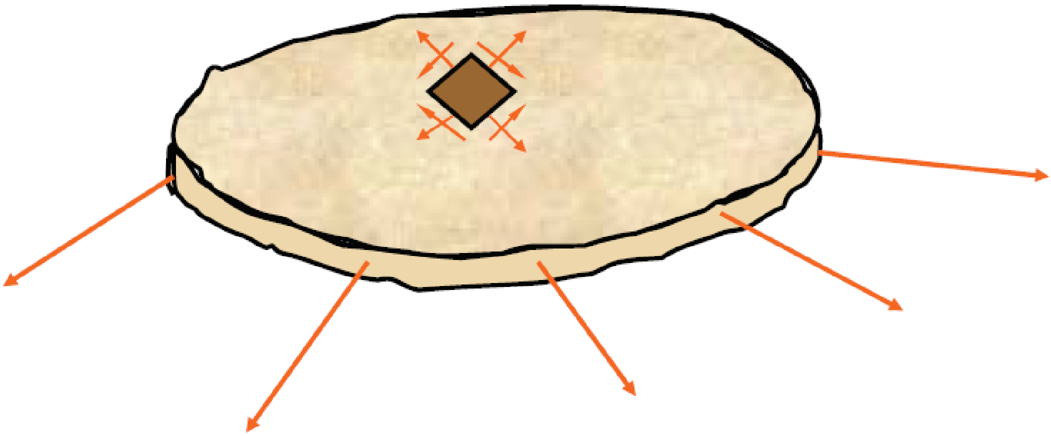
Spherical Pressure Vessels

Cylindrical Pressure Vessels

Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels

- This their walls offer **little resistance to bending**, it may be assumed that the internal forces exerted on a given portion of the wall are **tangent to the surface** of the vessel.

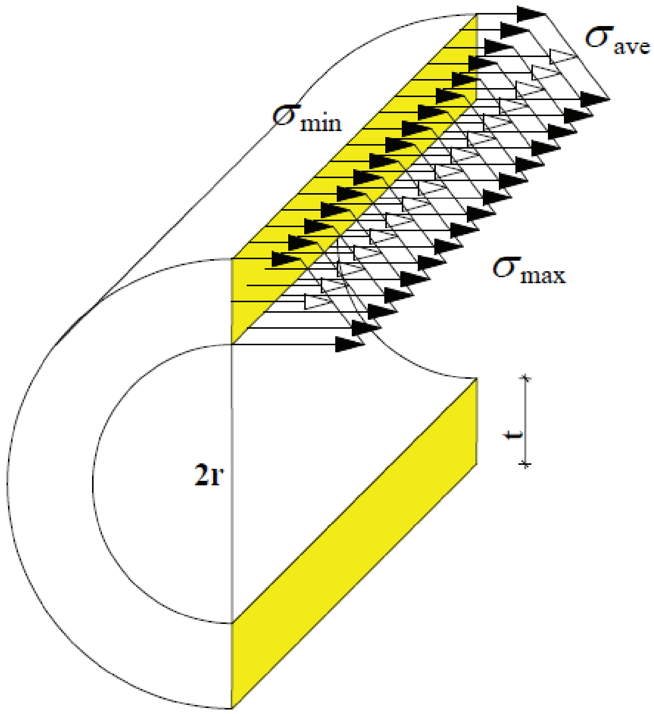


Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels

– The stress in thin-walled vessel varies from a maximum value at the inside surface to a minimum value at the outside surface of the vessel.

$$If \quad \frac{t}{r} \le 0.1 \quad \Rightarrow \quad \frac{\sigma_{max}}{\sigma_{min}} \le 0.05$$

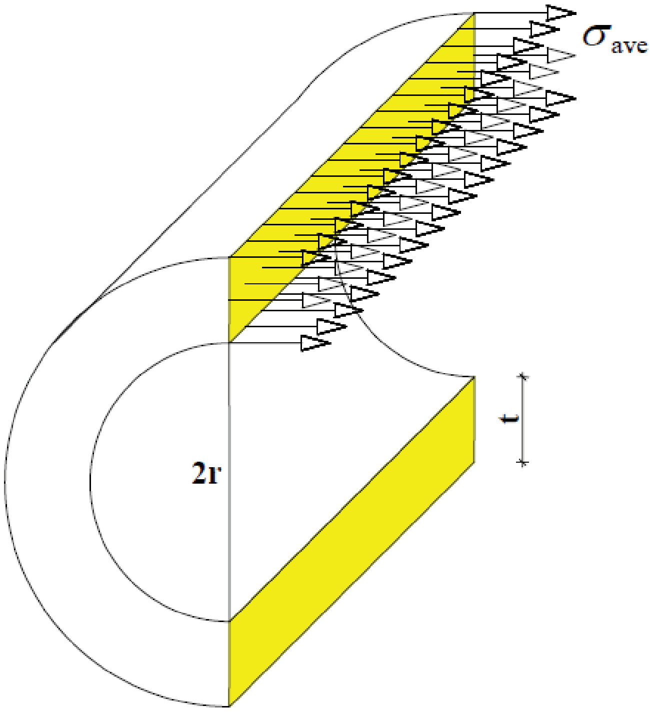


Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels

Definition

"A pressure vessel is defined as thin walled when the ratio of the wall thickness to the radius of the vessel is so small that the distribution of normal stress on a plane perpendicular to the surface of the vessel is essentially uniform throughout the thickness of the vessel."



