Mechanics of Materials

Ferdinand P.Beer, E.Russel Johnston, Jr., John T.Dewolf

Other Reference:
J.Wat Oler "Lectures notes on Mechanics od Materials"
Ibrahim A.Assakkaf "Lectures notes on Mechanics od Materials"

Introduction –Concept of Stress

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Introduction –Concept of Stress

- Main objectives:

Mechanics of Materials answers tow questions:

- ✓ ***Is the material strong enough?***
- ✓ ***Is the material stiff enough?***

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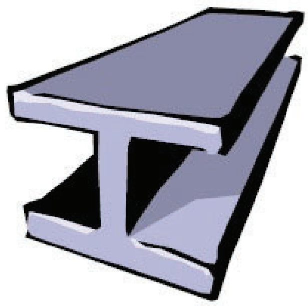
- Main objectives:
  - If the material is not strong enough, your design will break.
  - If the material isn't stiff enough, your design probably won't function the way it's intended to.



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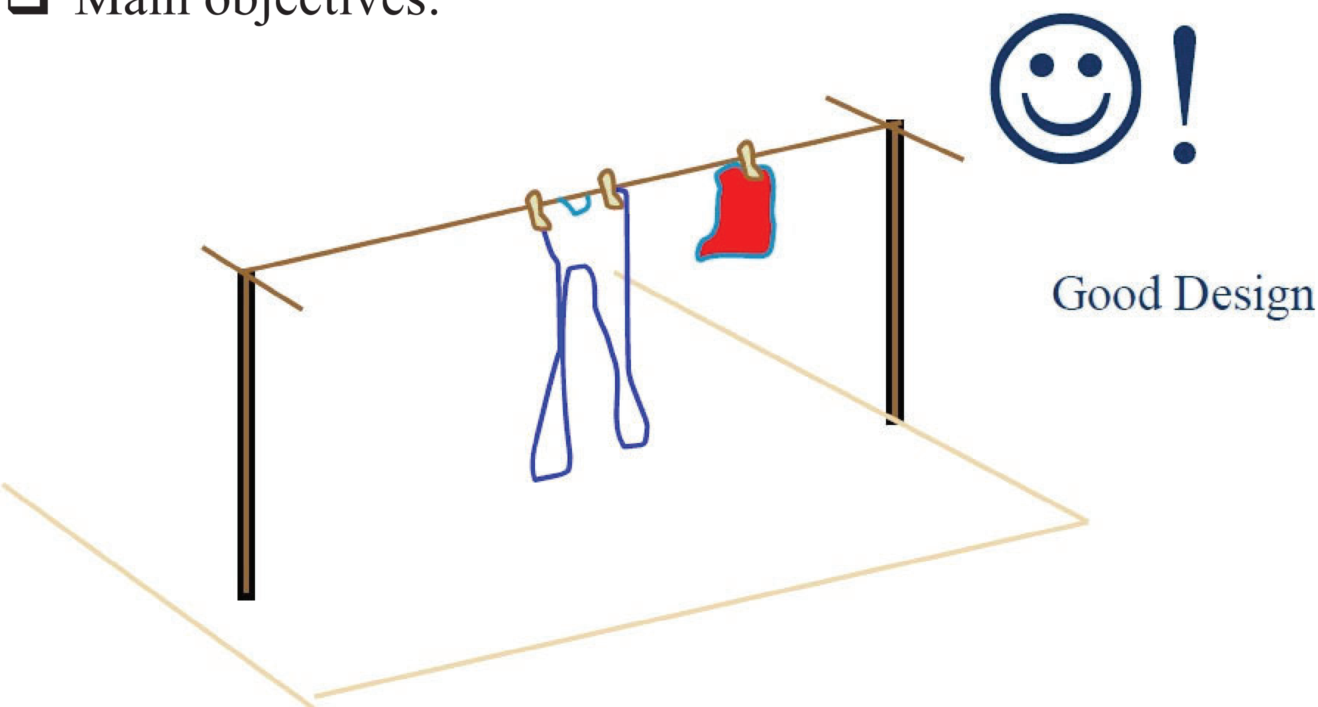
- Main objectives:



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- Main objectives:

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- Main objectives:

  - Provide the future engineer with the **means of analyzing and designing** various machines and load bearing structures.

  - Both the analysis and design of a given structure involve the **determination of *stresses* and *deformations*.** This chapter is devoted to the concept of stress.

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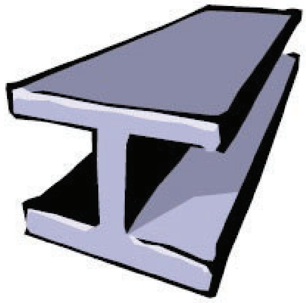
- Design Considerations:
  - Safety.
  - Economy.



Design of Engineering systems is usually trade off between **maximizing safety** and **minimizing cost**.

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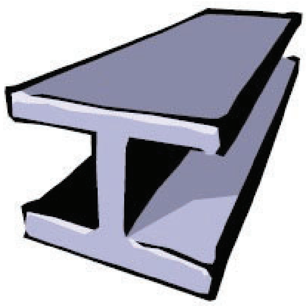
- Typical approach to Engineering solutions:
  - ✓ Identify the problem.
  - ✓ State the objective.
  - ✓ Develop alternative solutions.
  - ✓ Evaluate alternatives.
  - ✓ Use the best alternative.



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- Steps of Analysis

  - ✓ Equations of equilibrium are used for external forces.
  - ✓ Analysis of the effect of the external forces on the structures (machine) or any component of the structure (machine).
  - ✓ Behavior of the materials under the action of forces.

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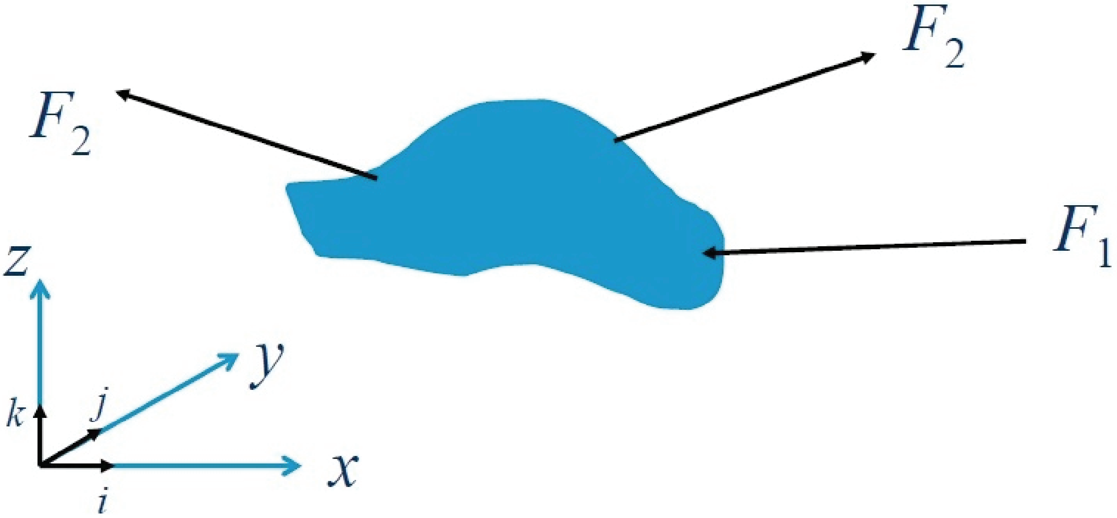
- Equations of Equilibrium

  - Rigid Body



❑ Equations of Equilibrium

- For a rigid body to be in equilibrium, both the resultant force **R** and a resultant moments (couples) **C** must vanish.

$$\mathbf{R} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = \mathbf{0}$$
$$\mathbf{C} = \sum M_x \mathbf{i} + \sum M_y \mathbf{j} + \sum M_z \mathbf{k} = \mathbf{0}$$

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❑ Equations of Equilibrium

- The two conditions can also be expressed in scalar form as:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

❑ Equilibrium in Two Dimensions

- The term “two dimensional” is used to describe problems in which the forces under consideration are contained in a plane (say the xy-plane)

❑ Equilibrium in Two Dimensions

- For two-dimensional problems, since a force in the xy-plane has no z-component and produces no moments about the x or y axes, hence

$$\mathbf{R} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} = \mathbf{0}$$
$$\mathbf{C} = \sum M_z \mathbf{k} = \mathbf{0}$$

❑ Equilibrium in Two Dimensions

- In scalar form, these conditions can be expressed as

$$\sum F_x = 0 \qquad \sum F_y = 0$$
$$\sum M_z = 0$$

❑ Internal Forces for Axially Loaded Members

- Analysis of Internal Forces

Example 1

Assume that: F1 = 2 k, F3 = 5 k, and F4 = 8 k

F2=?

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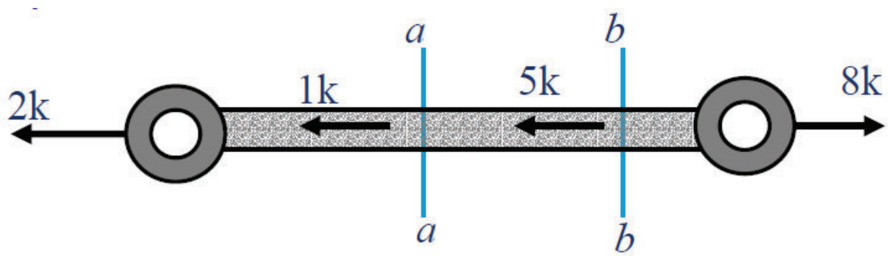
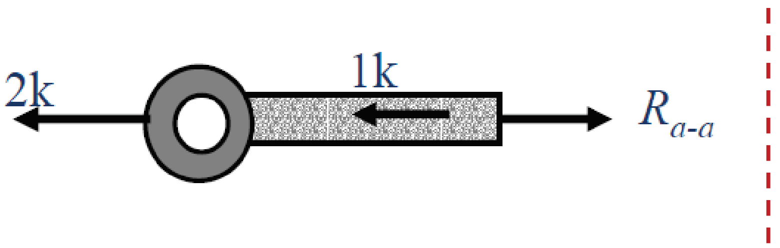
- ❑ Internal Forces for Axially Loaded Members
  - Analysis of Internal Forces

    -What is the internal force developed on plane a-a and b-b?

