Mechanics of Materials

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Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:
J.Wat Oler "Lectures notes on Mechanics od Materials"
Ibrahim A.Assakkaf "Lectures notes on Mechanics od Materials"

Pure Bending

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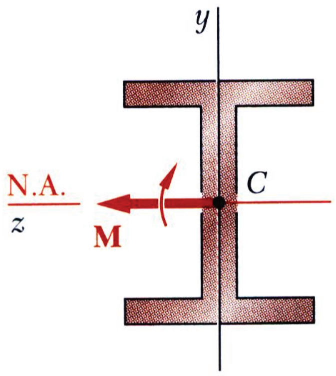
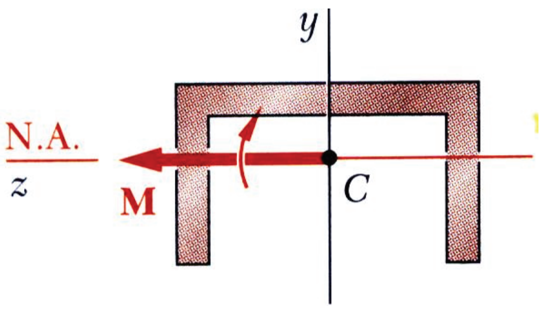
Pure Bending

- ***Symmetric Bending***

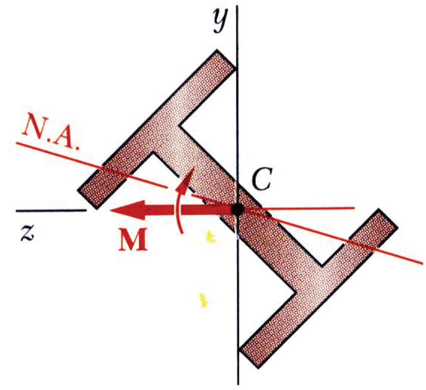




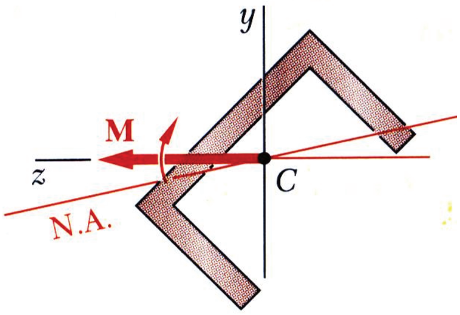
- Analysis of pure bending has been limited to members subjected to bending couples acting in a plane of symmetry.
- Members remain symmetric and bend in the plane of symmetry.
- The neutral axis of the cross section coincides with the axis of the couple

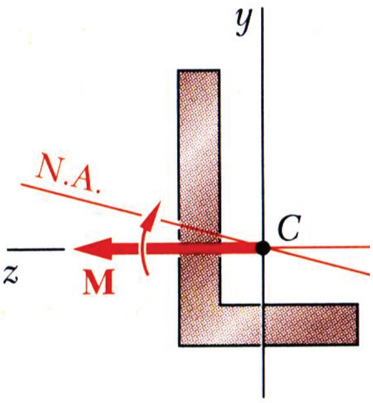
Pure Bending

❑ ***Unsymmetric Bending***



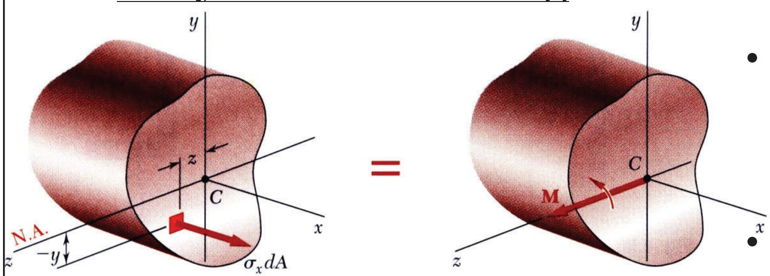
- Will now consider situations in which the bending couples do not act in a plane of symmetry.
- Cannot assume that the member will bend in the plane of the couples.
- In general, the neutral axis of the section will not coincide with the axis of the couple.





Pure Bending

❑ ***Unsymmetric Bending***



Wish to determine the conditions under which the neutral axis of a cross section of arbitrary shape coincides with the axis of the couple as shown.

- The resultant force and moment from the distribution of elementary forces in the section must satisfy

$$F_x = M_y = 0 \quad M_z = M = applied\ couple$$

- $$F_x = \int \sigma_x dA = 0 \quad \Rightarrow \quad \int y\, dA = 0$$

neutral axis passes through centroid

- $$M_z = -\int y\sigma_x dA = M \quad \Rightarrow \quad \sigma_x = -\frac{My}{I}$$

defines stress distribution

- $$M_y = \int z\sigma_x dA = 0 \quad \Rightarrow \quad \int z\left(-\frac{y}{c}\sigma_m\right)dA$$

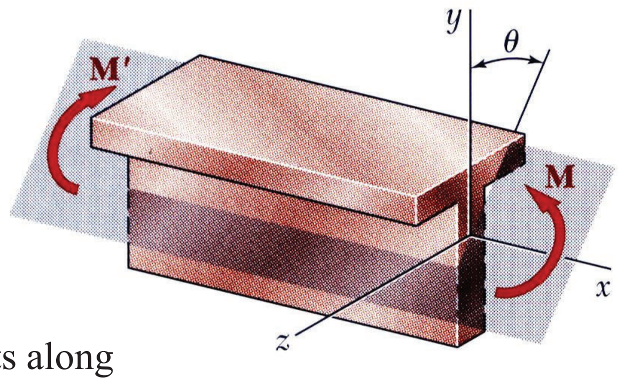
$$\Rightarrow \int yz\, dA = I_{yz} = product\ of\ inertia = 0$$

If one of y or z axis is principle then I_{yz}=0, so couple vector must be directed along a principal centroidal axis

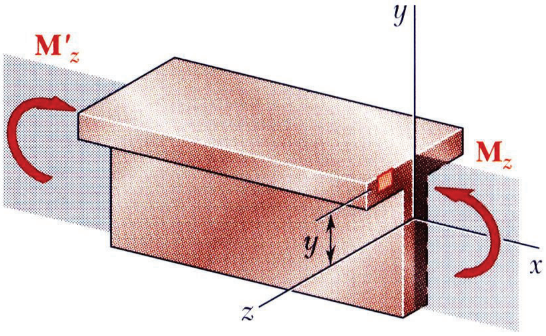
Pure Bending

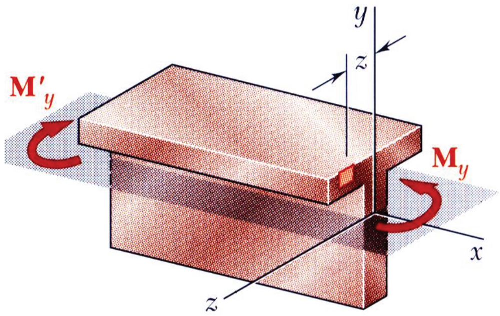
❑ ***Unsymmetric Bending***

Superposition is applied to determine stresses in the most general case of unsymmetric bending.



