DYNAMICS

- Vector Mechanics for Engineers: Dynamics, 10th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Dynamics, 7th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Dynamics"

Kinematics of Rigid Bodies

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Kinematics of Rigid Bodies

❑ Applications

A battering ram is an example of curvilinear translation – the ram stays horizontal as it swings through its motion.

Kinematics of Rigid Bodies

- **Applications**

How can we determine the velocity of the tip of a turbine blade?

Kinematics of Rigid Bodies

- **Applications**

Planetary gear systems are used to get high reduction ratios with minimum weight and space. How can we design the correct gear ratios?

Kinematics of Rigid Bodies

❑ Applications

Biomedical engineers must determine the velocities and accelerations of the leg in order to design prostheses.

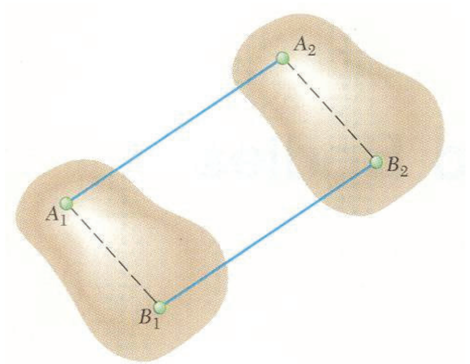
Kinematics of Rigid Bodies

❑ Introduction

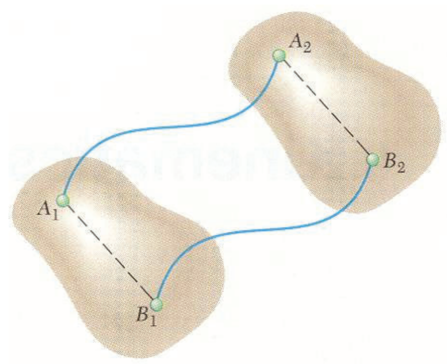
- Kinematics of rigid bodies: **relations** between **time** and the **positions**, **velocities**, and **accelerations** of the **particles forming a rigid body**.
- **Classification of rigid body motions:**

Translation

Rectilinear Translation



Curvilinear Translation

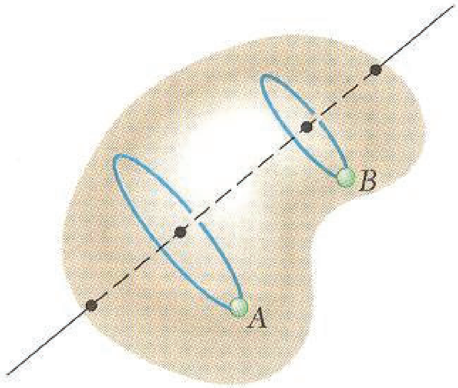


Kinematics of Rigid Bodies

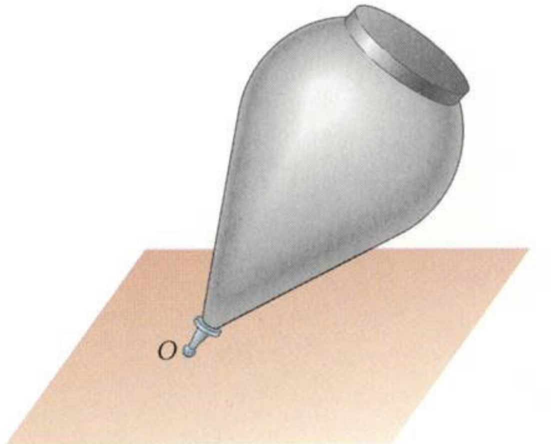
❑ Introduction

- Kinematics of rigid bodies: **relations** between **time** and the **positions**, **velocities**, and **accelerations** of the **particles forming a rigid body**.
- **Classification of rigid body motions:**

Rotation about a fixed axis



Motion about a fixed point

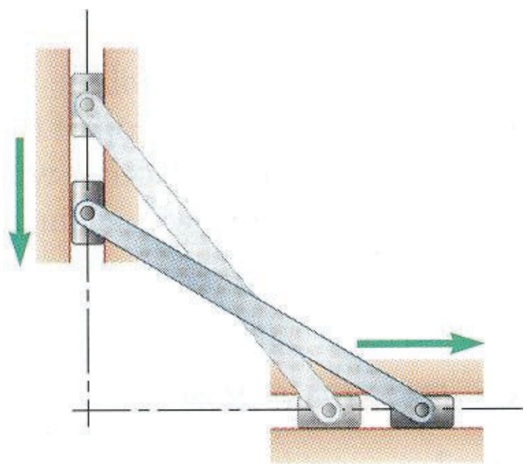


Kinematics of Rigid Bodies

❑ Introduction

- Kinematics of rigid bodies: **relations** between **time** and the **positions**, **velocities**, and **accelerations** of the **particles forming a rigid body**.
- **Classification of rigid body motions:**

General plane motion

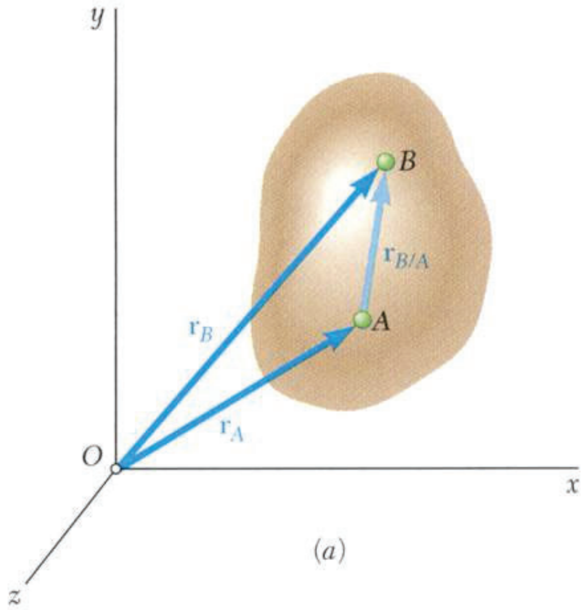


General motion

Kinematics of Rigid Bodies

❑ Translation

- Consider rigid body in translation:
  - Direction of any straight line inside the body is constant,
  - All particles forming the body move in parallel lines.



