Mechanics of Materials

دانشگاه کردستان
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Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:
J.Wat Oler "Lectures notes on Mechanics od Materials"
Ibrahim A.Assakkaf "Lectures notes on Mechanics od Materials"

Shearing Stresses in Beams and Thin-Walled Members

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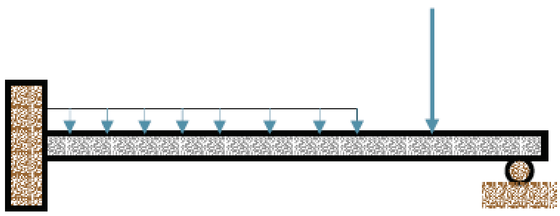
Shearing Stresses in Beams and Thin-Walled Members

- **Shear and Bending**

Pure Bending

Bending and Shear Force



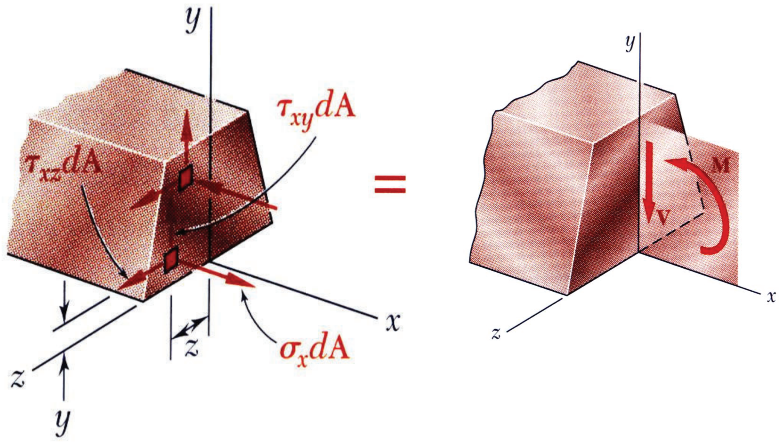
– Although it has been convenient to restrict the analysis of beams to pure bending, this type of loading is rarely encountered in an actual engineering problem.

– It is much common for the resultant internal forces to consist of a bending moment and shear force.

Shearing Stresses in Beams and Thin-Walled Members

❑ Shear and Bending

- Transverse loading applied to a beam results in normal and shearing stresses in transverse sections.

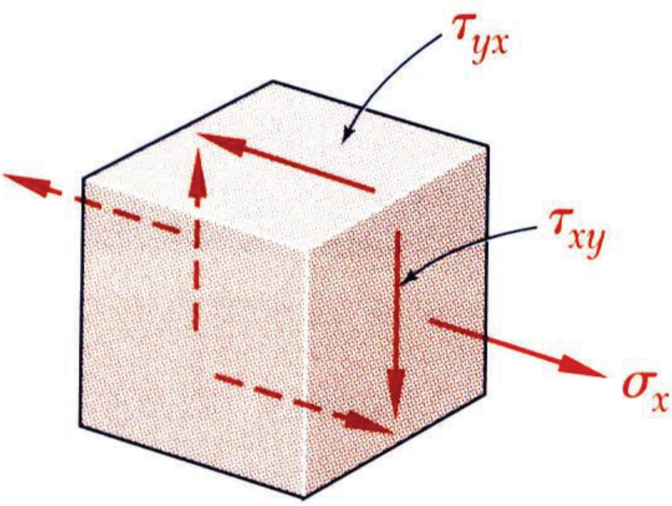


- Distribution of normal and shearing stresses satisfies

$$
\begin{aligned}
F_x &= \int \sigma_x dA = 0 & M_x &= \int (y\tau_{xz} - z\tau_{xy})dA = 0 \\
F_y &= \int \tau_{xy} dA = -V & M_y &= \int z\sigma_x dA = 0 \\
F_z &= \int \tau_{xz} dA = 0 & M_z &= \int (-y\sigma_x dA) = M
\end{aligned}
$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Shear and Bending



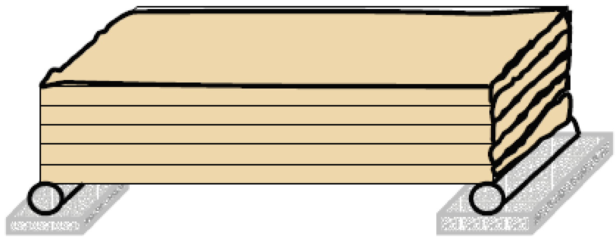
- When shearing stresses are exerted on the vertical faces of an element, equal stresses must be exerted on the horizontal faces

- Longitudinal shearing stresses must exist in any member subjected to transverse loading.

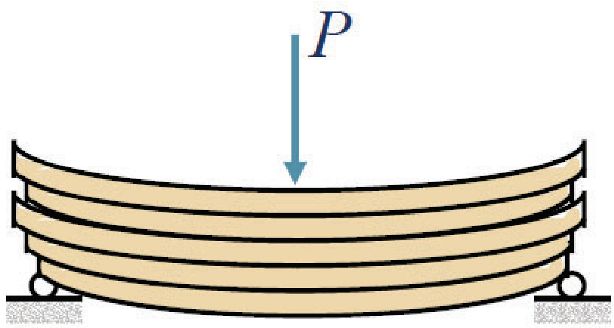
Shearing Stresses in Beams and Thin-Walled Members

❑ Shear and Bending

- When a bending load is applied, the stack will deform as shown in Figure.
- Since the slabs were free to slide on one another, the ends do not remain even but staggered.
- Each of the slabs behaves as independent beam, and the total resistance to bending of *n* slabs is approximately *n* times the resistance of one slab alone.



Unloaded Stack of Slabs

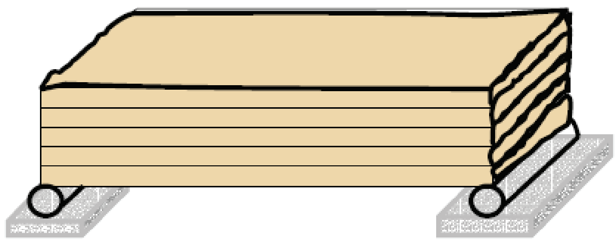


Unglued Slabs loaded

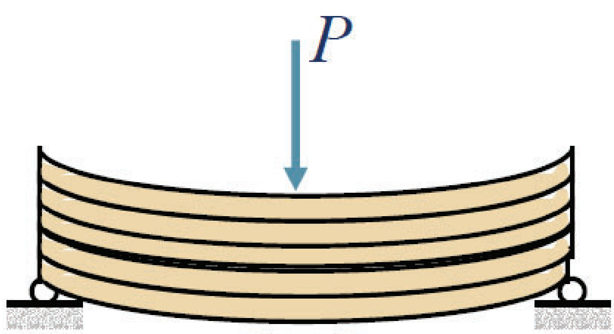
Shearing Stresses in Beams and Thin-Walled Members

❑ Shear and Bending

- If the slabs of Fig. 22b is fastened or glued, then the staggering or relative longitudinal movement of slabs would disappear under the action of the force. However, shear forced will develop between the slabs.
- In this case, the stack of slabs will act as a solid beam.
- The fact that this solid beam does not. exhibit this relative movement of longitudinal elements after the slabs are glued ***indicates the presence of shearing stresses on longitudinal planes.***



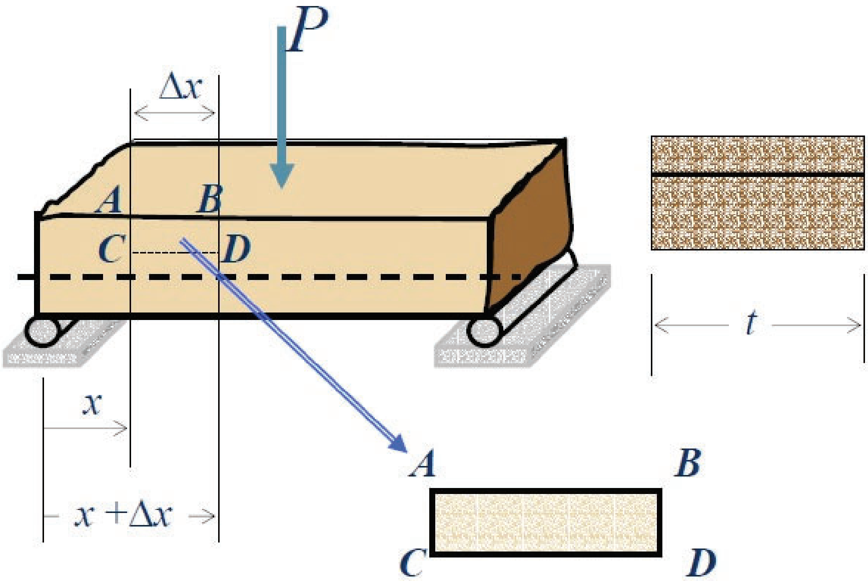
Glued Slabs Unloaded



Glued Slabs loaded

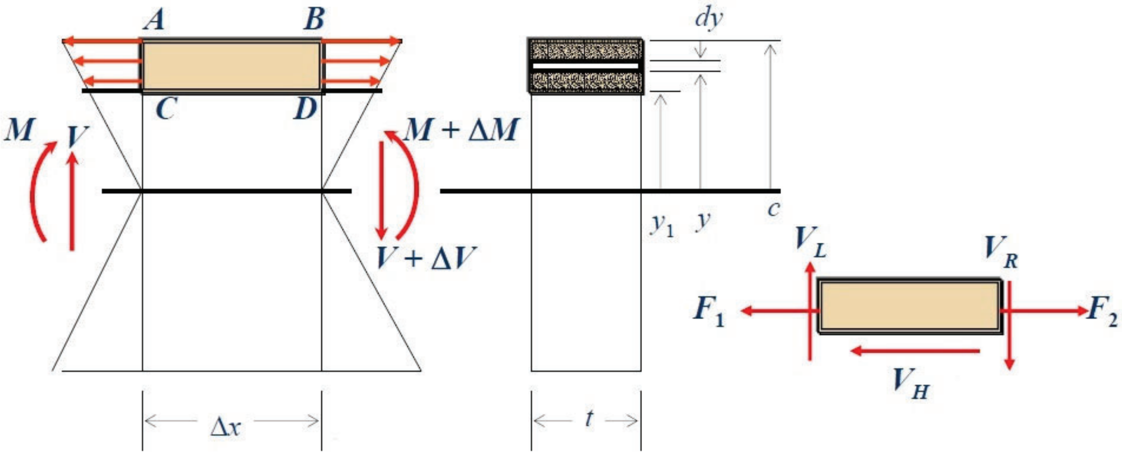
Shearing Stresses in Beams and Thin-Walled Members

- **Development of Shear Stress Formula**



Shearing Stresses in Beams and Thin-Walled Members

- **Development of Shear Stress Formula**



$$F_1 = \int \sigma \cdot dA = \int_{y_1}^{c} \left(-\frac{My}{I}\right)(t \cdot dy) = \boxed{-\frac{M}{I}\int_{y_1}^{c} y\,t\,dy} \qquad \boxed{F_2 = -\frac{M+\Delta M}{I}\int_{y_1}^{c} y\,t\,dy}$$

$$V_H = F_2 - F_1 = -\frac{M+\Delta M}{I}\int_{y_1}^{c} y\,t\,dy + \frac{M}{I}\int_{y_1}^{c} y\,t\,dy = \boxed{-\frac{\Delta M}{I}\int_{y_1}^{c} y\,t\,dy}$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

$$V_H = \frac{\Delta M}{I}\int_{y_1}^{c} y\,t\,dy$$

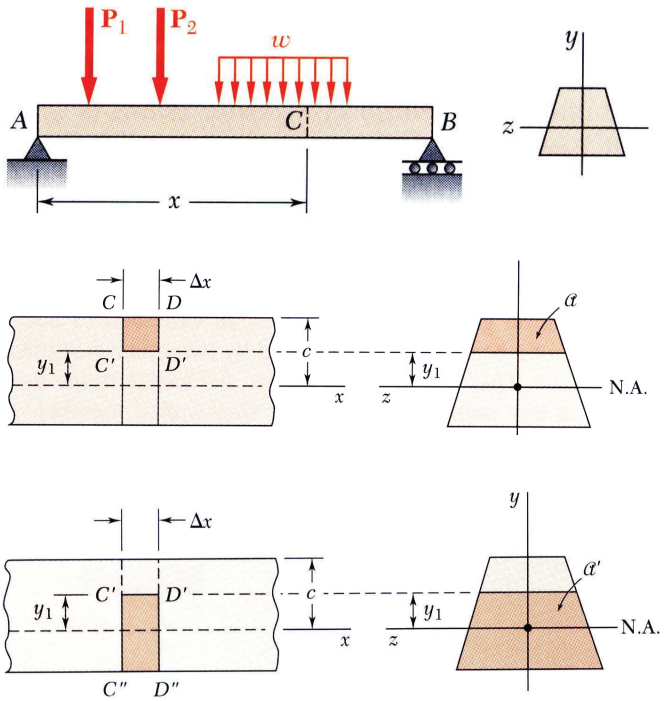
$$\Delta M = V\Delta x \qquad \Rightarrow \qquad V_H = \frac{VQ}{I}\Delta x$$

$$Q = \int_{y_1}^{c} y\,t\,dy$$

$$q = \frac{V_H}{\Delta x} = \frac{VQ}{I} = shear\ \ flow$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula



$Q + Q' = 0$: first moment with respect to neutral axis

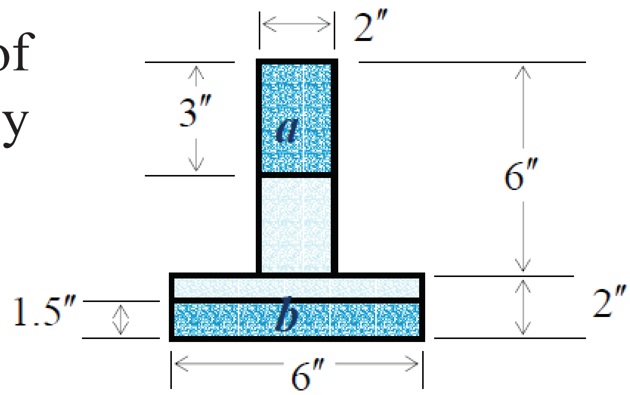
$$q = \frac{VQ}{I}$$

$$q' = -\frac{VQ'}{I}$$

❑ Development of Shear Stress Formula

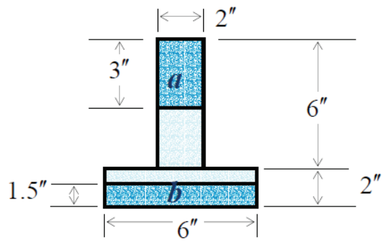
Example 1

Determine the first moment of area Q for the areas indicated by the shaded areas *a* and *b*

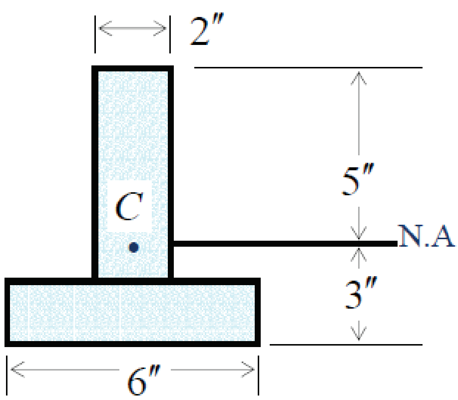


❑ Development of Shear Stress Formula

Example 1



First, we need to locate the neutral axis from the bottom edge:

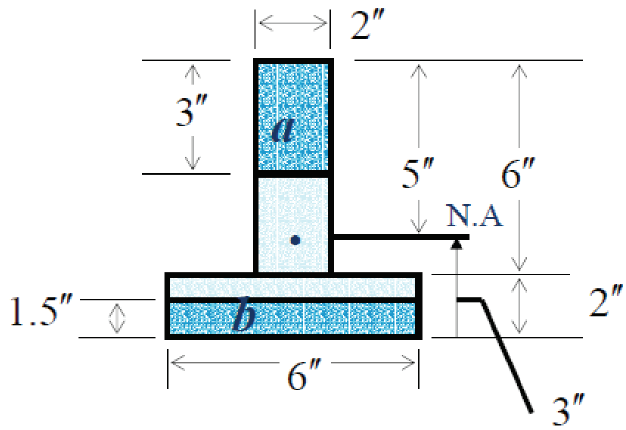


Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

Example 1

First, we need to locate the neutral axis from the bottom edge:

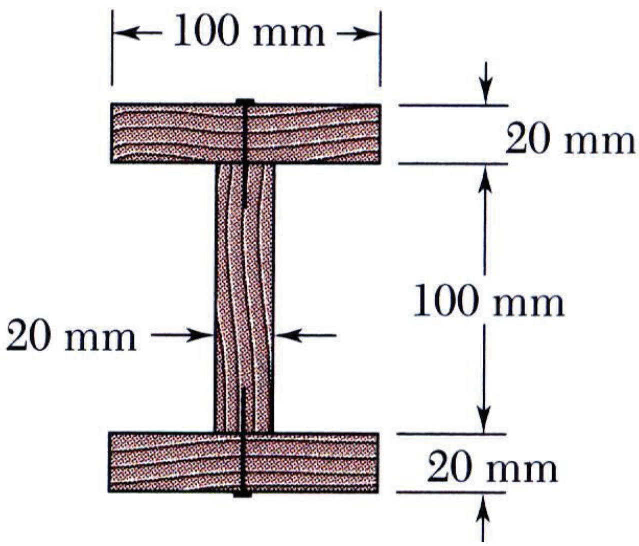


Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

Example 2

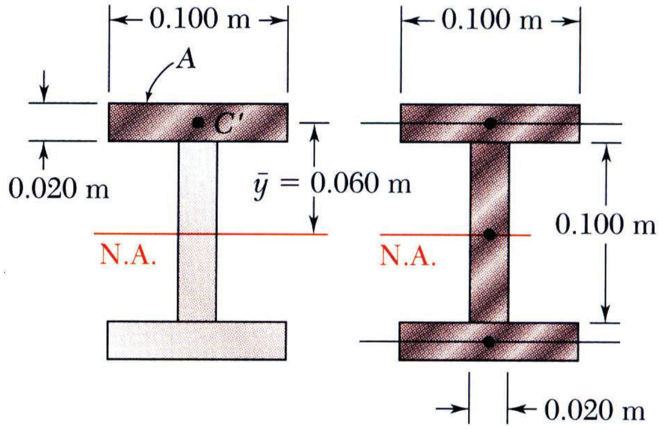
A beam is made of three planks, nailed together. Knowing that the spacing between nails is 25 mm and that the vertical shear in the beam is V = 500 N, determine the shear force in each nail.



Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

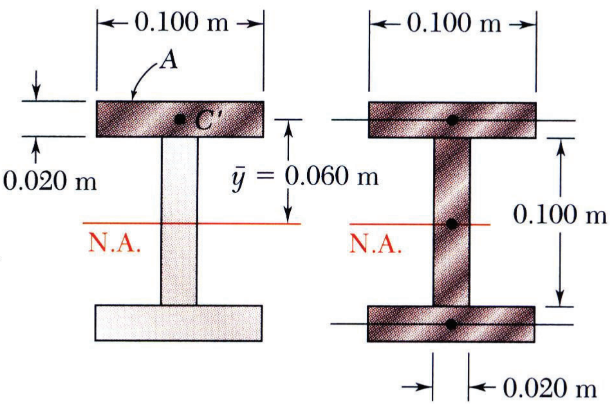
Example 2



Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

Example 2



$$Q = 120\times10^{-6}\,\text{m}^3$$
$$I = 16.20\times10^{-6}\,\text{m}^4$$

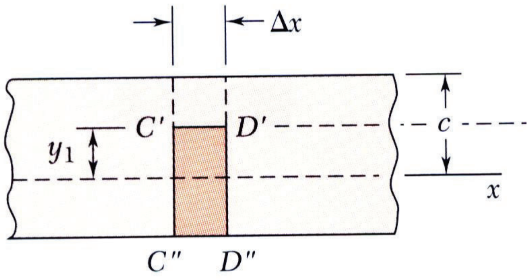
SOLUTION:

- Determine the horizontal force per unit length or shear flow q on the lower surface of the upper plank.

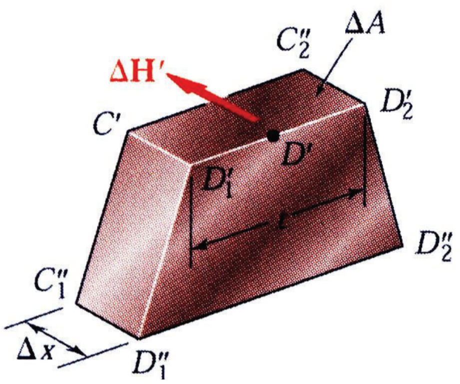
- Calculate the corresponding shear force in each nail for a nail spacing of 25 mm.

Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula



- **The *average* shearing stress** on the horizontal face of the element is obtained by dividing the shearing force on the element by the area of the face.

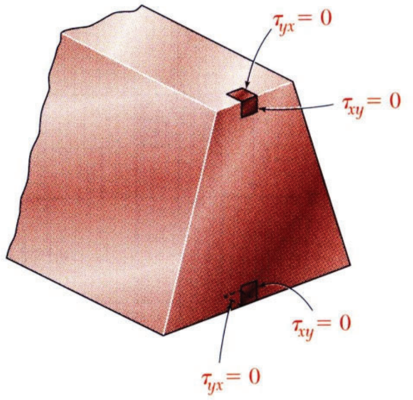


$$\tau_{ave} = \frac{\Delta H}{\Delta A} = \frac{q\,\Delta x}{\Delta A} = \frac{VQ}{I}\frac{\Delta x}{t\,\Delta x}$$

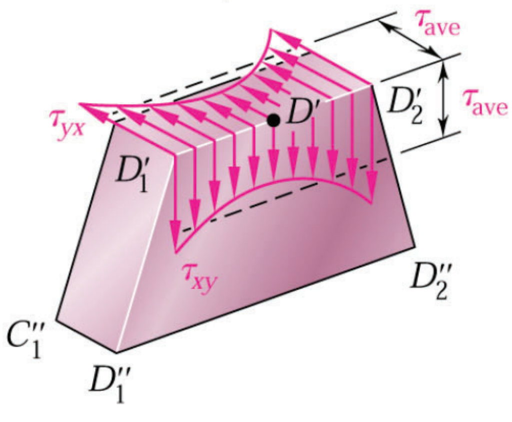
$$\Rightarrow \boxed{\tau_{ave} = \frac{VQ}{It}}$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula



- On the upper and lower surfaces of the beam, $\tau_{yx}= 0$. It follows that $\tau_{xy}= 0$ on the upper and lower edges of the transverse sections.

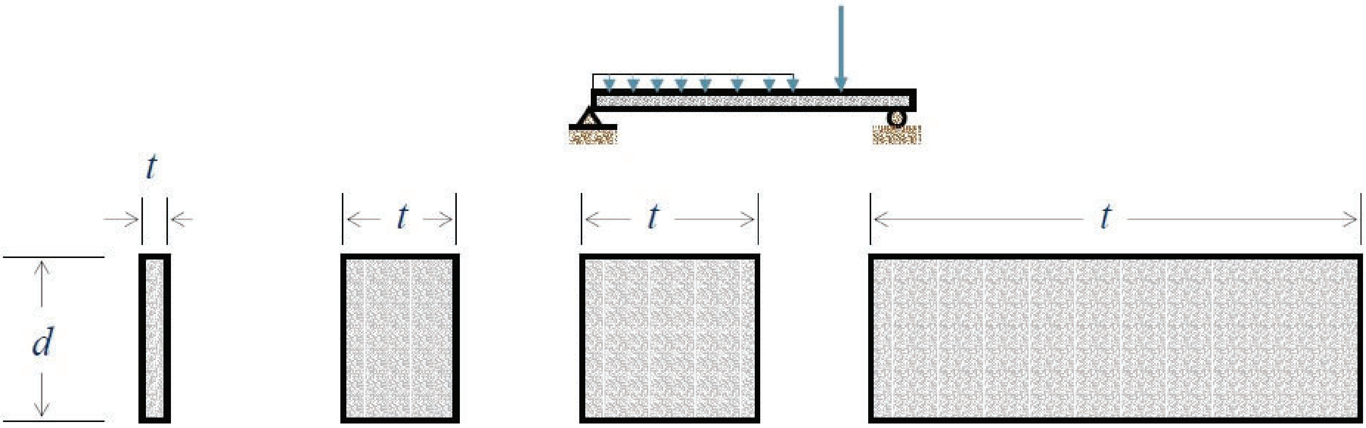


If the width of the beam is comparable or large relative to its depth, the shearing stresses at D_1 and D_2 are significantly higher than at D.

Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

How accurate is the shearing stress formula?



Great	3% error	12% error	100% error, worst case
t is small	$d = 2\,t$	$d = t$	$4d = t$

Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

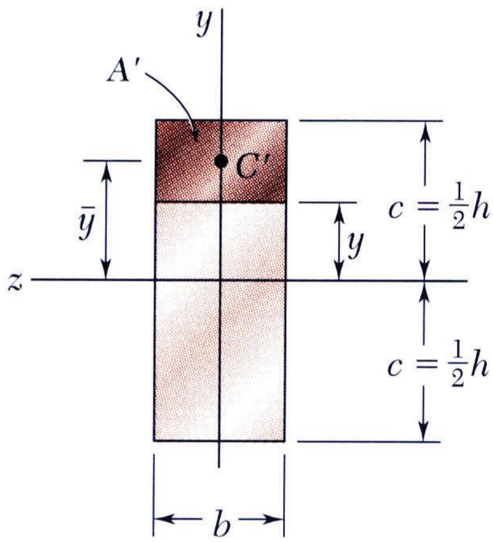
- Shear stress distribution in narrow rectangular beam,

$$Q = \bar{y}\cdot A = [\frac{h}{2} - \frac{1}{2}(\frac{h}{2} - y)]\cdot[b(\frac{h}{2} - y)] = \boxed{\frac{1}{2}b(\frac{h^2}{4} - y^2)}$$

$$I = \frac{1}{12}bh^3$$

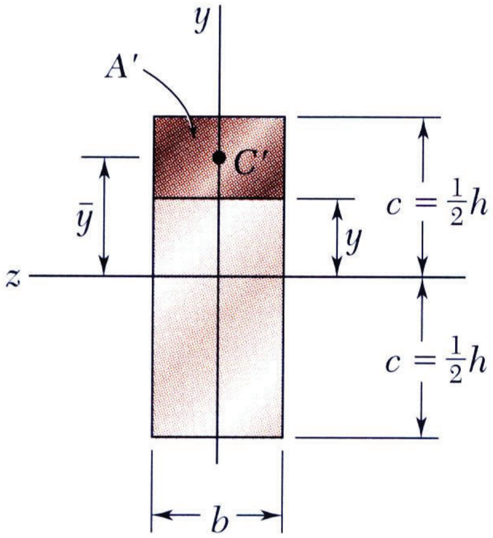
$$\tau_{xy} = \frac{VQ}{It} = \frac{V\times\frac{1}{2}b(\frac{h^2}{4} - y^2)}{\frac{1}{12}bh^3\times b} = \boxed{\frac{3}{2}\frac{V}{A}\left[1 - \left(\frac{2y}{h}\right)^2\right]}$$