Example 2





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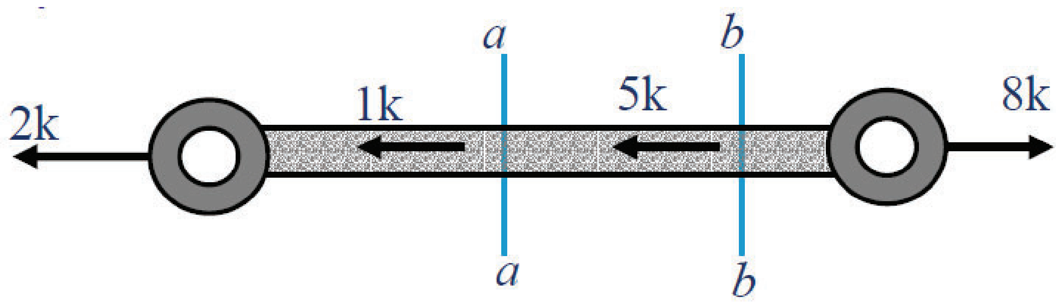
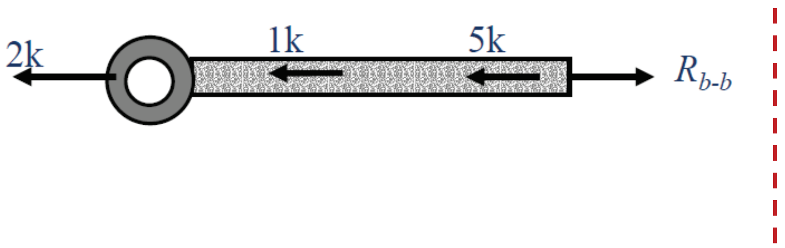
- ❑ Internal Forces for Axially Loaded Members
  - Analysis of Internal Forces

Example 2





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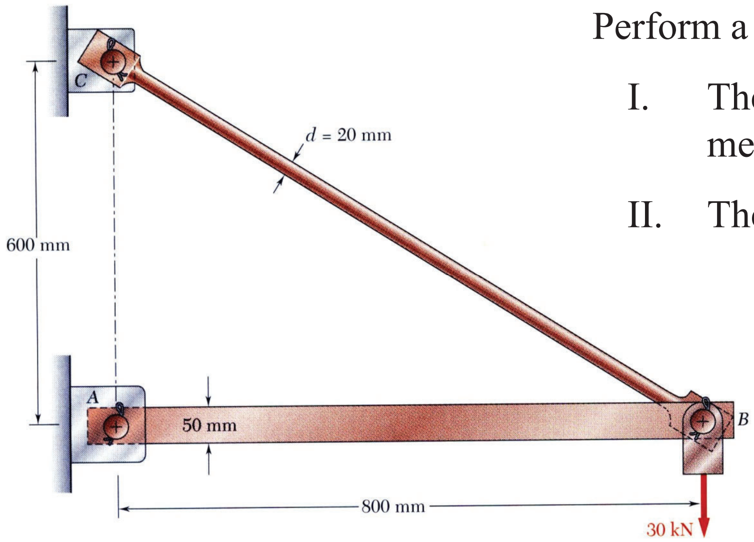
❑ Internal Forces for Axially Loaded Members

Example 3



Perform a static analysis to determine:

I. The internal force in each structural member

II. The reaction forces at the supports.

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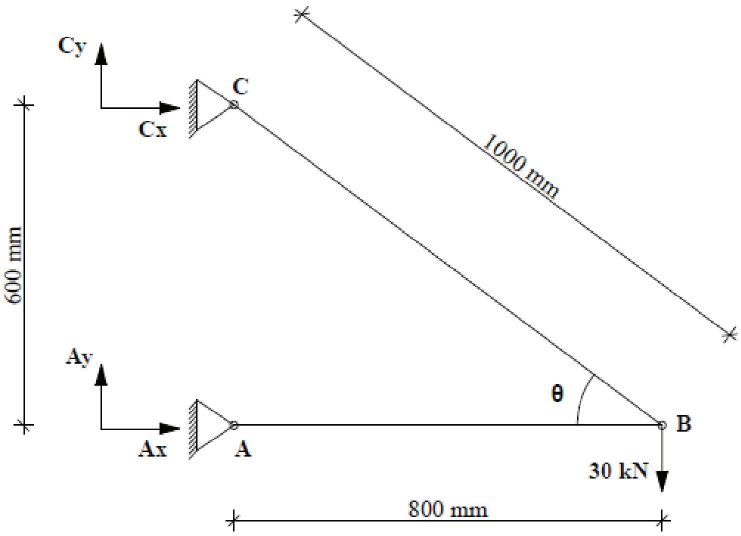
❑ Internal Forces for Axially Loaded Members

Example 3



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❑ Internal Forces for Axially Loaded Members

Example 3

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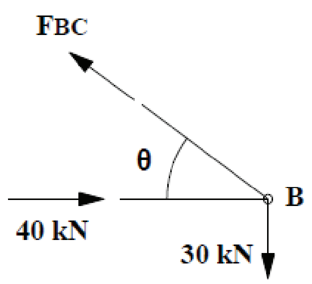
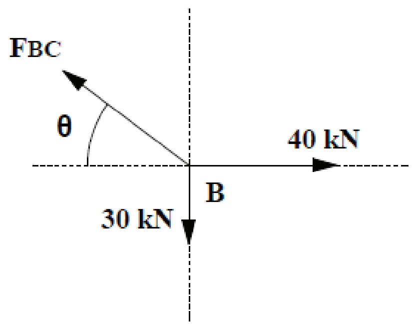
❑ Internal Forces for Axially Loaded Members

Example 3





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❑ Axial Loading: Normal Stress

- Stress
  - Stress is the intensity of internal force.
  - It can also be defined as force per unit area, or intensity of the forces distributed over a given section.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

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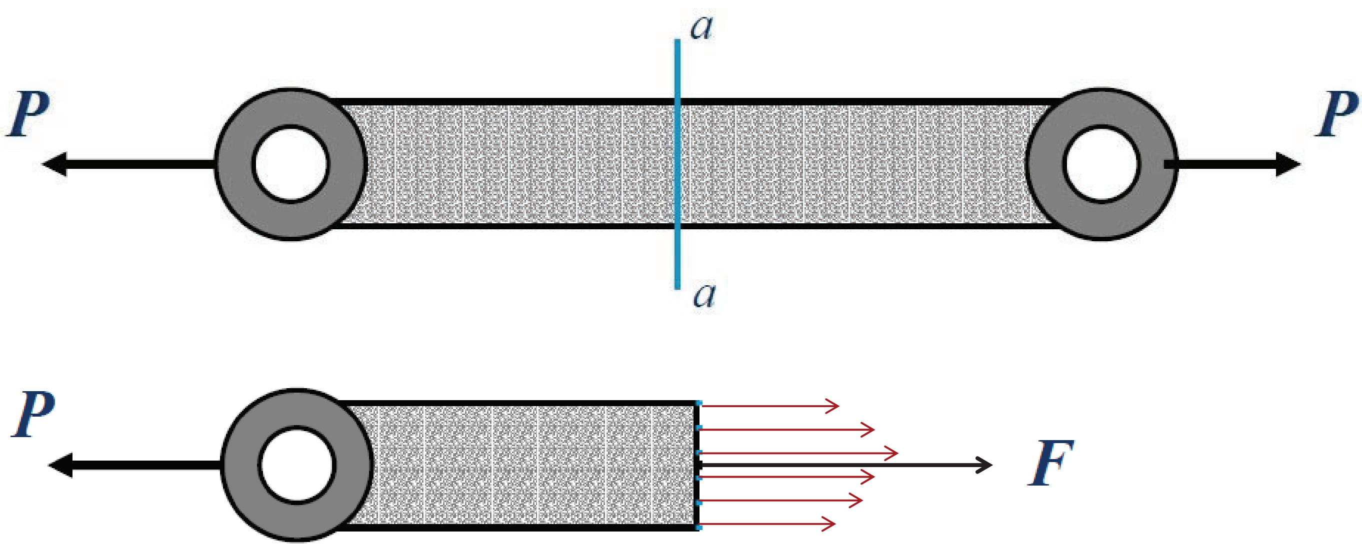
❑ Axial Loading: Normal Stress

- The resultant of the internal forces for an axially loaded member is *normal* to a section cut perpendicular to the member axis.



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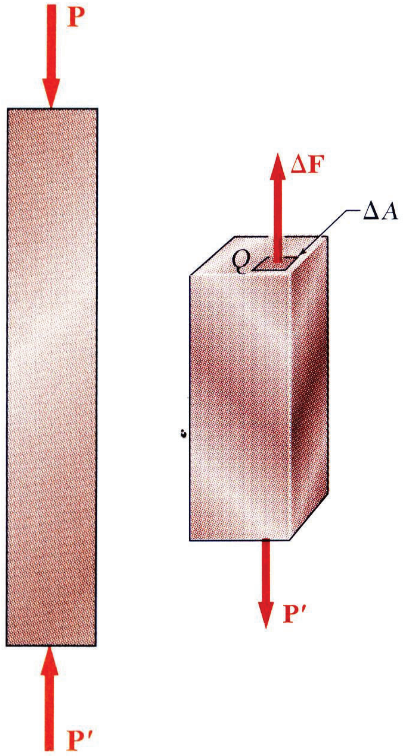
❑ Axial Loading: Normal Stress



- The force intensity on that section is defined as the normal stress.

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \qquad \sigma_{ave} = \frac{P}{A}$$

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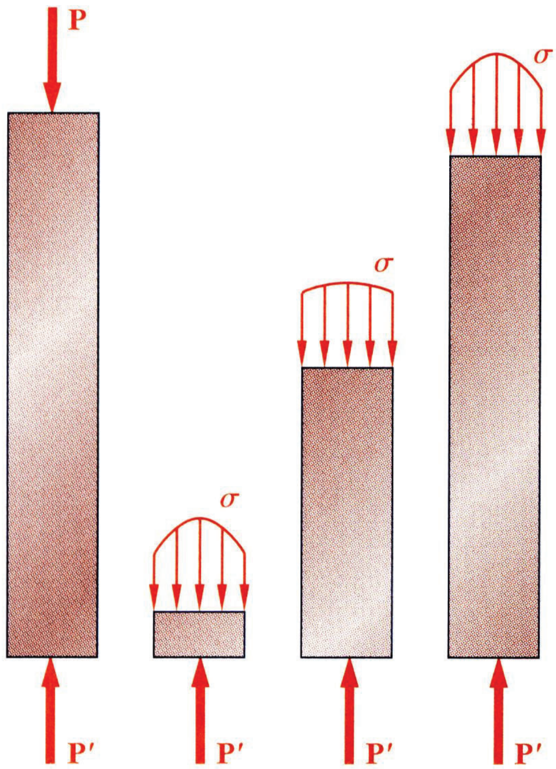
❑ Axial Loading: Normal Stress



- The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \quad \Rightarrow \quad \sigma = \frac{dF}{dA}$$

$$P = \int dF = \int_A \sigma \, dA$$

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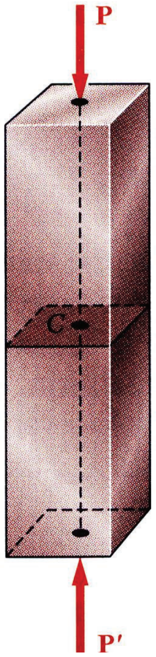
❑ Centric loading



- A uniform distribution of stress is only possible if the concentrated loads on the end sections of two-force members are applied at the section centroids. This is referred to as ***centric loading***.