(a)

- For any two particles in the body,

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

Kinematics of Rigid Bodies

❑ Translation

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Differentiating with respect to time,

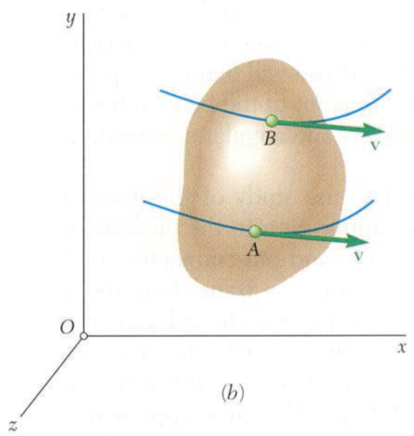
$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \underbrace{\dot{\vec{r}}_{B/A}}_{0} = \dot{\vec{r}}_A \quad \Rightarrow \quad \vec{v}_B = \vec{v}_A$$

All particles have the same velocity.

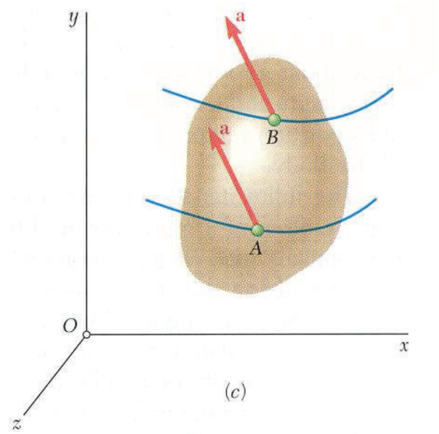
- Differentiating with respect to time again,

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \underbrace{\ddot{\vec{r}}_{B/A}}_{0} = \ddot{\vec{r}}_A \quad \Rightarrow \quad \vec{a}_B = \vec{a}_A$$

All particles have the same acceleration.



(b)



(c)

Kinematics of Rigid Bodies

❑ Rotation About a Fixed Axis. Velocity

- Consider rotation of rigid body about a fixed axis AA'

- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle P is tangent to the path with magnitude $v = ds/dt$

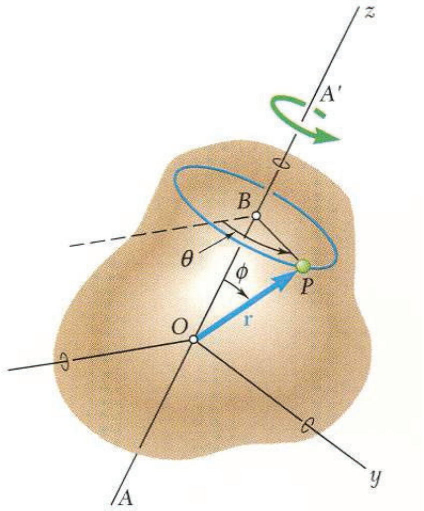
$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

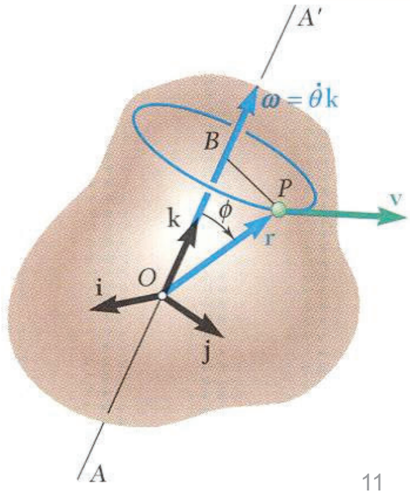
$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} \quad \Rightarrow \quad \boxed{v = r\dot{\theta}\sin\phi}$$

- The same result is obtained from

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega}\times\vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = \textit{angular velocity}$$





Kinematics of Rigid Bodies

❑ Concept Quiz

What is the direction of the velocity of point A on the turbine blade?

a) →

b) ←

c) ↑

d) ↓

Kinematics of Rigid Bodies

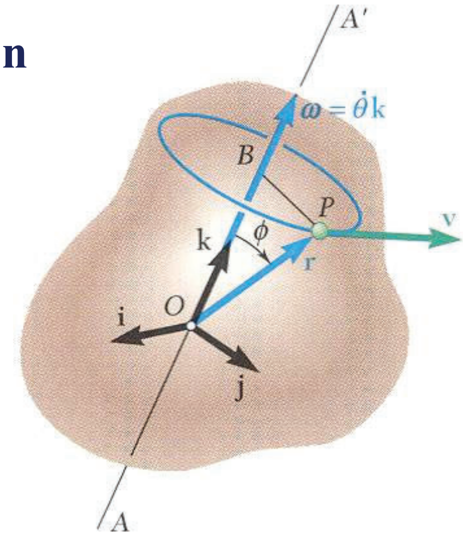
❑ Rotation About a Fixed Axis. Acceleration

