Mechanics of Materials

Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:
J.Wat Oler "Lectures notes on Mechanics od Materials"
Ibrahim A.Assakkaf "Lectures notes on Mechanics od Materials"

Loading combination

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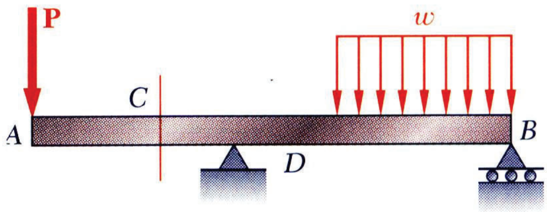
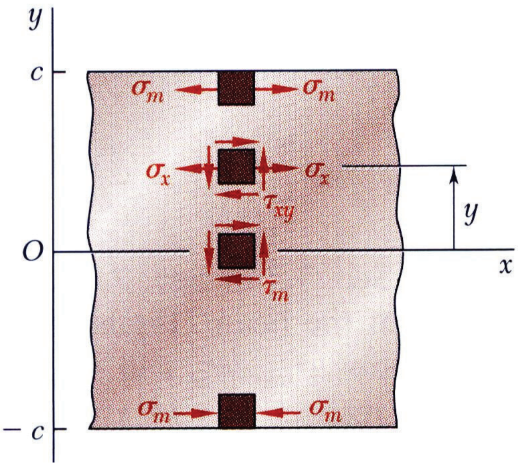
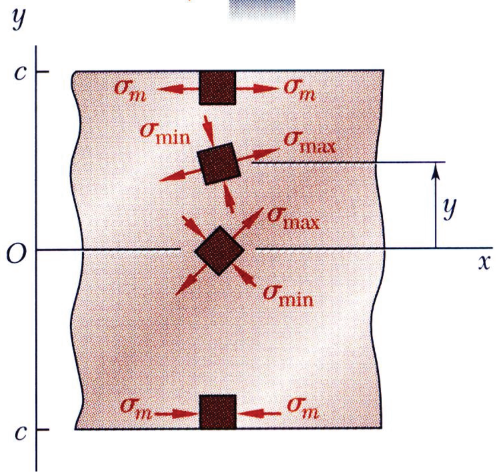
❑ Principle Stresses in a Beam

- Prismatic beam subjected to transverse loading







$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{Mc}{I}$$
$$\tau_{xy} = -\frac{VQ}{It} \qquad \tau_m = \frac{VQ}{It}$$

- The maximum normal stress within the cross-section can be larger than

$$\sigma_m = \frac{Mc}{I}$$

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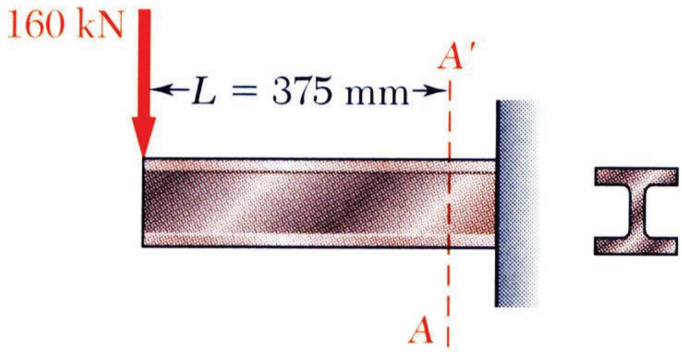
❑ Principle Stresses in a Beam

Example 1

A 160-kN force is applied at the end of a W200x52 rolled-steel beam.

Neglecting the effects of fillets and of stress concentrations, determine whether the normal stresses satisfy a design specification that they be equal to or less than 150 MPa at section *A-A'*.



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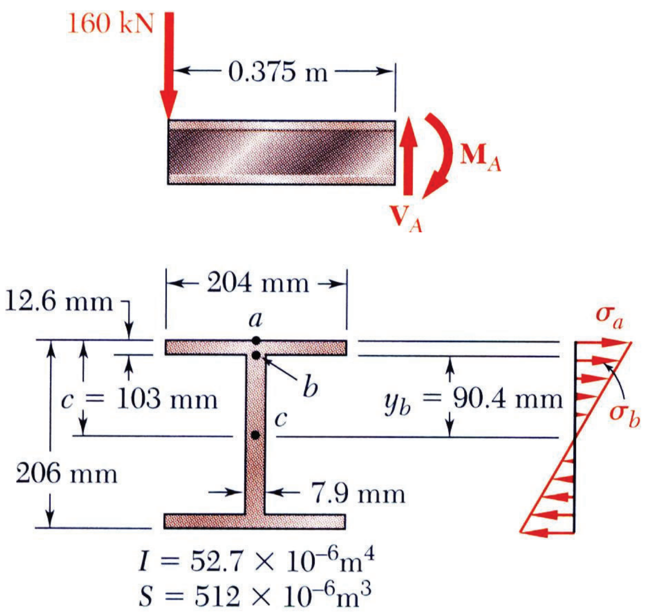
❑ Principle Stresses in a Beam

Example 1



SOLUTION:

- Determine shear and bending moment in Section *A-A'*
- Calculate the normal stress at top surface and at flange-web junction.

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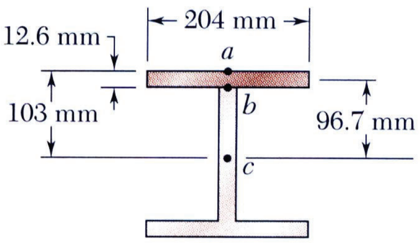
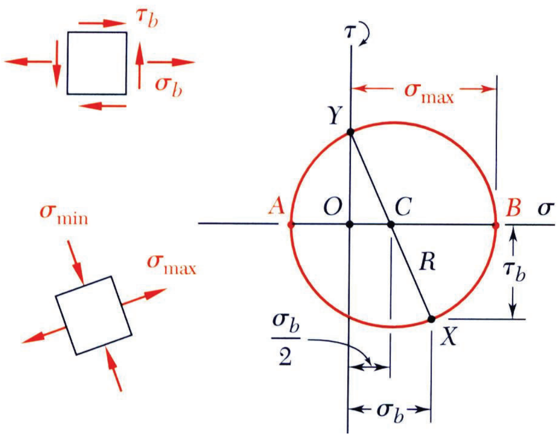
❑ Principle Stresses in a Beam

Example 1

- Evaluate shear stress at flange-web junction.





- Calculate the principal stress at flange-web junction

Design specification is not satisfied.

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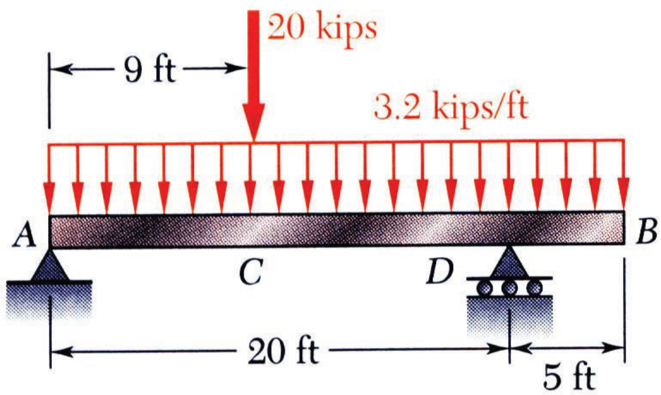
❑ Principle Stresses in a Beam

Example 2

The overhanging beam supports a uniformly distributed load and a concentrated load. Knowing that for the grade of steel to used

$\sigma_{all} = 24ksi$ and $\tau_{all} = 14.5ksi$

select the wide-flange beam which should be used.



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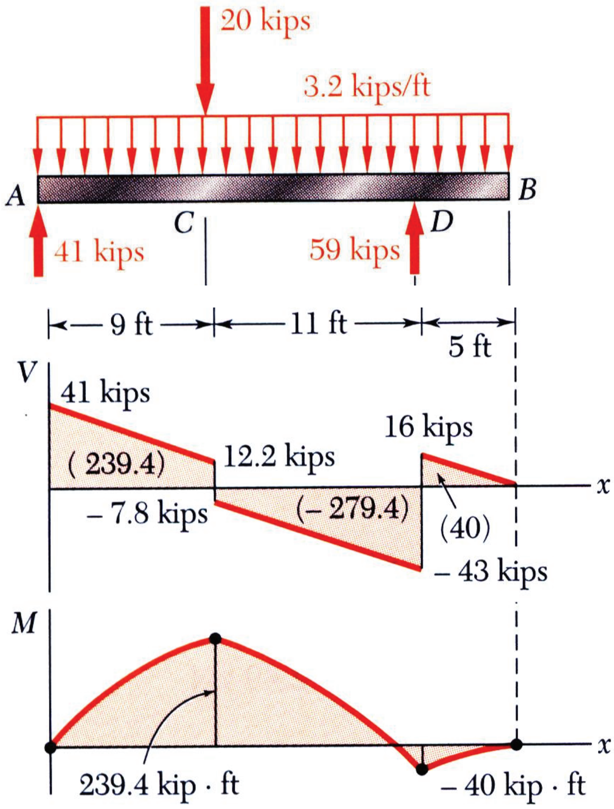
❑ Principle Stresses in a Beam

Example 2

SOLUTION:

- Determine reactions at *A* and *D.*

- Determine maximum shear and bending moment from shear and bending moment diagrams.



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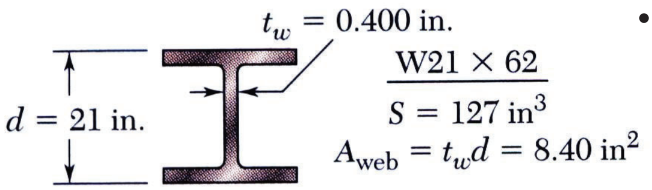
❑ Principle Stresses in a Beam

Example 2

Shape	*S* (in³)
W24 × 68	154
W21 × 62	127
W18 × 76	146
W16 × 77	134
W14 × 82	123
W12 × 96	131

- Calculate required section modulus and select appropriate beam section.



- Find maximum shearing stress.

  Assuming uniform shearing stress in web,

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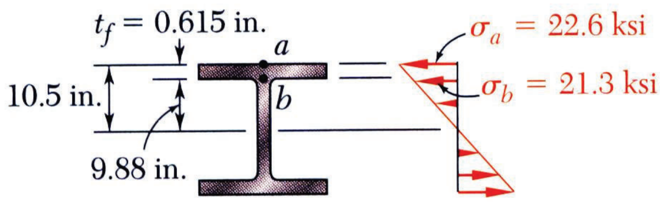
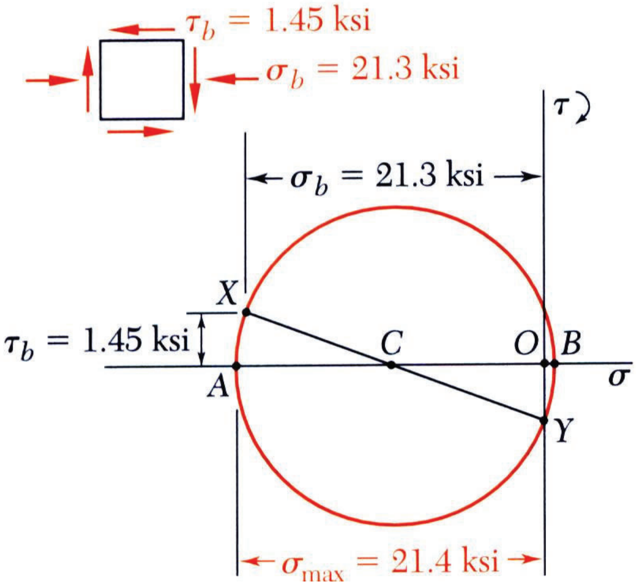
❑ Principle Stresses in a Beam

Example 2

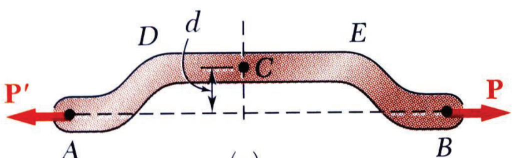
- Find maximum normal stress.