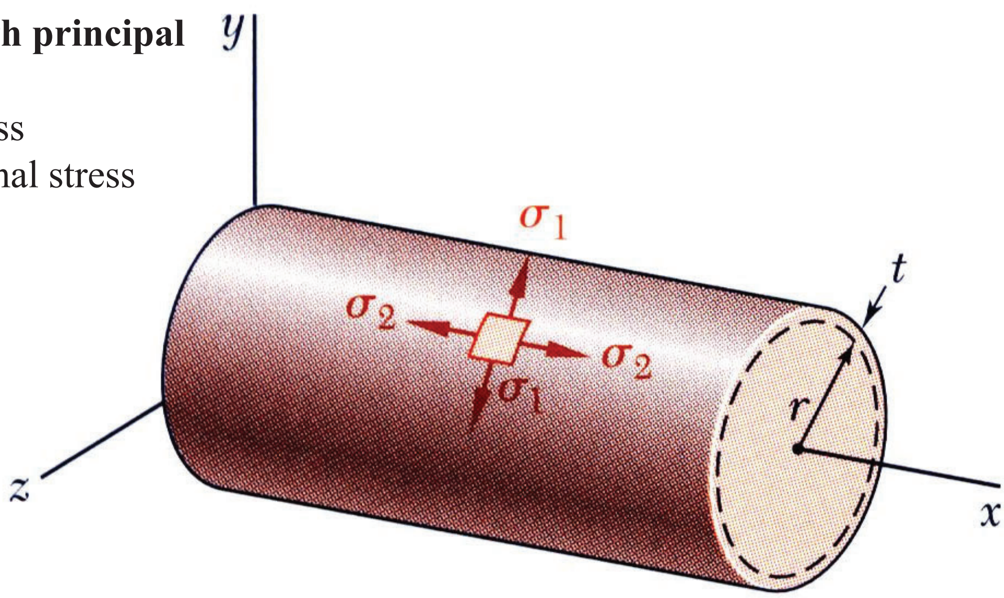
Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels

•Cylindrical vessel with principal stresses

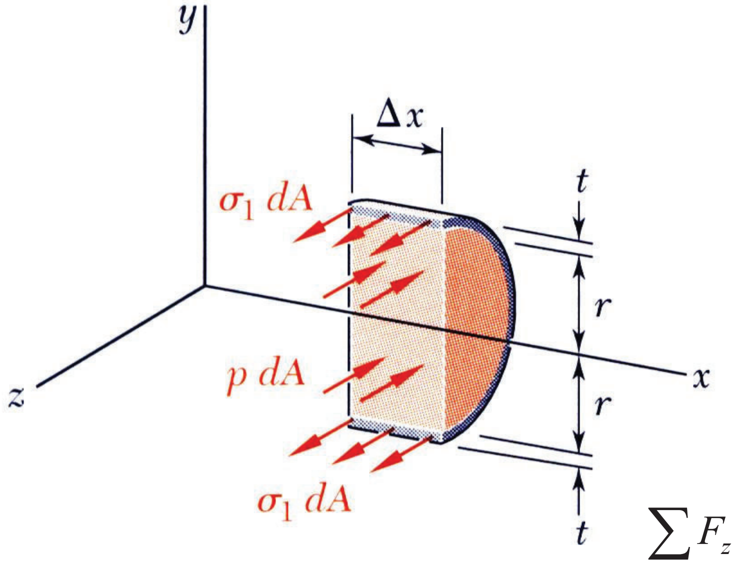
σ_1 = hoop stress

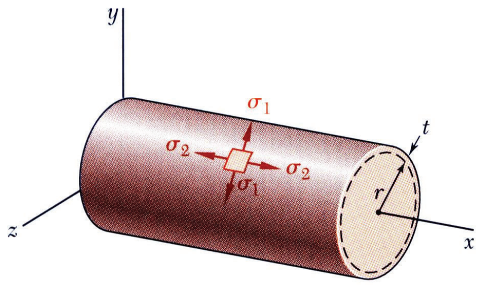
σ_2 = longitudinal stress



Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels





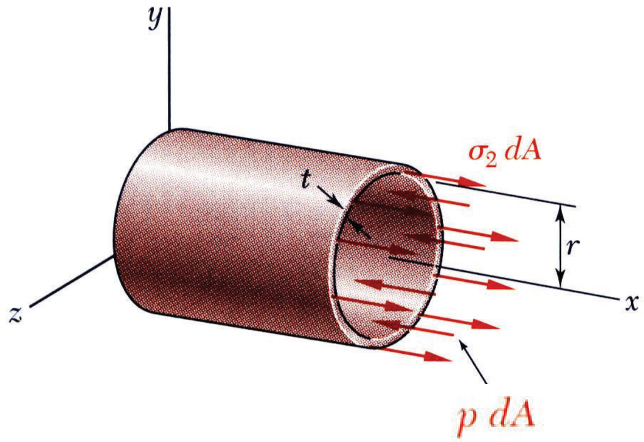
•Hoop stress:

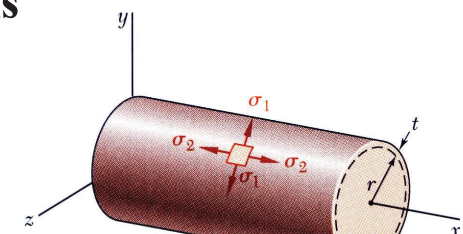
$$\sum F_z = 0 \quad \Rightarrow \quad 2\sigma_1(t \cdot \Delta x) - p(2r \cdot \Delta x) = 0$$

$$\Rightarrow \quad \boxed{\sigma_1 = \frac{p \cdot r}{t}}$$

Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels





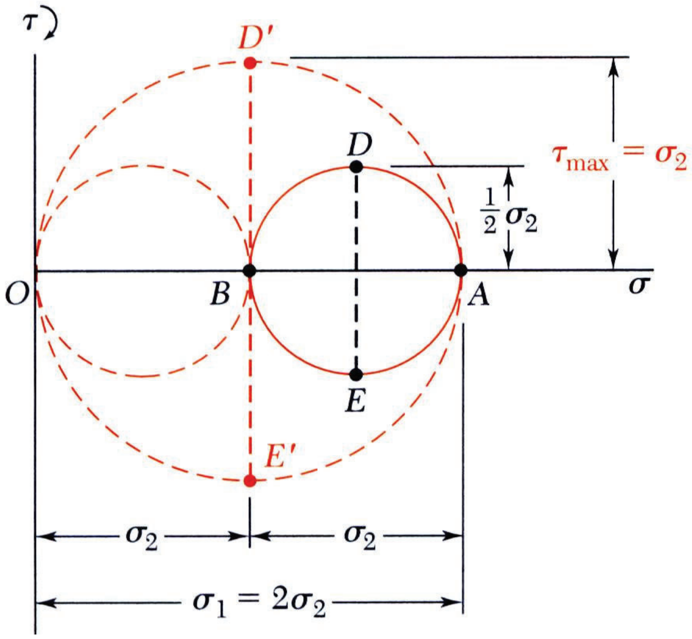
•Longitudinal stress:

$$\sum F_x = 0 \quad \Rightarrow \quad \sigma_2(2\pi r \cdot t) - p(\pi r^2) = 0$$

$$\Rightarrow \quad \boxed{\sigma_2 = \frac{p \cdot r}{2t}} \qquad \boxed{\sigma_1 = 2\sigma_2}$$

Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels

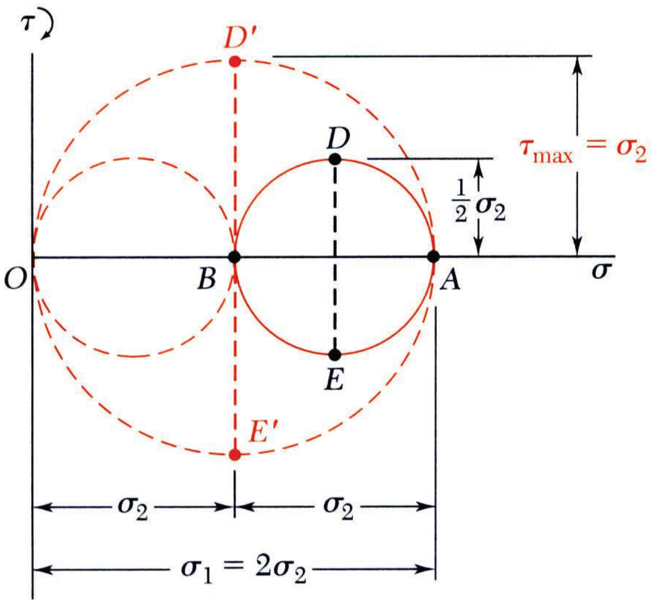


- Points A and B correspond to hoop stress, σ_1, and longitudinal stress, σ_2
- Maximum in-plane shearing stress:

$$\tau_{max(in-plane)} = \frac{1}{2}\sigma_2 = \frac{pr}{4t}$$

Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels



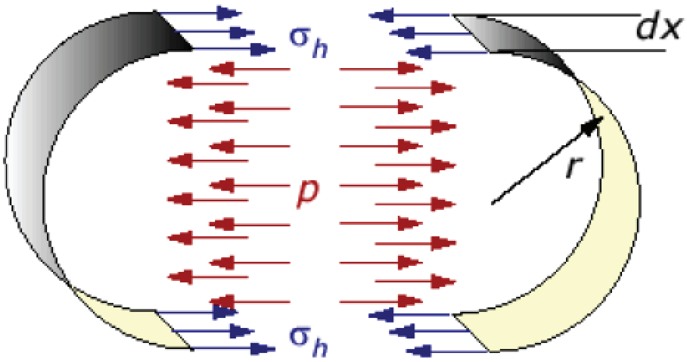
- Maximum out-of-plane shearing stress corresponds to a 45^{o} rotation of the plane stress element around a longitudinal axis

$$\tau_{max} = \sigma_2 = \frac{pr}{2t}$$

❑ Stresses in Thin-Walled Pressure Vessels

Example 1

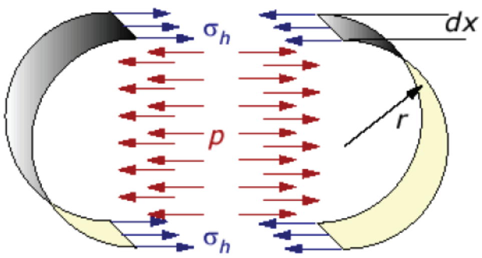
A steel pipe with inside diameter of 12 in. will be used to transmit steam under a pressure of 1000 psi. If the hoop stress in the pipe must be limited to 10 ksi because of a longitudinal weld in the pipe, determine the maximum satisfactory thickness for the pipe.