- Differentiating to determine the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega}\times\vec{r}) = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$$\Rightarrow \boxed{\vec{a} = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\vec{v}}$$

$$\frac{d\vec{\omega}}{dt} = \vec{\alpha} = \text{angular acceleration}$$
$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$



- Acceleration of P is combination of two vectors,

$$\boxed{\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})}$$

$\vec{\alpha}\times\vec{r}$: tangential acceleration component

$\vec{\omega}\times(\vec{\omega}\times\vec{r})$: radial acceleration component

Kinematics of Rigid Bodies

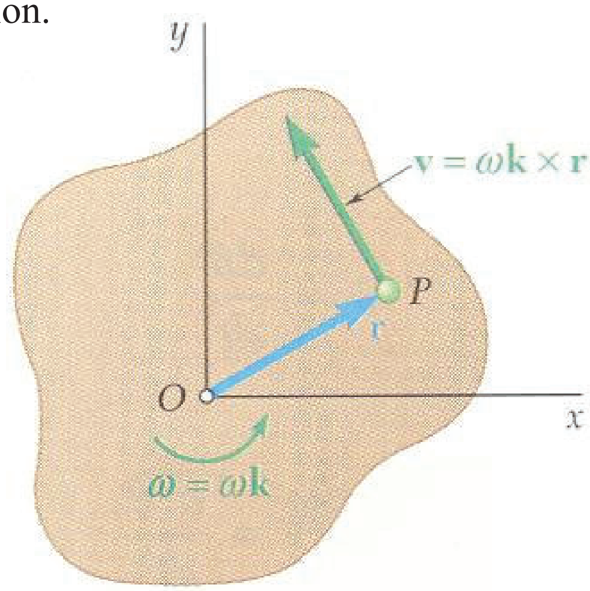
❑ Rotation About a Fixed Axis. Representative Slab

- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.

- Velocity of any point P of the slab,

$$\vec{v} = \vec{\omega}\times\vec{r} = \omega\vec{k}\times\vec{r}$$

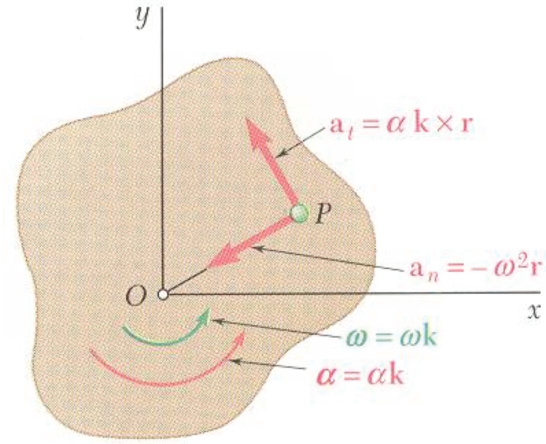
$$\boxed{v = r\omega \quad \& \quad v \perp r}$$



Kinematics of Rigid Bodies

❑ Rotation About a Fixed Axis. Representative Slab

- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.



- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) \quad \Rightarrow \quad \boxed{\vec{a} = \alpha\vec{k}\times\vec{r} - \omega^2\vec{r}}$$

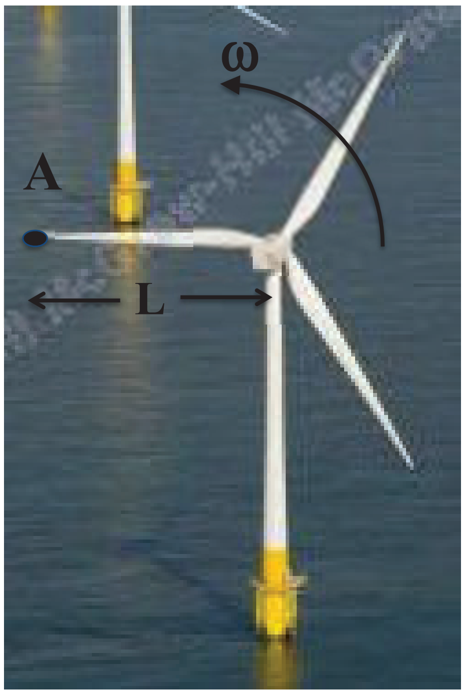
- Resolving the acceleration into tangential and normal components,

$$\boxed{\begin{array}{ll} \vec{a}_t = \alpha\vec{k}\times\vec{r} & a_t = r\alpha \\ \vec{a}_n = -\omega^2\vec{r} & a_n = r\omega^2 \end{array}}$$

Kinematics of Rigid Bodies

❑ Concept Quiz

What is the direction of the normal acceleration of point A on the turbine blade?



a) →

b) ←

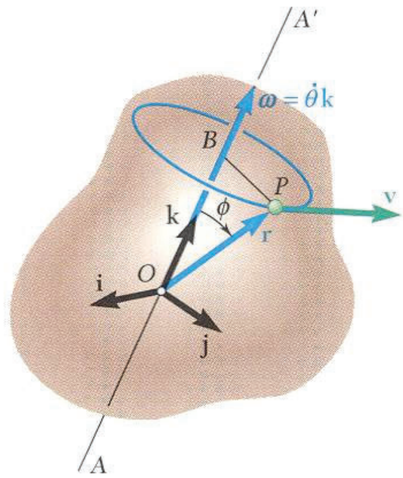
c) ↑

d) ↓

Kinematics of Rigid Bodies

❑ Equations Defining the Rotation of a Rigid Body About a Fixed Axis

- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.