- Resolve the couple vector into components along the principle centroidal axes.





$$M_z = M\cos\theta$$

$$M_y = M\sin\theta$$

Pure Bending

❑ ***Unsymmetric Bending***

- **Superpose the component stress distributions**

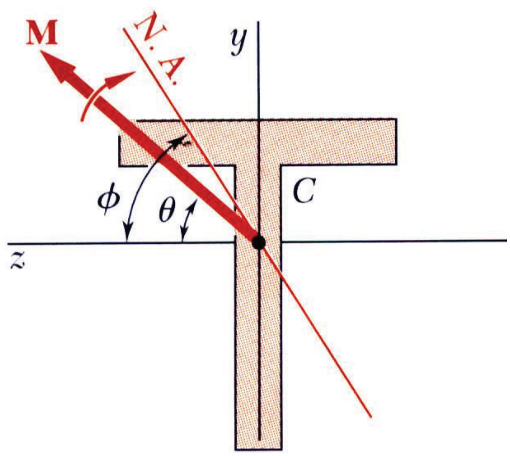
$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- **Along the neutral axis,**

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} = 0 \quad \Rightarrow$$

$$-\frac{(M\cos\theta)y}{I_z} + \frac{(M\sin\theta)z}{I_y} = 0$$

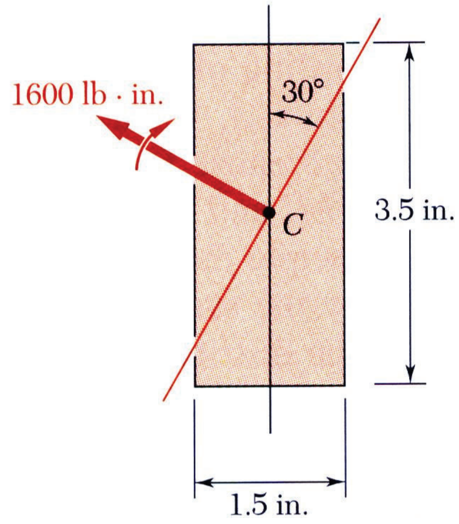
$$\Rightarrow \tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$



Pure Bending

Example 1

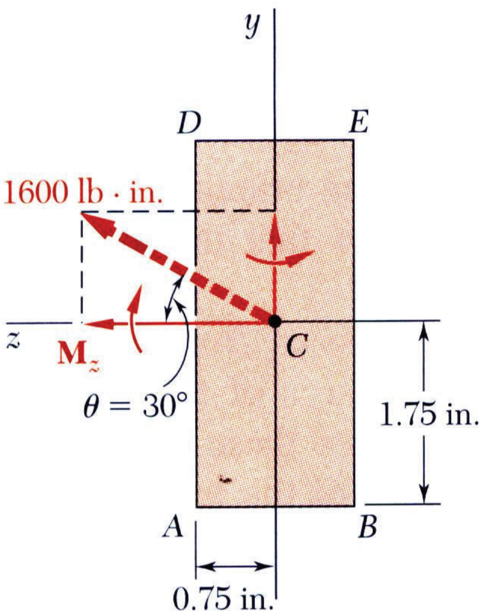
A 1600 lb-in couple is applied to a rectangular wooden beam in a plane forming an angle of 30 deg. with the vertical. Determine (a) the maximum stress in the beam, (b) the angle that the neutral axis forms with the horizontal plane.



Pure Bending

Example 1

- Resolve the couple vector into components and calculate the corresponding maximum stresses.

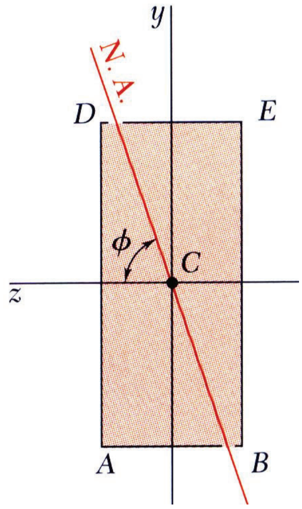


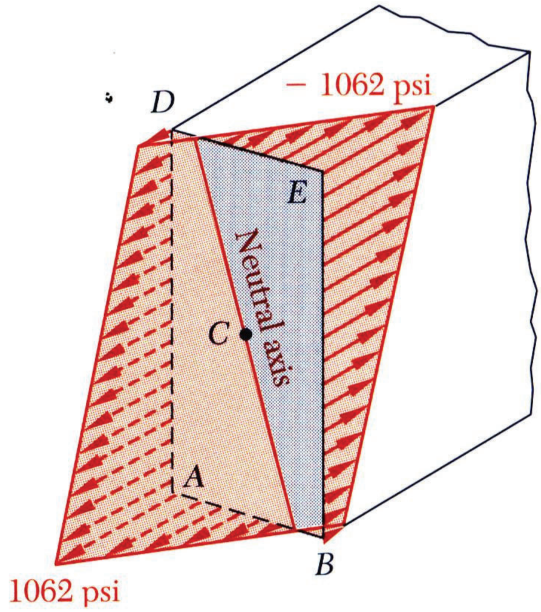
Pure Bending

Example 1

- The largest tensile stress due to the combined loading occurs at *A*.

- Determine the angle of the neutral axis.

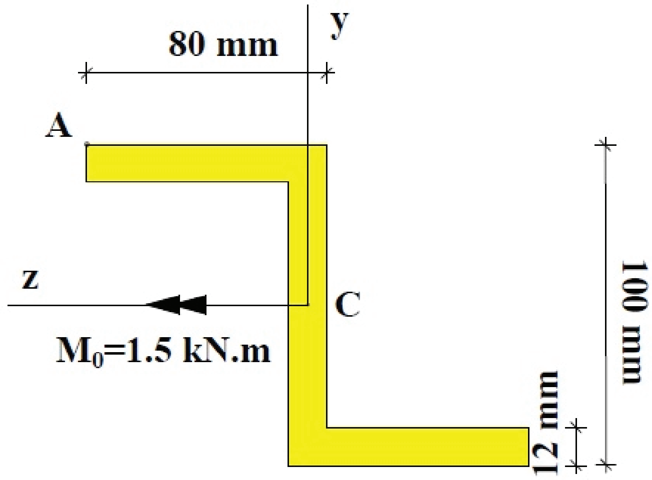




Pure Bending

Example 2

A couple of magnitude $M_0 = 1.5\ kN.m$ acting in a vertical plane is applied to a beam having the Z-shaped cross section shown. Determine (a) the stress at point A, (b) the angle that the neutral axis forms with the horizontal plane. The moments and product of inertia of the section with respect to the y and z axes have been computed and are as follows:



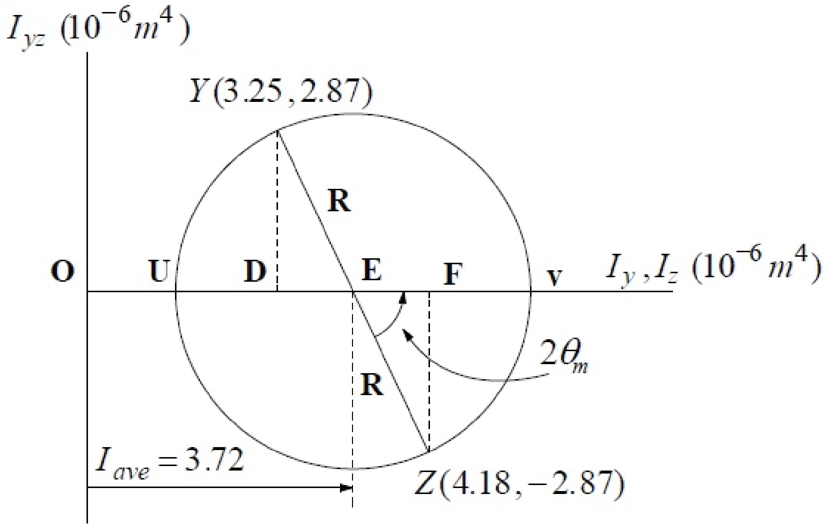
$$I_y = 3.25\times10^{-6}\,m^4$$

$$I_z = 4.18\times10^{-6}\,m^4$$

$$I_{yz} = 2.87\times10^{-6}\,m^4$$

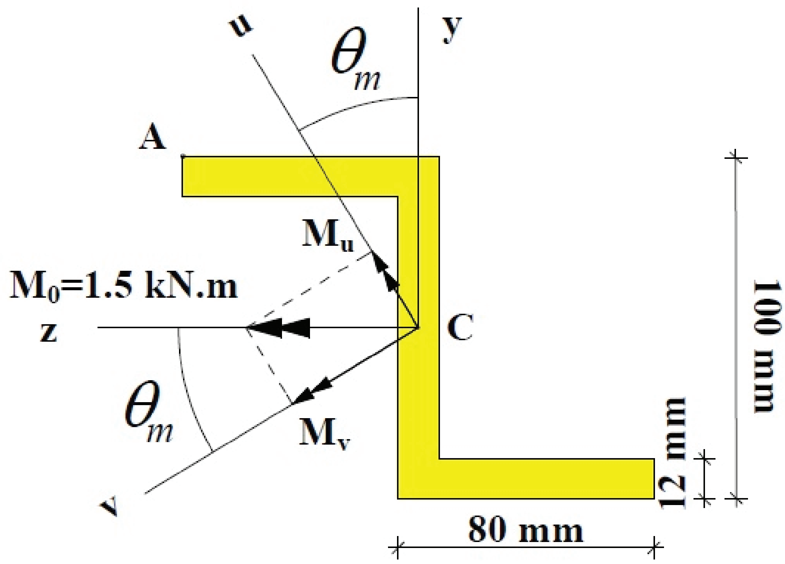
Pure Bending

Example 2



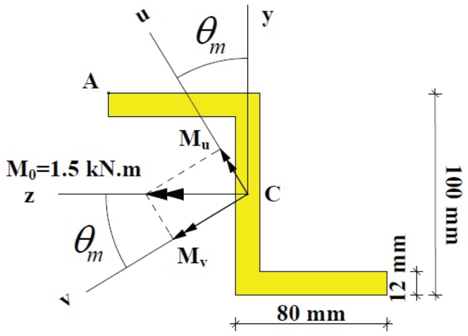
Pure Bending

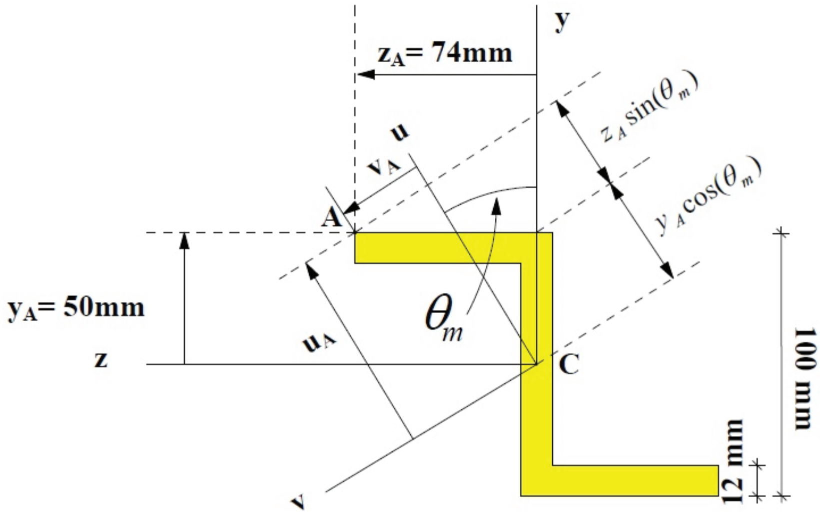
Example 2



Pure Bending

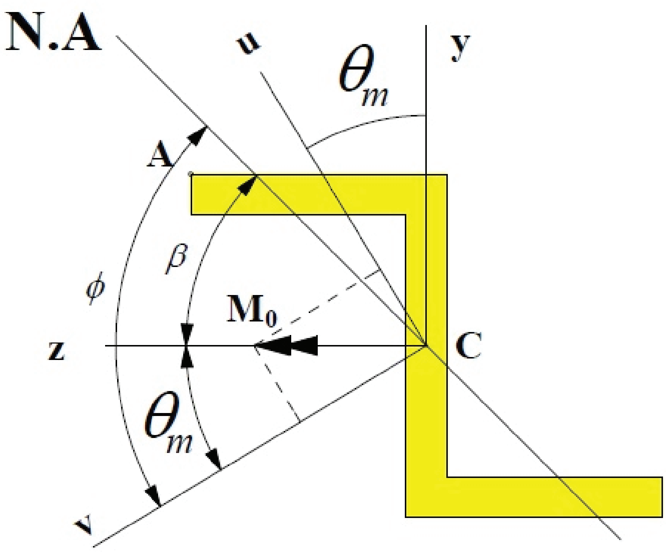
Example 2





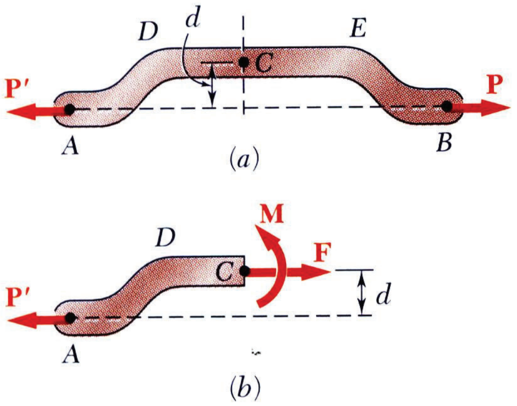
Pure Bending

Example 2



Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry



- Stress due to eccentric loading found by superposing the ***uniform stress due to a centric load*** and ***linear stress distribution due a pure bending moment***