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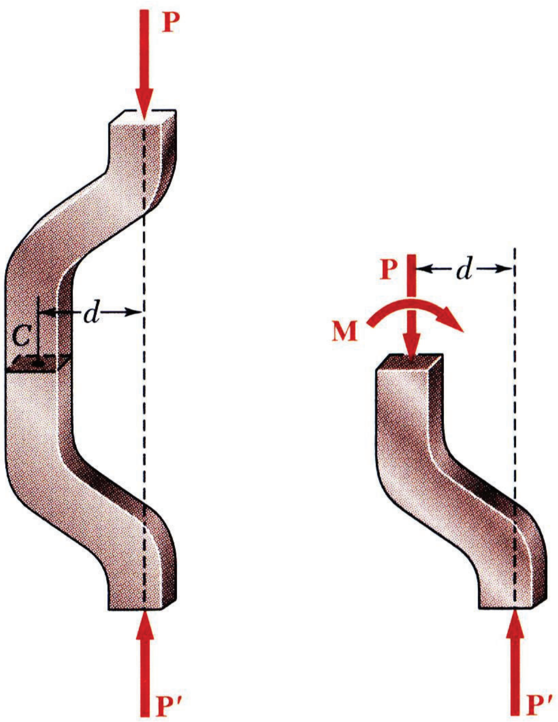
❑ Eccentrically loaded



- If a two-force member is ***eccentrically loaded***, then the resultant of the stress distribution in a section must yield an axial force and a moment.

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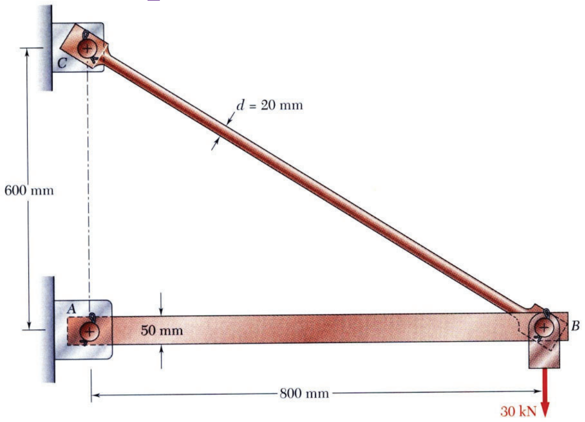
- ❑ Stress Analysis & Design Example

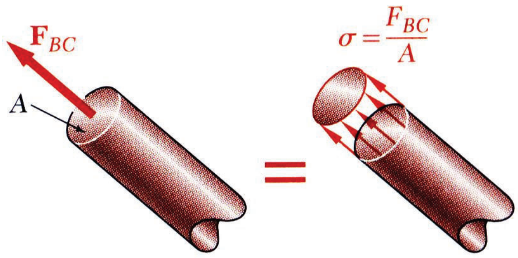
Example 4



Can the structure safely support the 30 kN load? Assume that, the element CB has enough capacity. $\sigma_{\text{all}\,AB} = 165\ \text{MPa}$

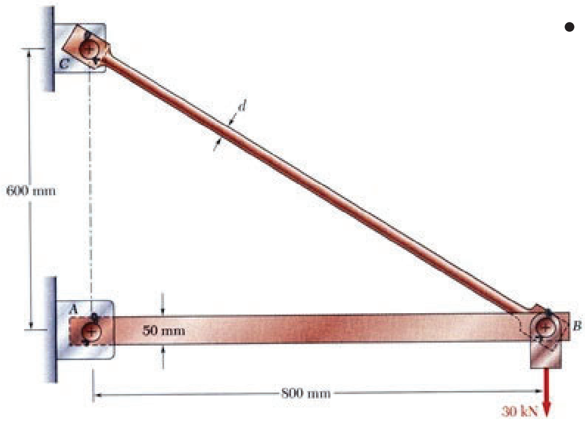
- From a statics analysis

  F_{AB} = 40 kN (compression)
  F_{BC} = 50 kN (tension)



- ❑ Stress Analysis & Design Example

Example 4



- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum ($\sigma_{\text{all}\,AB} = 100\ \text{MPa}$). What is an appropriate choice for the rod diameter?

- From a statics analysis

  F_{AB} = 40 kN (compression)
  F_{BC} = 50 kN (tension)

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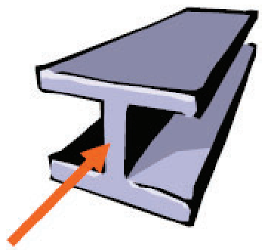
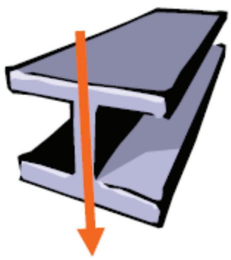
❑ Shearing Stress

- The internal forces discussed previously and the corresponding stresses were ***normal*** to the section considered.



- A different type of stress can occur in a **transverse** cross section of a member as shown in the next slide.



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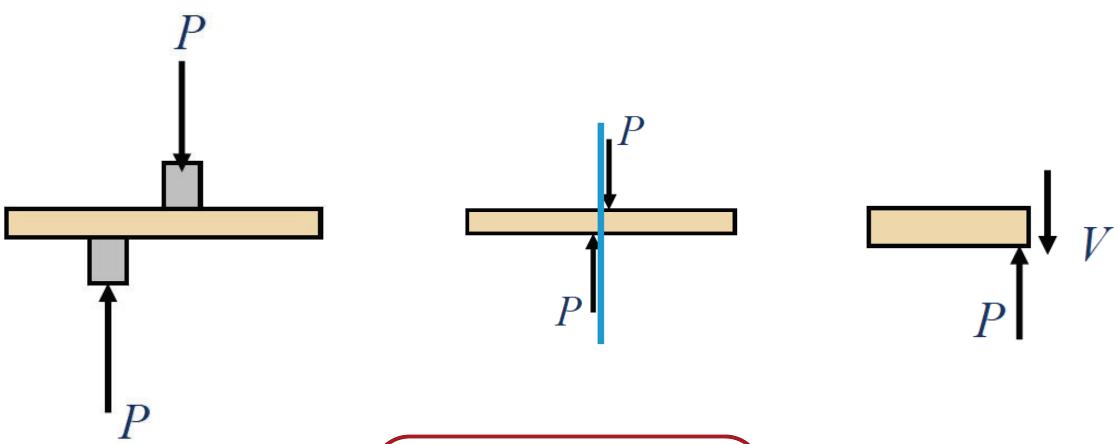
❑ Shearing Stress

- Illustration of Shearing Stress



$$\tau_{\text{ave}} = \frac{V}{A_s} = \frac{P}{A_s}$$

❑ Shearing Stress

- Shear

$$\tau = \lim_{\Delta A_s \to 0} \frac{\Delta V}{\Delta A_s} \quad \Rightarrow \quad \tau = \frac{dV}{dA_s}$$

$$P = \int dV = \int_{A_s} \tau \, dA_s$$

$A_s =$ cross-sectional area of bolt or rivet

- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.
- The shear stress distribution cannot be assumed to be uniform.

❑ Shearing Stress

Shearing Stress in Connection

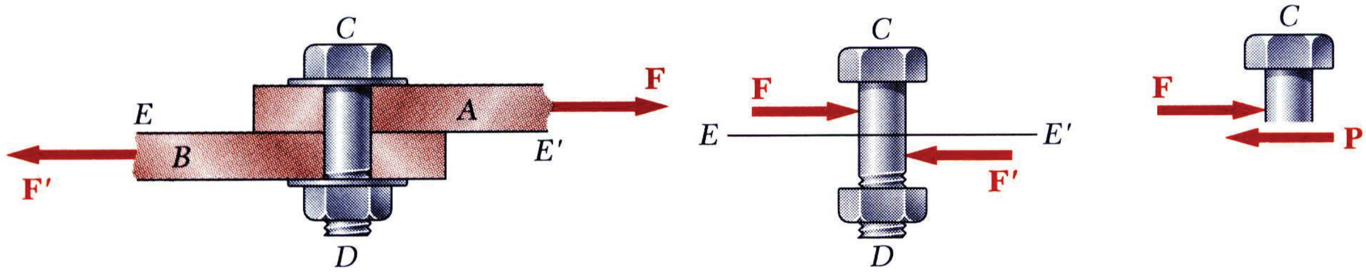
Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components.

Three Types of Shearing Stress

a) Single Shear
b) Double Shear
c) Punching Shear

❑ Shearing Stress Examples

- **Single Shear**

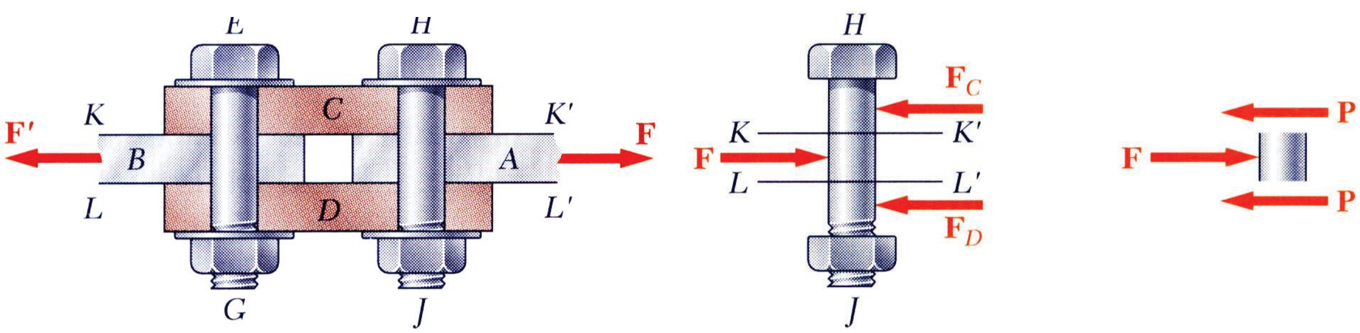


The bolt is said to in single shear P

$$\tau_{\text{ave}} = \frac{P}{A_s} = \frac{F}{A_s}$$

❑ Shearing Stress Examples

- **Double Shear**



The bolt is said to in double shear P

$$\tau_{\text{ave}} = \frac{P}{A_s} = \frac{F/2}{A_s} = \frac{F}{2A_s}$$

❑ Shearing Stress Examples

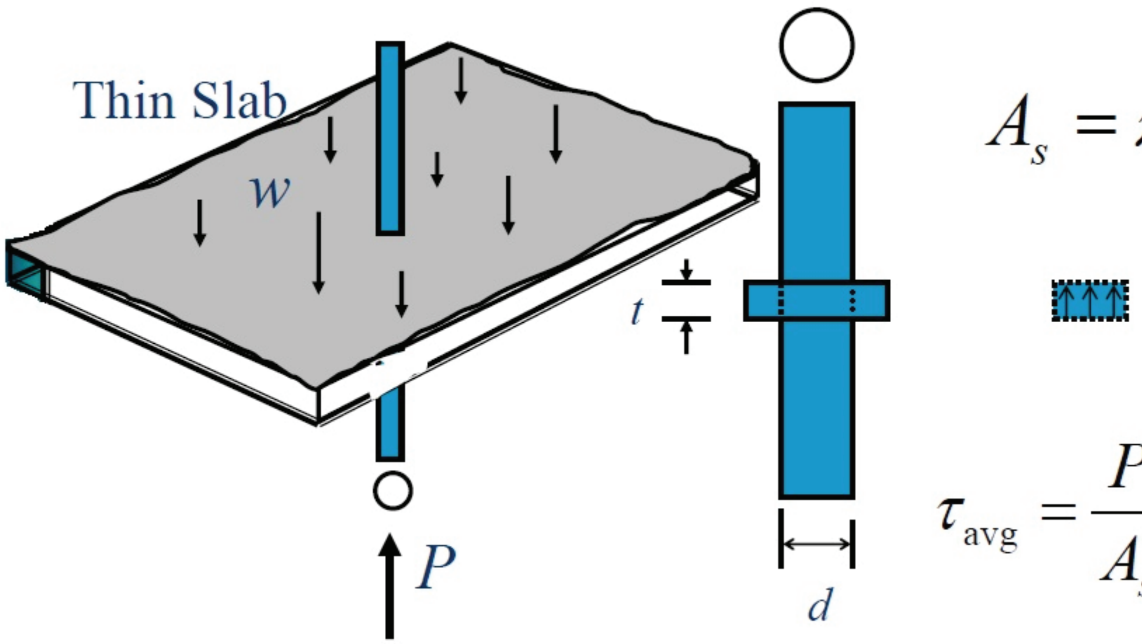
- **Punching Shear**

Examples of this type are:

- Action of punch in forming rivet hole in metal
- Tendency of building columns to punch trough footings
- Heavy thin-slab ceiling cause building columns to punch through slabs.