- Recall

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad \Rightarrow \qquad \alpha = \frac{d^2\theta}{dt^2} \quad or \quad \alpha = \omega\frac{d\omega}{d\theta}$$

- *Uniform Rotation,* $\alpha = 0$:

$$\theta = \theta_0 + \omega t$$

- *Uniformly Accelerated Rotation,* $\alpha =$ constant:

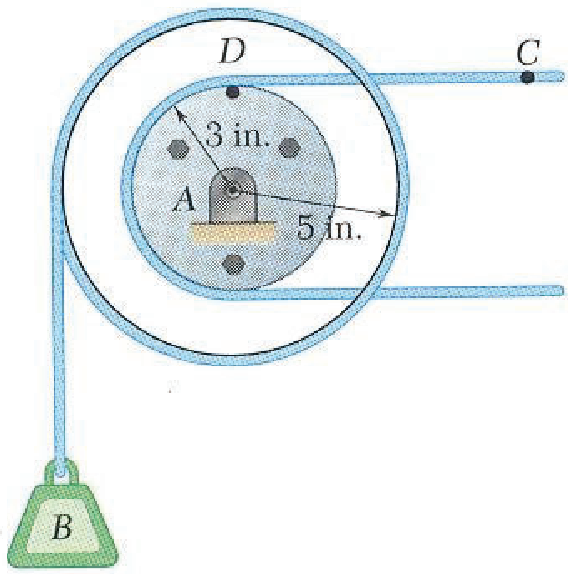
$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Kinematics of Rigid Bodies

❑ Sample Problem 01

Cable *C* has a constant acceleration of 9 in/s^2 and an initial velocity of 12 in/s, both directed to the right.

Determine *(a)* the number of revolutions of the pulley in 2 s, *(b)* the velocity and change in position of the load *B* after 2 s, and *(c)* the acceleration of the point *D* on the rim of the inner pulley at $t = 0$.

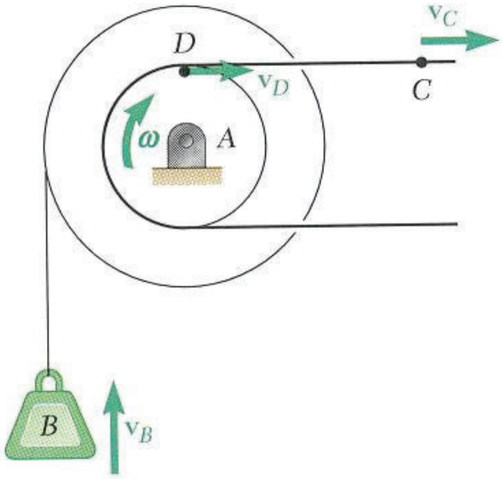


Kinematics of Rigid Bodies

❑ Sample Problem 01

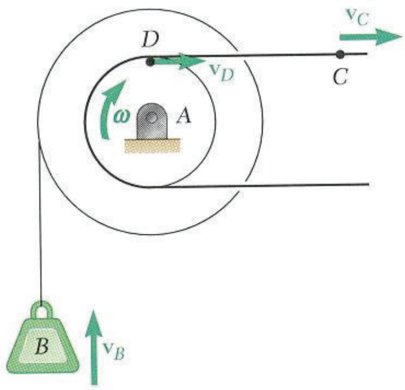
SOLUTION:

- The tangential velocity and acceleration of *D* are equal to the velocity and acceleration of *C.*



Kinematics of Rigid Bodies

❑ Sample Problem 01



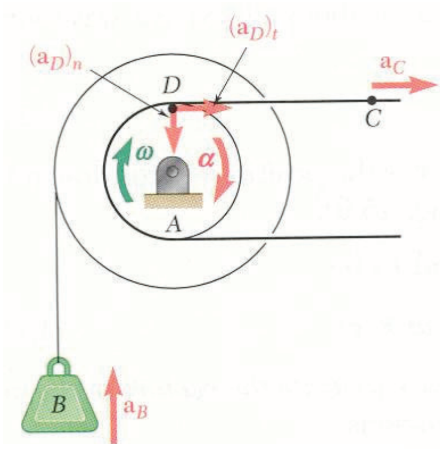
SOLUTION:

- Apply the relations for uniformly accelerated rotation to determine velocity and angular position of pulley after 2 s.

Kinematics of Rigid Bodies

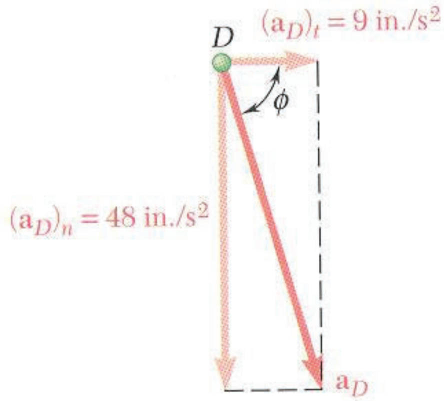
❑ Sample Problem 01

SOLUTION:



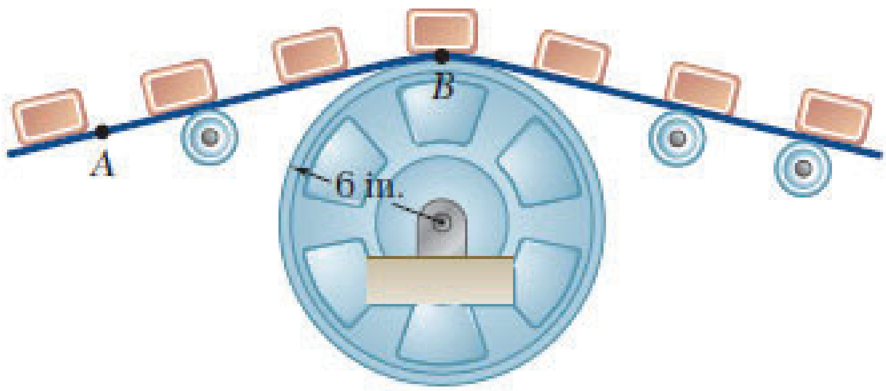
- Evaluate the initial tangential and normal acceleration components of D.