$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$

- Eccentric loading

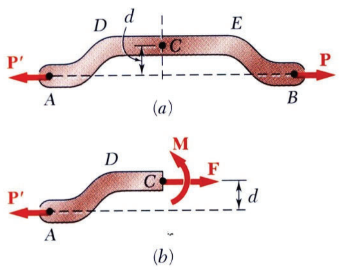
$$\sum F_x = 0 \quad \Rightarrow \quad F = P$$

$$\sum M_{/c} = 0 \quad \Rightarrow \quad M = Pd$$

$$\Rightarrow \quad \sigma_x = \frac{P}{A} - \frac{My}{I}$$

Loading combination

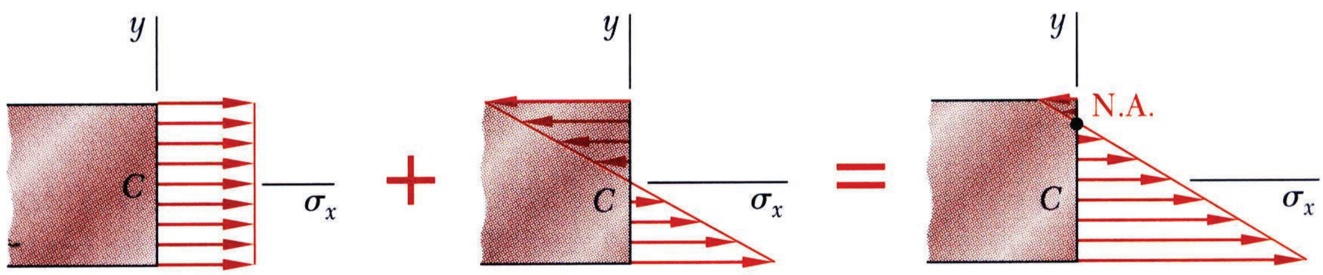
❑ Eccentric Axial Loading in a Plane of Symmetry



$$\sigma_x = \frac{P}{A} - \frac{My}{I}$$

Validity requires:

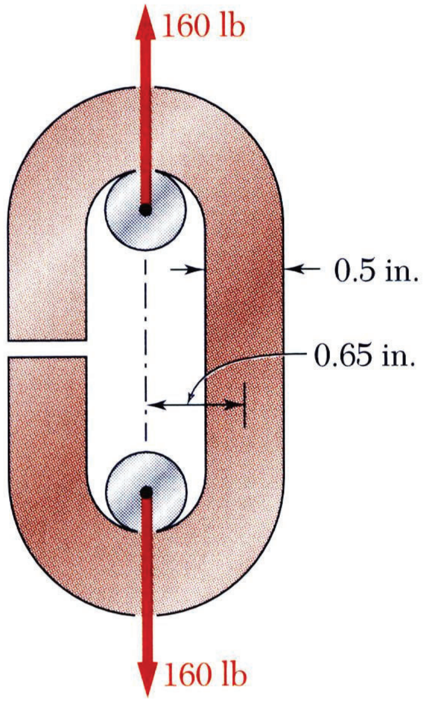
- Stresses below proportional limit
- Deformations have negligible effect on geometry
- Stresses not evaluated near points of load application.



Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 3

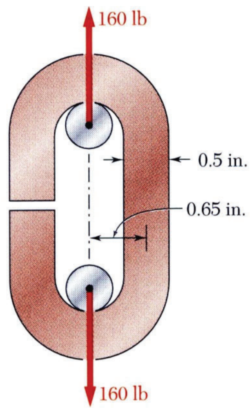


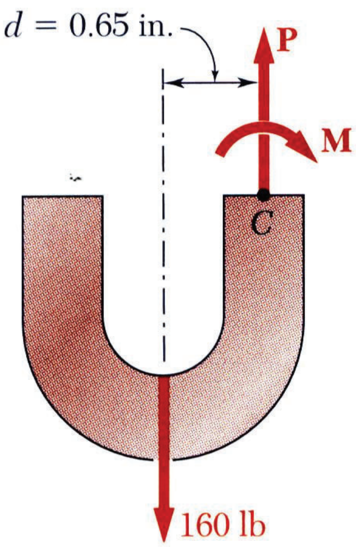
An open-link chain is obtained by bending low-carbon steel rods into the shape shown. For 160 lb load, determine (a) maximum tensile and compressive stresses, (b) distance between section centroid and neutral axis

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 3

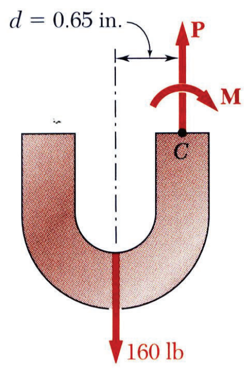




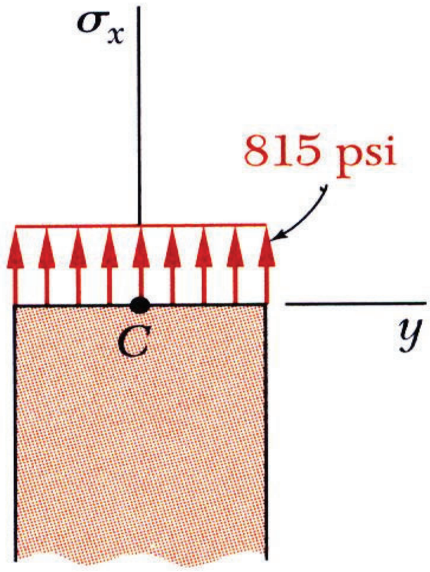
- Equivalent centric load and bending moment