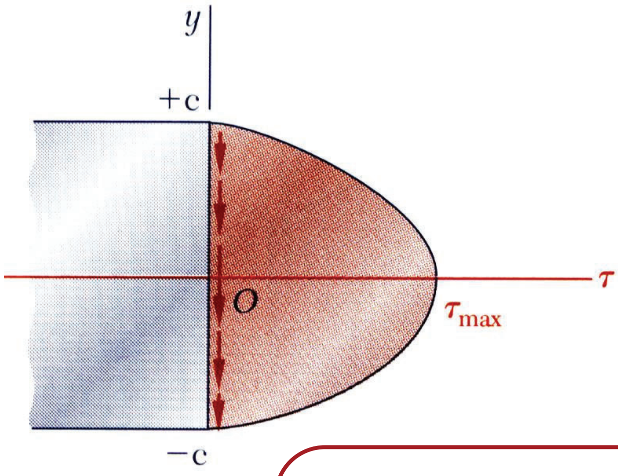
$$y = 0 \quad \Rightarrow \quad \boxed{\tau_{\max} = \frac{3}{2}\frac{V}{A}}$$



Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula





- For a narrow rectangular beam,

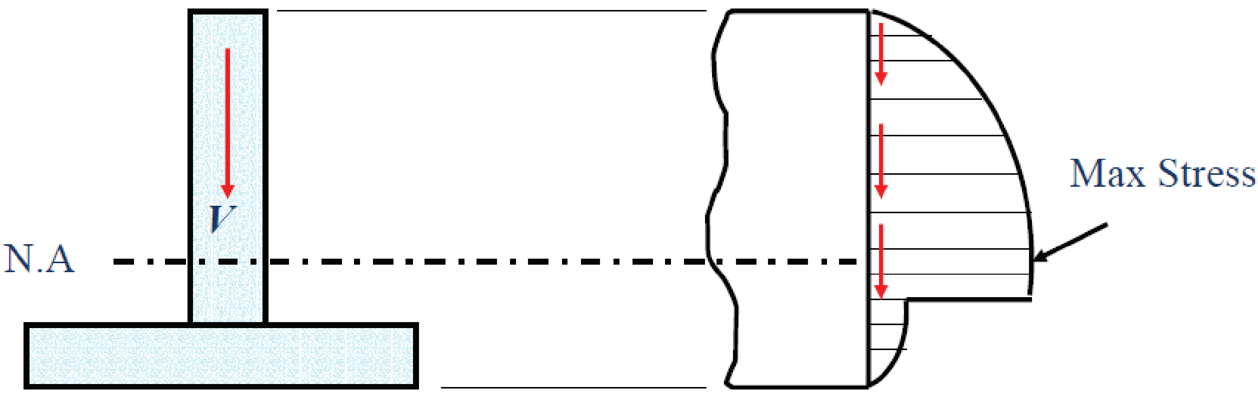
$$\tau_{xy} = \frac{3}{2}\frac{V}{A}\left[1-\left(\frac{2y}{h}\right)^2\right]$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$

Shearing Stresses in Beams and Thin-Walled Members

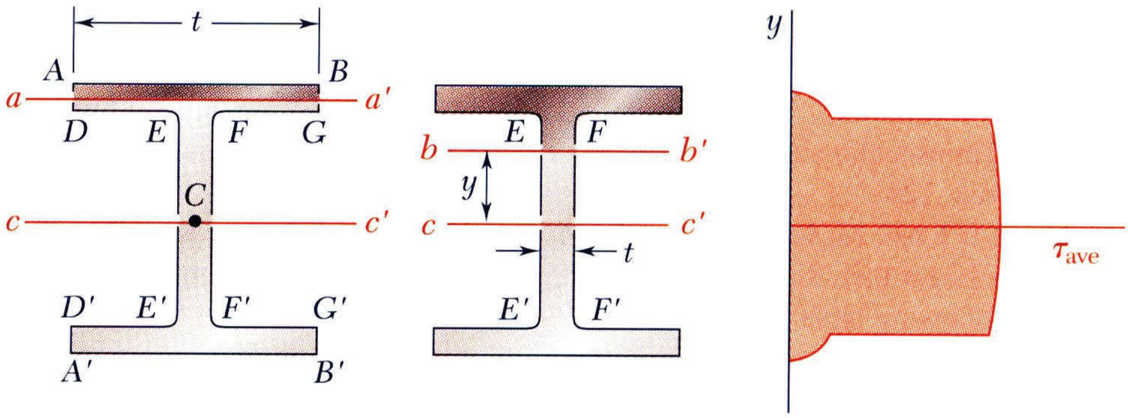
❑ Development of Shear Stress Formula

Variation of Vertical Shearing Stress in the Cross Section



Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula



- For American Standard (S-beam) and wide-flange (W-beam) beams

$$\tau_{ave} = \frac{VQ}{It}$$

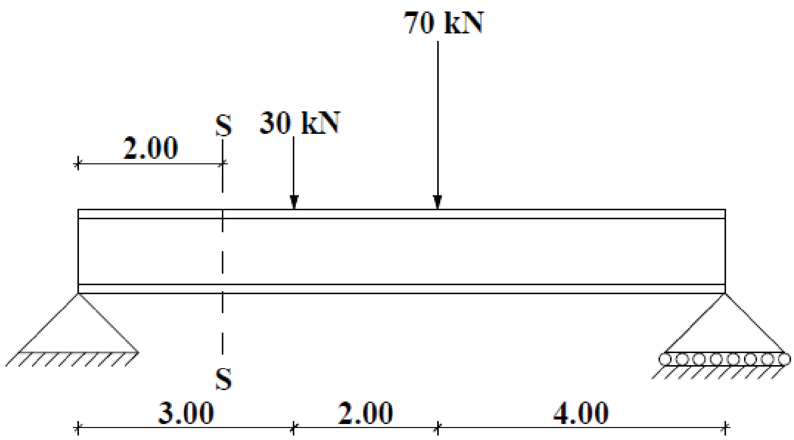
$$\tau_{max} = \frac{V}{A_{web}}$$

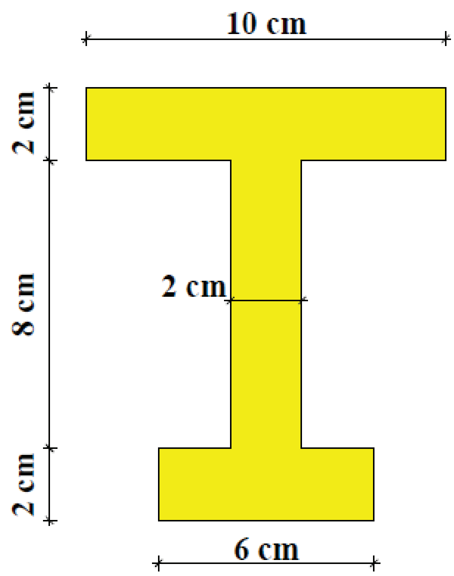
Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

Example 3

The simple supported beam is loaded as follows. Determine shear stress distribution on the cross section S-S.



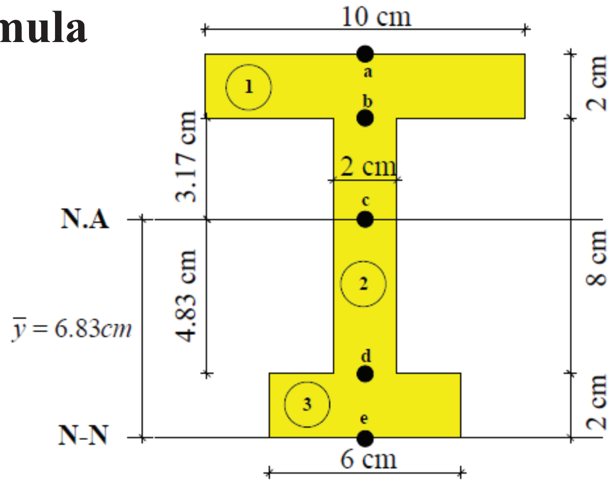


Shearing Stresses in Beams and Thin-Walled Members

Development of Shear Stress Formula

Example 3

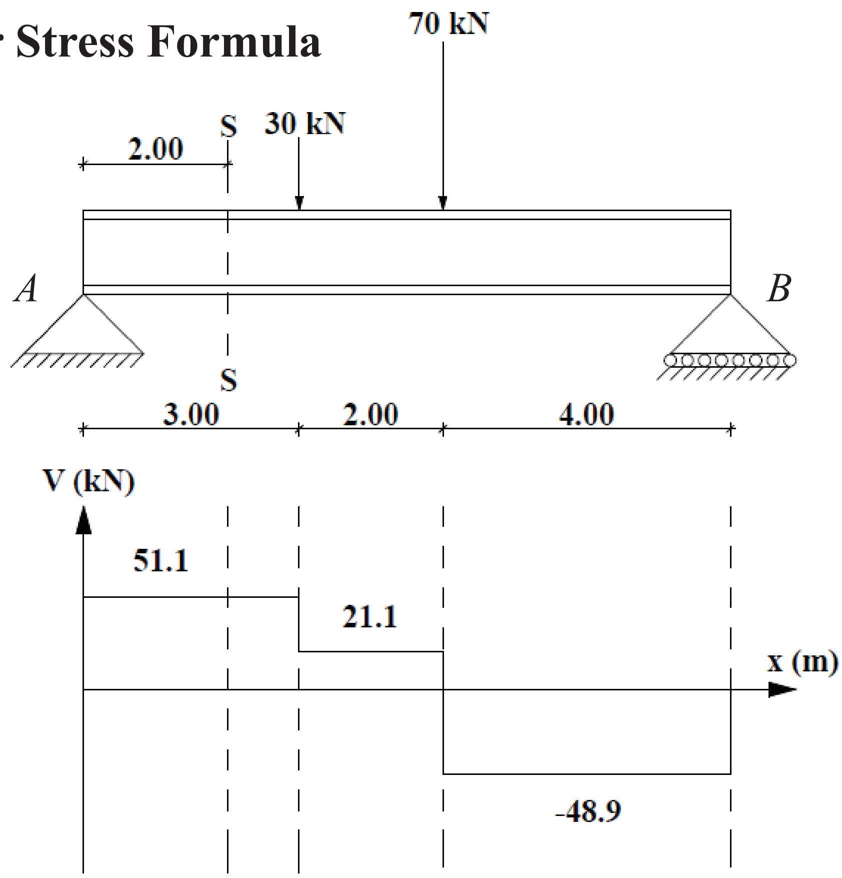
No	$\bar{y}_i$	A	$A\bar{y}_i$	$A\bar{y}_i^{\,2}$	I_g
1					
2					
3					
$\sum$					