Magnitude and direction of the total acceleration,



❑ Group Problem Solving

A series of small machine components being moved by a conveyor belt pass over a 6-in.-radius idler pulley. At the instant shown, the velocity of point A is 15 in./s to the left and its acceleration is 9 in./s^2 to the right. Determine (*a*) the angular velocity and angular acceleration of the idler pulley, (*b*) the total acceleration of the machine component at B.

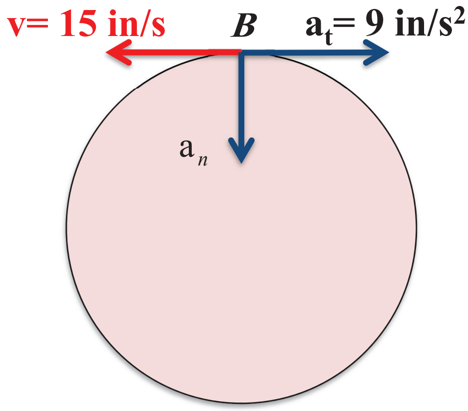


Kinematics of Rigid Bodies

❑ Group Problem Solving

SOLUTION:

Find the angular velocity of the idler pulley using the linear velocity at B.



Find the angular velocity of the idler pulley using the linear velocity at B.

Find the normal acceleration of point *B*.

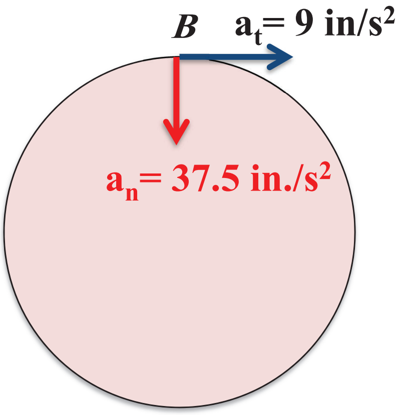
What is the direction of the normal acceleration of point *B*?

Kinematics of Rigid Bodies

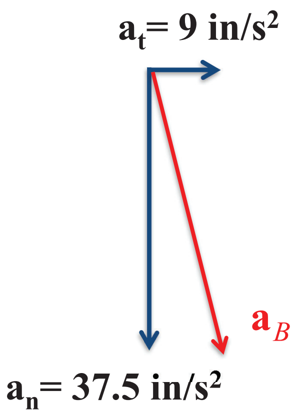
❑ Group Problem Solving

SOLUTION:

Find the total acceleration of the machine component at point *B*.



Calculate the magnitude

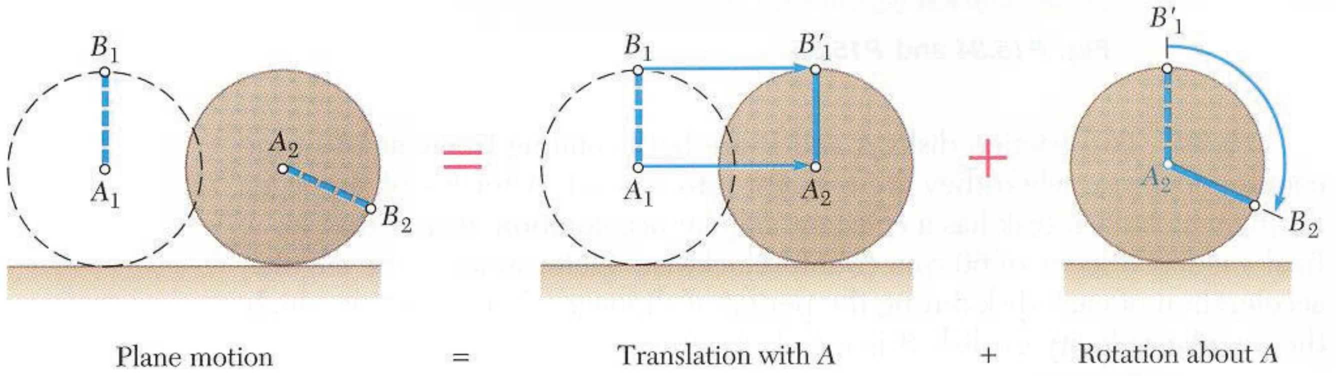


Calculate the angle from the horizontal

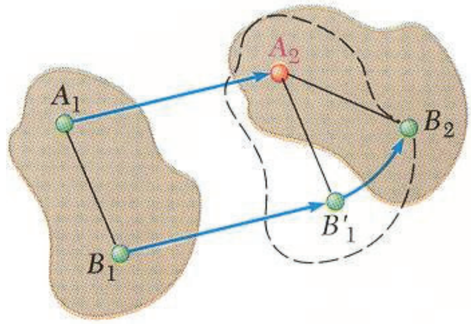
Kinematics of Rigid Bodies

❑ General Plane Motion

- *General plane motion* is ***neither a translation nor a rotation.***
- General plane motion can be considered as ***the sum of a translation and rotation.***