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry



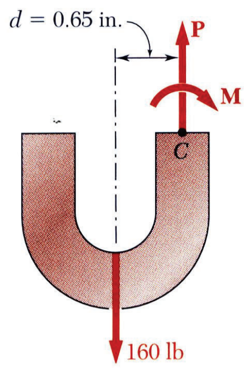
Example 3



- Normal stress due to a centric load

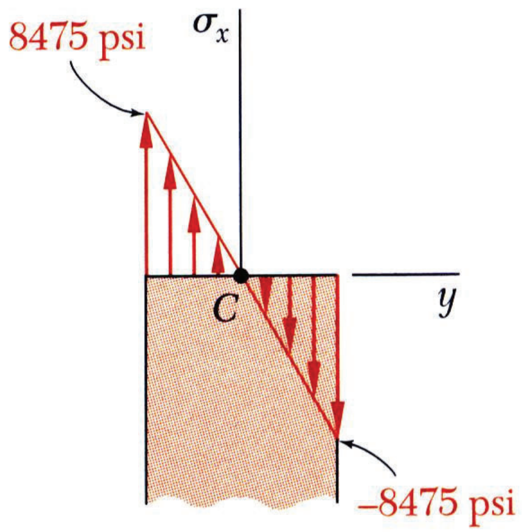
Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

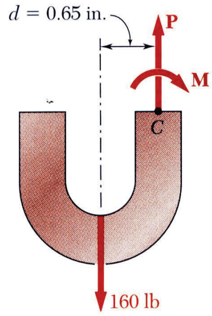


Example 3

- Normal stress due to bending moment

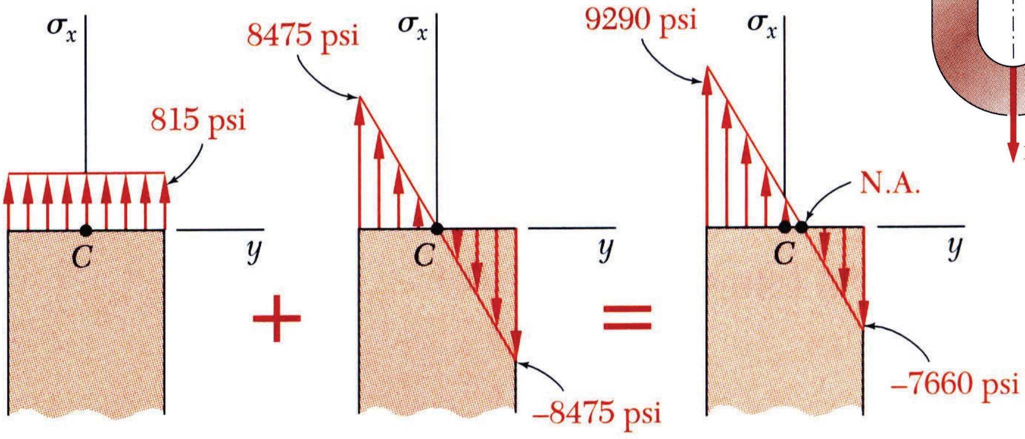


Loading combination



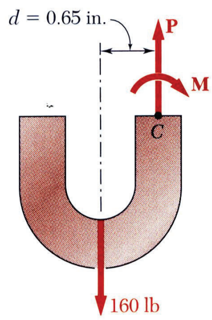
❑ Eccentric Axial Loading in a Plane of Symmetry

Example 3



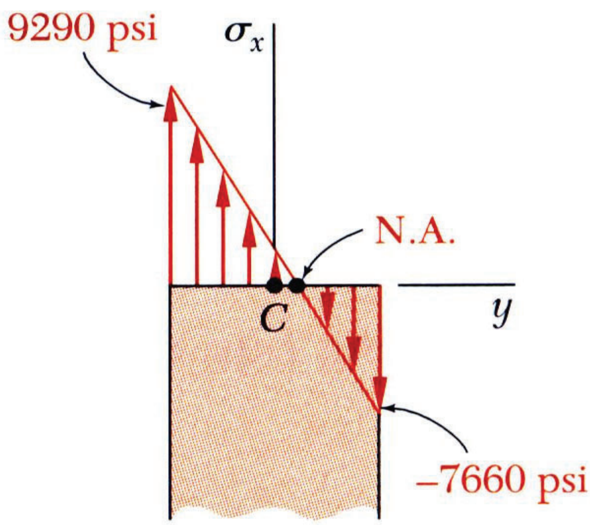
- Maximum tensile and compressive stresses

Loading combination



❑ Eccentric Axial Loading in a Plane of Symmetry

Example 3

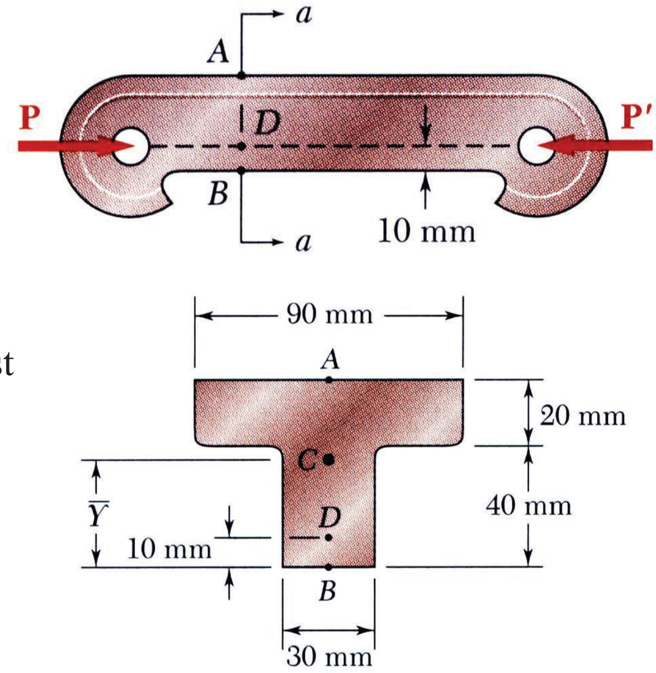


- Neutral axis location

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 4



Section *a–a*

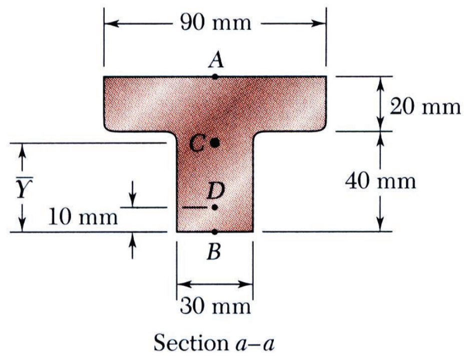
The largest allowable stresses for the cast iron link are 30 MPa in tension and 120 MPa in compression. Determine the largest force P which can be applied to the link.