Shearing Stresses in Beams and Thin-Walled Members

Development of Shear Stress Formula

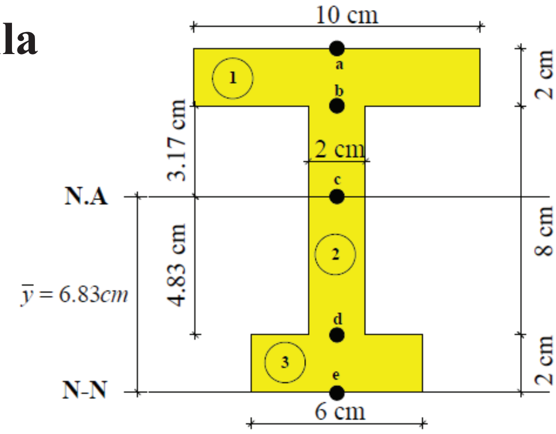
Example 3



Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

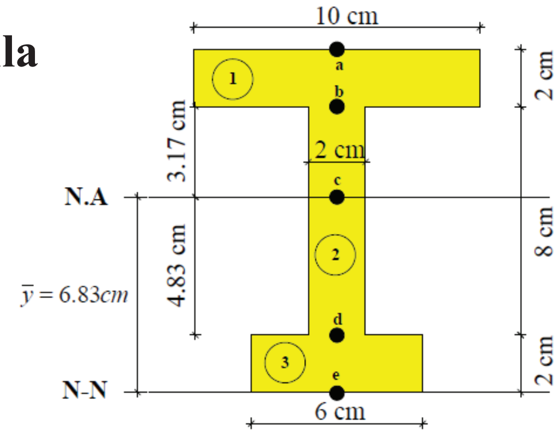
Example 3



Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

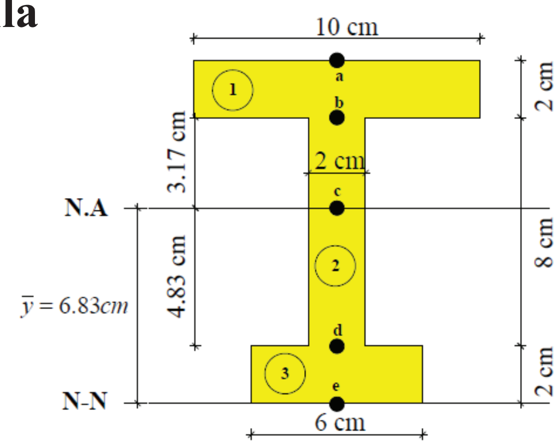
Example 3



Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

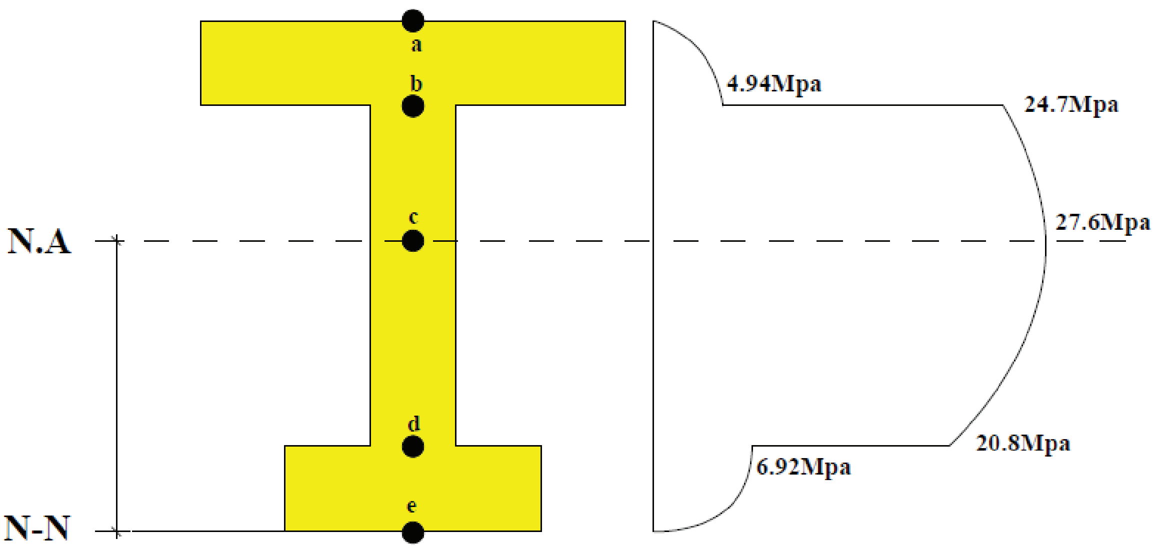
Example 3



Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

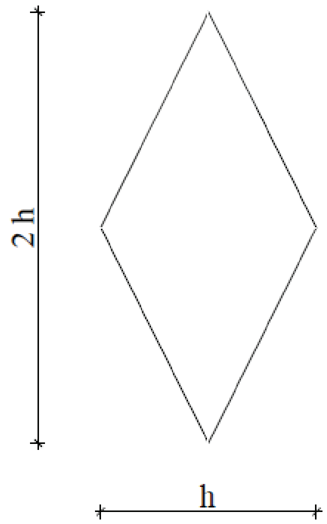
Example 3



- **Development of Shear Stress Formula**

Example 4

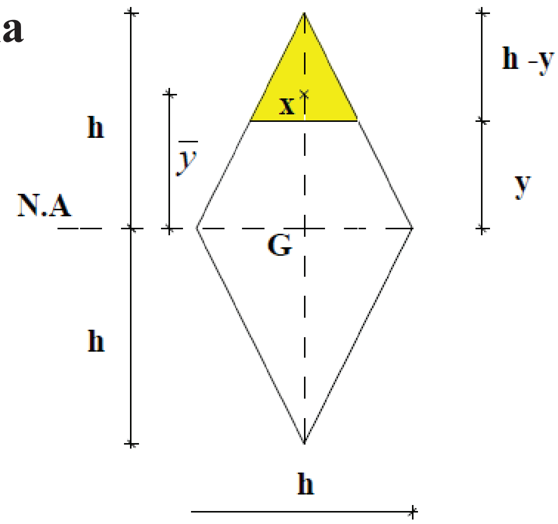
Determine shear stress distribution on the rhombic cross section as shown in following figure.



- **Development of Shear Stress Formula**

Example 4

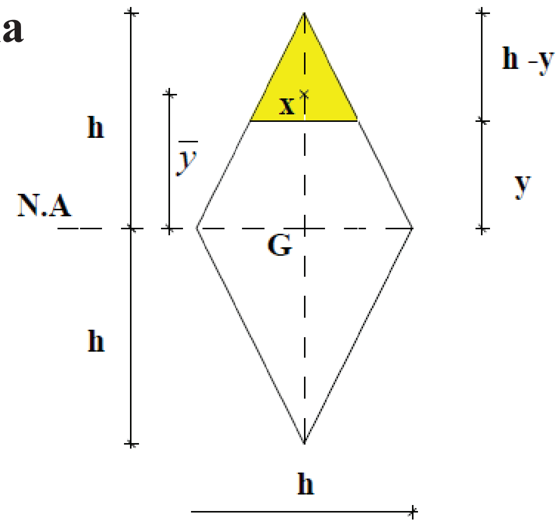
Thales' theorem:



- **Development of Shear Stress Formula**

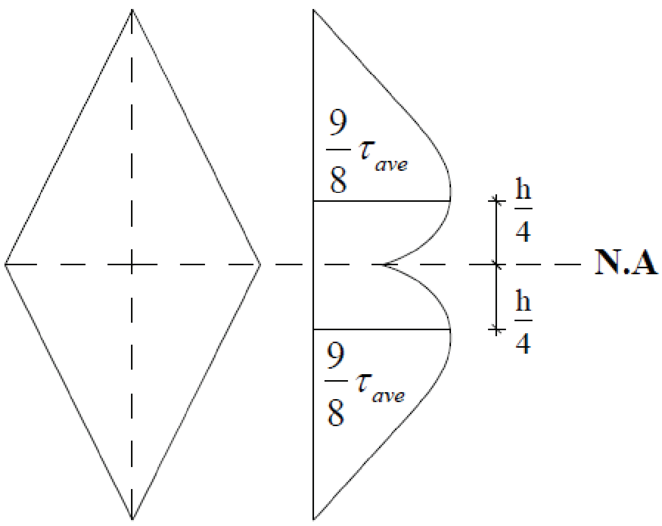
Example 4

Determine maximum shear stress



- **Development of Shear Stress Formula**

Example 4



Shearing Stresses in Beams and Thin-Walled Members

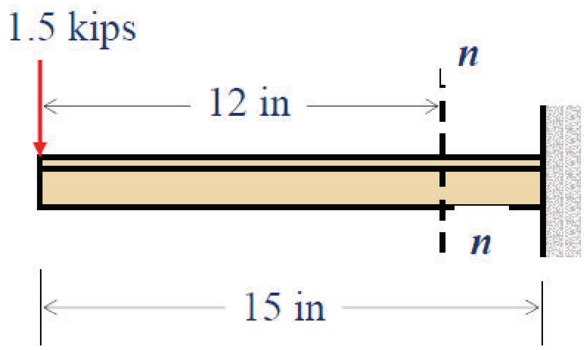
❑ Development of Shear Stress Formula

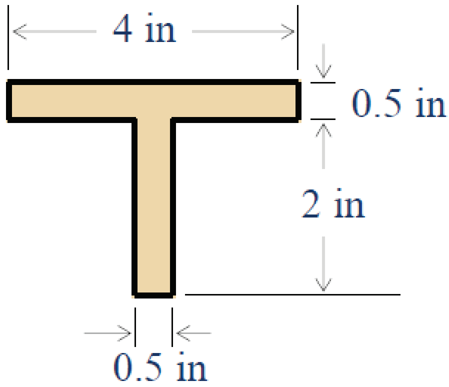
Example 5

A machine part has a T-shaped cross section and is acted upon in its plane of symmetry by the single force shown. Determine:

(a) The maximum compressive stress at section n-n and

(b) The maximum shearing stress.

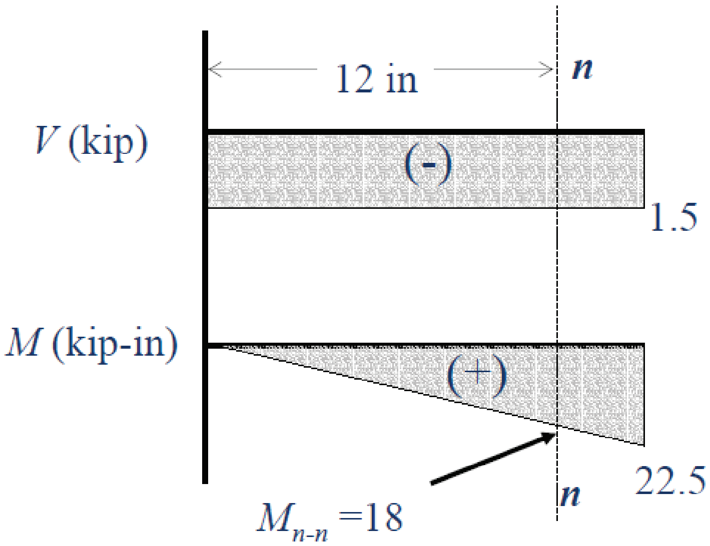


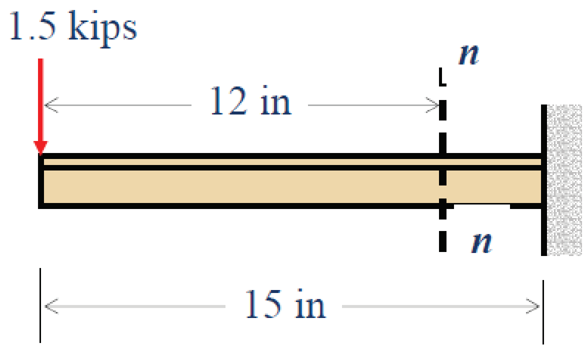


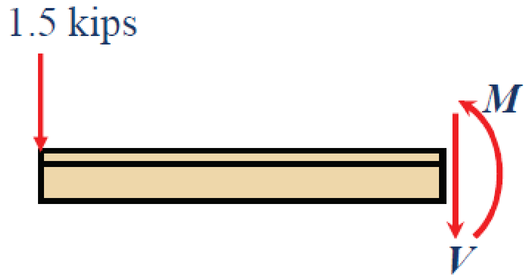
Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

Example 5



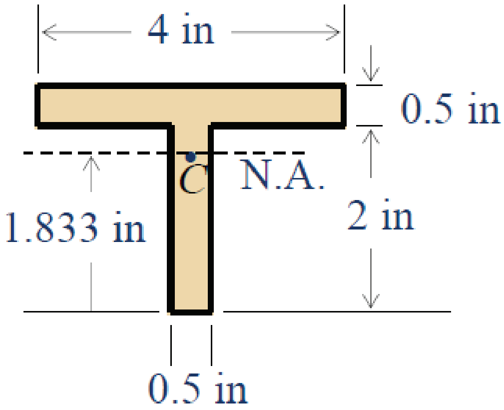


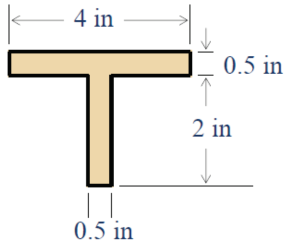


Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

Example 5





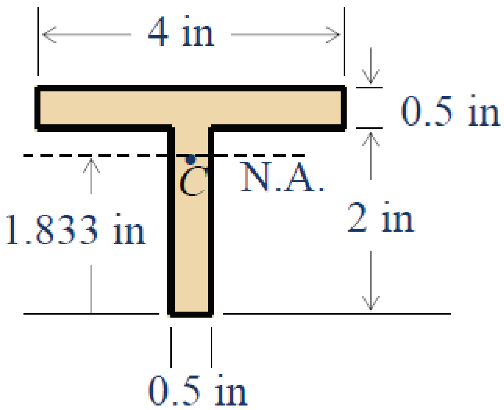
First, we need to locate the neutral axis. Let's make our reference from the bottom edge.

Next find the moment of inertia about the neutral axis:

Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

Example 5

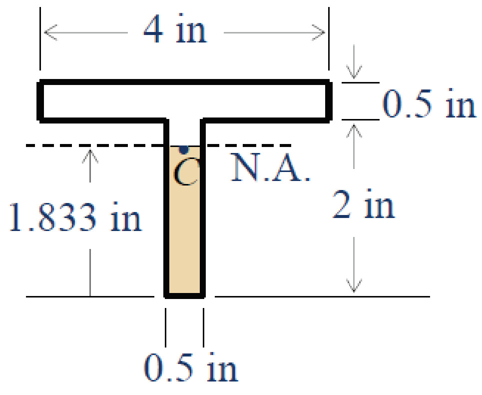


(a) Maximum normal stress is a compressive stress:

Shearing Stresses in Beams and Thin-Walled Members

❑ Development of Shear Stress Formula

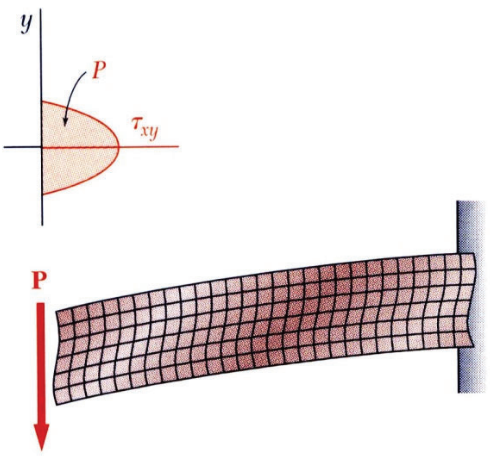
Example 5



(b) Maximum shearing stress:
The maximum value of Q occurs at the neutral axis. Since in this cross section the width t is minimum at the neutral axis, the maximum shearing stress will occur there. Choosing the area below a-a at the neutral axis, we have

Shearing Stresses in Beams and Thin-Walled Members

❑ Further Discussion of the Distribution of Stresses in a Narrow Rectangular Beam



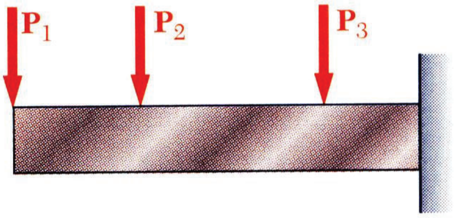
- Consider a narrow rectangular cantilever beam subjected to load P at its free end:

$$\tau_{xy} = \frac{3}{2}\frac{P}{A}\left(1 - \frac{y^2}{c^2}\right) \qquad \sigma_x = +\frac{Pxy}{I}$$

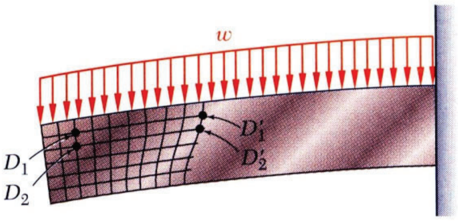
- Shearing stresses are independent of the distance from the point of application of the load.
- Normal strains and normal stresses are unaffected by the shearing stresses.