❑ Shearing Stress Examples

- **Punching Shear**



$$A_s = \pi d t$$

$$\tau_{avg} = \frac{P}{A_s} = \frac{P}{\pi d t}$$

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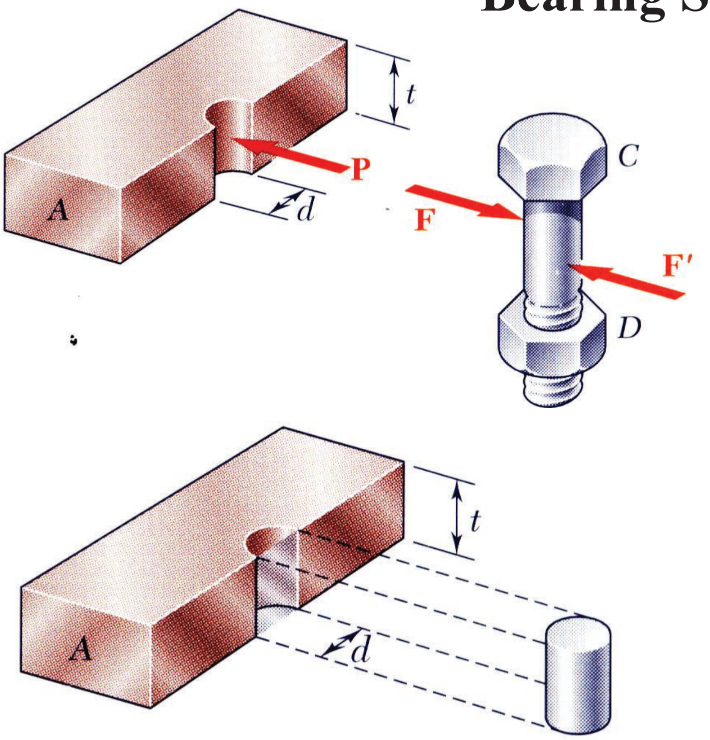
❑ Shearing Stress Examples

▪ Bearing Stress in Connections



- Bolts, rivets, and pins create stresses on the points of contact or ***bearing surfaces*** of the members they connect.

- Corresponding average force intensity is called the **bearing stress**,

$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

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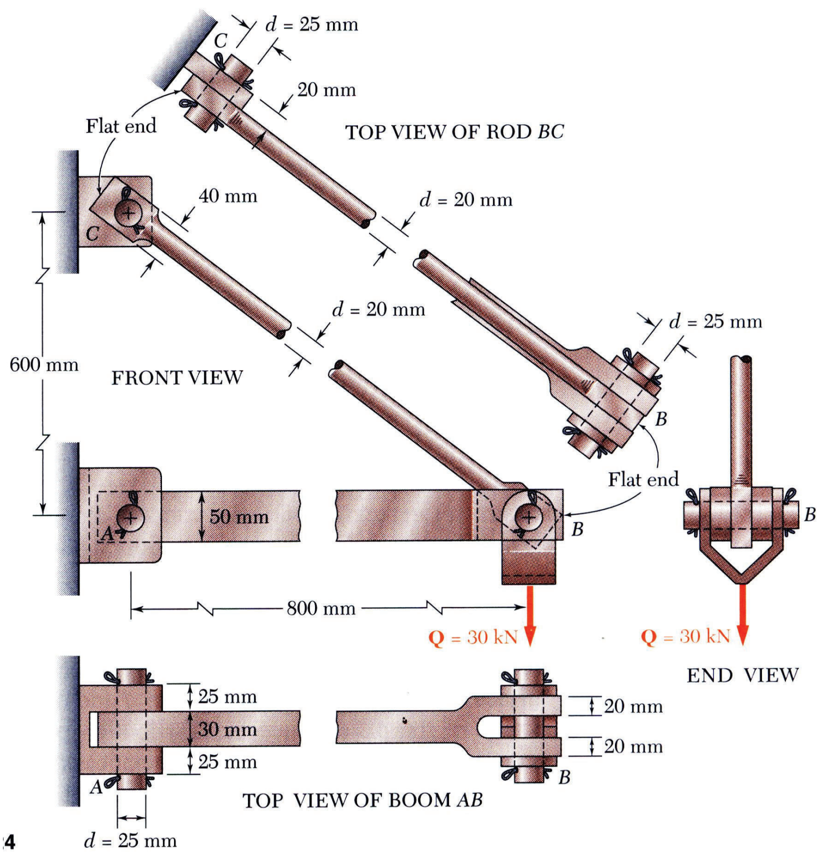
Example 5



❑ Stress Analysis & Design Example

- Determine the stresses in the members and connections of the structure shown.

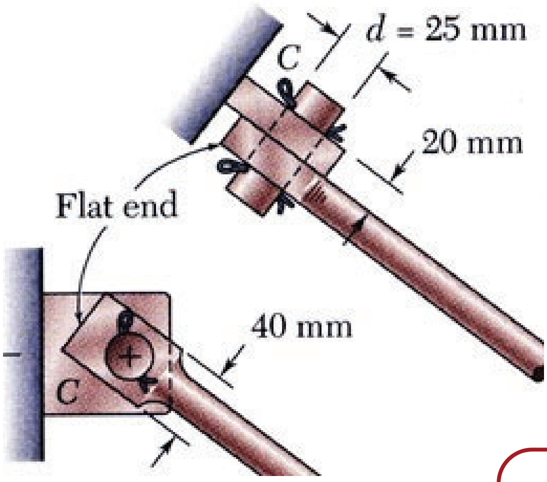
- From a statics analysis

  F_{AB} = 40 kN (compression)

  F_{BC} = 50 kN (tension)

❑ Rod & Boom Normal Stresses

Example 5



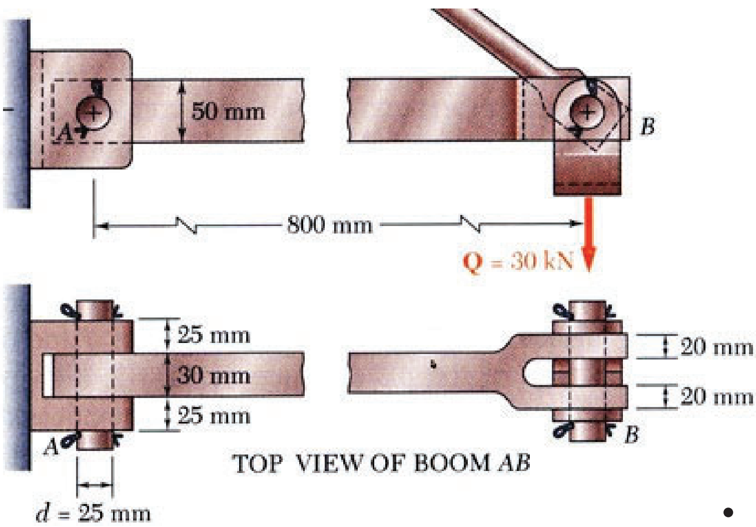
F_{BC} = 50 kN (tension)

- At the rod center, the average normal stress in the circular cross-section.

- At the flattened rod ends, the smallest cross-sectional area occurs at the pin centerline.

❑ Rod & Boom Normal Stresses

Example 5



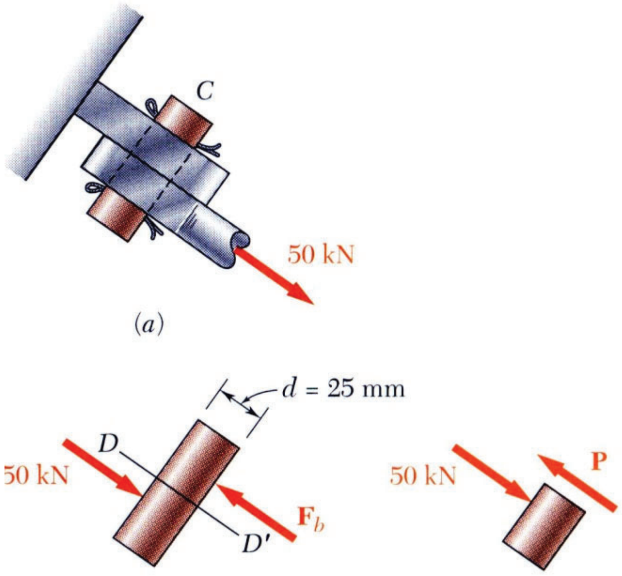
F_{AB} = 40 kN (compression)

- The average normal stress in the center boom.

- The minimum area sections at the boom ends are unstressed since the boom is in compression.