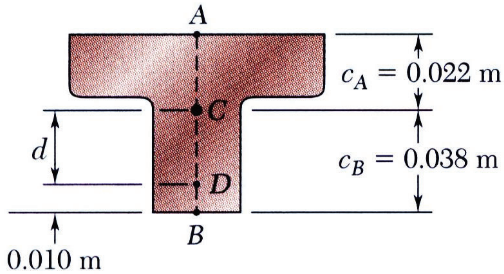
Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 4

Parts	A_i	$\bar{y}_i$	$A_i\bar{y}_i$	$A_i\bar{y}_i^2$	I_{g_i}
1					
2					

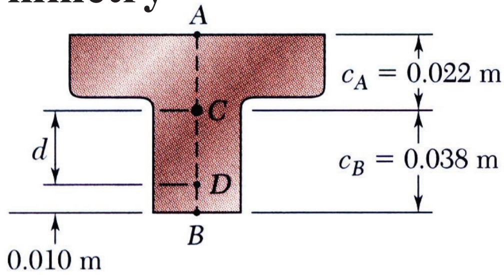




Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 4



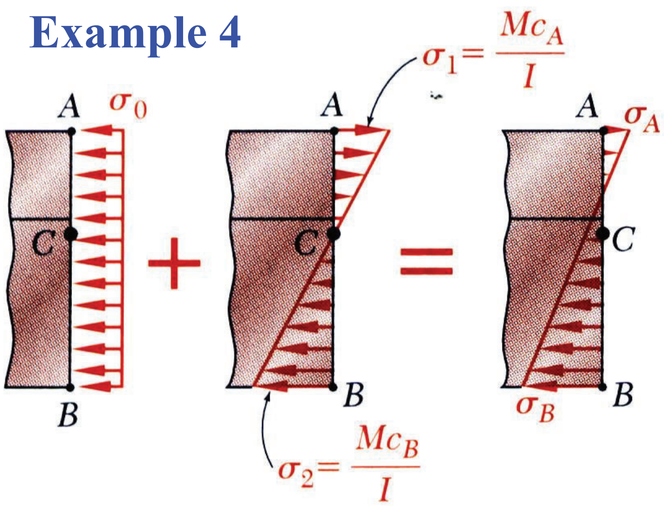
- Determine an equivalent centric and bending loads.

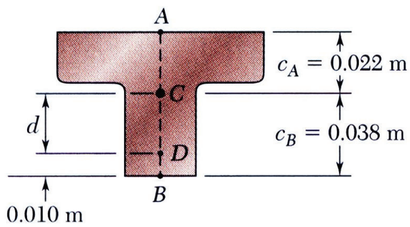
- Superpose stresses due to centric and bending loads

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 4



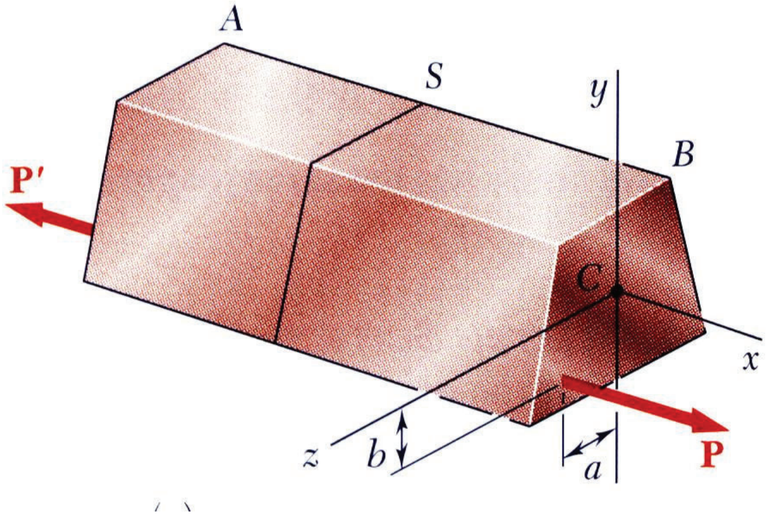


- Evaluate critical loads for allowable stresses.

- The largest allowable load

Loading combination

❑ General Case of Eccentric Axial Loading



- Consider a straight member subject to equal and opposite eccentric forces.

$$P = \text{Centric force}$$

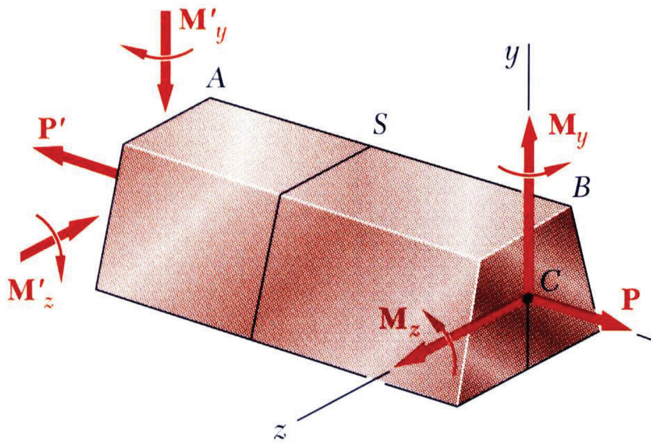
- The eccentric force is equivalent to the system of a centric force and two couples.

$$M_y = Pa$$

$$M_z = Pb$$

Loading combination

❑ General Case of Eccentric Axial Loading



- By the principle of superposition, the combined stress distribution is

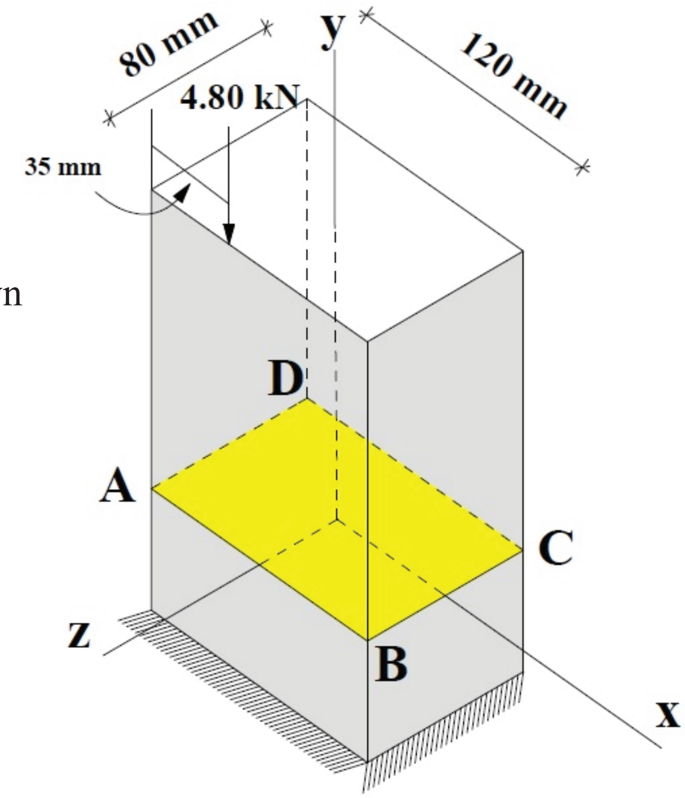
$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- If the neutral axis lies on the section, it may be found from