Shearing Stresses in Beams and Thin-Walled Members

❑ Further Discussion of the Distribution of Stresses in a Narrow Rectangular Beam



- From Saint-Venant's principle, effects of the load application mode are negligible except in immediate vicinity of load application points.



- Stress/strain deviations for distributed loads are negligible for typical beam sections of interest.

Shearing Stresses in Beams and Thin-Walled Members

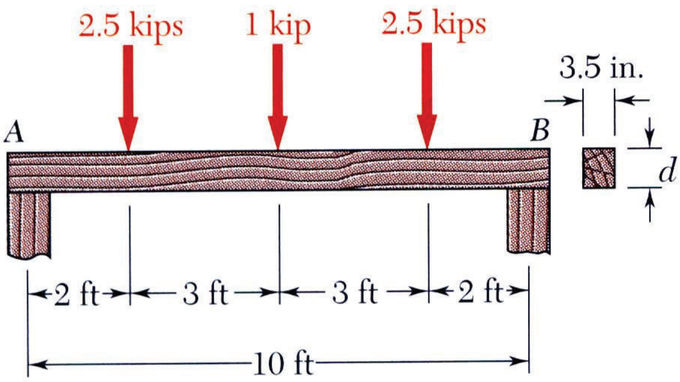
❑ Shearing Stress in Beams

Example 6

A timber beam is to support the three concentrated loads shown. Knowing that for the grade of timber used,

$$\sigma_{all} = 1800\,\text{psi} \qquad \tau_{all} = 120\,\text{psi}$$

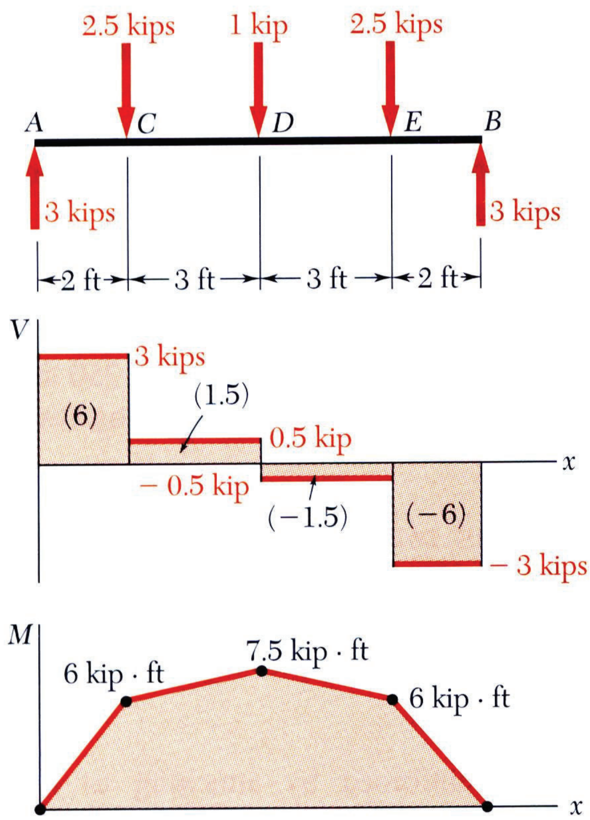
determine the minimum required depth d of the beam.



Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stress in Beams

Example 6



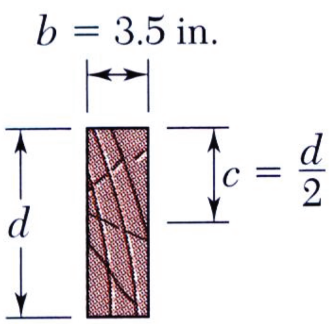
SOLUTION:

Develop shear and bending moment diagrams. Identify the maximums.

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stress in Beams

Example 6



- Determine the beam depth based on allowable normal stress.

- Determine the beam depth based on allowable shear stress.

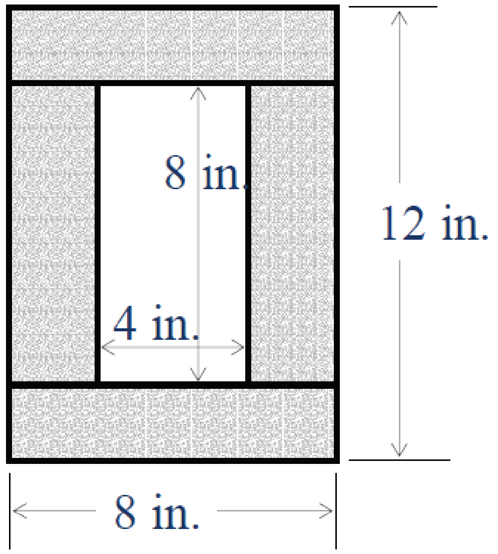
- Required beam depth is equal to the larger of the two.

❑ Shearing Stress in Beams

Example 7

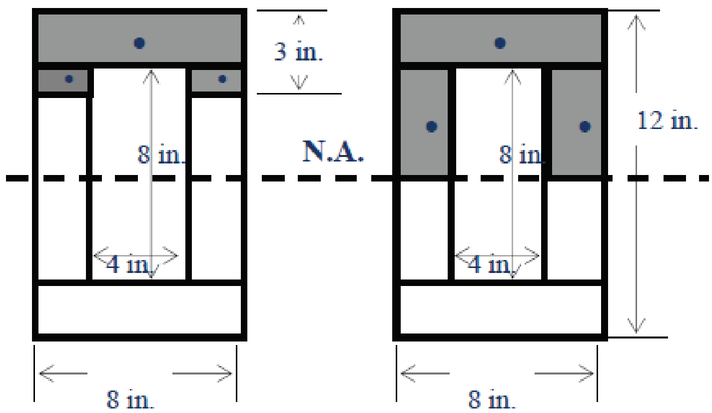
The transverse shear V at a certain section of a timber beam is 6000 lb. If the beam has the cross section shown in the figure, determine:

(a) The vertical shearing stress 3 in. below the top of the beam, and
(b) The maximum vertical stress on the cross section.



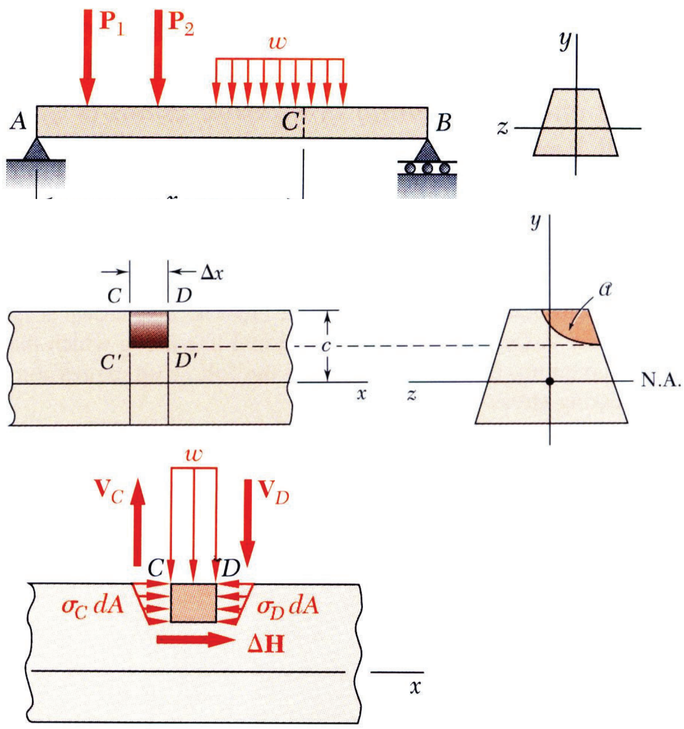
❑ Shearing Stress in Beams

Example 7



Shearing Stresses in Beams and Thin-Walled Members

❑ Longitudinal Shear on a Beam Element of Arbitrary Shape



- We have examined the distribution of the vertical components τ_{xy} on a transverse section of a beam. We now wish to consider the horizontal components τ_{zx} of the stresses.
- Consider prismatic beam with an element defined by the curved surface CDD'C'.

$$\sum F_x = 0 = \Delta H + \int_a (\sigma_D - \sigma_C)\, dA$$

- Except for the differences in integration areas, this is the same result obtained before which led to

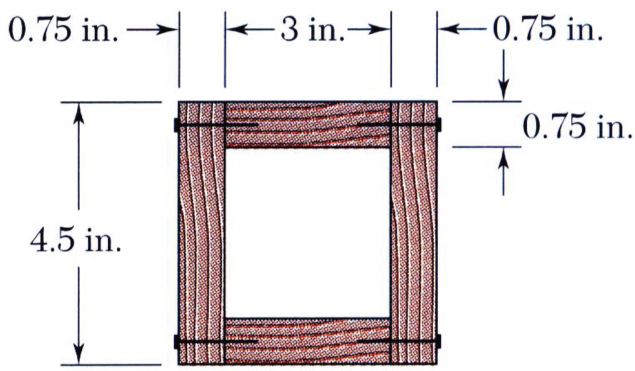
$$\Delta H = \frac{VQ}{I}\Delta x \qquad q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Longitudinal Shear on a Beam Element of Arbitrary Shape

Example 8

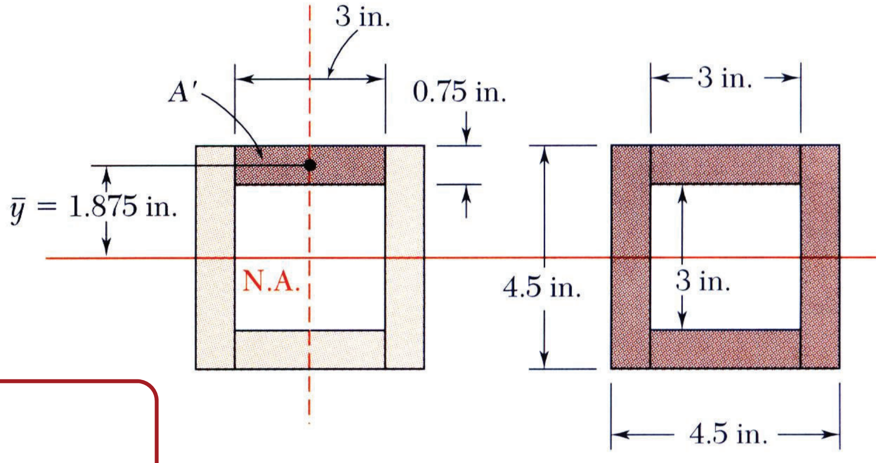
A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.75 in. and the beam is subjected to a vertical shear of magnitude $V = 600$ lb, determine the shearing force in each nail.



Shearing Stresses in Beams and Thin-Walled Members

❑ Longitudinal Shear on a Beam Element of Arbitrary Shape

Example 8



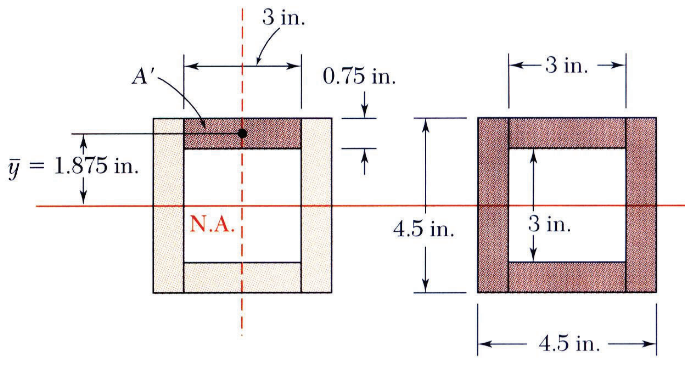
For the upper plank,

For the overall beam cross-section,

Shearing Stresses in Beams and Thin-Walled Members

❑ Longitudinal Shear on a Beam Element of Arbitrary Shape

Example 8



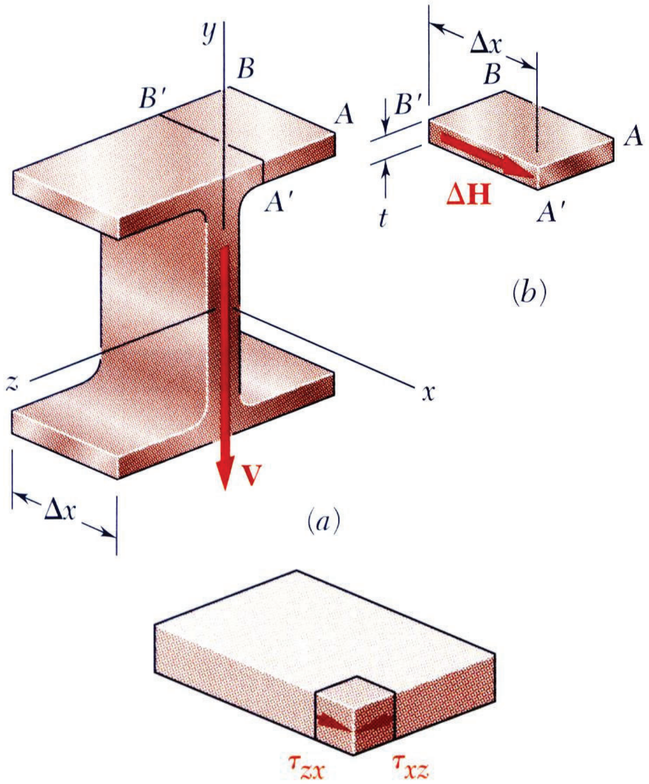
SOLUTION:

- Determine the shear force per unit length along each edge of the upper plank.

- Based on the spacing between nails, determine the shear force in each nail.