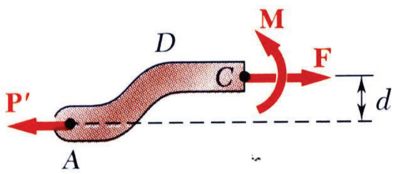


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry



(*a*)



(*b*)

- Eccentric loading

$$\sum F_x = 0 \quad \Rightarrow \quad F = P$$
$$\sum M_{/c} = 0 \quad \Rightarrow \quad M = Pd$$

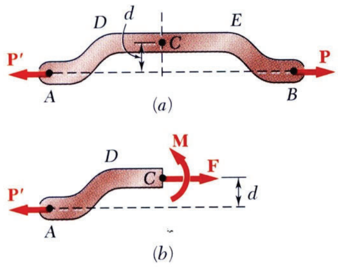
- Stress due to eccentric loading found by superposing the ***uniform stress due to a centric load*** and ***linear stress distribution due a pure bending moment***

$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$

$$\Rightarrow \quad \sigma_x = \frac{P}{A} - \frac{My}{I}$$

Loading combination

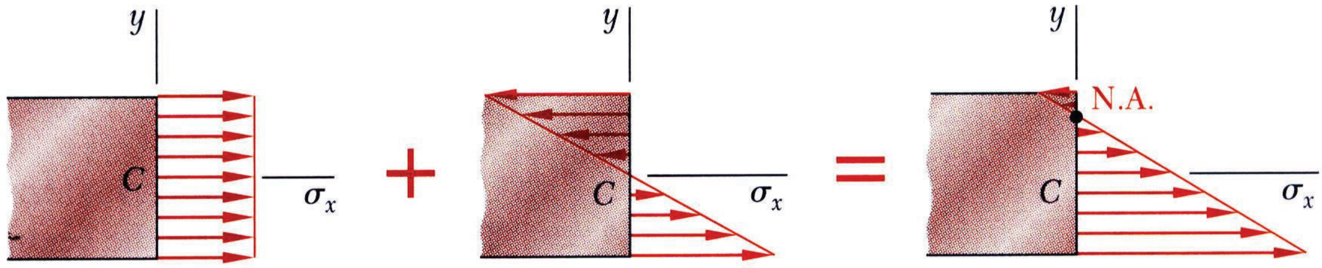
❑ Eccentric Axial Loading in a Plane of Symmetry



$$\sigma_x = \frac{P}{A} - \frac{My}{I}$$

Validity requires:

- Stresses below proportional limit
- Deformations have negligible effect on geometry
- Stresses not evaluated near points of load application.

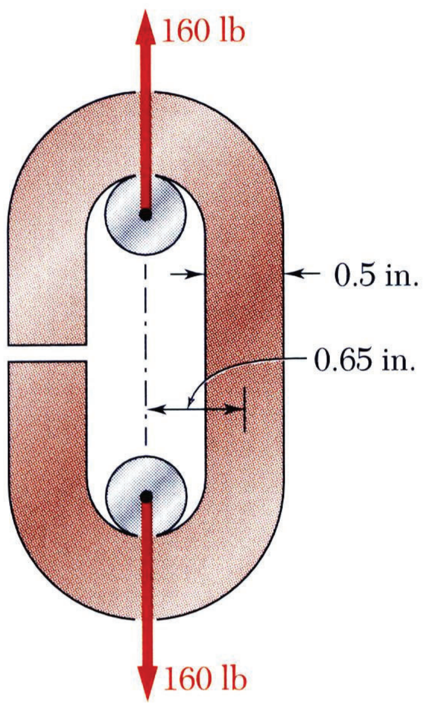


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

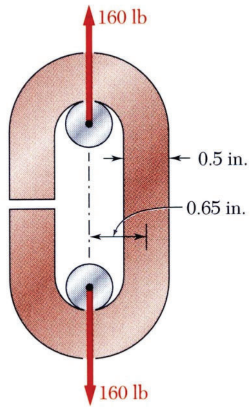
Example 3

An open-link chain is obtained by bending low-carbon steel rods into the shape shown. For 160 lb load, determine (a) maximum tensile and compressive stresses, (b) distance between section centroid and neutral axis

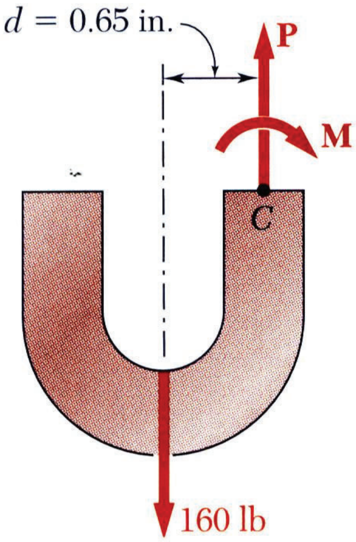


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry



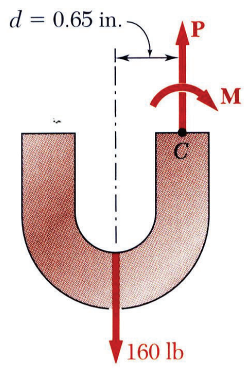
Example 3



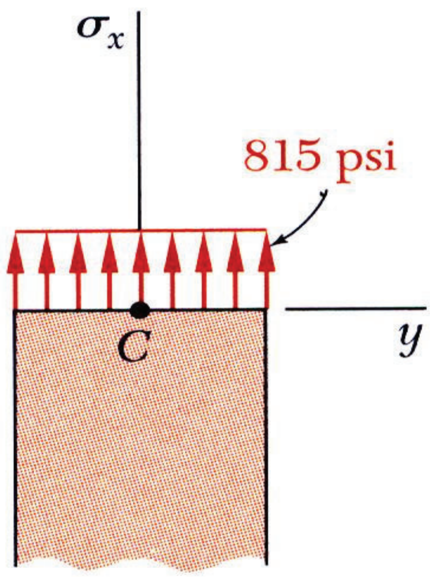
- Equivalent centric load and bending moment

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry



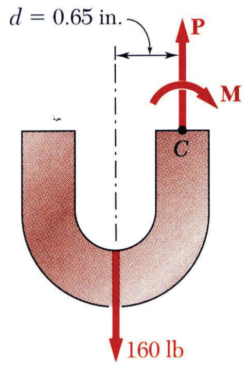
Example 3



- Normal stress due to a centric load

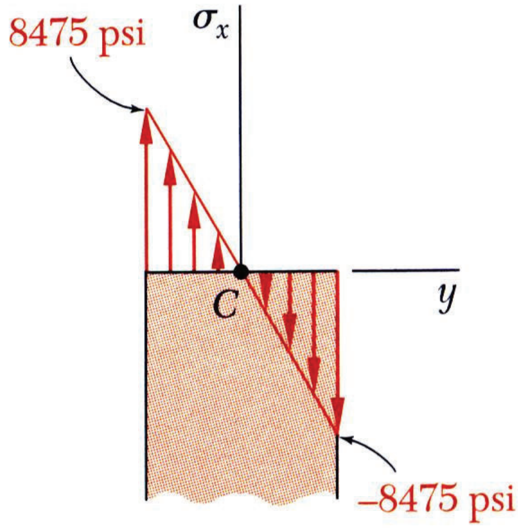
Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry



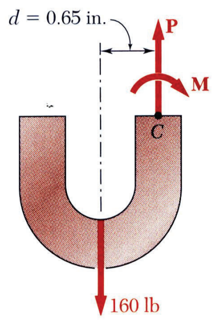
Example 3

- Normal stress due to bending moment

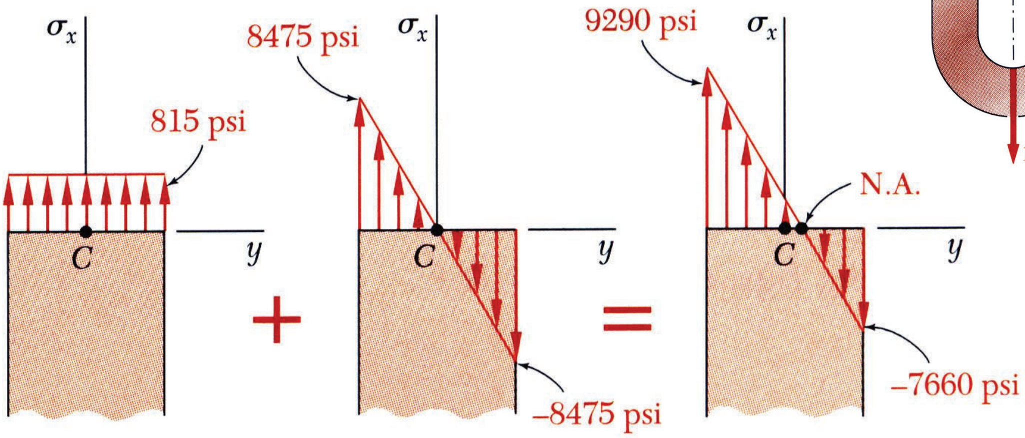


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry



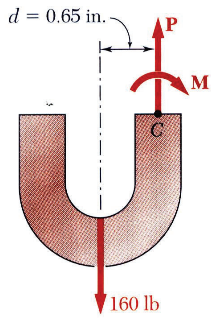
Example 3



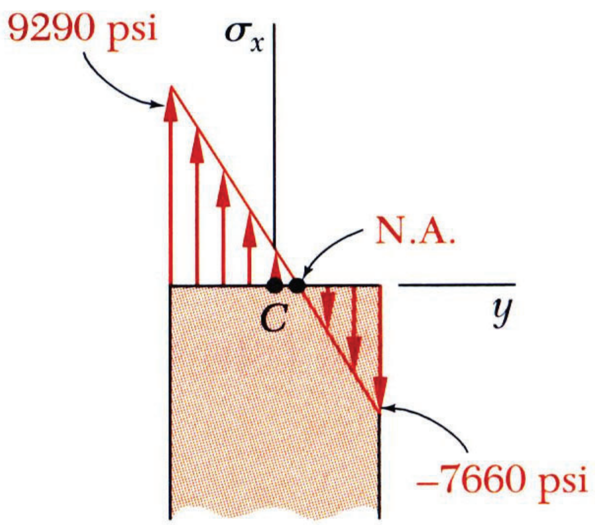
- Maximum tensile and compressive stresses

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry



Example 3

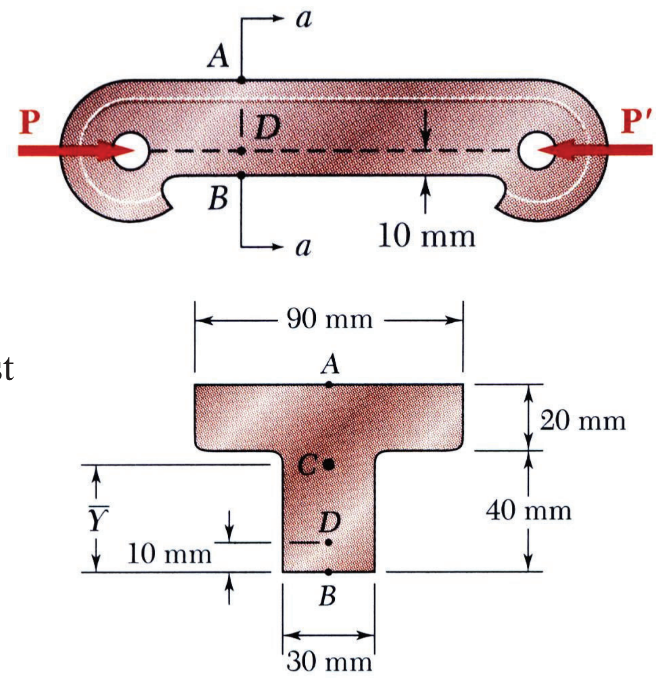


- Neutral axis location

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 4



Section a–a

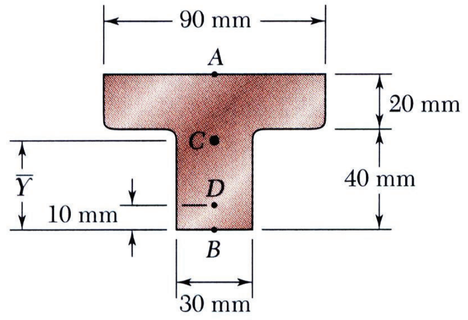
The largest allowable stresses for the cast iron link are 30 MPa in tension and 120 MPa in compression. Determine the largest force P which can be applied to the link.