$$\frac{M_z}{I_z} y - \frac{M_y}{I_y} z = \frac{P}{A}$$

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5

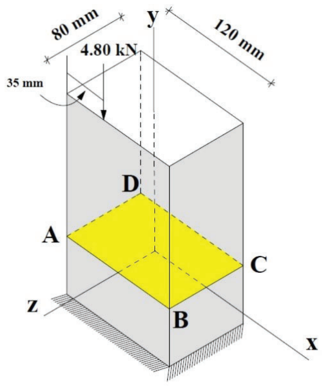


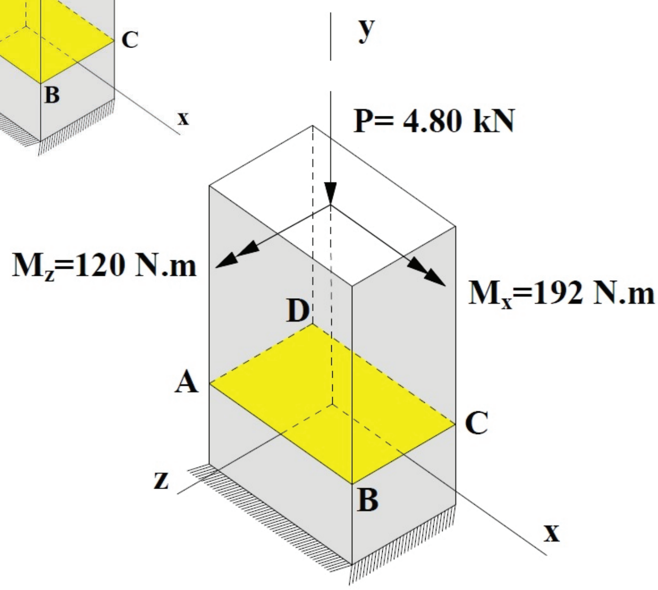
A vertical 4.80 kN load is applied as shown on a wooden post of rectangular cross section, 80 by 120 mm. (a) Determine the stress at points A, B, C, and D. (b) Locate the neutral axis of the cross section.