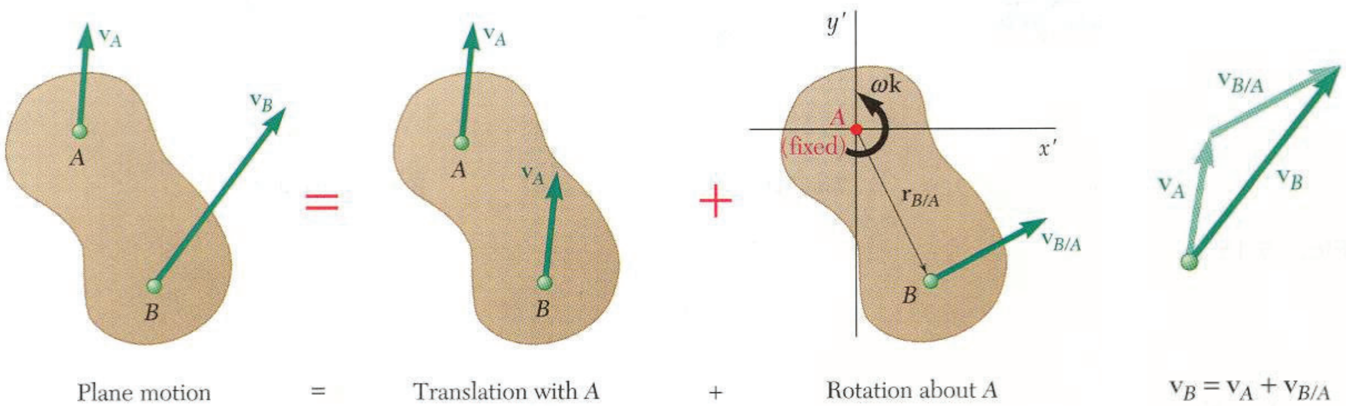
- Displacement of particles A and B to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B_1'
  - rotation of B_1' about A_2 to B_2



Kinematics of Rigid Bodies

❑ Absolute and Relative Velocity in Plane Motion

- Any plane motion can be replaced by a translation of an arbitrary reference point A and a simultaneous rotation about A.



$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{B/A}$$

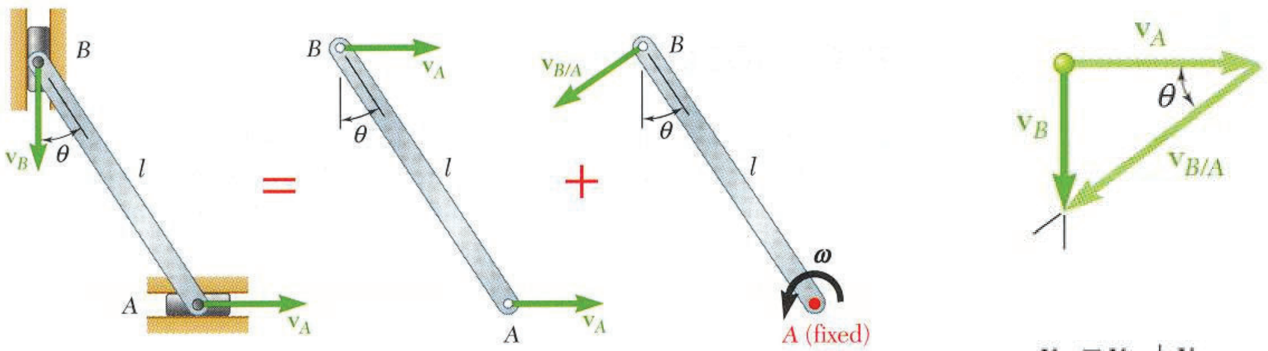
$$v_{B/A} = r\omega$$

$$\Rightarrow \quad \vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$

Kinematics of Rigid Bodies

❑ Absolute and Relative Velocity in Plane Motion

- Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

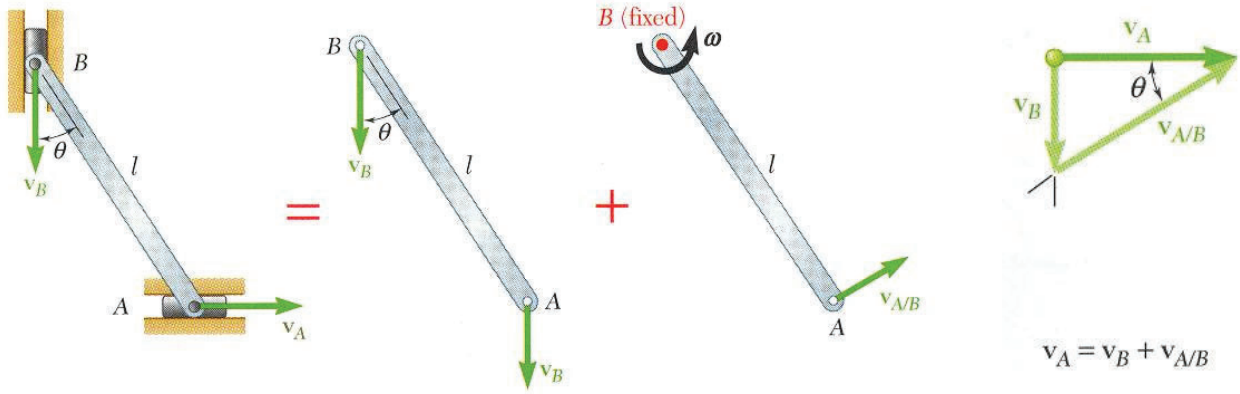
$$\frac{v_B}{v_A} = \tan\theta \quad \Rightarrow \quad \boxed{v_B = v_A \tan\theta}$$

$$\cos\theta = \frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} \quad \Rightarrow \quad \boxed{\omega = \frac{v_A}{l\cos\theta}}$$

Kinematics of Rigid Bodies

❑ Absolute and Relative Velocity in Plane Motion

- Selecting point B as the reference point and solving for the velocity v_A of end A and the angular velocity ω leads to an equivalent velocity triangle.