Loading combination

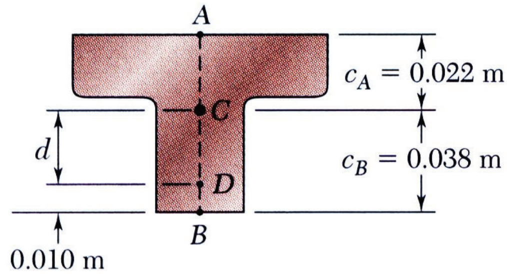
❑ Eccentric Axial Loading in a Plane of Symmetry

Example 4

Parts	A_i	$\bar{y}_i$	$A_i\bar{y}_i$	$A_i\bar{y}_i^{\,2}$	I_{g_i}
1					
2					



Section a–a

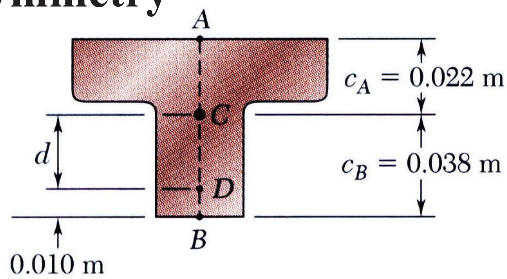


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 4

- Determine an equivalent centric and bending loads.

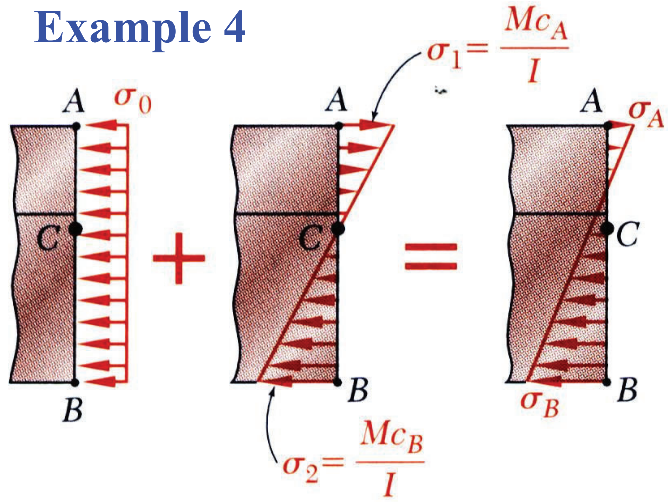


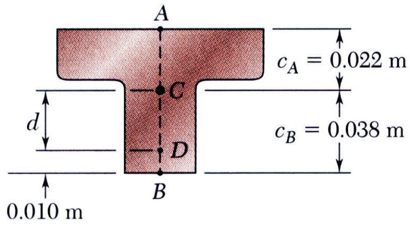
- Superpose stresses due to centric and bending loads

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 4



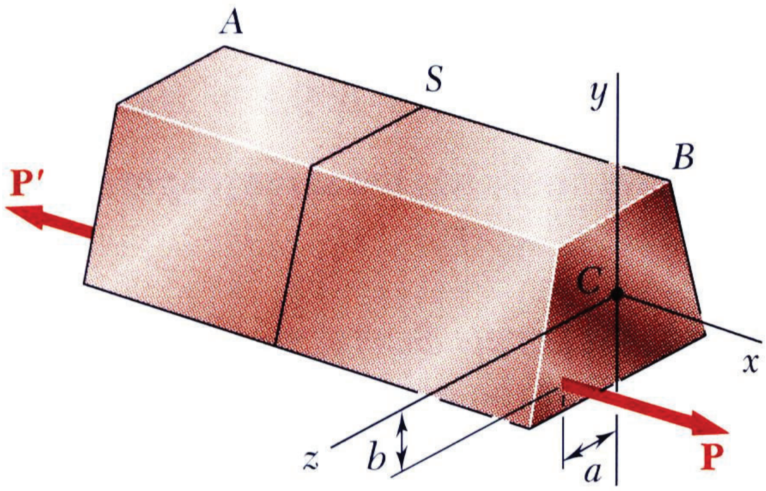


- Evaluate critical loads for allowable stresses.

- The largest allowable load

Loading combination

❑ General Case of Eccentric Axial Loading



- Consider a straight member subject to equal and opposite eccentric forces.

- The eccentric force is equivalent to the system of a centric force and two couples.

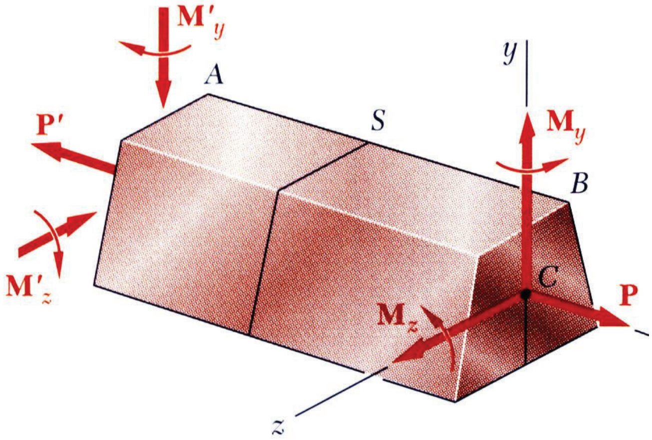
$P = \text{Centric force}$

$M_y = Pa$

$M_z = Pb$

Loading combination

❑ General Case of Eccentric Axial Loading



- By the principle of superposition, the combined stress distribution is

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- If the neutral axis lies on the section, it may be found from

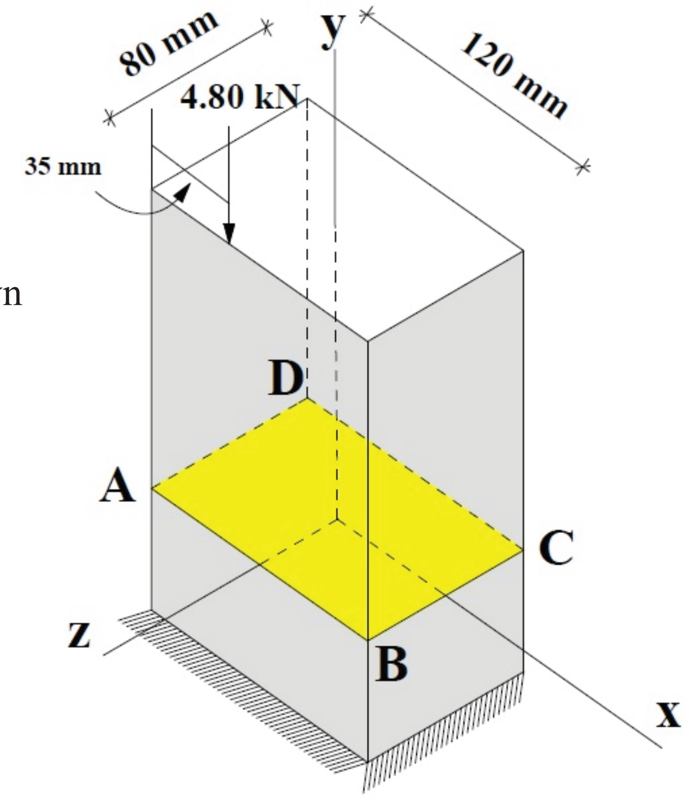
$$\frac{M_z}{I_z} y - \frac{M_y}{I_y} z = \frac{P}{A}$$

Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5

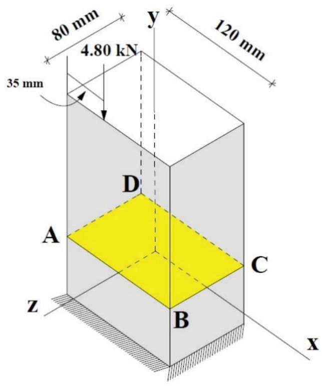
A vertical 4.80 kN load is applied as shown on a wooden post of rectangular cross section, 80 by 120 mm. (a) Determine the stress at points A, B, C, and D. (b) Locate the neutral axis of the cross section.