❑ Pin Shearing Stresses

Example 5



- The cross-sectional area for pins at *A*, *B*, and *C*,

- The force on the pin at *C* is equal to the force exerted by the rod *BC*,

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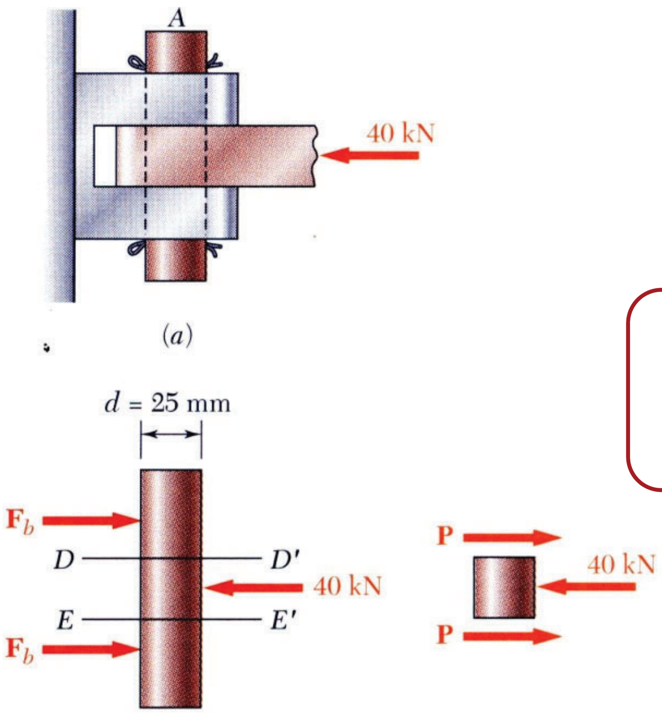
❑ Pin Shearing Stresses

Example 5



- The pin at *A* is in double shear with a total force equal to the force exerted by the boom *AB*,

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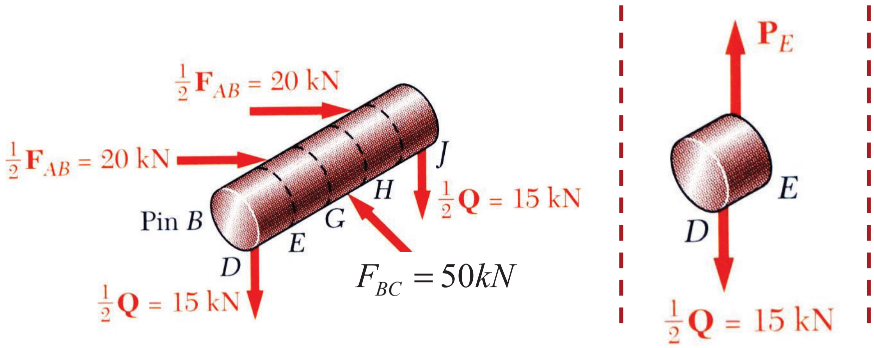
Example 5

❑ Pin Shearing Stresses



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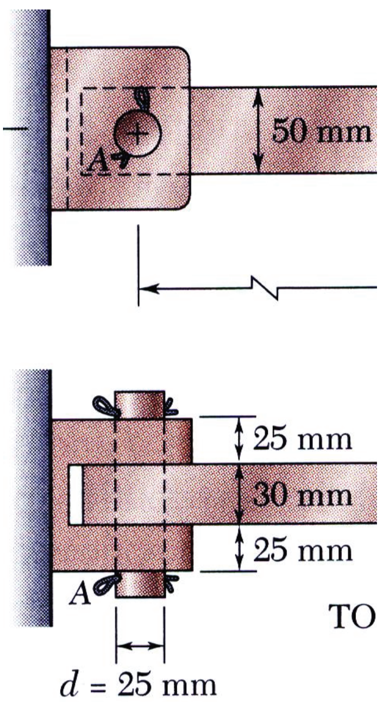
Example 5

❑ Pin Shearing Stresses



- To determine the bearing stress at A in the boom AB, we have $t = 30$ mm and $d = 25$ mm,

- To determine the bearing stress at A in the bracket, we have $t = 2(25 \text{ mm}) = 50$ mm and $d = 25$ mm,

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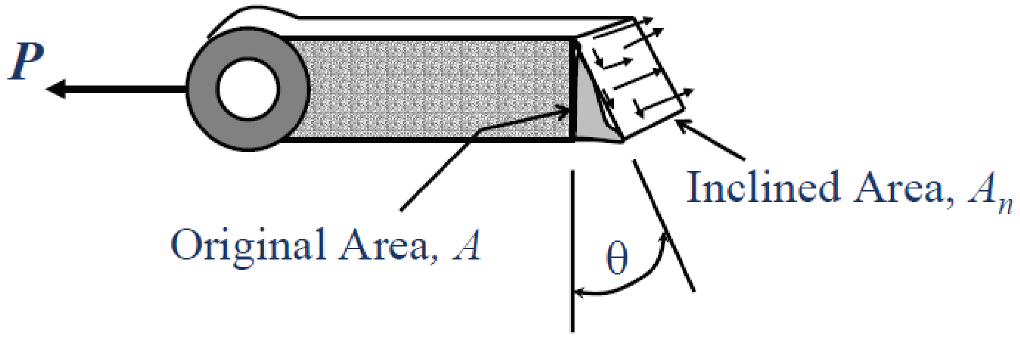
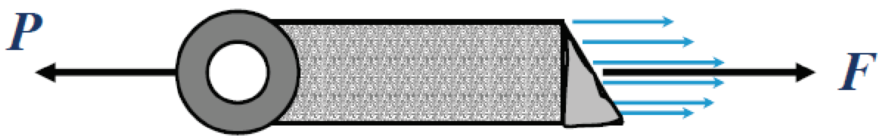
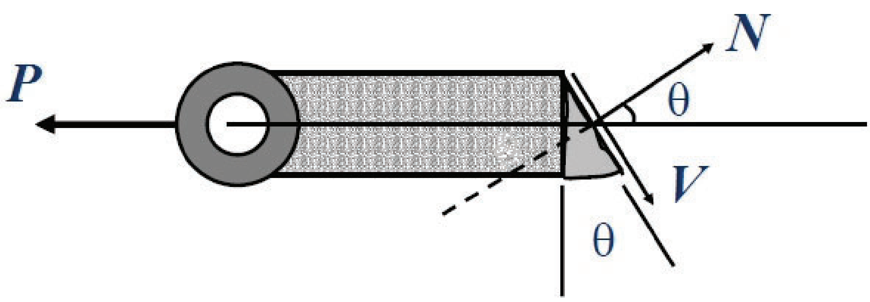
- Stresses on an Inclined Plane in an Axially Loaded Member
  - Illustration





Introduction –Concept of Stress

- Stresses on an Inclined Plane in an Axially Loaded Member
  - Illustration



$$V = P\sin(\theta)$$
$$N = P\cos(\theta)$$
$$A_n = \frac{A}{\cos(\theta)}$$

$$\sigma_n = \frac{N}{A_n} = \frac{P\cos(\theta)}{A/\cos(\theta)} = \frac{P}{A}\cos^2(\theta) = \frac{P}{2A}\left(1+\cos(2\theta)\right)$$

$$\tau_n = \frac{V}{A_n} = \frac{P\sin(\theta)}{A/\cos(\theta)} = \frac{P}{A}\sin(\theta)\cos(\theta) = \frac{P}{2A}\sin(2\theta)$$

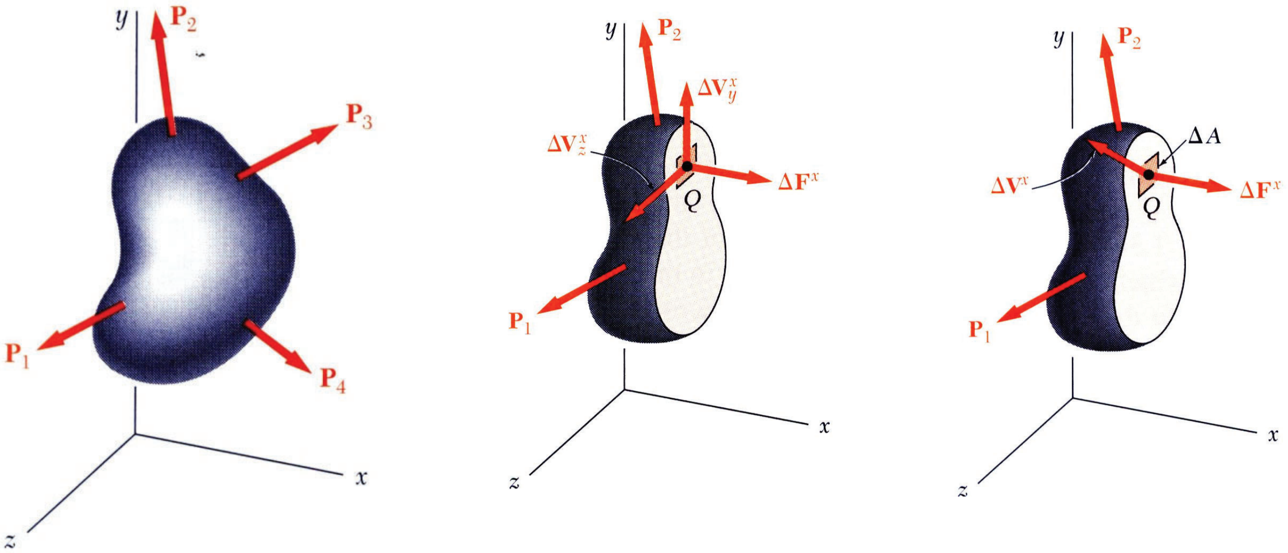
- Stresses on an Inclined Plane in an Axially Loaded Member
  - Maximum Normal and Shear Stresses

$$\theta = 0^\circ \; or \; 180^\circ \Rightarrow \sigma_n \; is \; \max imum \quad \Rightarrow \quad \sigma_{n\,Max} = \frac{P}{A}$$

$$\theta = 45^\circ \; or \; 135^\circ \Rightarrow \tau_n \; is \; \max imum \quad \Rightarrow \quad \tau_{n\,Max} = \frac{P}{2A}$$

$$\Rightarrow \tau_{n\,Max} = \frac{\sigma_{n\,Max}}{2}$$

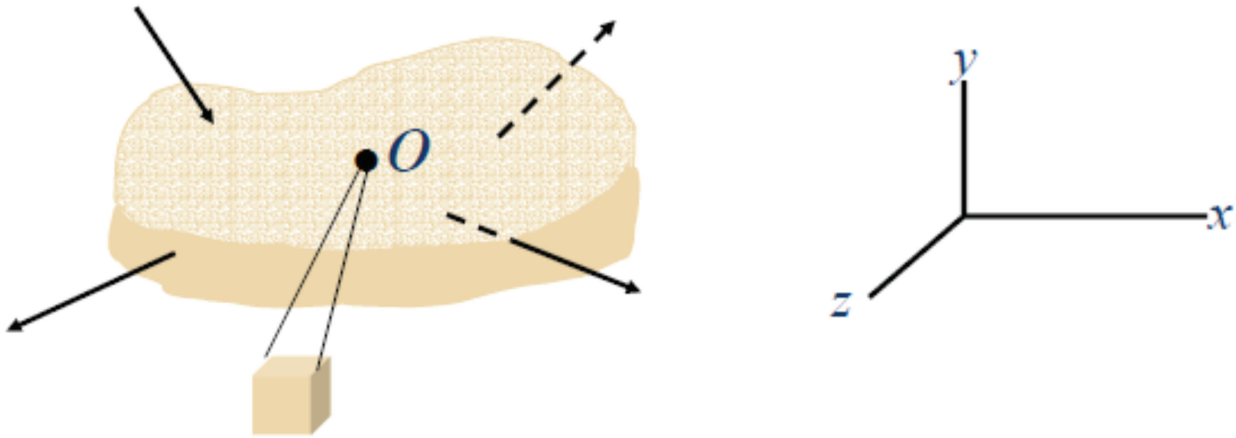
- Stress Under General Loadings



$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A} \qquad \tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

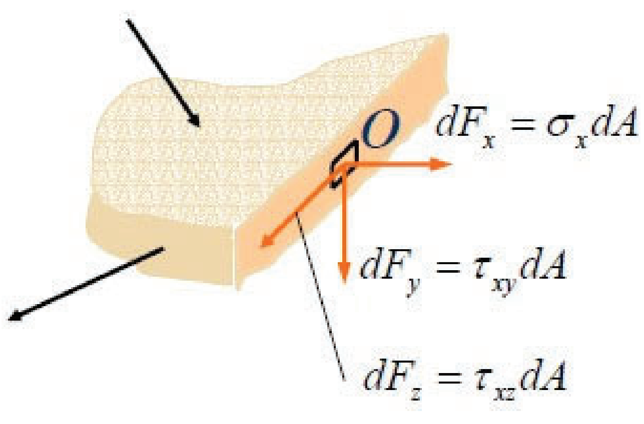
❑ State of Stress

Stress at a point in a material body has been defined as a force per unit area. But this definition is somewhat ambiguous since it depends upon ***what area we consider at the point***.