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5

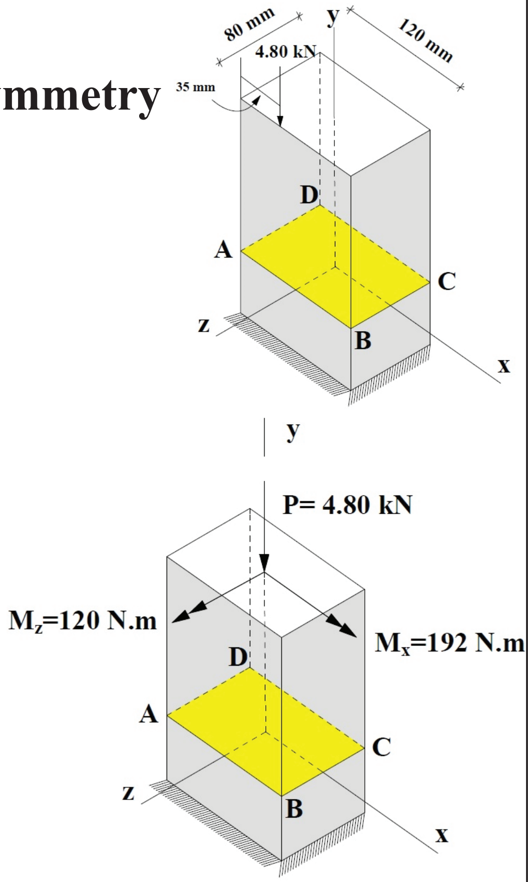




Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

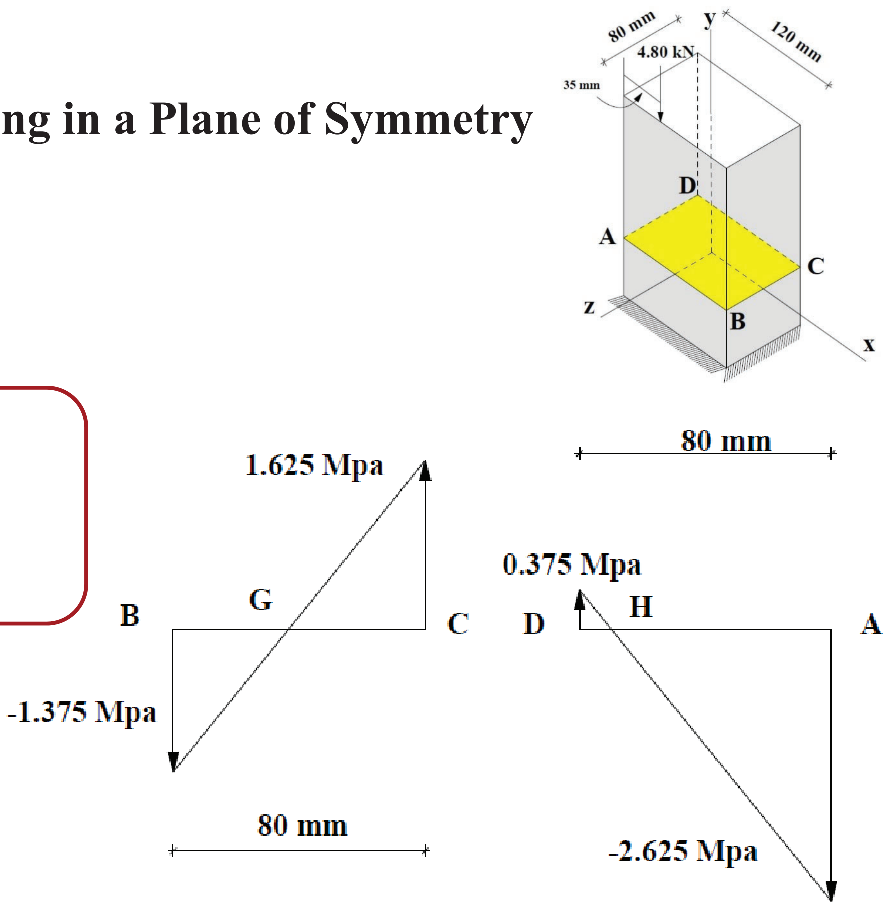
Example 5



Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5

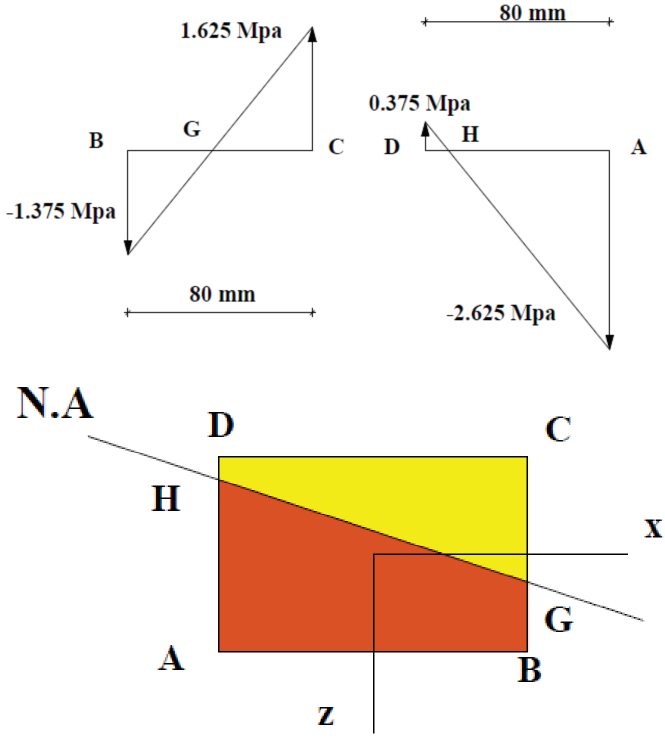


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5

Neutral Axis

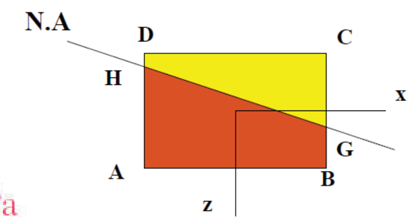


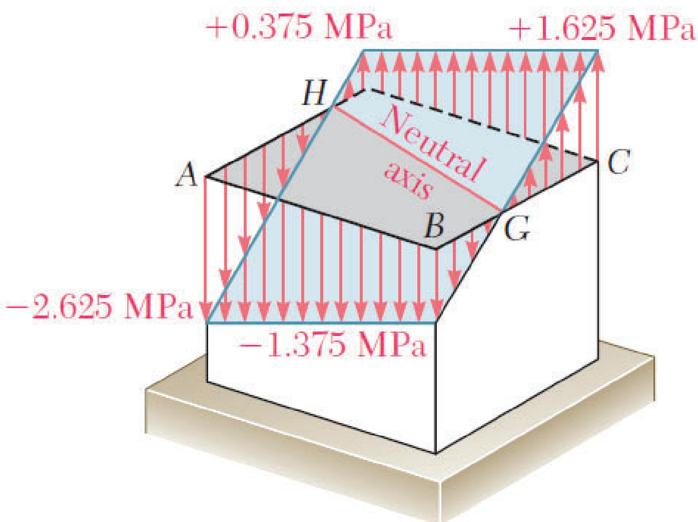
Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5

Neutral Axis



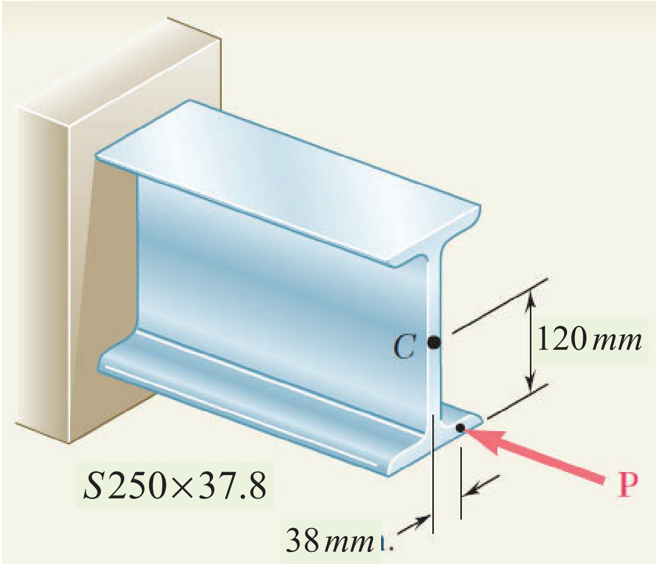


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 6

A horizontal load P is applied as shown to a short section of an S250 X 37.8 rolled-steel member. Knowing that the compressive stress in the member is not to exceed 82 MPa, determine the largest permissible load P.



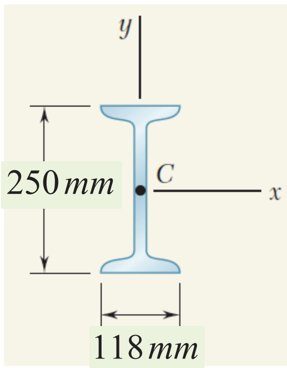
$A = 4820 mm^2$

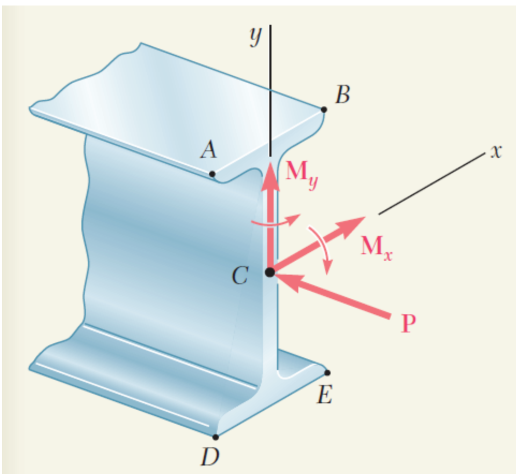
$S_x = 402\times10^3\ mm^3 \quad S_y = 47.5\times10^3\ mm^3$

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 6

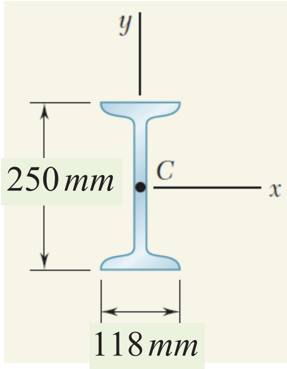


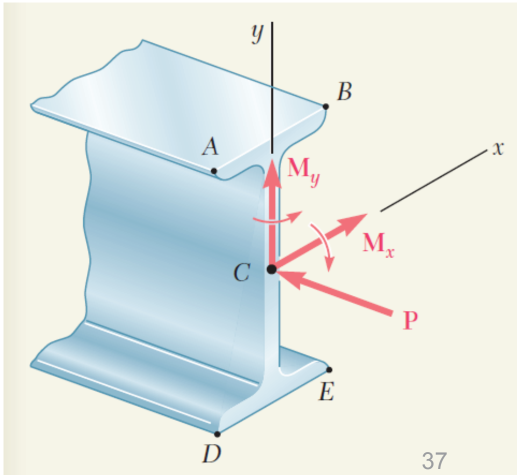


Loading combination

- ❑ **Eccentric Axial Loading in a Plane of Symmetry**

Example 6





Loading combination

- ❑ **Eccentric Axial Loading in a Plane of Symmetry**

Example 6