❑ Stresses in Thin-Walled Pressure Vessels

Example 1

For the cylinder, the hoop stress is given by



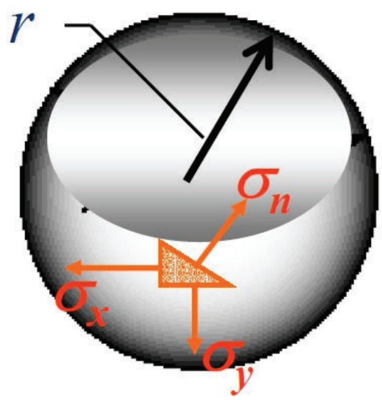
Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels

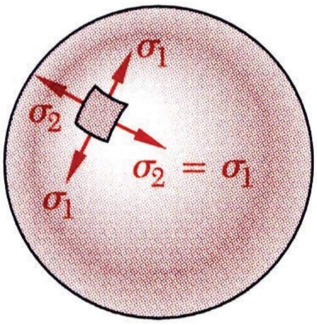
Spherical pressure vessel:

- If the weights of the gas and vessel are negligible (in most cases), symmetry of loading and geometry requires that stresses on sections that pass through the center of the sphere be equal.

$$\sigma_x = \sigma_y = \sigma_n$$

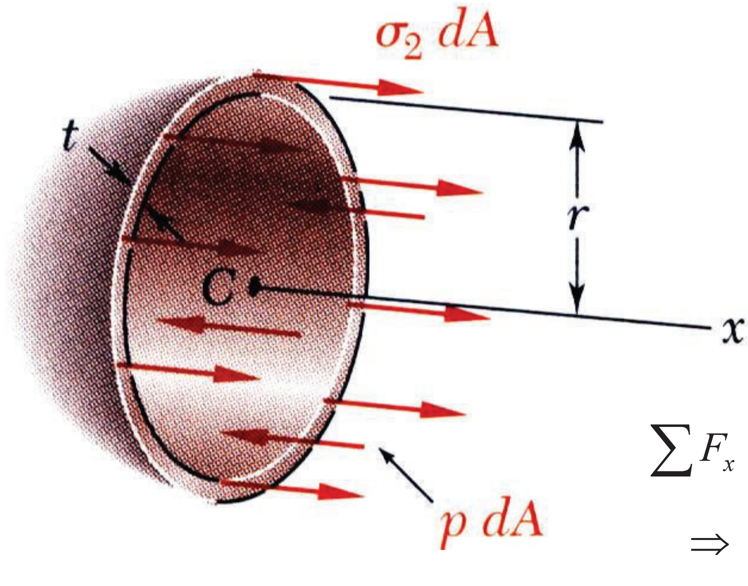


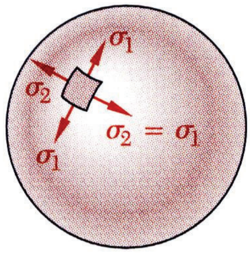
- σ_1 = hoop stress
  σ_2 = longitudinal stress



Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels



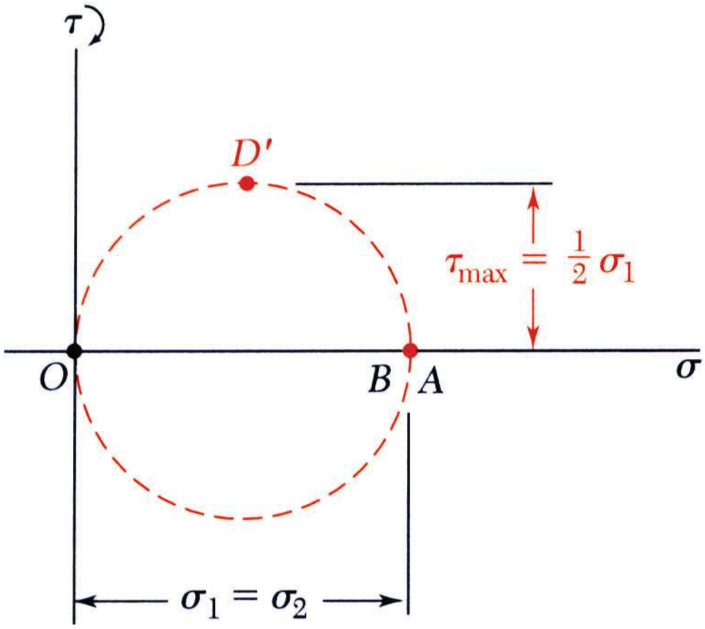


$$\sum F_x = 0 \quad \Rightarrow \quad \sigma_1(2\pi r \cdot t) - p(\pi r^2) = 0$$

$$\Rightarrow \quad \sigma_1 = \sigma_2 = \frac{p \cdot r}{2t}$$

Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels



•Mohr's circle for in-plane transformations reduces to a point

$$\sigma = \sigma_1 = \sigma_2 = cte$$
$$\tau_{\max(in-plane)} = 0$$

•Maximum out-of-plane shearing stress

$$\tau_{\max} = \frac{1}{2}\sigma_1 = \frac{pr}{4t}$$

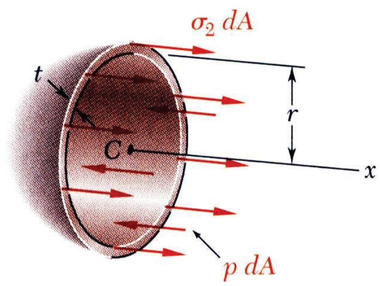
Thin & Thick Walled Vessels

❑ Stresses in Thin-Walled Pressure Vessels

Example 2

A steel pressure vessel of spherical shape has the following specifications:

– Inside radius *r* of 36 inches
– Thickness *t* of 3/16"
– Allowable yield stress is 50 ksi
– Modulus of elasticity *E* of 29,000 ksi
– Poisson's ratio ν of 0.25