❑ State of Stress

let us pass a cutting plane through point O perpendicular to the x axis.
If dA is the area, then by definition



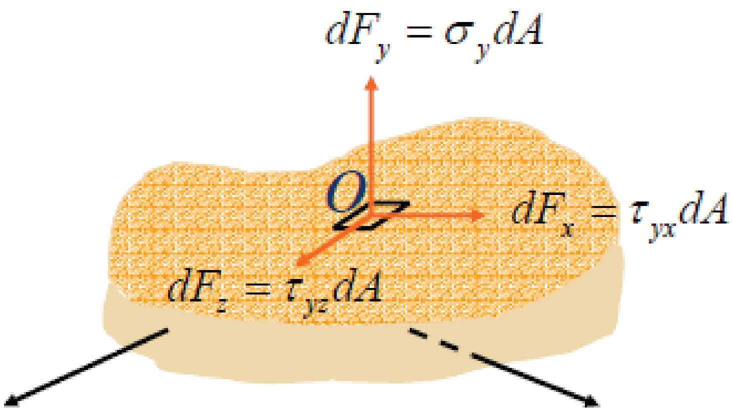
$$\sigma_x = \frac{dF_x}{dA}$$

$$\tau_{xy} = \frac{dF_y}{dA}$$

$$\tau_{xz} = \frac{dF_z}{dA}$$

❑ State of Stress

let us pass a cutting plane through point *O* perpendicular to the *y* axis.
If *dA* is the area, then by definition



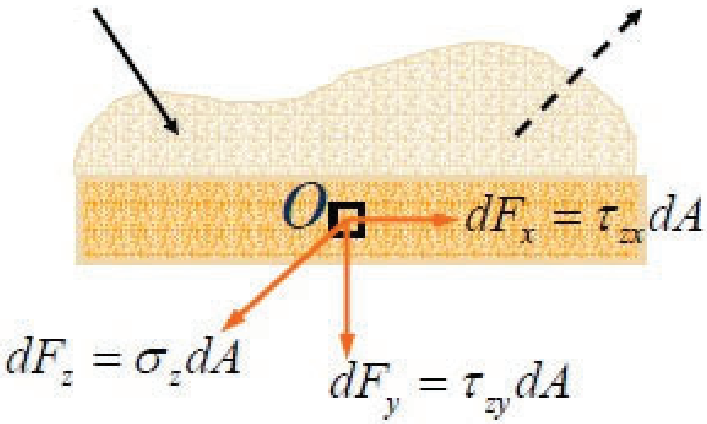
$$\sigma_y = \frac{dF_y}{dA}$$

$$\tau_{yx} = \frac{dF_x}{dA}$$

$$\tau_{yz} = \frac{dF_z}{dA}$$

❑ State of Stress

let us pass a cutting plane through point *O* perpendicular to the *z* axis.
If *dA* is the area, then by definition

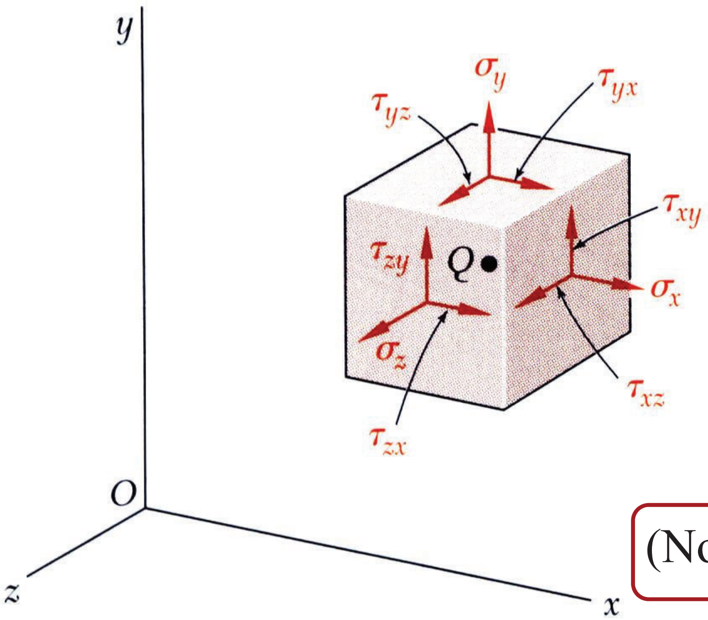


$$\sigma_z = \frac{dF_z}{dA}$$

$$\tau_{zx} = \frac{dF_x}{dA}$$

$$\tau_{zy} = \frac{dF_y}{dA}$$

❑ General or Triaxial State of stress



- **Normal Stresses**

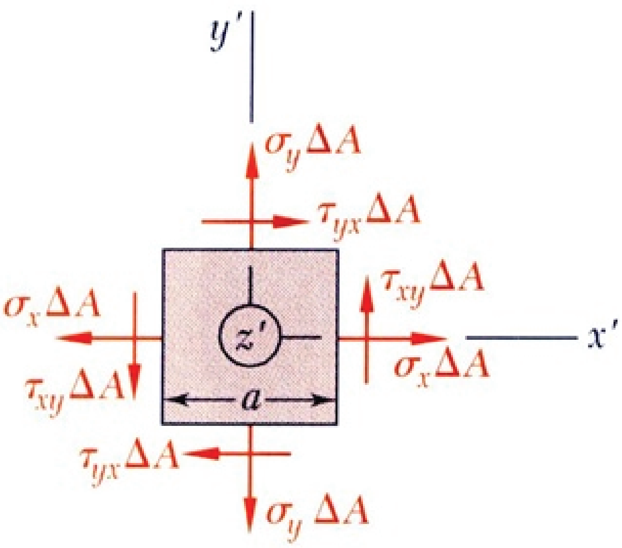
$$\sigma_x, \sigma_y, \sigma_z$$

- **Shear Stress**

$$\tau_{xy}, \tau_{yz}, \tau_{zx}$$

$$(\text{Note}: \tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xz})$$

❑ Stress Under General Loadings



- Consider the moments about the z axis:

$$\sum M_z = 0 = (\tau_{xy} \Delta A)\, a - (\tau_{yx} \Delta A)\, a = 0$$

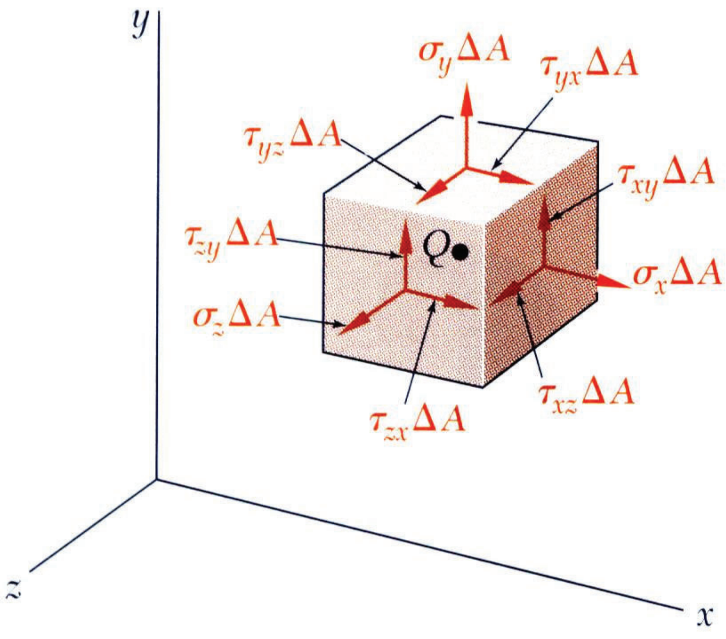
$$\Rightarrow \tau_{xy} = \tau_{yx}$$

similarly, $\tau_{yz} = \tau_{zy}$ and $\tau_{yz} = \tau_{zy}$

- It follows that only **6** components of stress are required to define the complete state of stress

Introduction –Concept of Stress

❑ Stress Under General Loadings



- **9** stress components are unknown.

- Stress components are defined for the planes cut parallel to the *x*, *y* and *z* axes. For equilibrium, equal and opposite stresses are exerted on the hidden planes.

- The combination of forces generated by the stresses must satisfy the conditions for equilibrium:

$$\sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum M_x = \sum M_y = \sum M_z = 0$$

Introduction –Concept of Stress

❑ Factor of Safety

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

FS = Factor of safety

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Factor of safety considerations:

- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to structures integrity
- risk to life and property
- influence on machine function

- Design Loads, Working Stresses, and Factor of Safety (FS)

Design Approaches:

- Deterministic, e.g., working stress or allowable stress design (ASD)

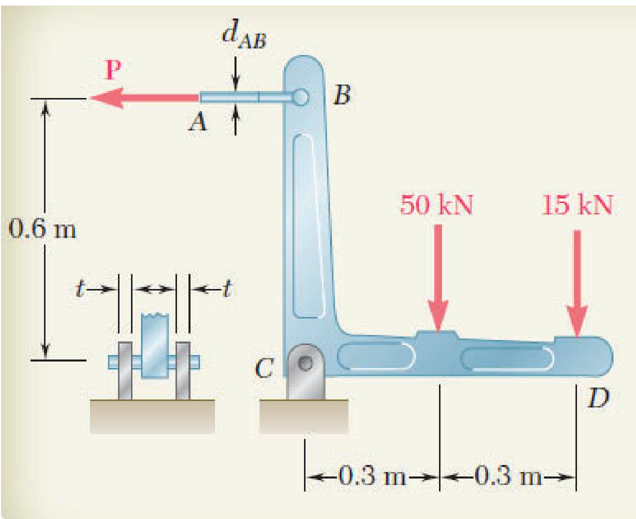
$$\frac{R_n}{FS} \geq \sum_{i=1}^{m} L_i$$