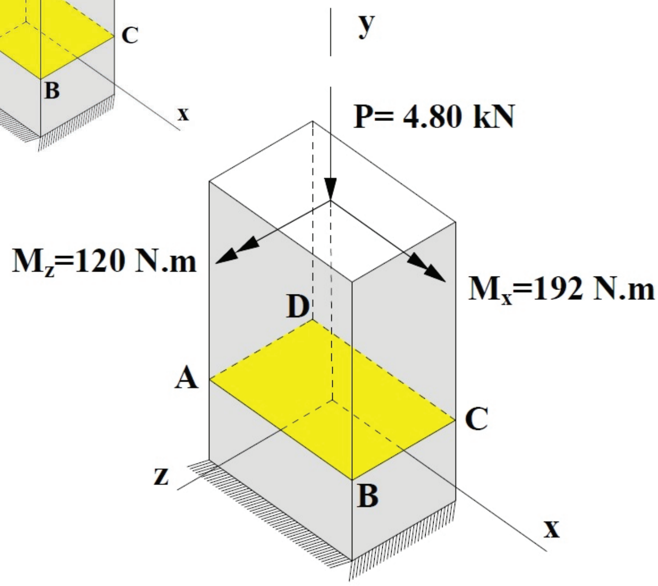


Loading combination

❑ **Eccentric Axial Loading in a Plane of Symmetry**

Example 5

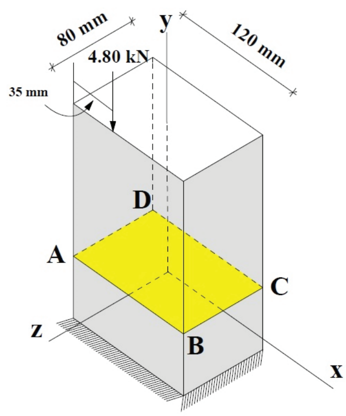


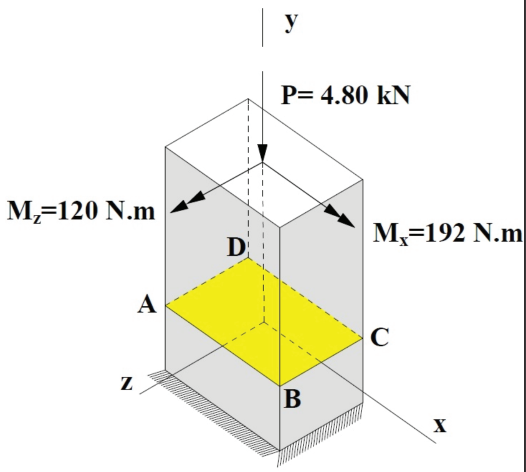


Loading combination

❑ **Eccentric Axial Loading in a Plane of Symmetry**

Example 5

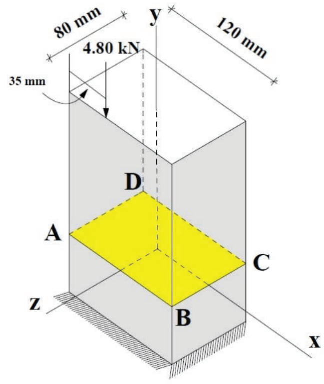


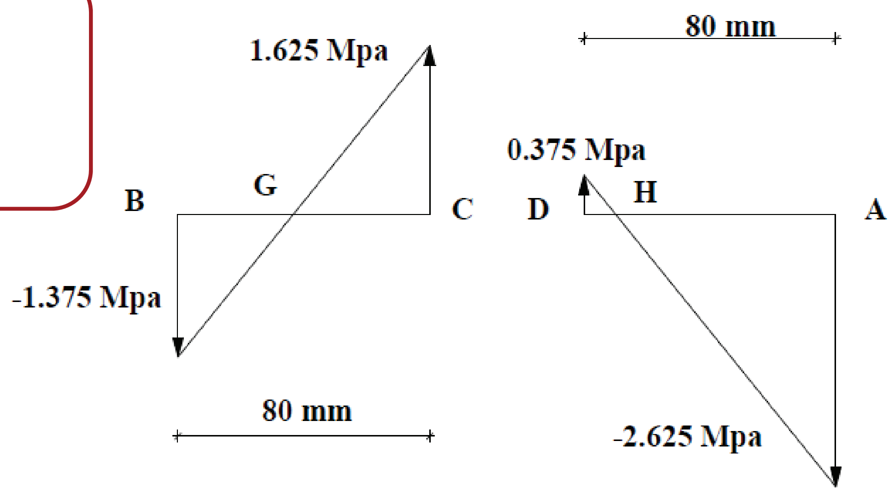


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5



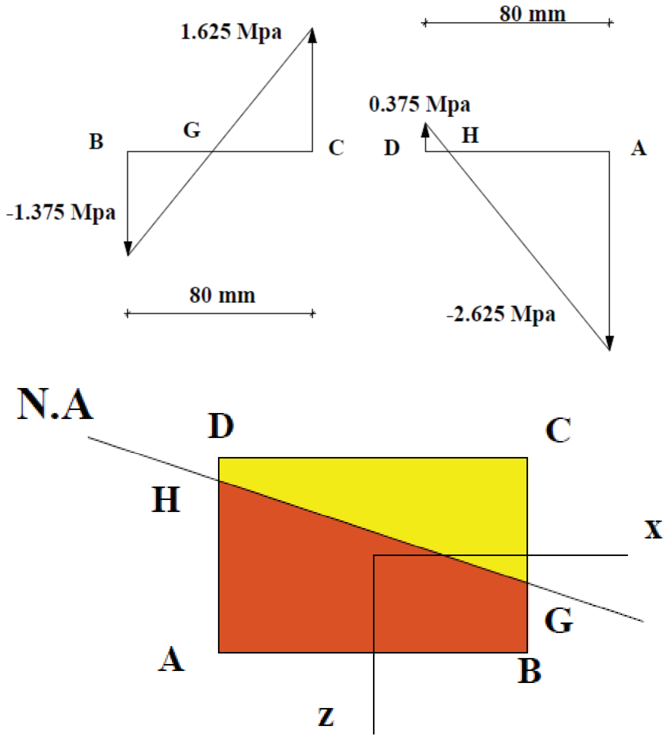


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5

Neutral Axis

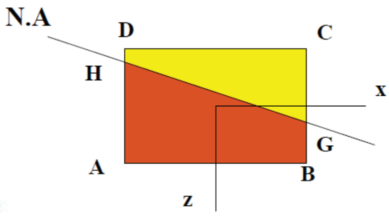


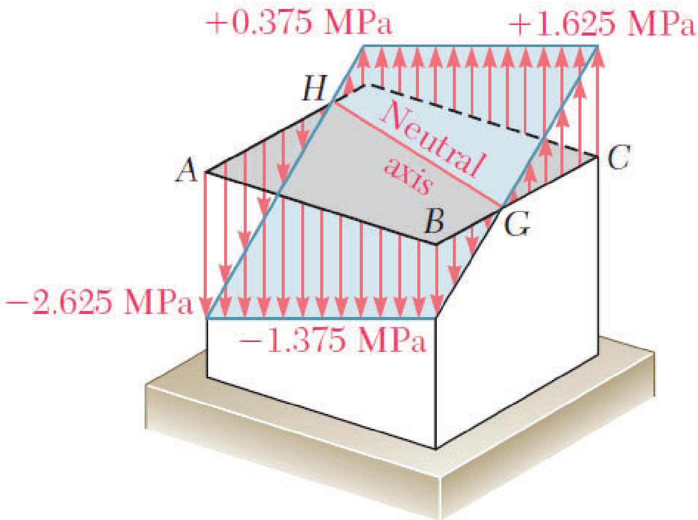
Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 5

Neutral Axis



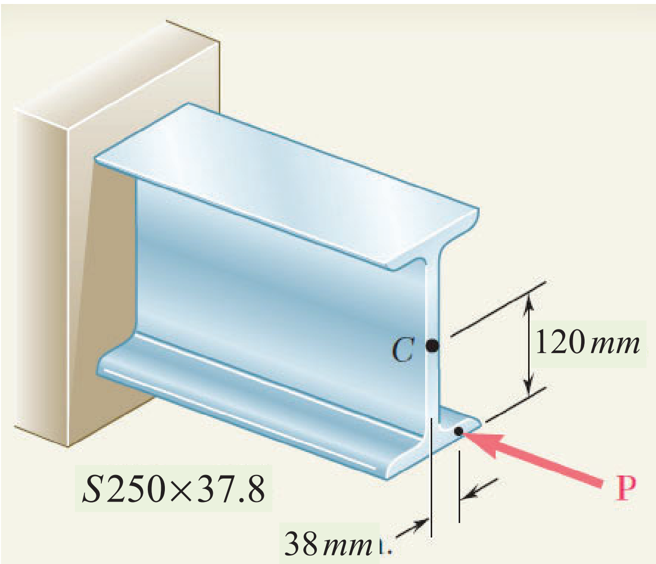


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

Example 6

A horizontal load P is applied as shown to a short section of an S250 X 37.8 rolled-steel member. Knowing that the compressive stress in the member is not to exceed 82 MPa, determine the largest permissible load P.



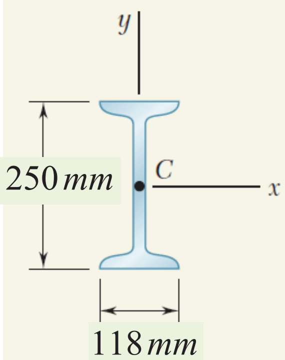
$A = 4820 mm^2$

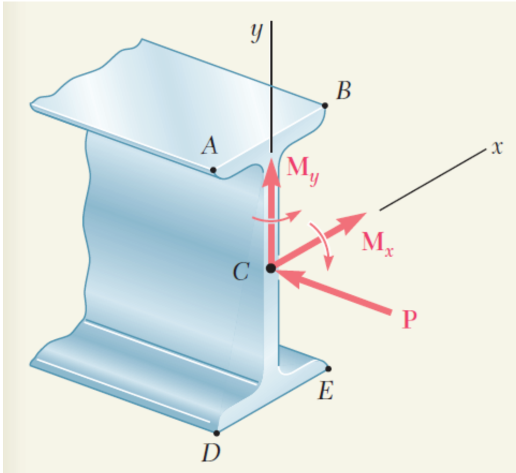
$S_x = 402 \times 10^3 mm^3 \quad S_y = 47.5 \times 10^3 mm^3$

Loading combination

❑ **Eccentric Axial Loading in a Plane of Symmetry**

Example 6

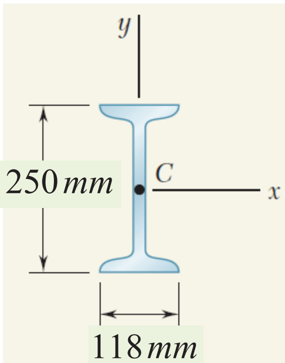


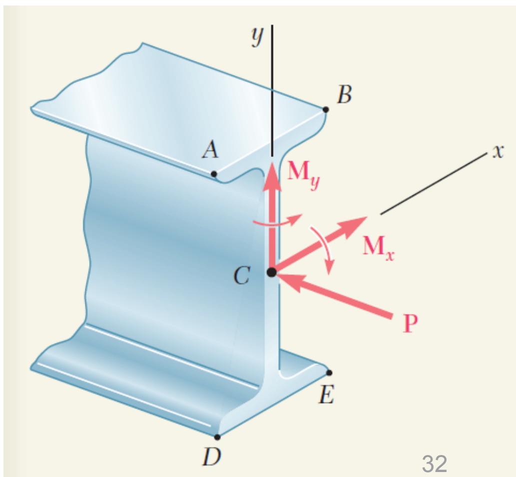


Loading combination

❑ **Eccentric Axial Loading in a Plane of Symmetry**

Example 6

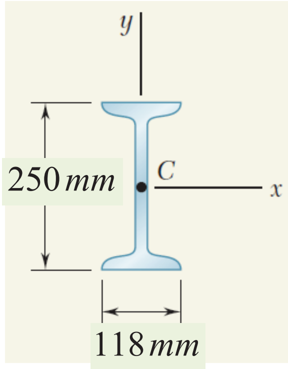


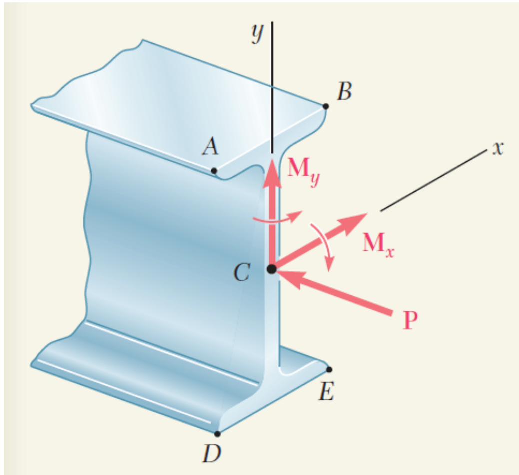


Loading combination

❑ Eccentric Axial Loading in a Plane of Symmetry

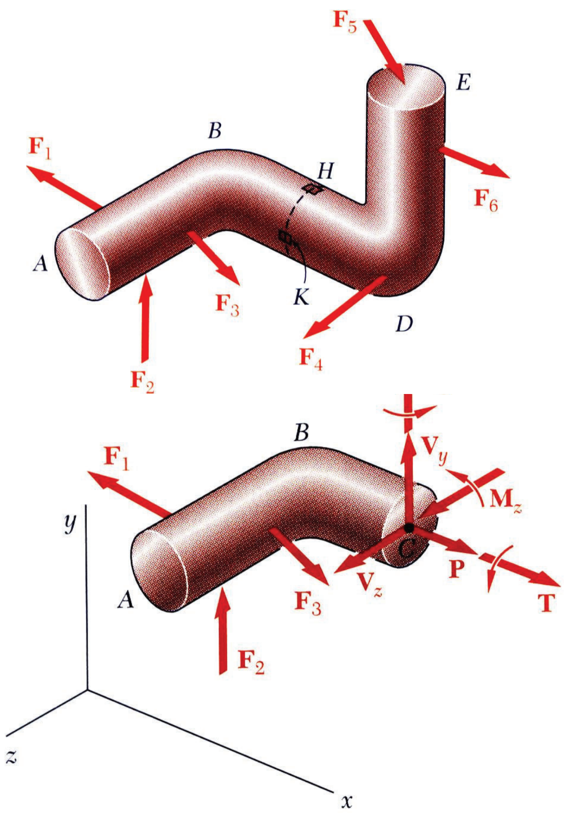
Example 6





Loading combination

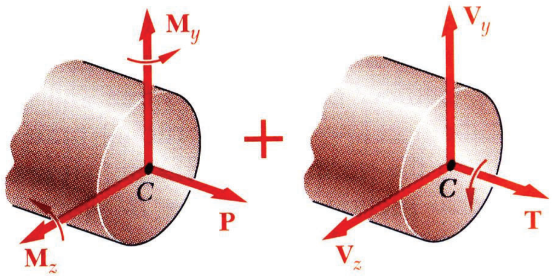
❑ Stresses Under Combined Loadings



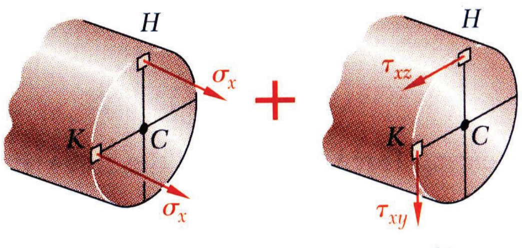
- Wish to determine stresses in slender structural members subjected to arbitrary loadings.
- Pass section through points of interest. Determine force-couple system at centroid of section required to maintain equilibrium.
- System of internal forces consist of three force components and three couple vectors.
- Determine stress distribution by applying the superposition principle.

Loading combination

❑ Stresses Under Combined Loadings



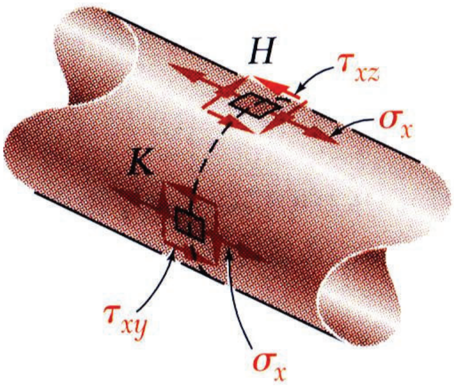
- Axial force and in-plane couple vectors contribute to normal stress distribution in the section.