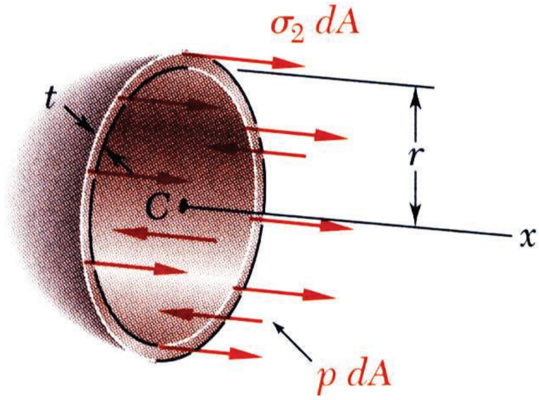


a) What is the maximum pressure p carried by the tank before yielding occurs?
b) If *p* = 100 psi, what is the new outer radius of the tank?

❑ Stresses in Thin-Walled Pressure Vessels

Example 2

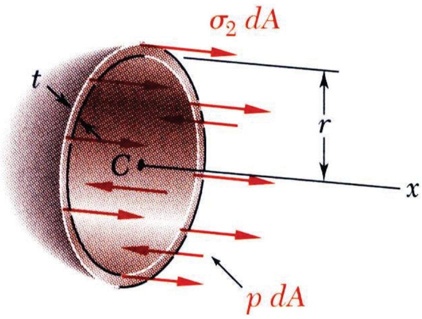
Normal in-plane stresses are given by following equation. Rewrite the equation to solve for the maximum *p*



❑ Stresses in Thin-Walled Pressure Vessels

Example 2

First find the normal in-plane stress in the shell:



Now apply Hooke's law for plane stress:

❑ Remarks of Thin-Walled Pressure Vessels

I. The above formulas are good for thin walled pressure vessels. Generally, a pressure vessel is considered to be "thin walled“ if its radius r is larger than 5 times its wall thickness t ($r > 5 \cdot t$).

Thin & Thick Walled Vessels

❑ Remarks of Thin-Walled Pressure Vessels

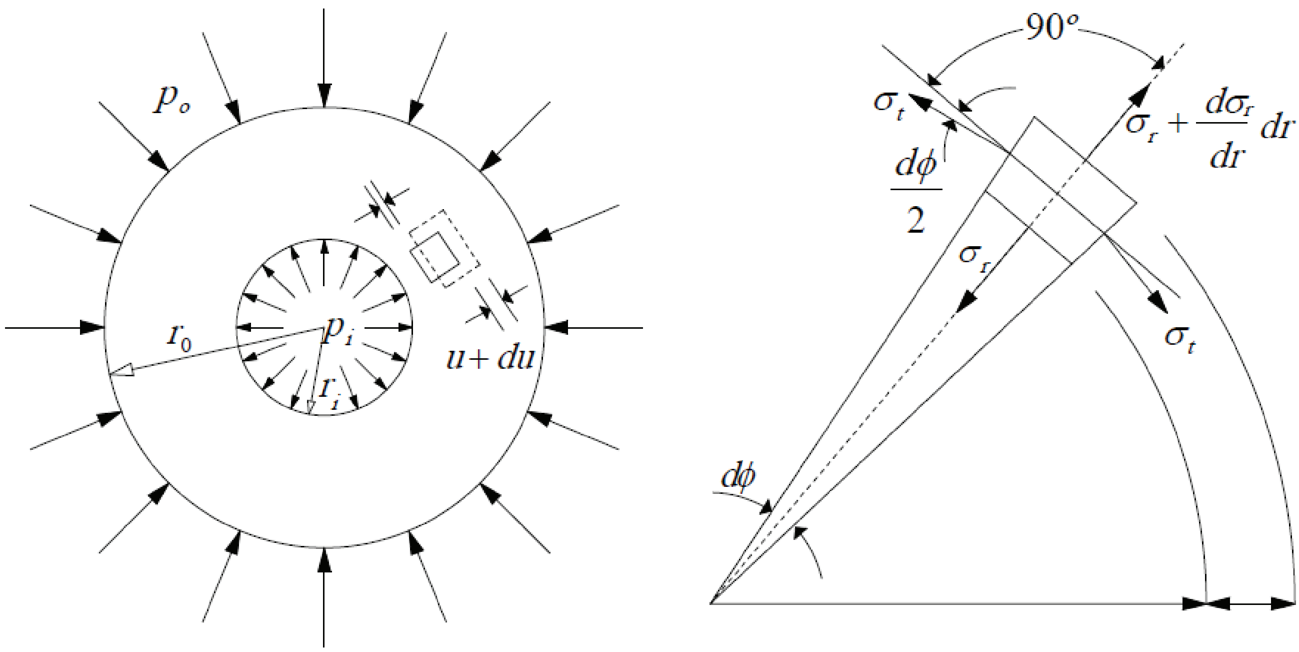
II. When a pressure vessel is subjected to external pressure, the above formulas are still valid. However, the stresses are now negative since the wall is now in compression instead of tension.

❑ Remarks of Thin-Walled Pressure Vessels

III. The hoop stress is twice as much as the longitudinal stress for the cylindrical pressure vessel. This is why an overcooked hotdog usually cracks along the longitudinal direction first (i.e. its skin fails from hoop stress, generated by internal steam pressure).