- Probability-Based, e.g., load and resistance factor design (LRFD)

$$\phi R_n \geq \sum_{i=1}^{m} \gamma_i L_i$$

Example 6

- Stress Analysis & Design Example



a) Determine $d_{AB} = ?$

$\sigma_U = 600\ MPa \quad \& \quad FS = 3.3$

b) Determine $d_C = ?$

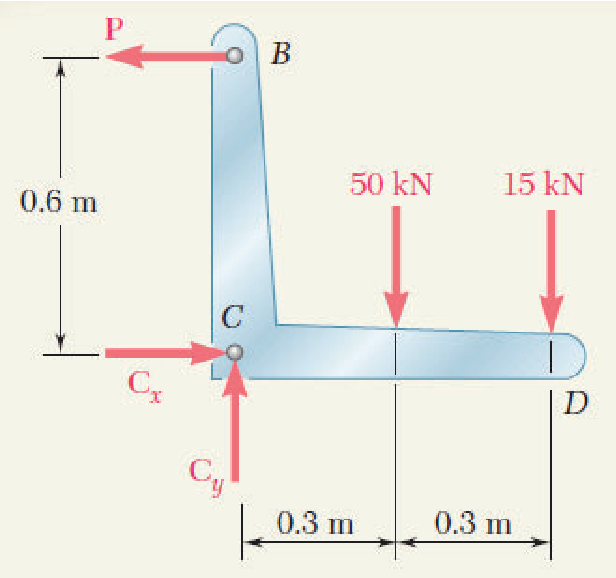
$\tau_U = 350\ MPa \quad \& \quad FS = 3.3$

c) Determine $t = ?$

$\sigma_{all} = 300\ MPa$

Example 6

❑ Stress Analysis & Design Example

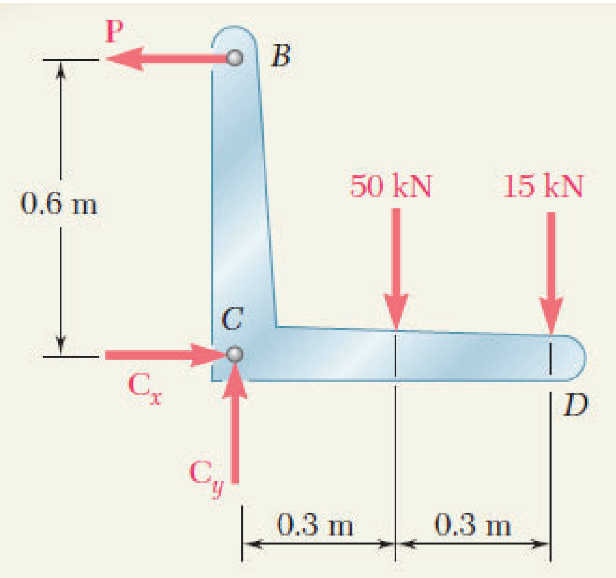


Introduction –Concept of Stress

Example 6

❑ Stress Analysis & Design Example

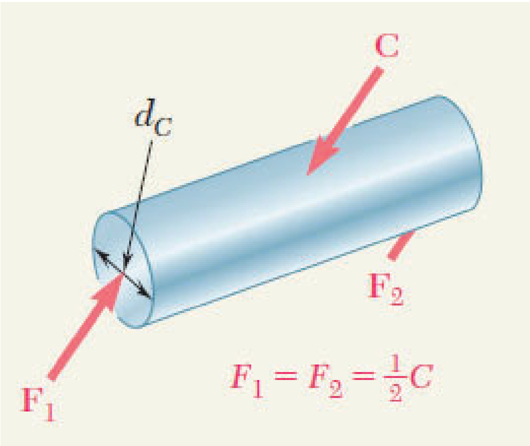
a)



Example 6

❑ Stress Analysis & Design Example

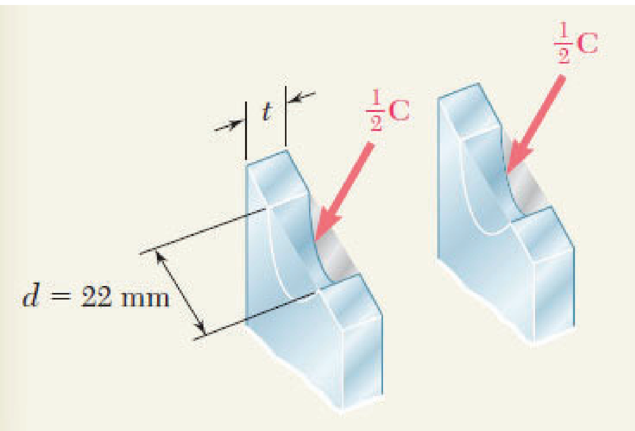
b)



Example 6

❑ Stress Analysis & Design Example

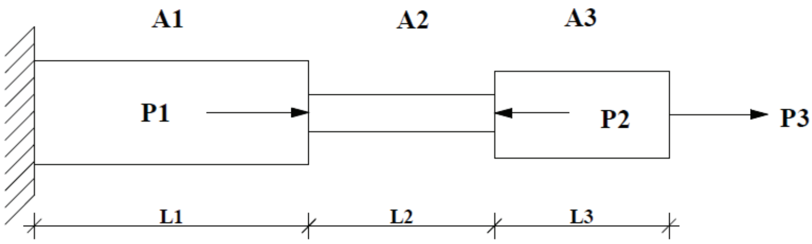
C)



- Stress Analysis & Design Example

Example 7

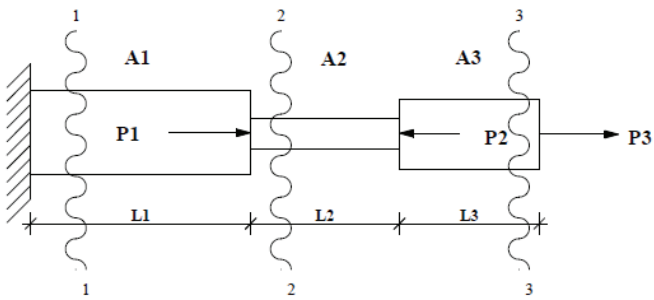
Determine axial forces and normal stresses in the shown bar. Also draw the variations of axial forces and normal stresses.



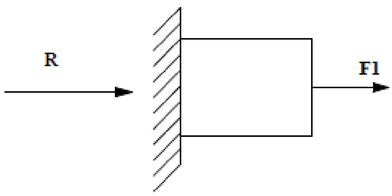
$$A_1 = 200\ mm^2 \qquad P_1 = 4\ kN$$
$$A_2 = 100\ mm^2 \qquad P_2 = -2\ kN$$
$$A_3 = 150\ mm^2 \qquad P_3 = 3\ kN$$

- Stress Analysis & Design Example

Example 7



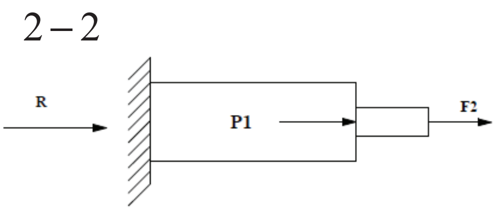
$1-1$

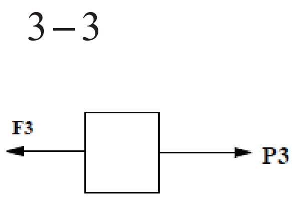


Introduction –Concept of Stress

- Stress Analysis & Design Example

Example 7

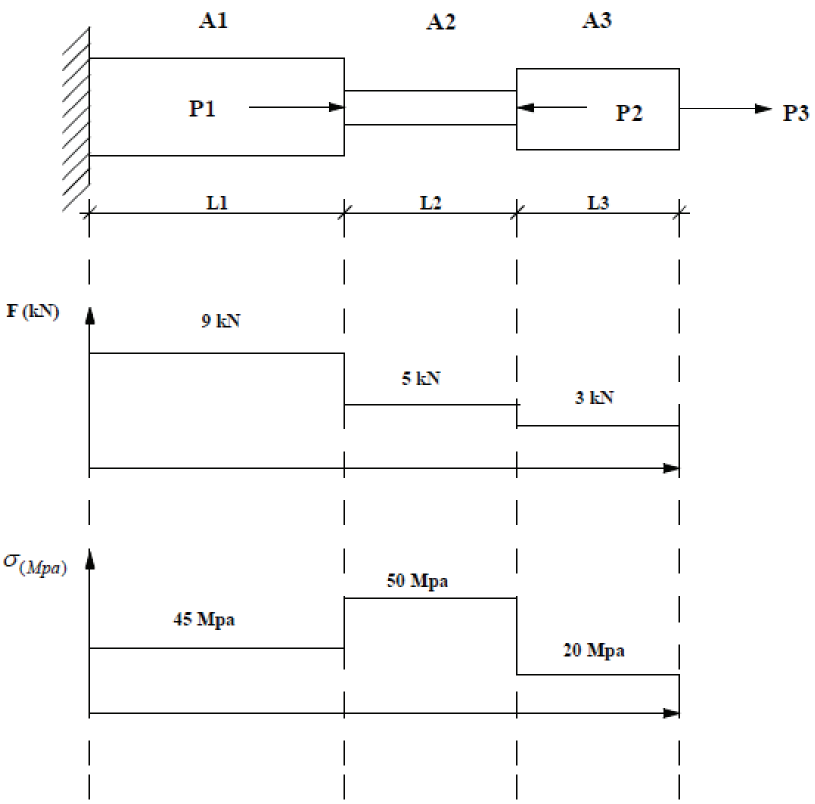




Introduction –Concept of Stress

- Stress Analysis & Design Example

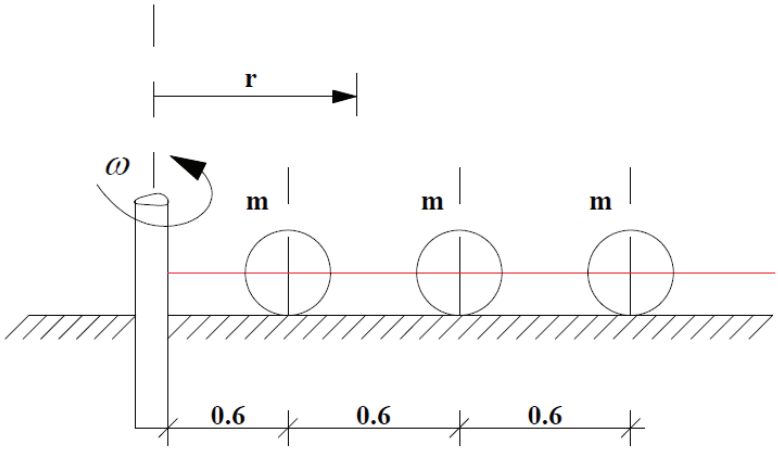
Example 7



- Stress Analysis & Design Example

Example 8

Three lumped masses rotate around a beam. These masses are connected to each other using a wire with diameter ***d***. determine axial stresses in parts of wire and draw the results based on r using a diagram. There is no friction between the surface and masses.