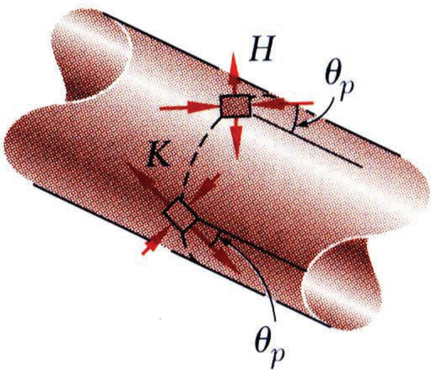
- Shear force components and twisting couple contribute to shearing stress distribution in the section.

Loading combination

❑ Stresses Under Combined Loadings



- Normal and shearing stresses are used to determine principal stresses, maximum shearing stress and orientation of principal planes.



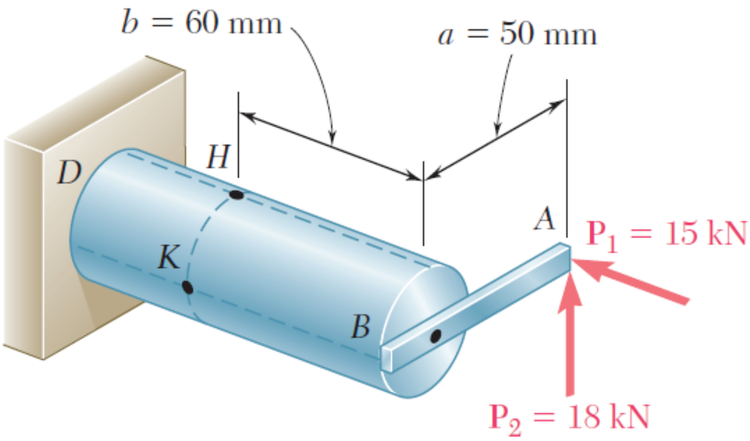
- Analysis is valid only to extent that conditions of applicability of superposition principle and Saint-Venant’s principle are met.

Loading combination

❑ Stresses Under Combined Loadings

Example 7

Two forces are applied as shown to the end A of bar AB, which is welded to a cylindrical member BD of radius c = 20 mm. Assuming that all stresses remain below the proportional limit of the material, determine the normal and shearing stresses at point H and K



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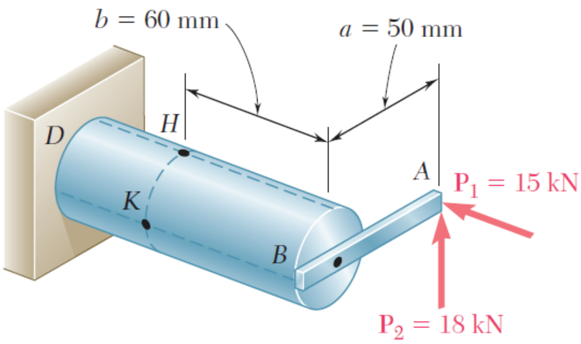
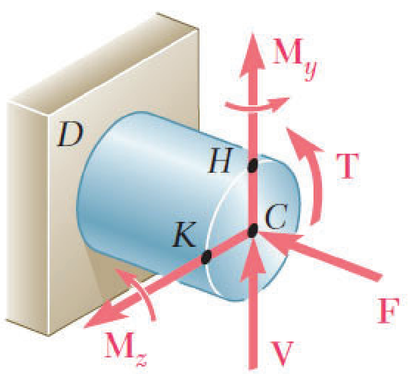
❑ Stresses Under Combined Loadings

Example 7





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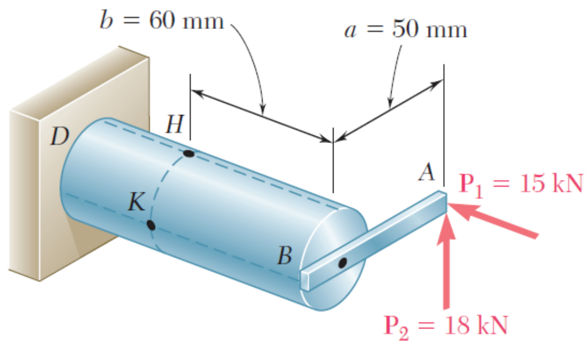
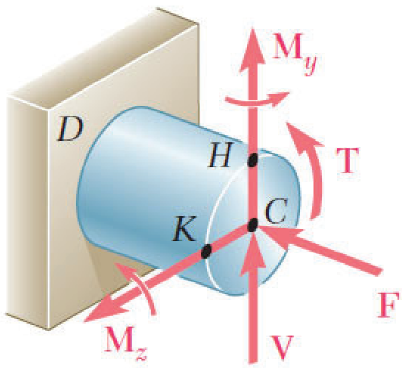
❑ **Stresses Under Combined Loadings**

Example 7

Geometric Properties of the Section.





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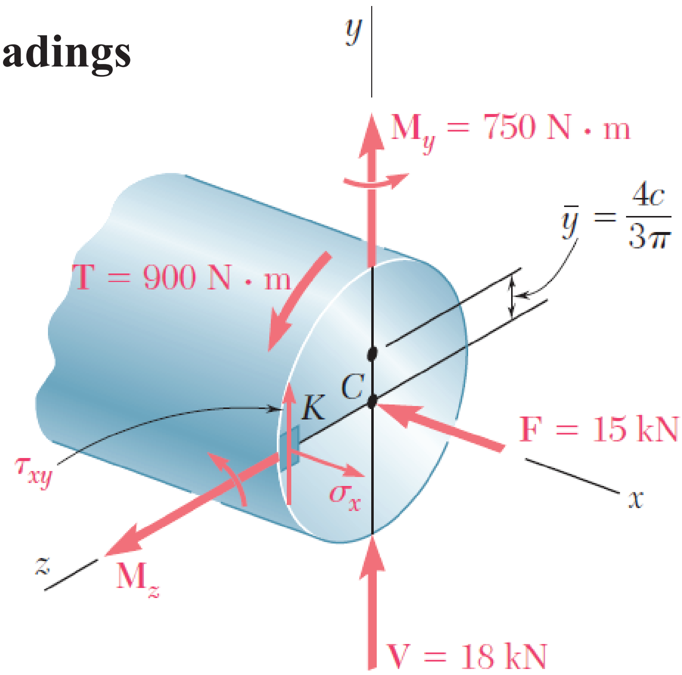
❑ **Stresses Under Combined Loadings**

Example 7

Stress at point H



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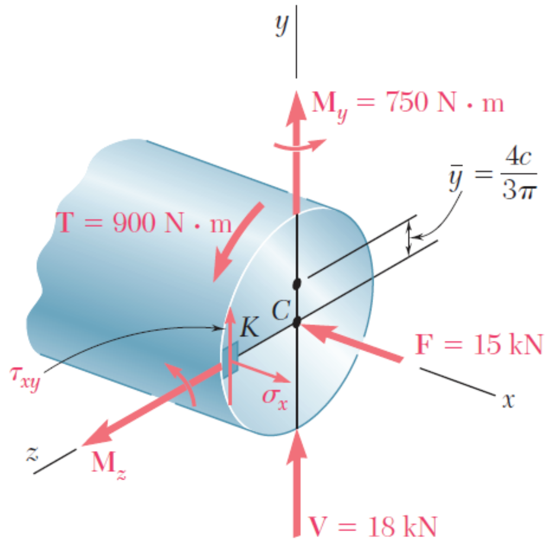
❑ Stresses Under Combined Loadings

Example 7

Stress at point K



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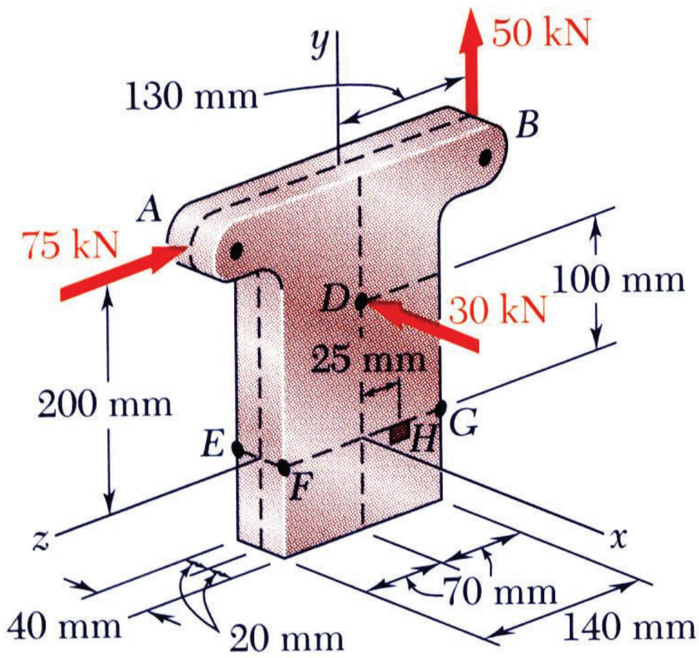
❑ Stresses Under Combined Loadings

Example 8

Three forces are applied as shown at points A, B, and D of a short steel post. Knowing that the horizontal cross section of the post is a 40 X 140-mm rectangle. Determine:

a) The normal and shear stress at point H.

b) The principle stresses, principal planes.



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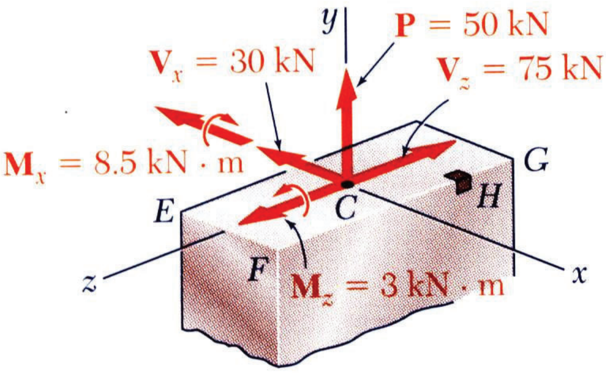
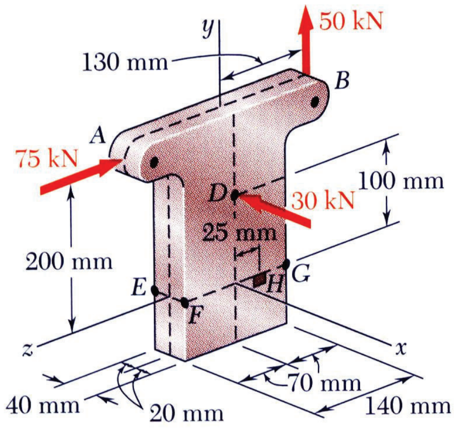
❑ Stresses Under Combined Loadings

Example 8





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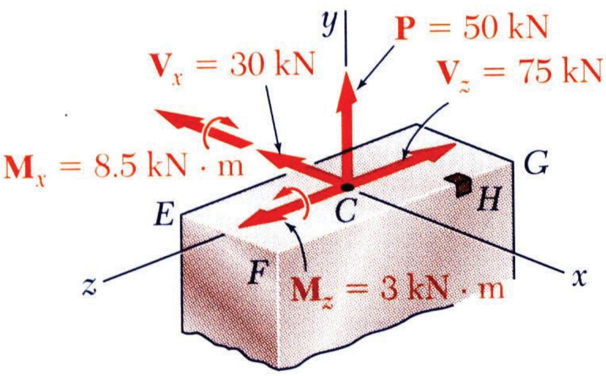
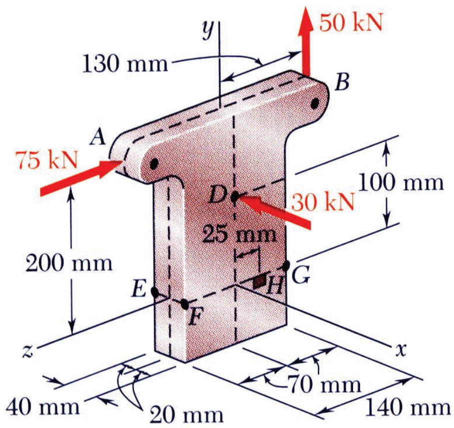
❑ Stresses Under Combined Loadings