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members



- Consider a segment of a wide-flange beam subjected to the vertical shear V.
- The longitudinal shear force on the element is

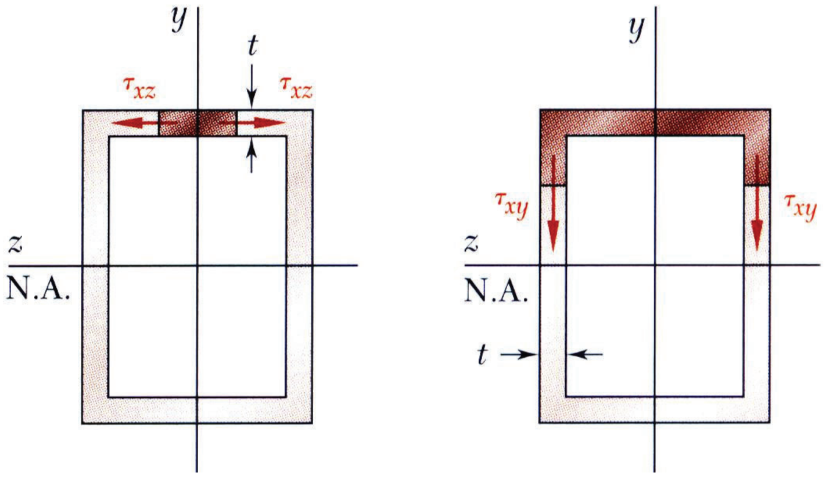
$$\Delta H = \frac{VQ}{I}\Delta x$$

- The corresponding shear stress is

$$\tau_{zx} = \tau_{xz} \approx \frac{\Delta H}{t\,\Delta x} = \frac{VQ}{It}$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

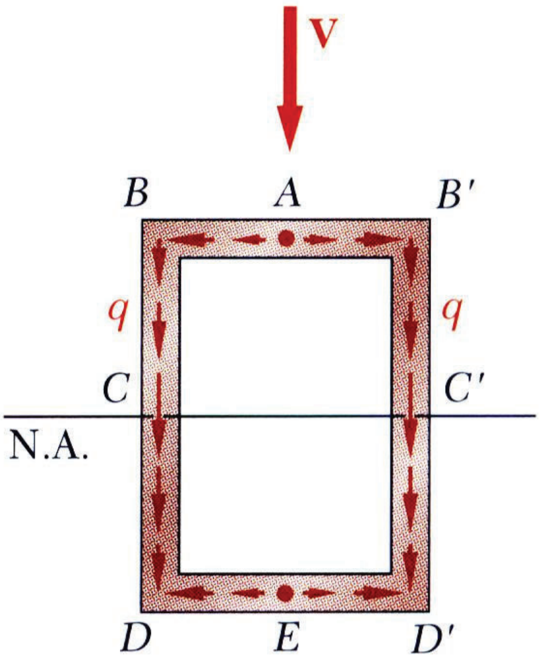


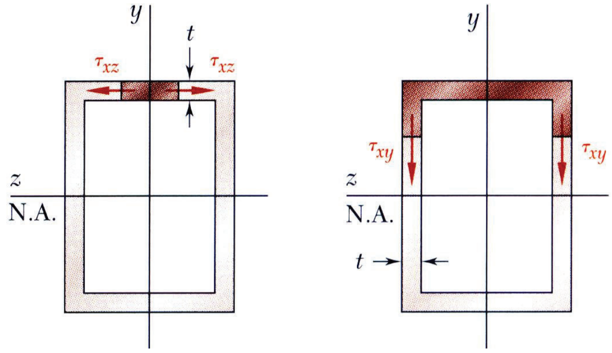
- The variation of shear flow across the section depends only on **the variation of the first moment**.

$$q = \tau t = \frac{VQ}{I}$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

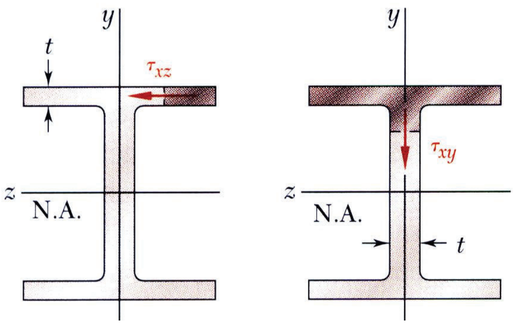




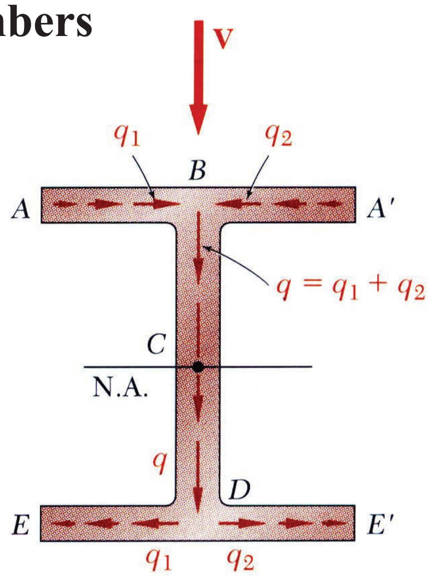
- For a box beam, q grows smoothly from zero at A to a maximum at C and C' and then decreases back to zero at E.

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members



- For a wide-flange beam, the shear flow increases symmetrically from zero at A and A', reaches a maximum at C and the decreases to zero at E and E'.

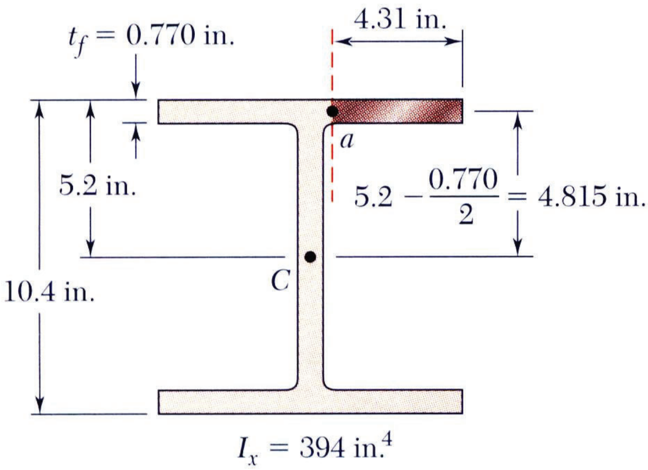


- The continuity of the variation in q and the merging of q from section branches suggests an analogy to fluid flow.

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 9

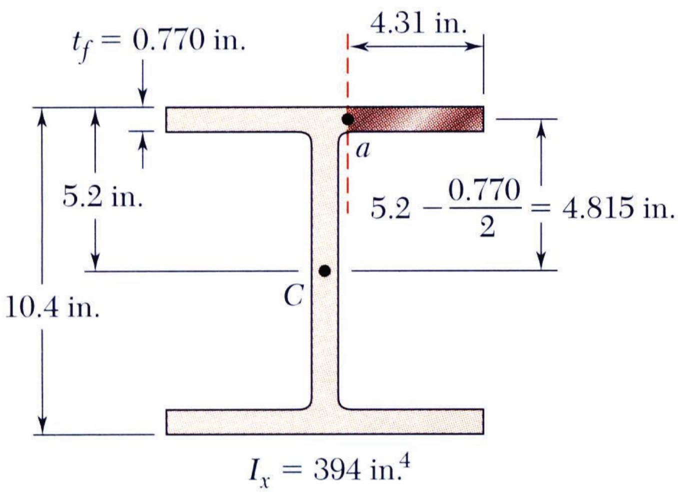


Knowing that the vertical shear is 50 kips in a W10x68 rolled-steel beam, determine the horizontal shearing stress in the top flange at the point *a*.

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 9



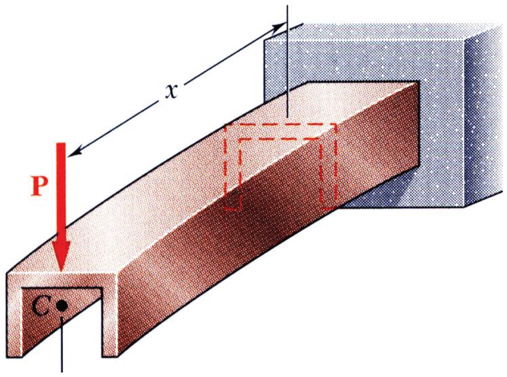
SOLUTION:

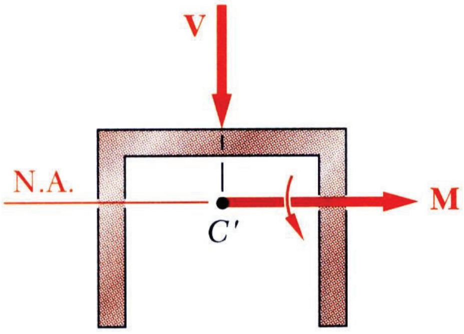
- For the shaded area,

- The shear stress at *a*,

Shearing Stresses in Beams and Thin-Walled Members

❑ Unsymmetric Loading of Thin-Walled Members



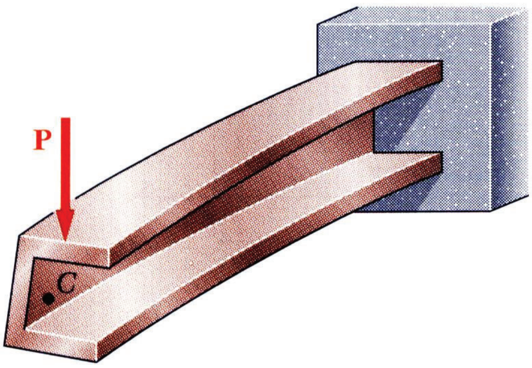


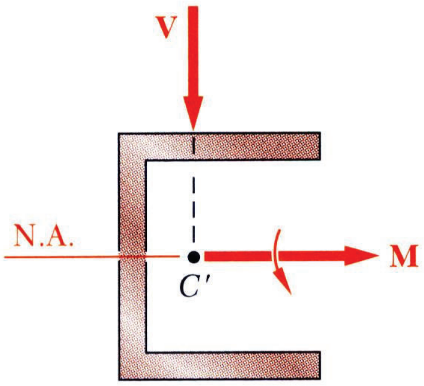
- Beam loaded in a vertical plane of symmetry deforms in the symmetry plane without twisting.

$$\sigma_x = -\frac{My}{I} \qquad \tau_{ave} = \frac{VQ}{It}$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Unsymmetric Loading of Thin-Walled Members



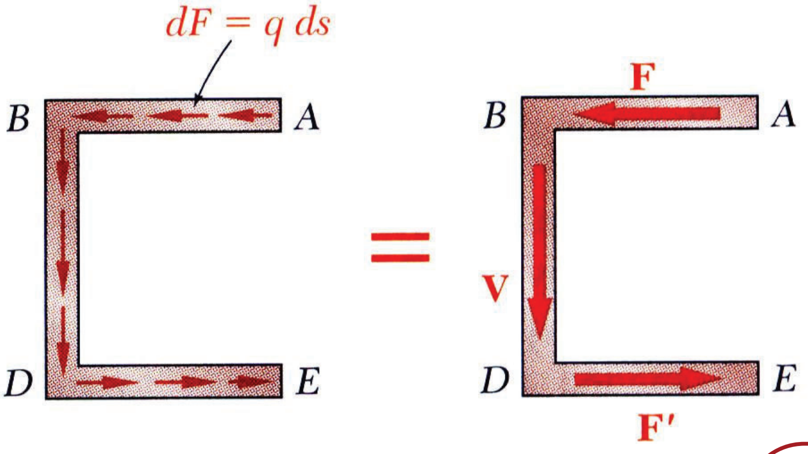


- Beam without a vertical plane of symmetry bends and twists under loading.

$$\sigma_x = -\frac{My}{I} \qquad \tau_{ave} \neq \frac{VQ}{It}$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Unsymmetric Loading of Thin-Walled Members



- If the shear load is applied such that the beam does not twist, then the shear stress distribution satisfies

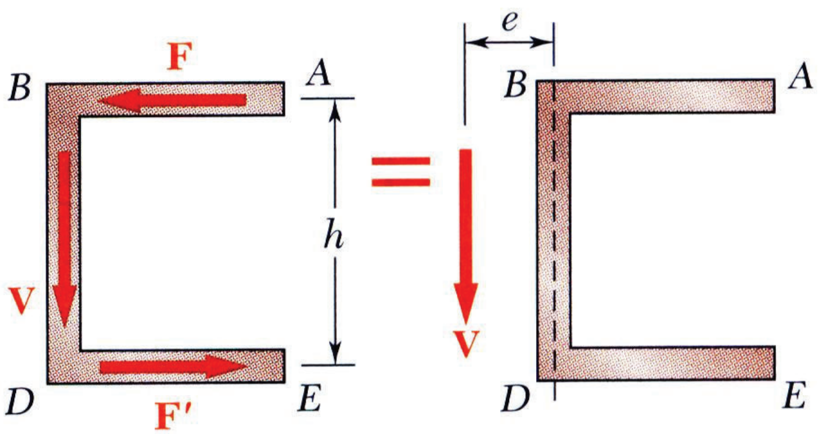
$$\tau_{ave} = \frac{VQ}{It}$$

$$V = \int_B^D q\, ds$$

$$F = \int_A^B q\, ds = -\int_D^E q\, ds = -F'$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Unsymmetric Loading of Thin-Walled Members

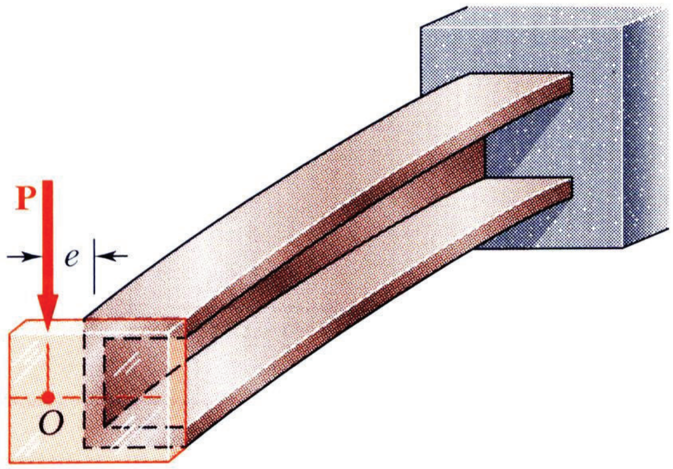


- *F* and *F'* indicate a couple *Fh* and the need for the application of a torque as well as the shear load.

$$Fh = Ve$$

Shearing Stresses in Beams and Thin-Walled Members

❑ Unsymmetric Loading of Thin-Walled Members



- When the force P is applied at a distance e to the left of the web centerline, the member bends in a vertical plane without twisting.