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

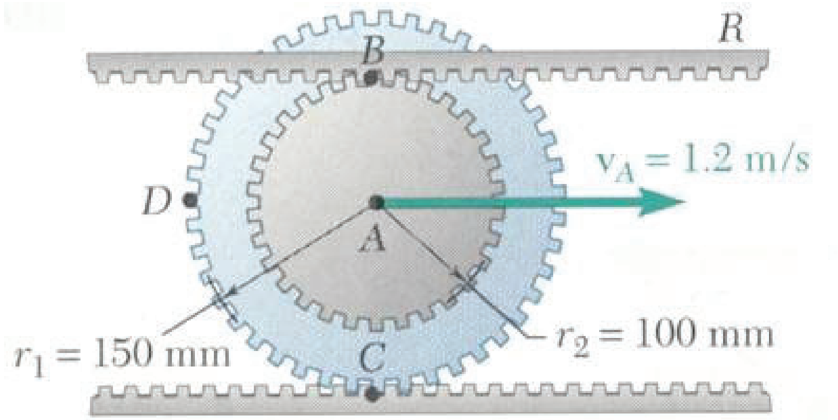
- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. ***The sense of the relative velocity is dependent on the choice of reference point.***
- Angular velocity ω of the rod in its rotation about B is the same as its rotation about A. ***Angular velocity is not dependent on the choice of reference point.***

Kinematics of Rigid Bodies

❑ Sample Problem 02

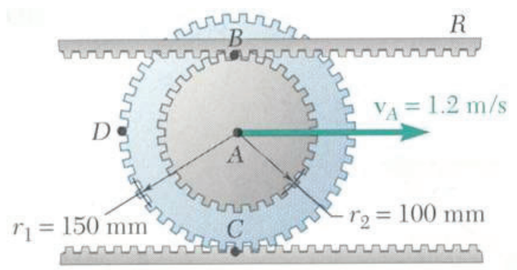
The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s.

Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack *R* and point *D* of the gear.



Kinematics of Rigid Bodies

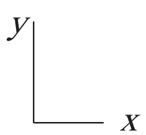
❑ Sample Problem 02



SOLUTION:

- The displacement of the gear center in one revolution is equal to the outer circumference.

  For $x_A > 0$ (***moves to right***), $\omega < 0$ (***rotates clockwise***).



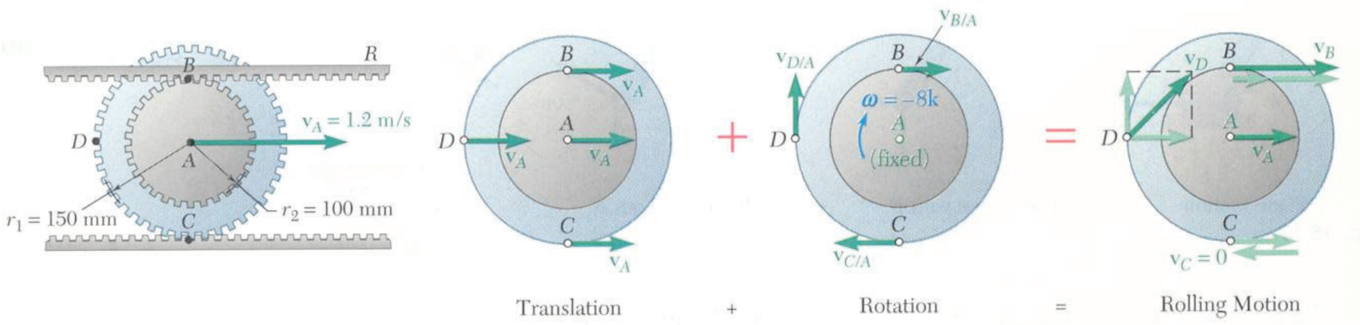
Differentiate to relate the translational and angular velocities.

Kinematics of Rigid Bodies

❑ Sample Problem 02

SOLUTION:

- For any point P on the gear,



Velocity of the upper rack is equal to velocity of point B:

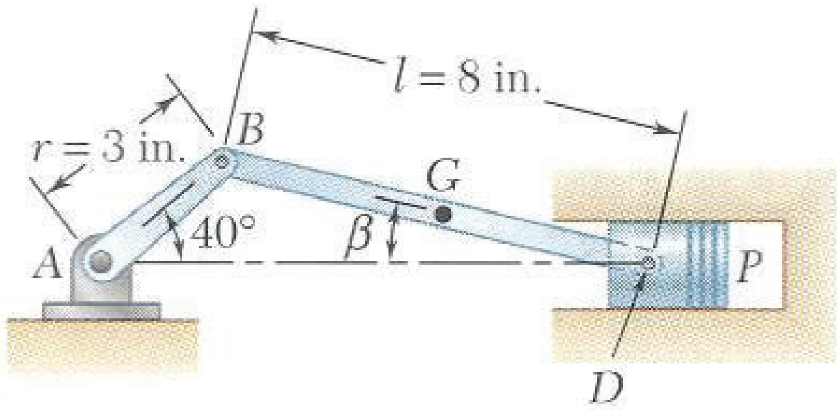
Velocity of the point D:

Kinematics of Rigid Bodies

❑ Sample Problem 03

The crank AB has a constant clockwise angular velocity of 2000 rpm.