Example 8





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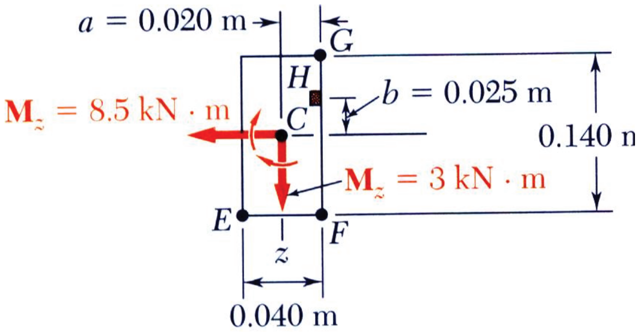
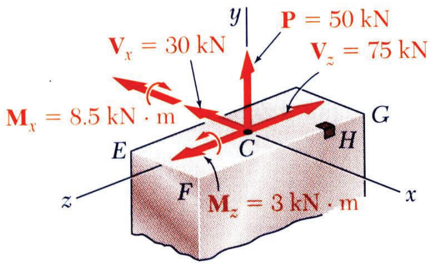
❑ **Stresses Under Combined Loadings**

Example 8





Normal stress at point H

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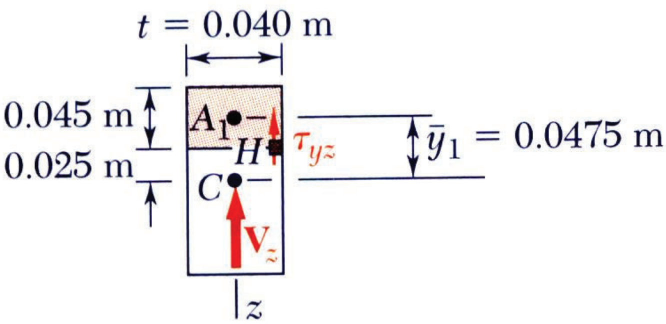
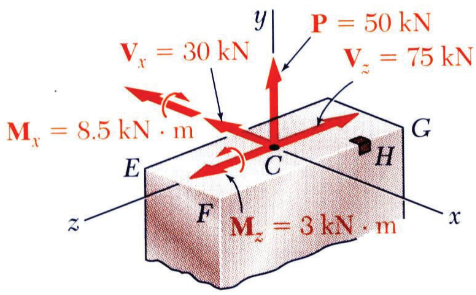
❑ **Stresses Under Combined Loadings**

Example 8





Shear stress at point H

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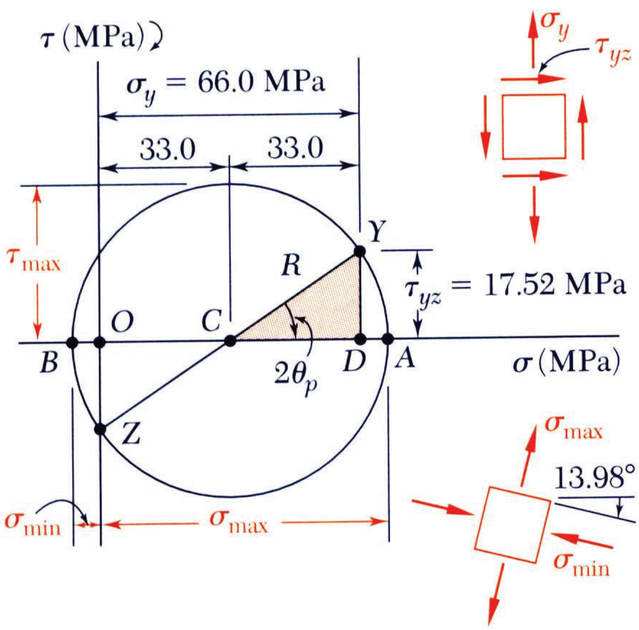
❑ Stresses Under Combined Loadings

Example 8

- Calculate principal stresses and maximum shearing stress. Determine principal planes.



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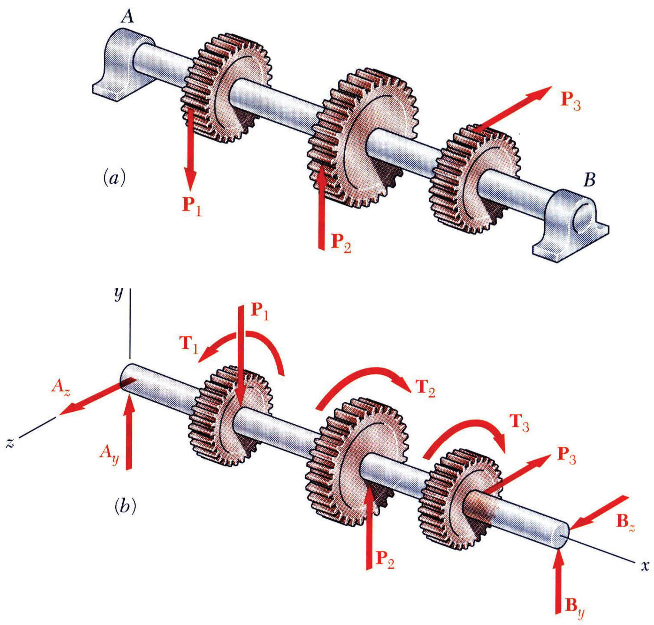
❑ Design of a Transmission Shaft



- If power is transferred to and from the shaft by gears or sprocket wheels, the shaft is subjected to transverse loading as well as shear loading.
- Normal stresses due to transverse loads may be large and should be included in determination of maximum shearing stress.
- Shearing stresses due to transverse loads are usually small and contribution to maximum shear stress may be neglected.

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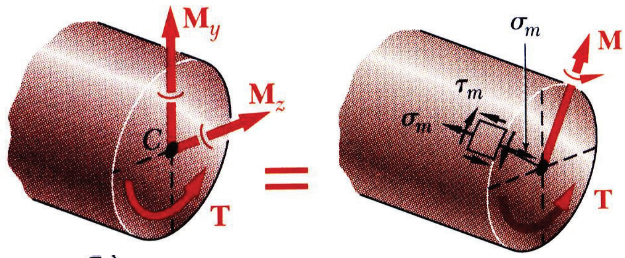
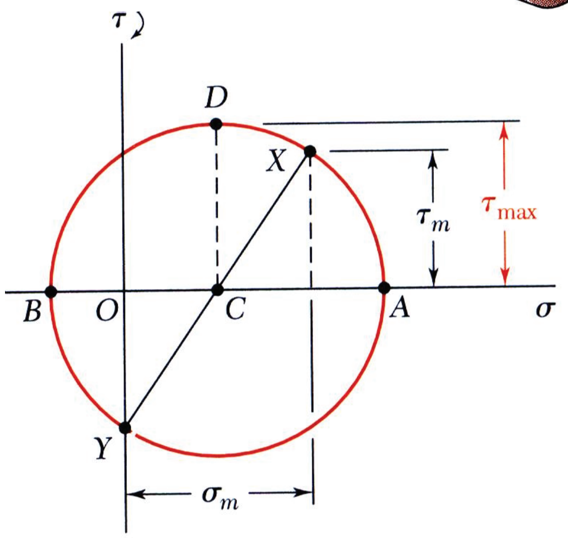
❑ Design of a Transmission Shaft





- At any section,

$$\sigma_m = \frac{Mc}{I} \quad \text{where} \quad M^2 = M_y^2 + M_z^2$$

$$\tau_m = \frac{Tc}{J}$$

- Maximum shearing stress,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + (\tau_m)^2} = \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{J}\right)^2}$$

for a circular or annular cross - section, $2I = J$

$$\tau_{max} = \frac{c}{J}\sqrt{M^2 + T^2}$$

- Shaft section requirement,

$$\left(\frac{J}{c}\right)_{min} = \frac{\left(\sqrt{M^2 + T^2}\right)_{max}}{\tau_{all}}$$

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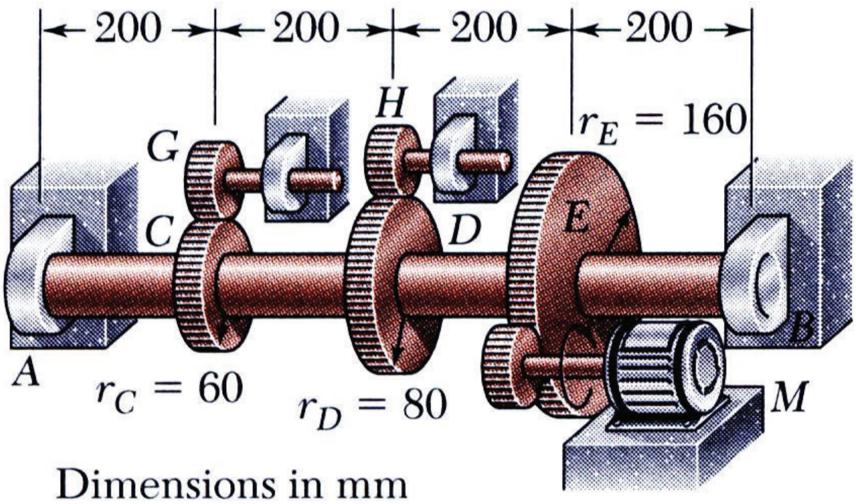
❑ Design of a Transmission Shaft

Example 9



Solid shaft rotates at 480 rpm and transmits 30 kW from the motor to gears *G* and *H*; 20 kW is taken off at gear *G* and 10 kW at gear *H*. Knowing that s_{all} = 50 MPa, determine the smallest permissible diameter for the shaft.

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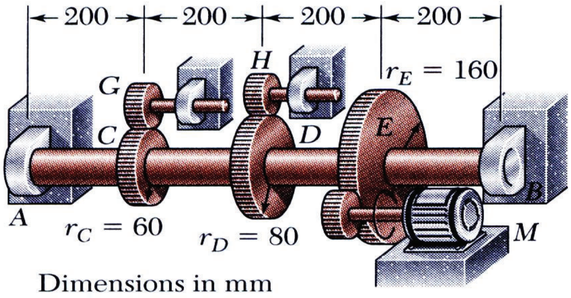
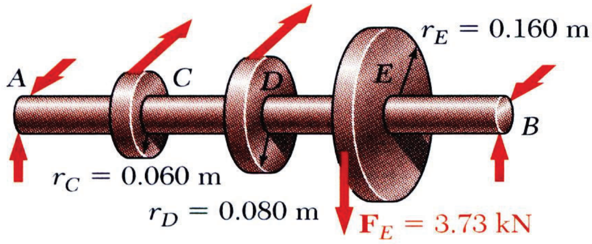
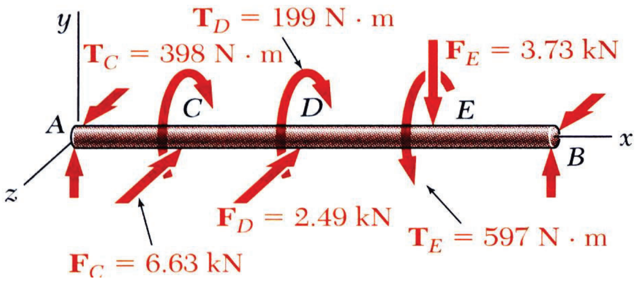
❑ Design of a Transmission Shaft

Example 9

- Determine the gear torques and corresponding tangential forces.