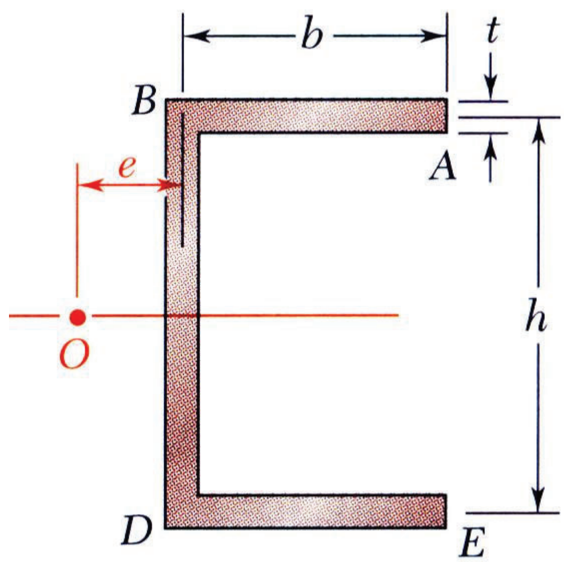
Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 10

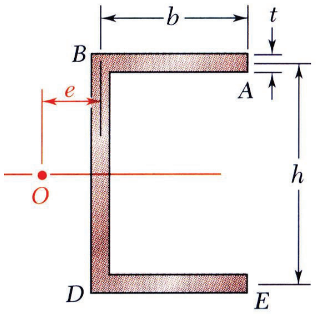
- Determine the location for the shear center of the channel section with

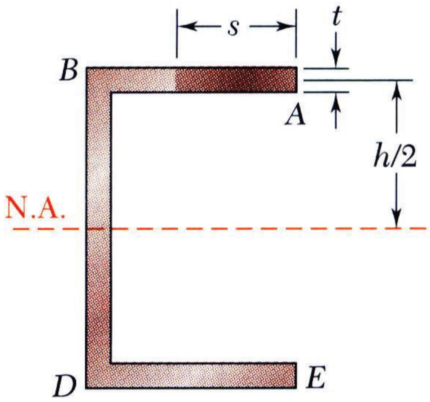
  $b = 4$ in., $h = 6$ in., and $t = 0.15$ in.



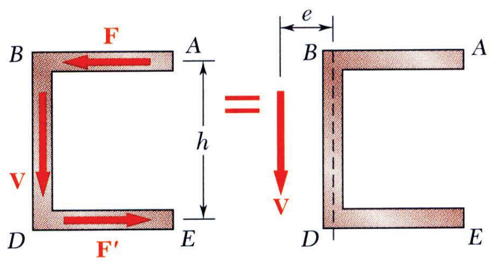
❑ Shearing Stresses in Thin-Walled Members

Example 10





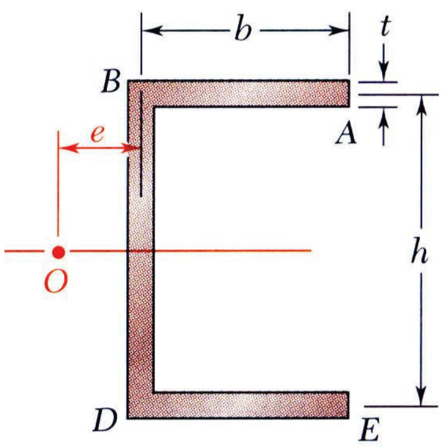
$$e = \frac{F\,h}{V}$$



- where

❑ Shearing Stresses in Thin-Walled Members

Example 10



- Combining,

Shearing Stresses in Beams and Thin-Walled Members

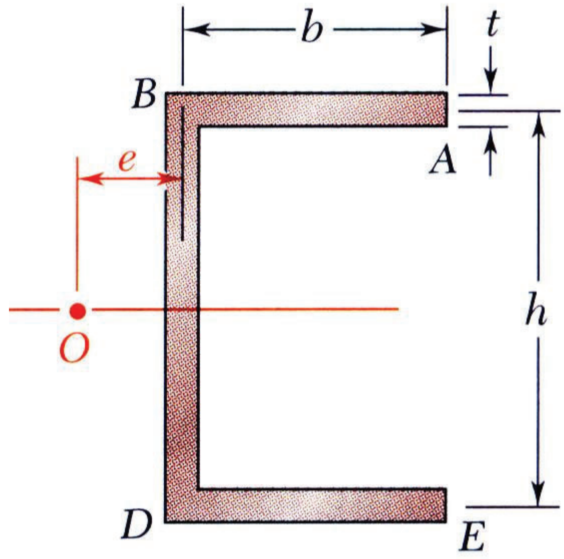
❑ Shearing Stresses in Thin-Walled Members

Example 11

- Determine the shear stress distribution for $V = 2.5$ kips.

  $b = 4$ in., $h = 6$ in., and $t = 0.15$ in.

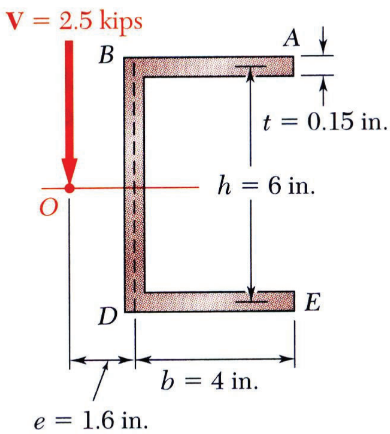
  $e = 1.6\,\text{in.}$

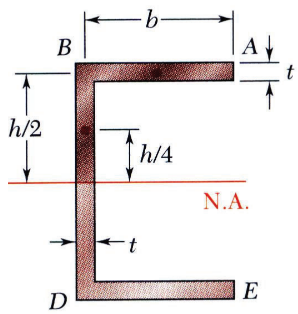


Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 11



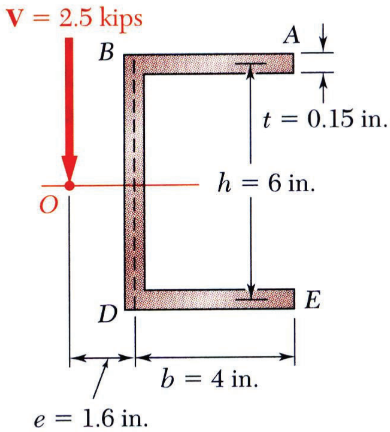


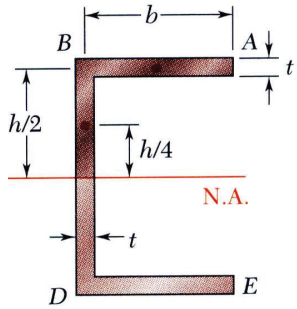
- Shearing stresses in the flanges,

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 11



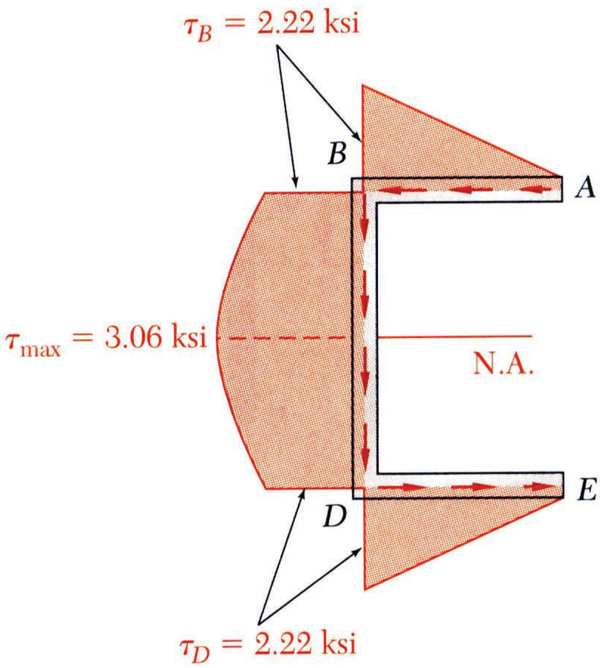


- Shearing stress in the web,

Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

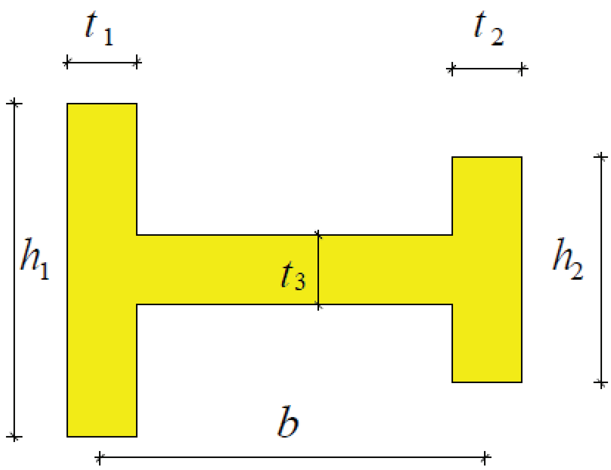
Example 11



❑ Shearing Stresses in Thin-Walled Members

Example 12

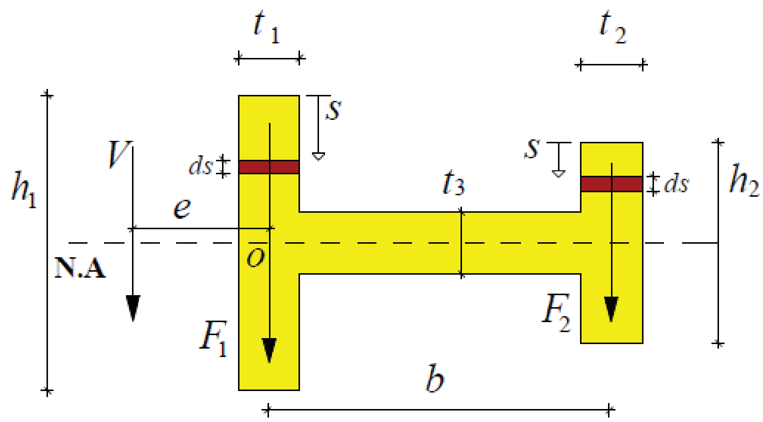
- Determine the location for the shear center of the channel section with



Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

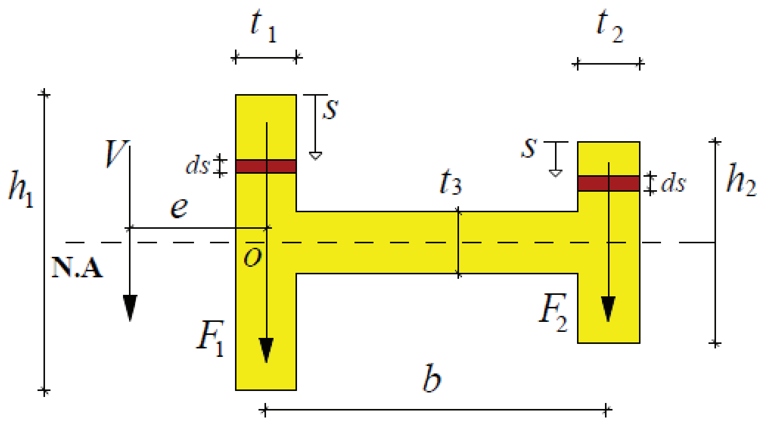
Example 12



Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 12

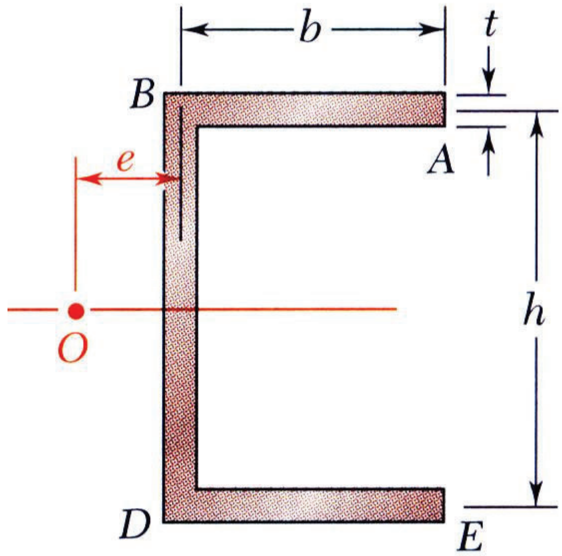


Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

For the cross section which is made only by horizontal and vertical elements, the shear center is determined using the following equation:

$$e = \frac{\sum I_i \bar{x}_i}{\sum I_i}$$



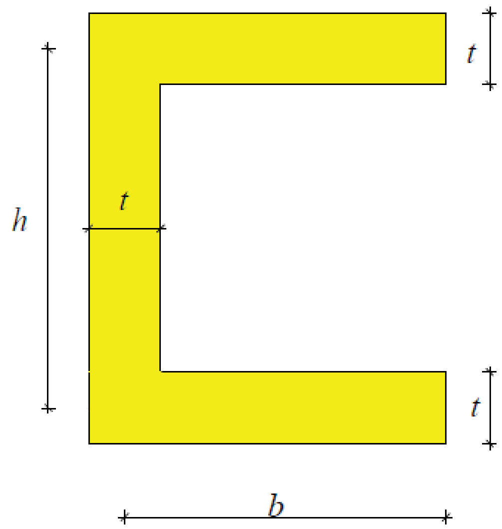
I_i : Moment inertia of the ith element refer to the neutral axis of the whole cross section

$\bar{x}_i$: Distance between area center of the ith element refer to the reference axis.