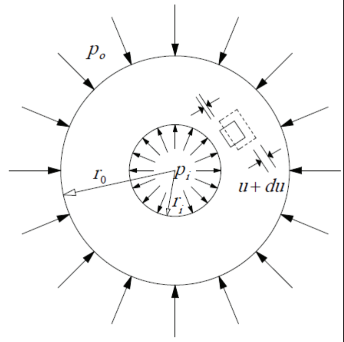
❑ Stresses in Thick-Walled Pressure Vessels with ends fixed

Consider an element in cylindrical coordinate



Thin & Thick Walled Vessels

❑ Stresses in Thick-Walled Pressure Vessels



Establish the static equilibrium equation for the element

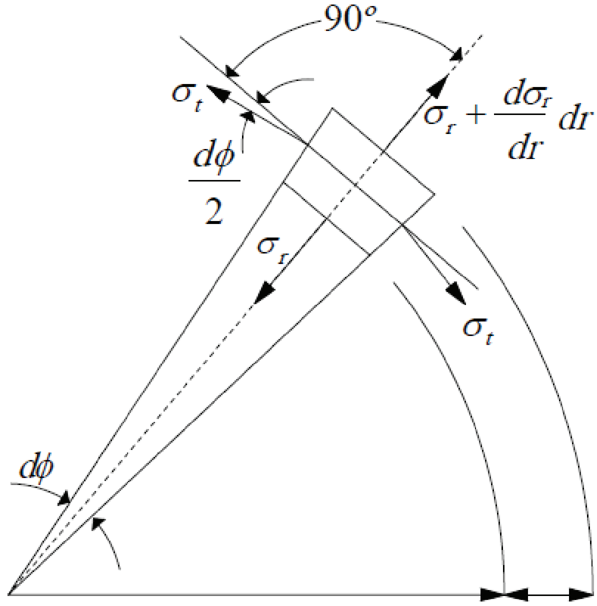
$$\sum F_r = 0 \quad \Rightarrow$$

$$\sigma_r(r\cdot d\phi\times 1)+2\sigma_t(dr\times 1)\sin(\frac{d\phi}{2})$$

$$-(\sigma_r+\frac{d\sigma_r}{dr}dr)[(r+dr)\cdot d\phi\times 1]=0$$

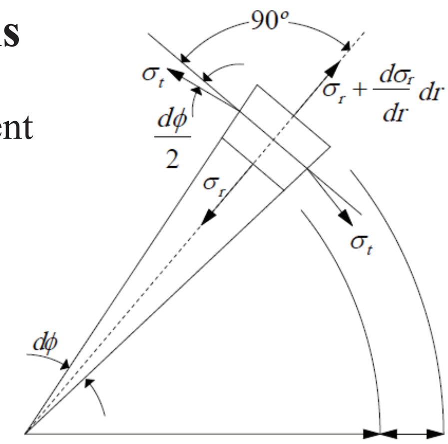
$$\Rightarrow \quad \sigma_r\cdot r\cdot d\phi+2\sigma_t\cdot dr\cdot\frac{d\phi}{2}$$

$$-(\sigma_r+\frac{d\sigma_r}{dr}dr)\cdot(r+dr)\cdot d\phi=0$$



Thin & Thick Walled Vessels

❑ Stresses in Thick-Walled Pressure Vessels



Establish the static equilibrium equation for the element

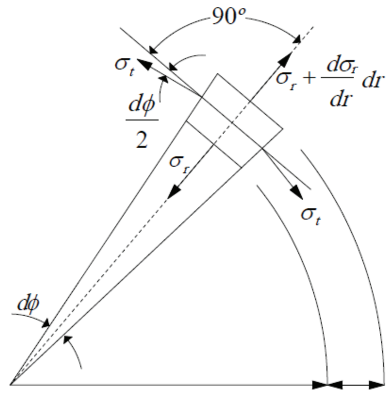
$$\Rightarrow \quad \sigma_r\cdot r\cdot d\phi+2\sigma_t\cdot dr\cdot\frac{d\phi}{2}$$

$$-(\sigma_r+\frac{d\sigma_r}{dr}dr)\cdot(r+dr)\cdot d\phi=0$$

$$if \quad dr^2\approx 0 \quad \& \quad dr d\phi\approx 0 \quad \Rightarrow \qquad \frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_t}{r}=0 \qquad (1)$$

$$\sigma_r\ ,\sigma_t=?$$

Thin & Thick Walled Vessels



❑ Stresses in Thick-Walled Pressure Vessels

Consider the consistent deformation relationship
u: radios displacement

A section with radius r $\Rightarrow$ $displacement = u$

A section with radius $r + dr$ $\Rightarrow$ $displacement = u + \dfrac{du}{dr}dr$

Radius strain:

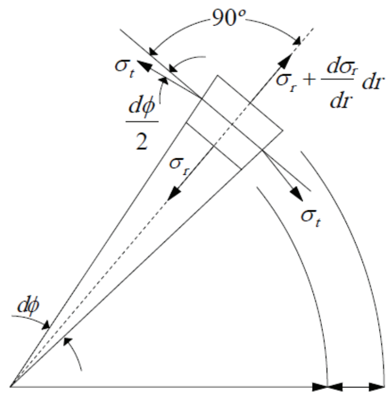
$$\varepsilon_r = \frac{\left(u + \dfrac{du}{dr}dr\right) - u}{dr} = \frac{du}{dr} \qquad (2)$$