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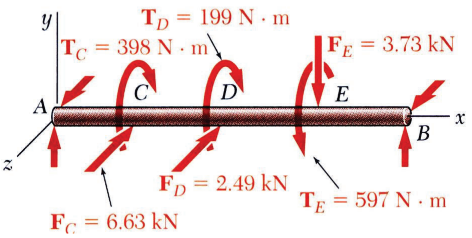
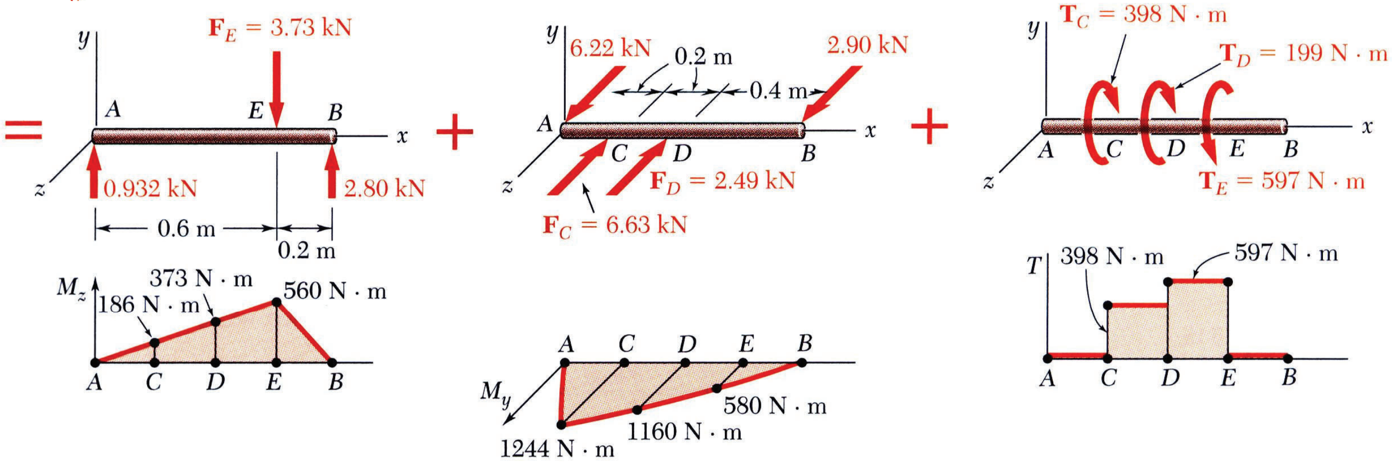
❑ Design of a Transmission Shaft

Example 9

- Identify critical shaft section from torque and bending moment diagrams.





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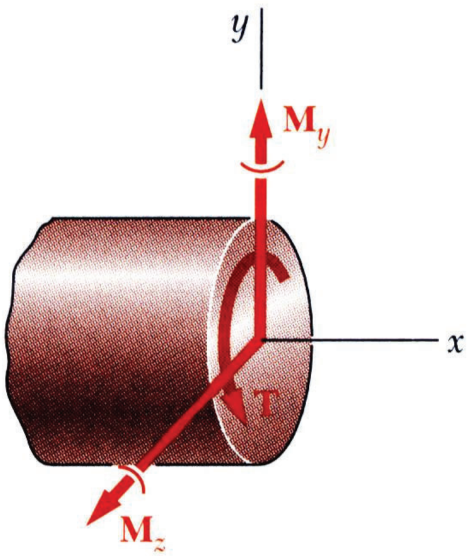
❑ Design of a Transmission Shaft

Example 9

- Calculate minimum allowable shaft diameter.



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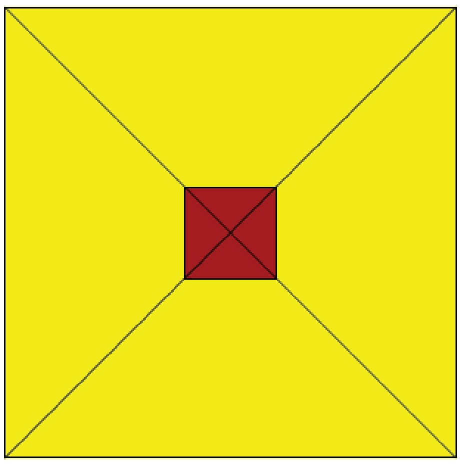
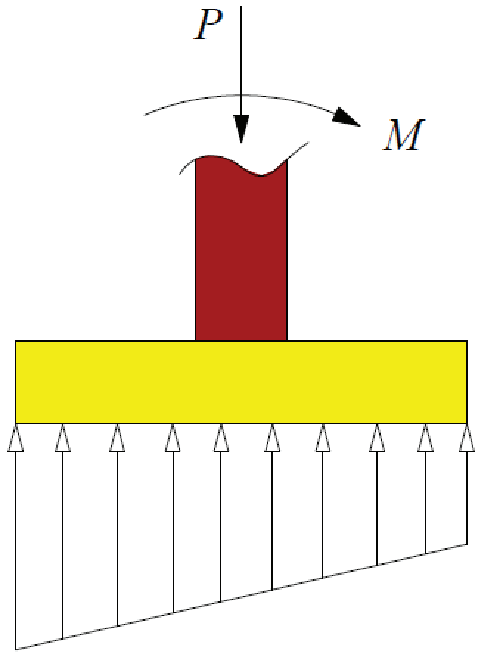
❑ Middle section

- In civil engineering, the Middle Third/quarter Rules state that no tension is developed in the section if the compressive resultant force lies within this zone of the section.





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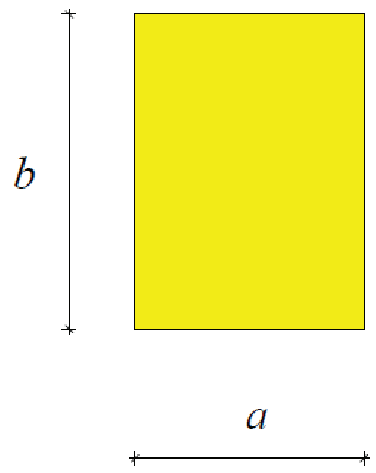
❑ Middle section

Example 10

Determine the middle section of the following shape.



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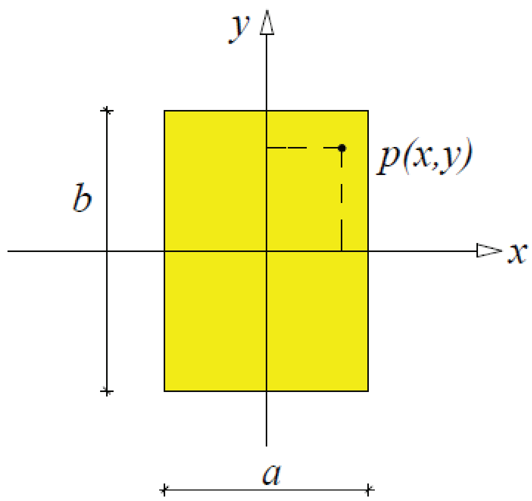
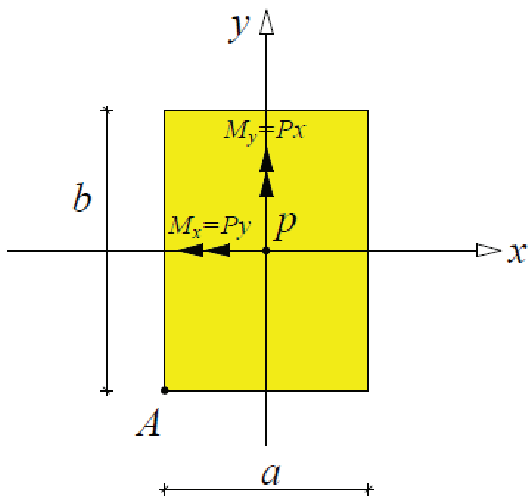
❑ Middle section

Example 10

Obviously, the maximum tensile stress is occur at point A.





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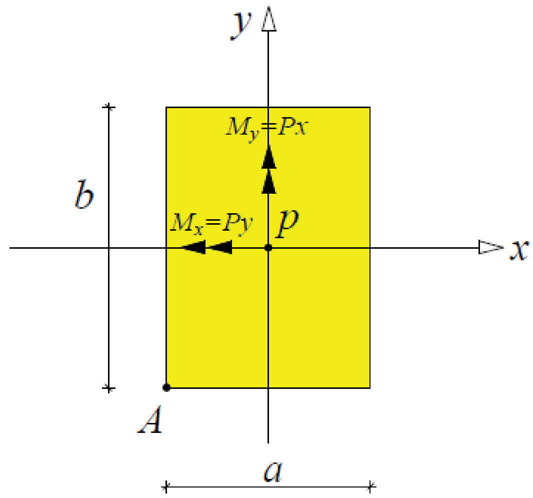
❑ Middle section

Example 10



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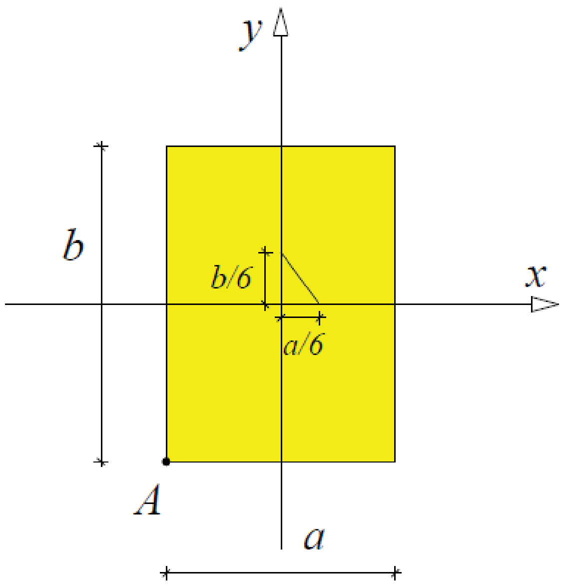
❑ Middle section

Example 10



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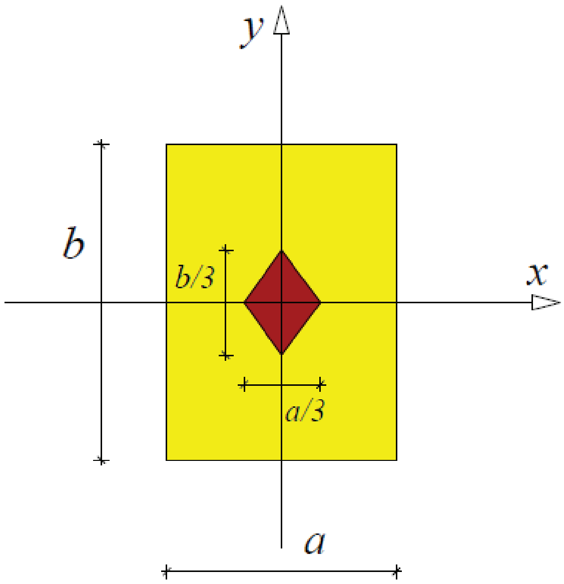
❑ Middle section

Example 10



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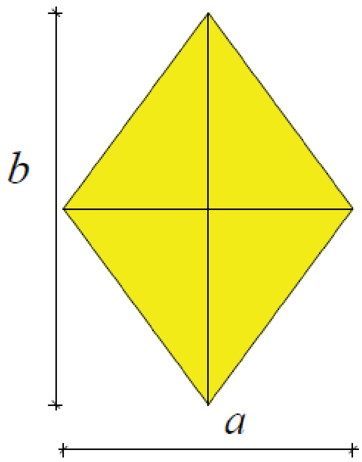
❑ Middle section

Example 11

Determine the middle section of the following shape.



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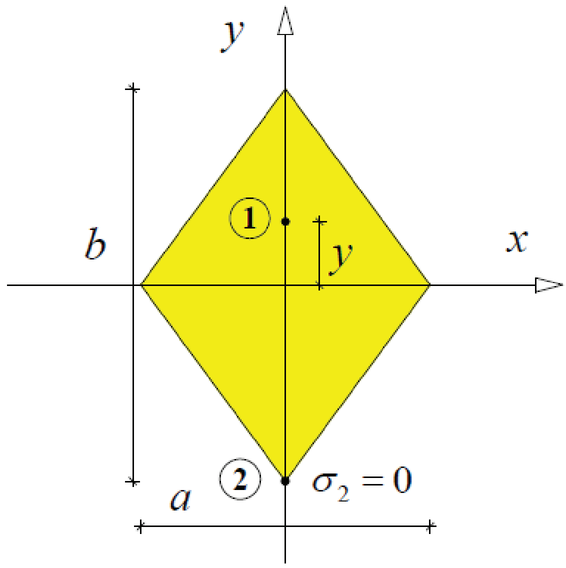
❑ Middle section

Example 11



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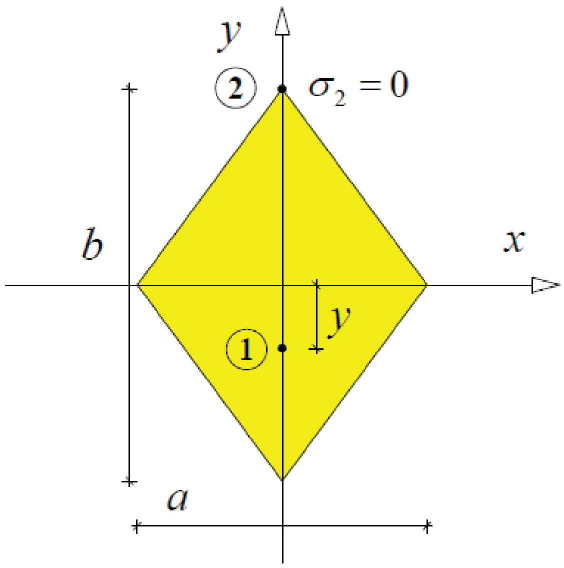
❑ Middle section