❑ Shearing Stresses in Thin-Walled Members

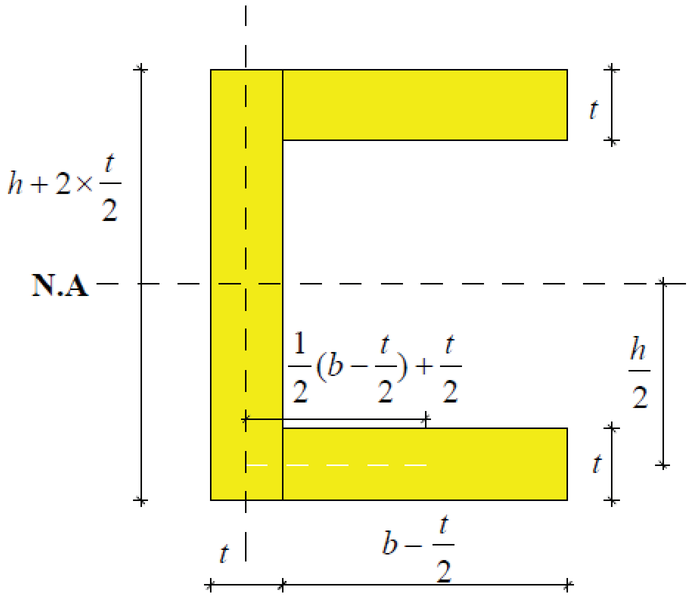
Example 13

- Determine the location for the shear center of the channel section with



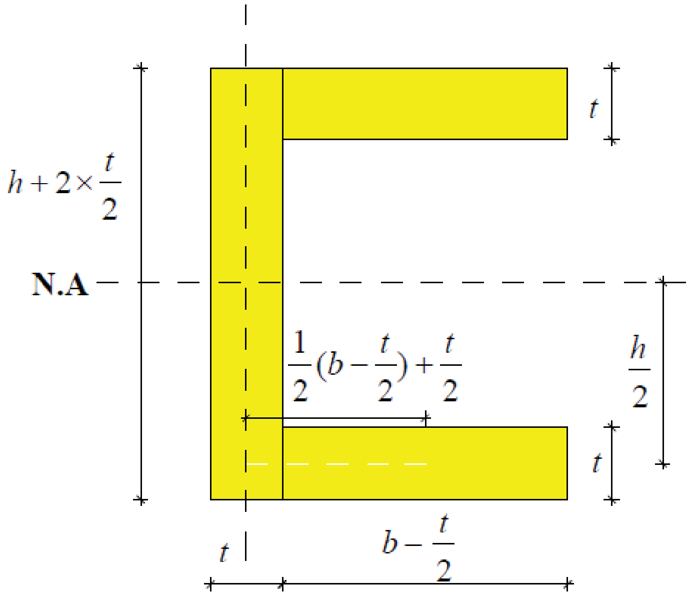
❑ Shearing Stresses in Thin-Walled Members

Example 13



❑ Shearing Stresses in Thin-Walled Members

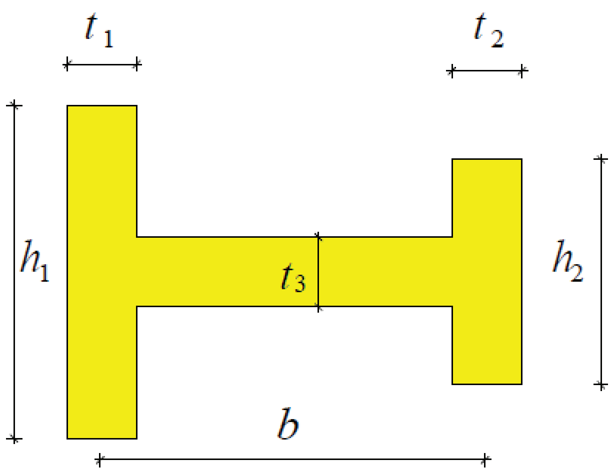
Example 13



❑ Shearing Stresses in Thin-Walled Members

Example 14

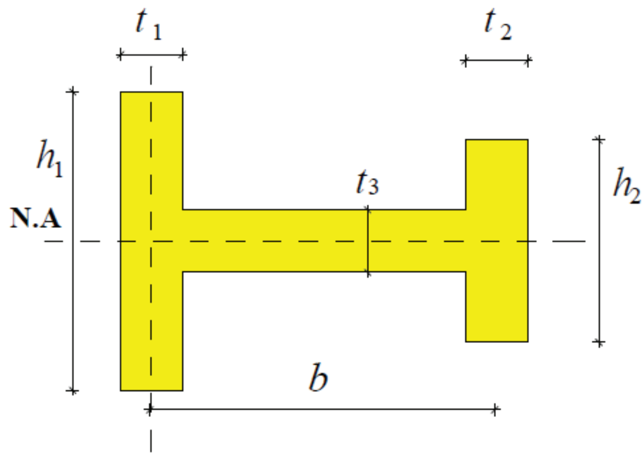
- Determine the location for the shear center of the channel section with



Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 14

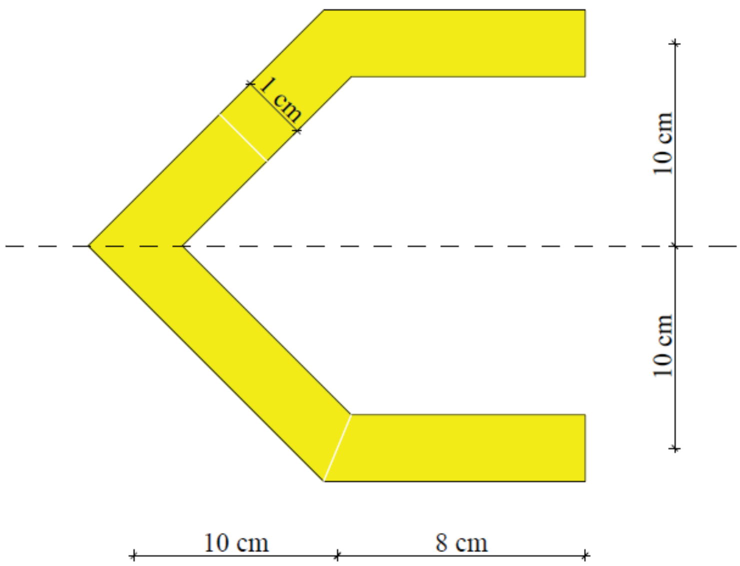


Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 15

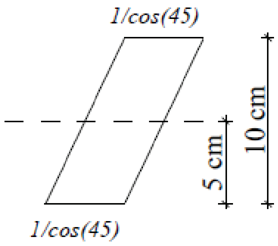
- Determine the location for the shear center of the channel section with

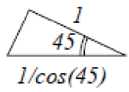


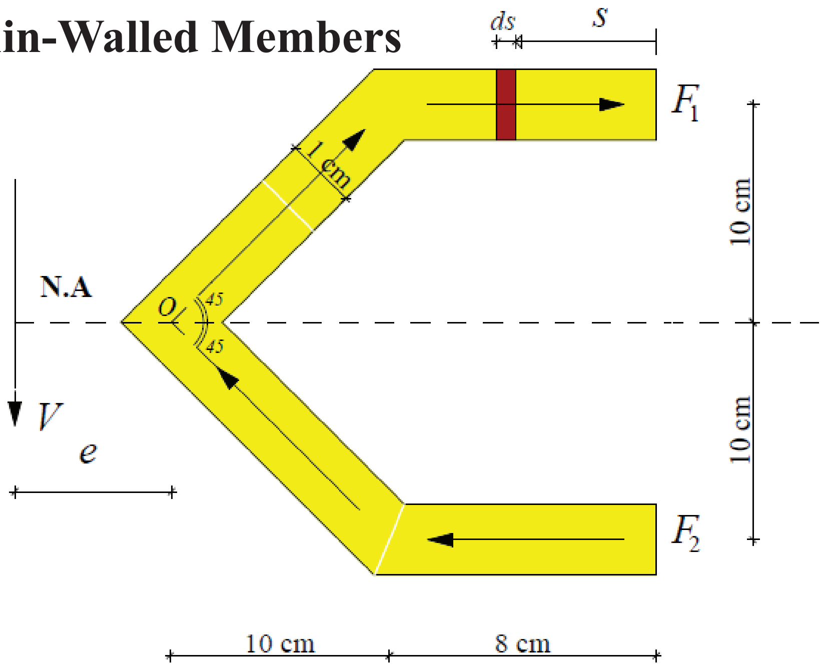
Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 15



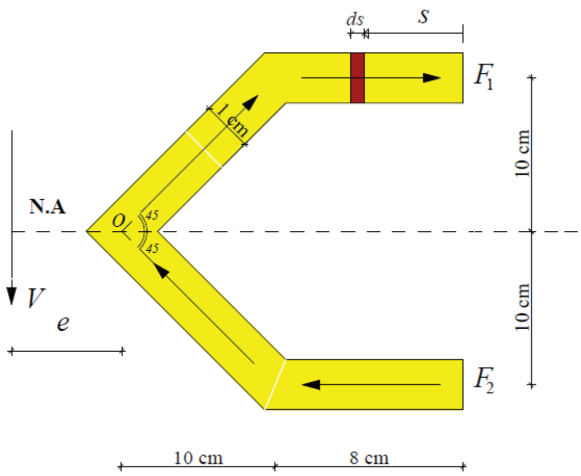




Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

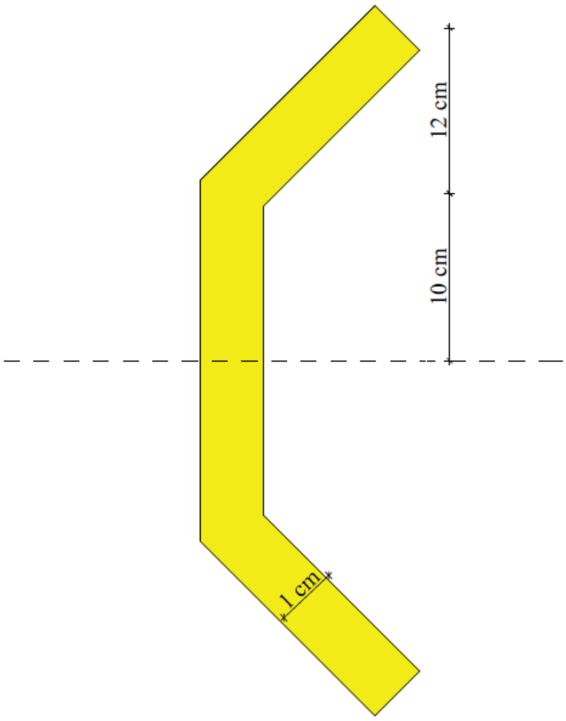
Example 15



Shearing Stresses in Thin-Walled Members

Example 16

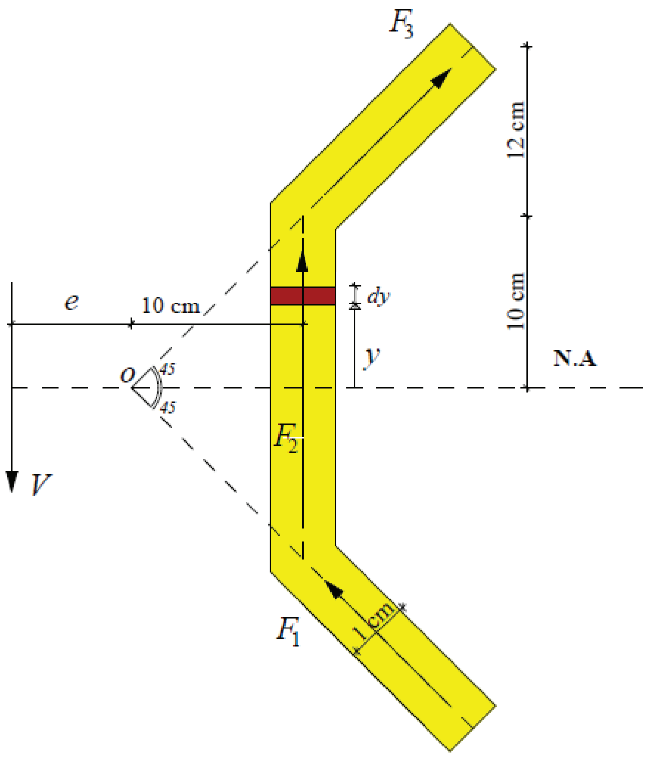
- Determine the location for the shear center of the channel section with



Shearing Stresses in Beams and Thin-Walled Members

Shearing Stresses in Thin-Walled Members

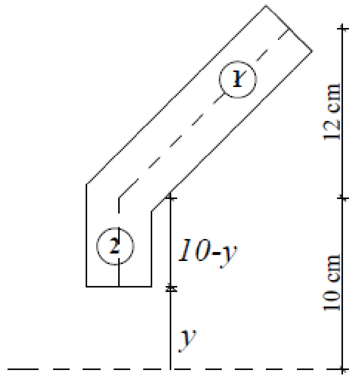
Example 16

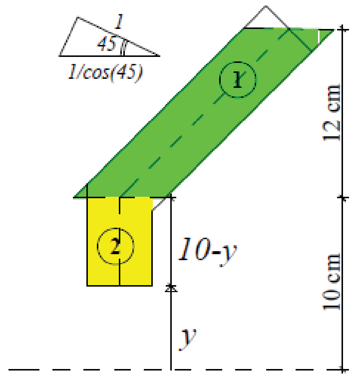


Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 16





Shearing Stresses in Beams and Thin-Walled Members

❑ Shearing Stresses in Thin-Walled Members

Example 16