Tangent strain:

$$\varepsilon_t = \frac{2\pi(r+u) - 2\pi r}{2\pi r} = \frac{u}{r} \qquad (3)$$

$$\Rightarrow \quad \varepsilon_r \ \& \ \varepsilon_t = f(u)$$

Thin & Thick Walled Vessels



❑ Stresses in Thick-Walled Pressure Vessels

Establish the Hooke's law

$$\varepsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_t - \nu\sigma_x) \qquad (4)$$

$$\varepsilon_t = \frac{1}{E}(\sigma_t - \nu\sigma_r - \nu\sigma_x) \qquad (5)$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_r - \nu\sigma_t) \qquad (6)$$

with end fixed $\Rightarrow$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_r - \nu\sigma_t) = 0$$

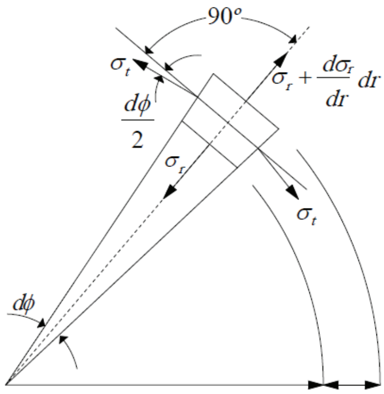
$$\Rightarrow \boxed{\sigma_x = \nu(\sigma_r + \sigma_t)} \qquad (7)$$

By substituting equation (7) in to (4) and (5):

$$\sigma_r = \frac{E}{(1+\nu)+(1-2\nu)}[(1-\nu)\varepsilon_r + \nu\varepsilon_t]$$

$$\sigma_t = \frac{E}{(1+\nu)+(1-2\nu)}[(1-\nu)\varepsilon_t + \nu\varepsilon_r] \qquad (8)$$

Thin & Thick Walled Vessels



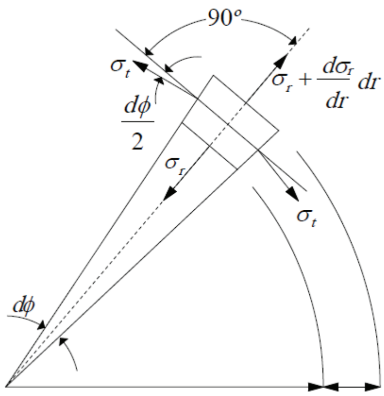
❑ Stresses in Thick-Walled Pressure Vessels

Using equation (2) and (3): $\varepsilon_r = \frac{du}{dr}$ $\varepsilon_t = \frac{u}{r}$

(8) $\Rightarrow$

$$\sigma_r = \frac{E}{(1+\nu)+(1-2\nu)}[(1-\nu)\frac{du}{dr}+\nu\frac{u}{r}]$$
$$\sigma_t = \frac{E}{(1+\nu)+(1-2\nu)}[(1-\nu)\frac{u}{r}+\nu\frac{du}{dr}] \quad (9)$$

Thin & Thick Walled Vessels



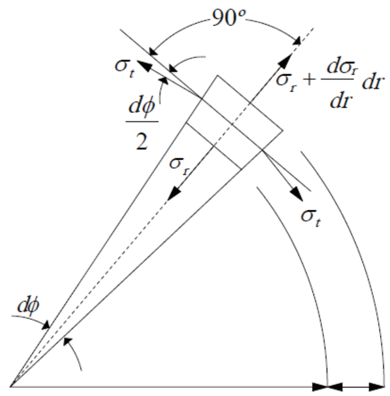
❑ Stresses in Thick-Walled Pressure Vessels

By substituting equation (9)
in to (1) :

$\Rightarrow$ ***Differentiate equation***

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}=0 \quad (10)$$

Thin & Thick Walled Vessels



❑ Stresses in Thick-Walled Pressure Vessels

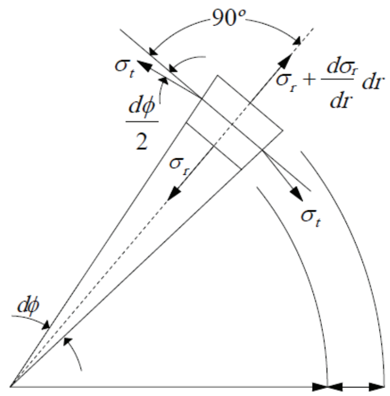
$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0$$

Answer is: $u = A_1 r + \frac{A_2}{r}$ $\Rightarrow$ $\frac{du}{dr} = A_1 - \frac{A_2}{r^2}$

Using consistent conditions:

$$r = r_i \;\Rightarrow\; \sigma_r = -p_i$$
$$r = r_o \;\Rightarrow\; \sigma_r = -p_o$$

Thin & Thick Walled Vessels



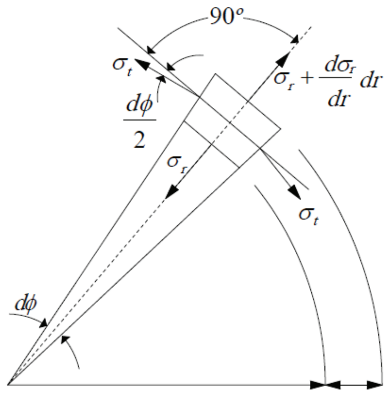
❑ Stresses in Thick-Walled Pressure Vessels

$$\frac{du}{dr} = A_1 - \frac{A_2}{r^2}$$

$$if \quad r = r_i \;\Rightarrow \sigma_r = \frac{E}{(1+\nu)+(1-2\nu)}[A_1 - (1-2\nu)\frac{A_2}{r_i^2}] = -p_i$$
$$if \quad r = r_o \;\Rightarrow \sigma_r = \frac{E}{(1+\nu)+(1-2\nu)}[A_1 - (1-2\nu)\frac{A_2}{r_o^2}] = -p_o \qquad (10)$$