Example 11



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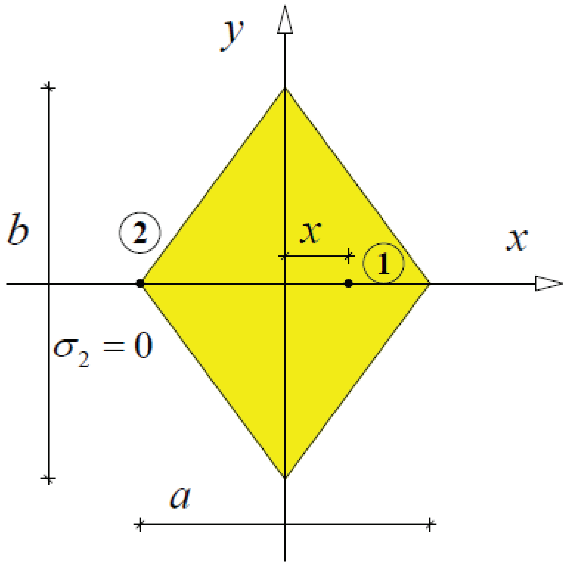
❑ **Middle section**

Example 11



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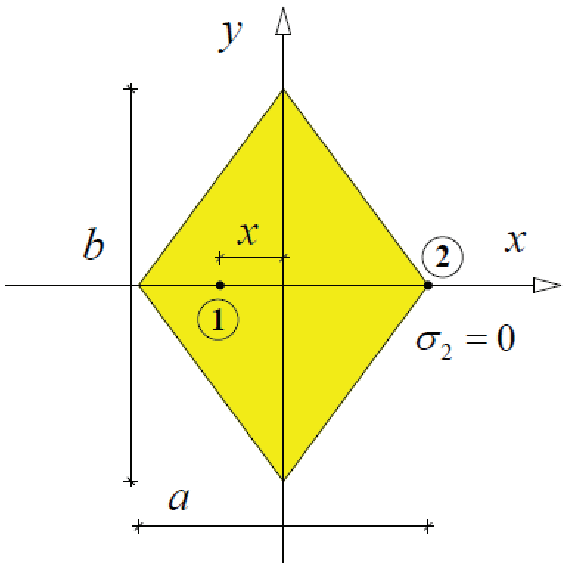
❑ **Middle section**

Example 11



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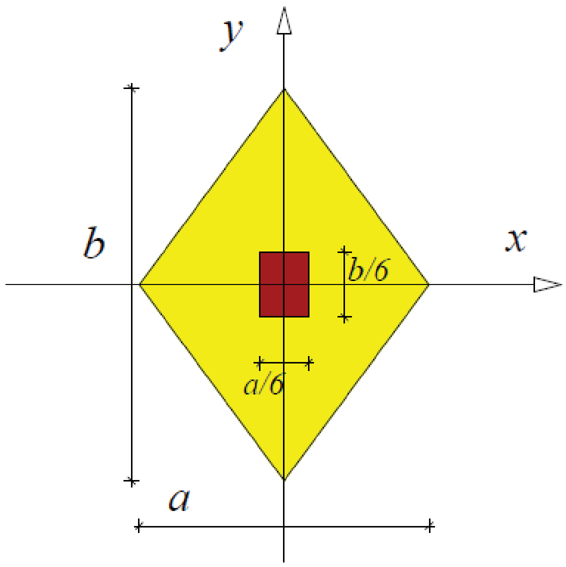
❑ Middle section

Example 11



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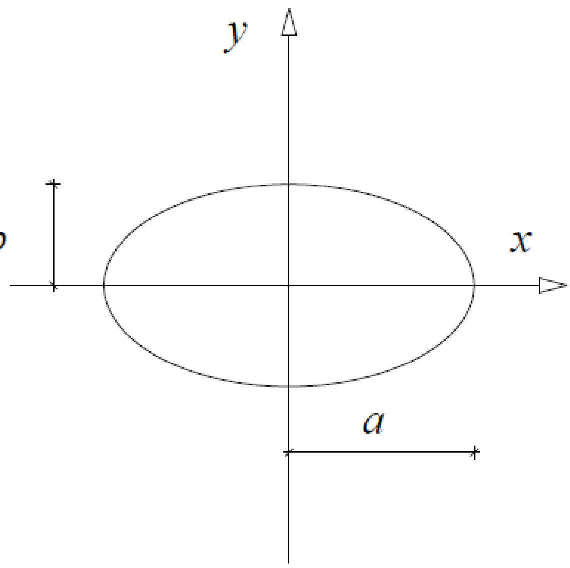
❑ Middle section

Example 12

Determine the middle section of the following shape.



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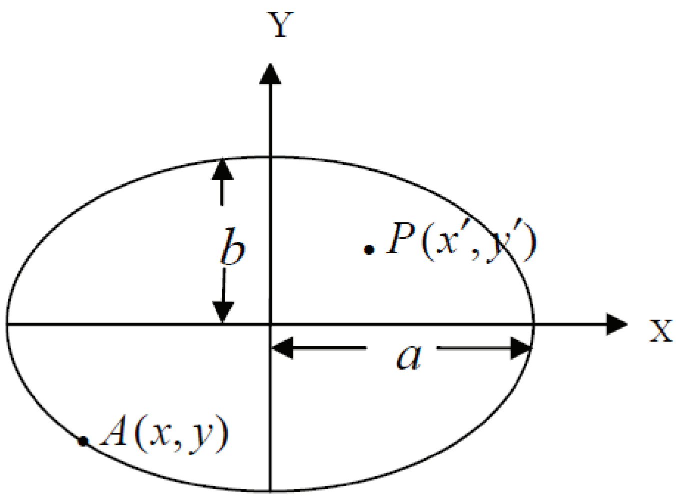
❑ Middle section

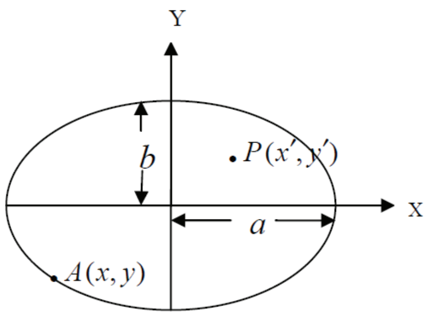
Example 12



Obviously, the maximum tensile stress is occur at point A.

Loading combination

❑ Middle section



Example 12

By Substituting y in stress equation

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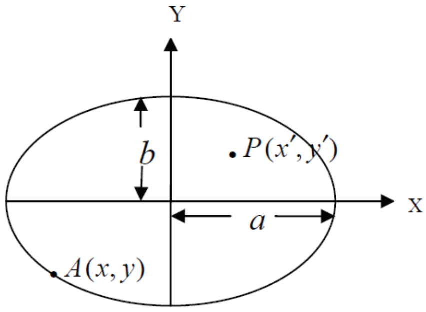
❑ Middle section



Example 12

Coordinate point A is determined

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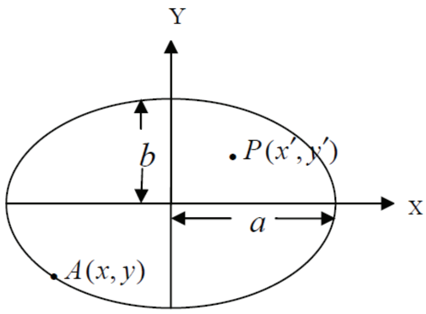
❑ Middle section



Example 12

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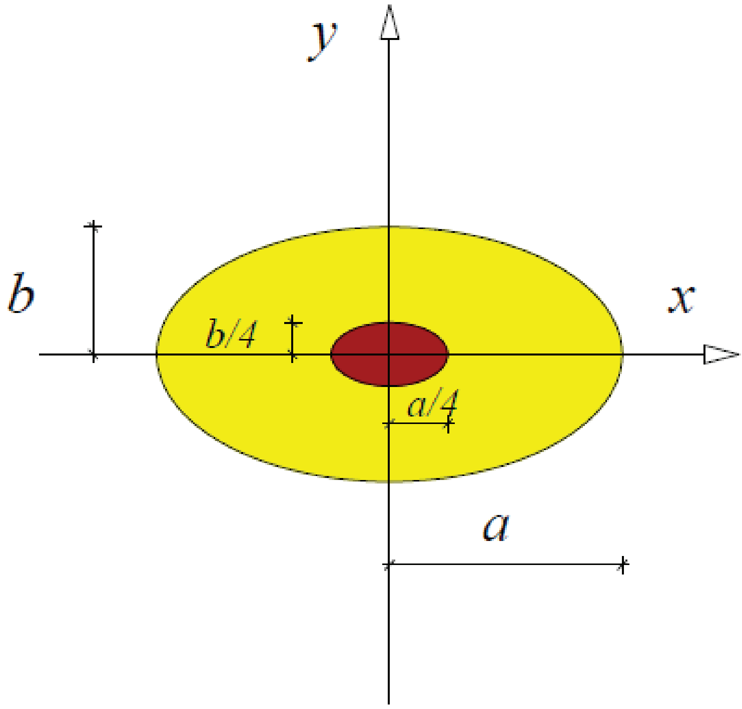
❑ Middle section

Example 12

$$\frac{x'^2}{\left(\frac{a}{4}\right)^2}+\frac{y'^2}{\left(\frac{b}{4}\right)^2}=1$$



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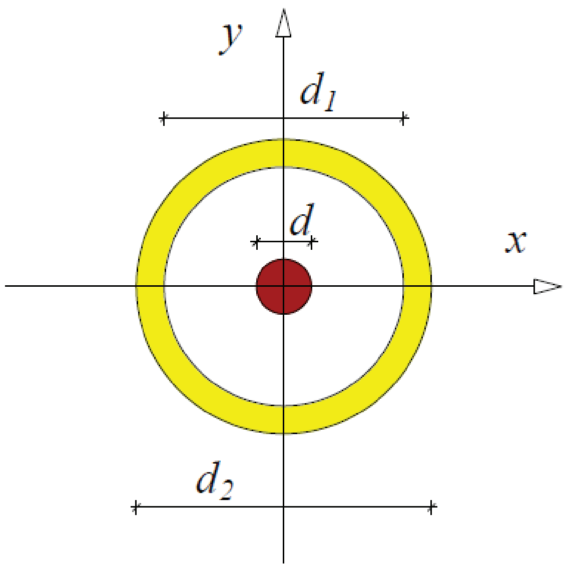
❑ Middle section

$$d=\frac{d_1^{\ 2}+d_2^{\ 2}}{4d_2}$$



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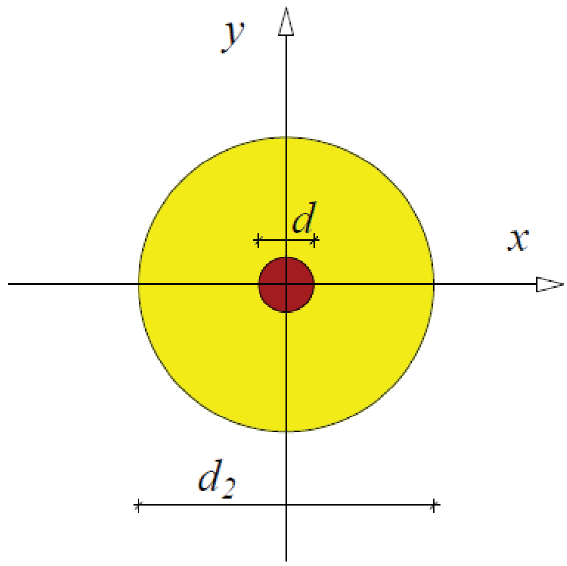
❑ **Middle section**

$$d = \frac{d_1^{\,2} + d_2^{\,2}}{4d_2}$$

$$If \quad d_1 = 0 \quad \Rightarrow \quad \boxed{d = \frac{d_2}{4}}$$



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❑ **Middle section**

$$\boxed{\frac{a}{4} \quad \& \quad \frac{b}{4}}$$