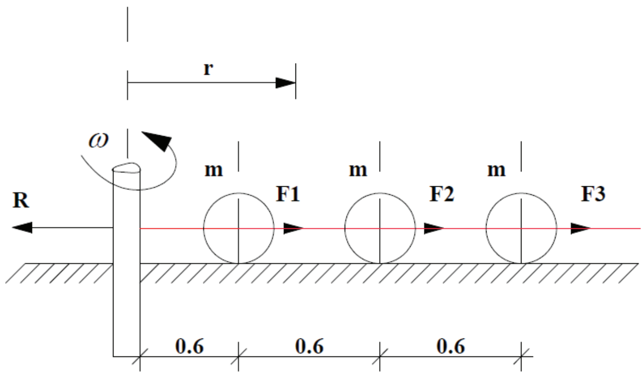


$$m = 0.5\ kg \quad \upsilon = 4\ Hz \quad d = 10\ mm$$

Introduction –Concept of Stress

- Stress Analysis & Design Example

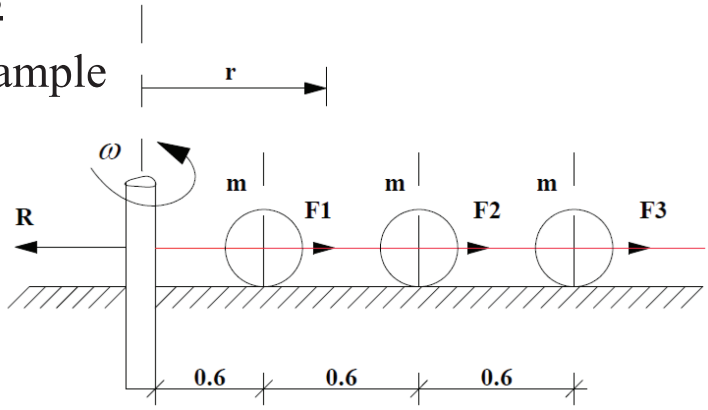
Example 8



Introduction –Concept of Stress

- Stress Analysis & Design Example

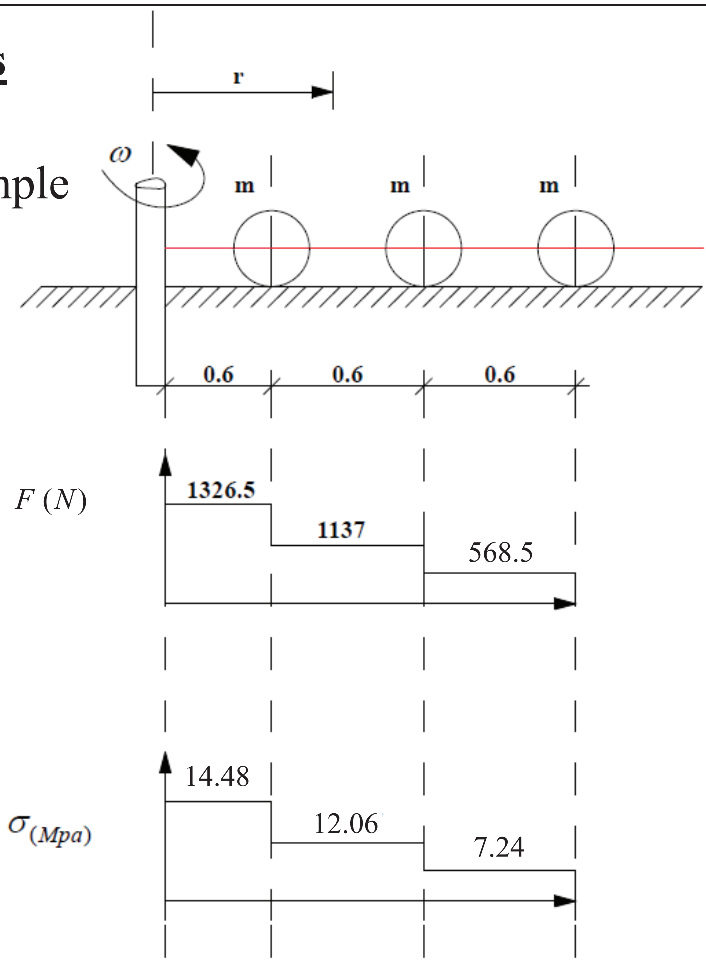
Example 8



Introduction –Concept of Stress

- Stress Analysis & Design Example

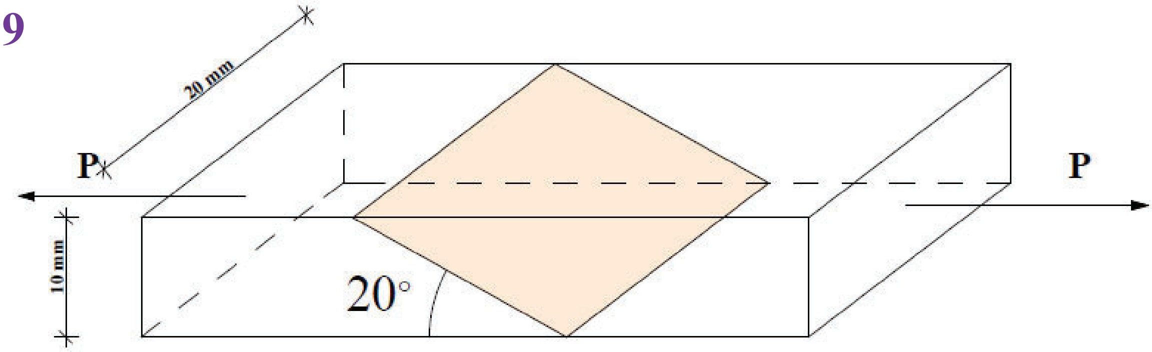
Example 8



Introduction –Concept of Stress

- ❑ Stress Analysis & Design Example

Example 9



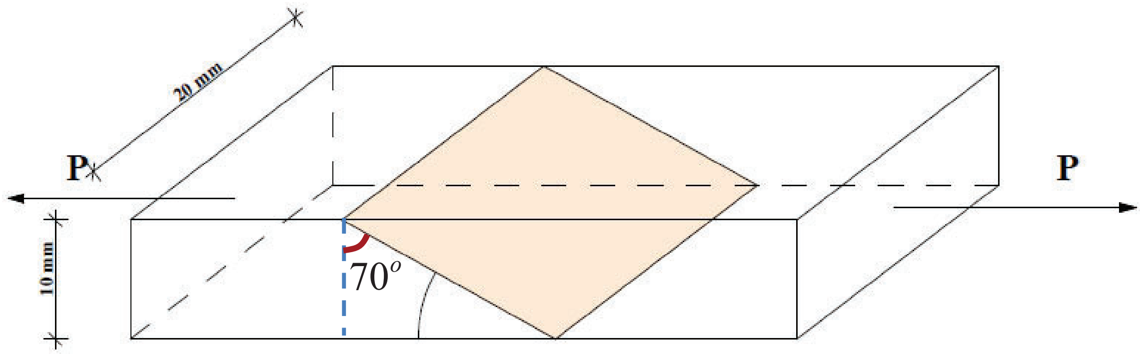
Two pieces of the wood beam are connected with glue connection. Determine the maximum safe axial load.

$$\tau_{all} = 10\ Mpa$$

Introduction –Concept of Stress

- ❑ Stress Analysis & Design Example

Example 9



UNITS CONVERSION TABLES

Table 1: Multiples and Submultiples of SI units

Prefix	Symbol	Multiplying Factor	
exa	E	10^{18}	1 000 000 000 000 000 000
peta	P	10^{15}	1 000 000 000 000 000
tera	T	10^{12}	1 000 000 000 000
giga	G	10^{9}	1 000 000 000
mega	M	10^{6}	1 000 000
kilo	k	10^{3}	1 000
hecto*	h	10^{2}	100
deca*	da	10	10
deci*	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	u	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001
femto	f	10^{-15}	0.000 000 000 000 001
atto	a	10^{-18}	0.000 000 000 000 000 001

* these prefixes are not normally used

UNITS CONVERSION TABLES

Table 2: Length Units

Millimeters	Centimeters	Meters	Kilometers	Inches	Feet	Yards	Miles
mm	cm	m	km	in	ft	yd	mi
1	0.1	0.001	0.000001	0.03937	0.003281	0.001094	6.21e-07
10	1	0.01	0.00001	0.393701	0.032808	0.010936	0.000006
1000	100	1	0.001	39.37008	3.28084	1.093613	0.000621
1000000	100000	1000	1	39370.08	3280.84	1093.613	0.621371
25.4	2.54	0.0254	0.000025	1	0.083333	0.027778	0.000016
304.8	30.48	0.3048	0.000305	12	1	0.333333	0.000189
914.4	91.44	0.9144	0.000914	36	3	1	0.000568
1609344	160934.4	1609.344	1.609344	63360	5280	1760	1

Table 3: Area Units

Millimeter square	Centimeter square	Meter square	Inch square	Foot square	Yard square
mm^2	cm^2	m^2	in^2	ft^2	yd^2
1	0.01	0.000001	0.00155	0.000011	0.000001
100	1	0.0001	0.155	0.001076	0.00012
1000000	10000	1	1550.003	10.76391	1.19599
645.16	6.4516	0.000645	1	0.006944	0.000772
92903	929.0304	0.092903	144	1	0.111111
836127	8361.274	0.836127	1296	9	1

UNITS CONVERSION TABLES

Table 4: Volume Units

Centimeter cube	Meter cube	Liter	Inch cube	Foot cube	US gallons	Imperial gallons	US barrel (oil)
cm^3	m^3	ltr	in^3	ft^3	US gal	Imp. gal	US brl
1	0.000001	0.001	0.061024	0.000035	0.000264	0.00022	0.000006
1000000	1	1000	61024	35	264	220	6.29
1000	0.001	1	61	0.035	0.264201	0.22	0.00629
16.4	0.000016	0.016387	1	0.000579	0.004329	0.003605	0.000103
28317	0.028317	28.31685	1728	1	7.481333	6.229712	0.178127
3785	0.003785	3.79	231	0.13	1	0.832701	0.02381
4545	0.004545	4.55	277	0.16	1.20	1	0.028593
158970	0.15897	159	9701	6	42	35	1

Table 5: Mass Units

Grams	Kilograms	Metric tonnes	Short ton	Long ton	Pounds	Ounces
g	kg	tonne	shton	Lton	lb	oz
1	0.001	0.000001	0.000001	9.84e-07	0.002205	0.035273
1000	1	0.001	0.001102	0.000984	2.204586	35.27337
1000000	1000	1	1.102293	0.984252	2204.586	35273.37
907200	907.2	0.9072	1	0.892913	2000	32000
1016000	1016	1.016	1.119929	1	2239.859	35837.74
453.6	0.4536	0.000454	0.0005	0.000446	1	16
28	0.02835	0.000028	0.000031	0.000028	0.0625	1

UNITS CONVERSION TABLES

Table 10: High Pressure Units

Bar	Pound/square inch	Kilopascal	Megapascal	Kilogram force/ centimeter square	Millimeter of mercury	Atmospheres
bar	psi	kPa	MPa	kgf/cm^2	mm Hg	atm
1	14.50326	100	0.1	1.01968	750.0188	0.987167
0.06895	1	6.895	0.006895	0.070307	51.71379	0.068065
0.01	0.1450	1	0.001	0.01020	7.5002	0.00987
10	145.03	1000	1	10.197	7500.2	9.8717
0.9807	14.22335	98.07	0.09807	1	735.5434	0.968115
0.001333	0.019337	0.13333	0.000133	0.00136	1	0.001316
1.013	14.69181	101.3	0.1013	1.032936	759.769	1

Table 16: Temperature Conversion Formulas

Degree Celsius (°C)	(°F - 32) x 5/9
	(K - 273.15)
Degree Fahrenheit (°F)	(°C x 9/5) + 32
	(1.8 x K) - 459.67
Kelvin (K)	(°C + 273.15)
	(°F + 459.67) ÷ 1.8