Thin & Thick Walled Vessels

❑ Stresses in Thick-Walled Pressure Vessels

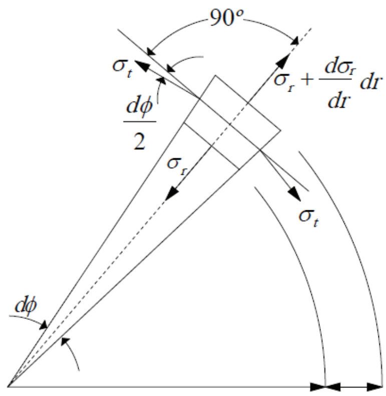


(10) $\Rightarrow$

$$A_1 = \frac{(1+\nu)+(1-2\nu)}{E}\frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$
$$A_2 = \frac{(1+\nu)}{E}\frac{(p_i - p_o) r_i^2 r_o^2}{r_o^2 - r_i^2} \qquad (11)$$

Thin & Thick Walled Vessels

❑ Stresses in Thick-Walled Pressure Vessels



$$\sigma_r = C_1 - \frac{C_2}{r^2} \quad , \quad \sigma_t = C_1 + \frac{C_2}{r^2}$$

$$C_1 = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} \qquad C_2 = \frac{(p_i - p_o) r_i^2 r_o^2}{r_o^2 - r_i^2}$$

$$\sigma_r + \sigma_t = cte \quad \Rightarrow \quad \sigma_x = \nu(\sigma_r + \sigma_t) = cte$$

❑ Stresses in Thick-Walled Pressure Vessels

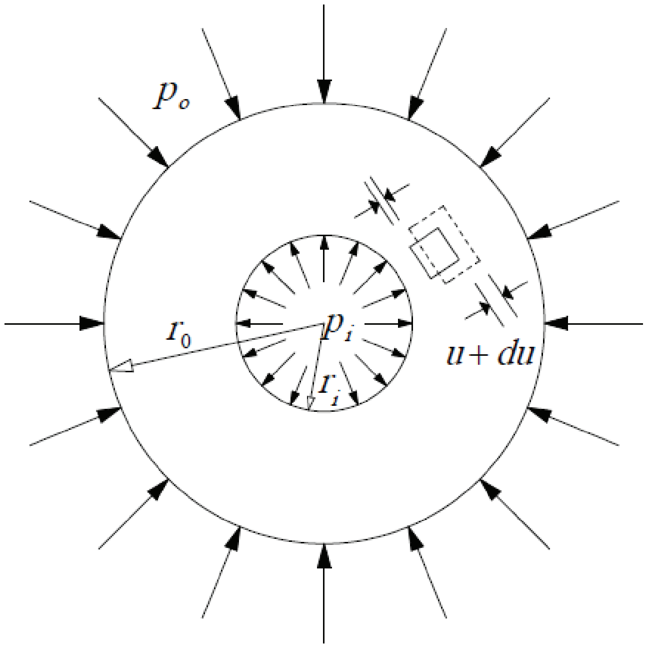
Example 3

Determine the tangent stress distribution in cylindrical vessel under p_i pressure if :

$a)\ r_o = 1.1 r_i$

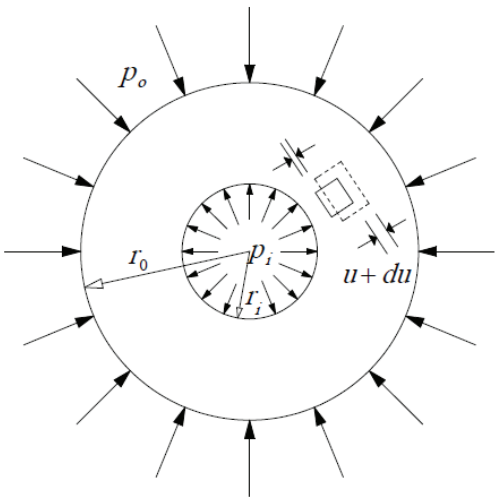
$b)\ r_o = 4 r_i$

Compare the results with thin wall pressure formula

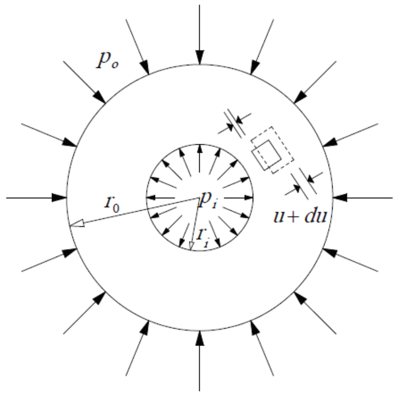


❑ Stresses in Thick-Walled Pressure Vessels

Example 3



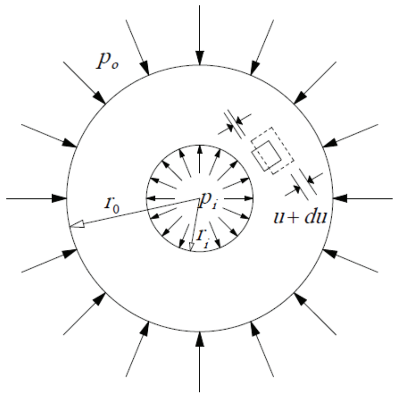
❑ Stresses in Thick-Walled Pressure Vessels



Example 3

Thin & Thick Walled Vessels

❑ Stresses in Thick-Walled Pressure Vessels



Example 3