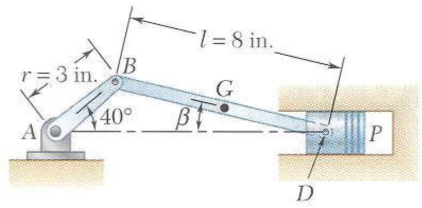
For the crank position indicated, determine *(a)* the angular velocity of the connecting rod BD, and *(b)* the velocity of the piston P.



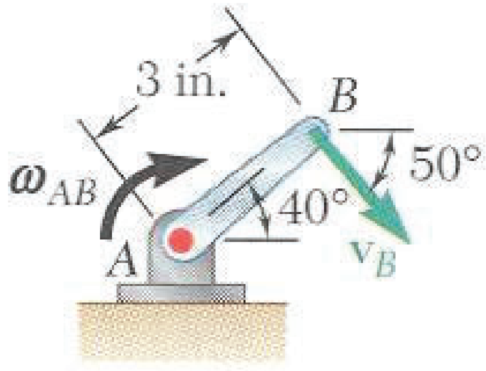
Kinematics of Rigid Bodies

❑ Sample Problem 03

SOLUTION:



- Will determine the absolute velocity of point *D* with



- The velocity $\vec{v}_B$ is obtained from the crank rotation data.

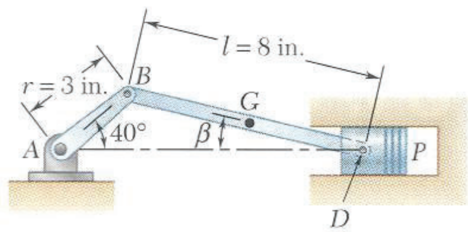
The velocity direction is as shown.

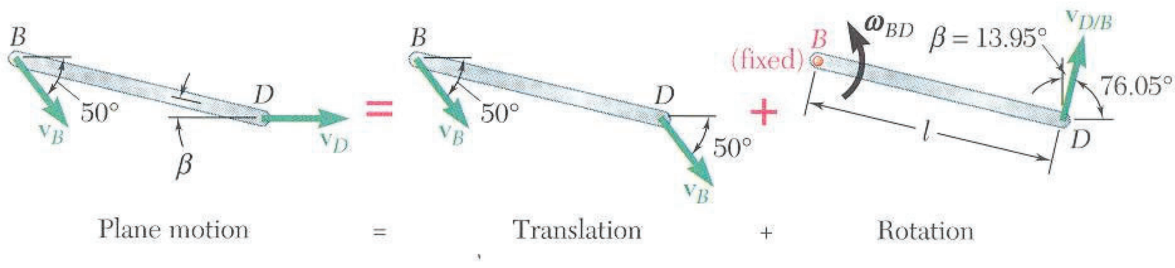
Kinematics of Rigid Bodies

❑ Sample Problem 03

SOLUTION:

- The direction of the absolute velocity $\vec{v}_D$ is ***horizontal***. The direction of the relative velocity $\vec{v}_{D/B}$ is ***perpendicular to BD***. Compute the angle between the horizontal and the connecting rod from the law of sines.

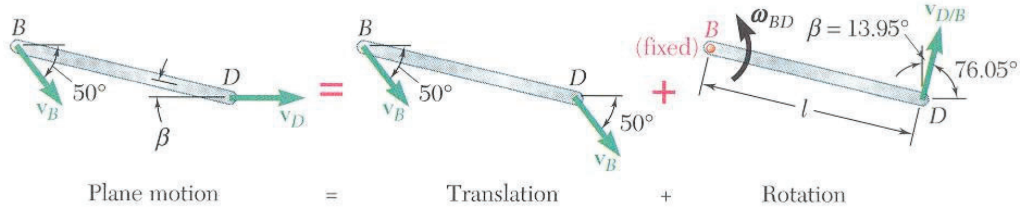




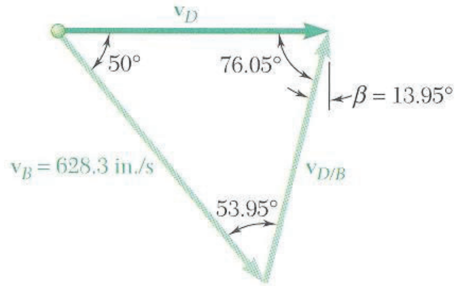
Kinematics of Rigid Bodies

❑ Sample Problem 03

SOLUTION:



- Determine the velocity magnitudes v_D and $v_{D/B}$ from the vector triangle.

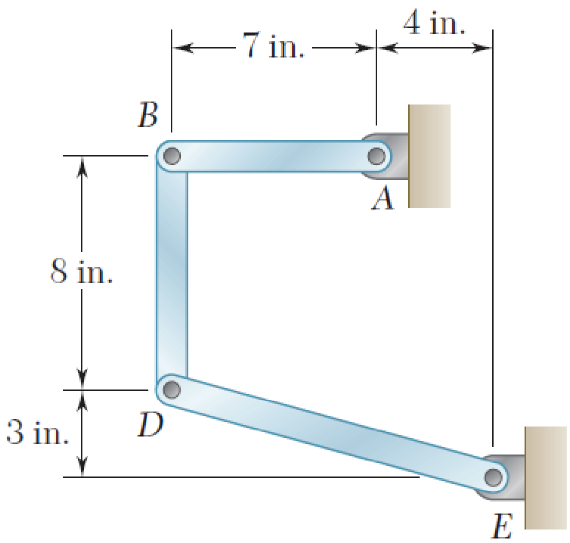


$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

Kinematics of Rigid Bodies

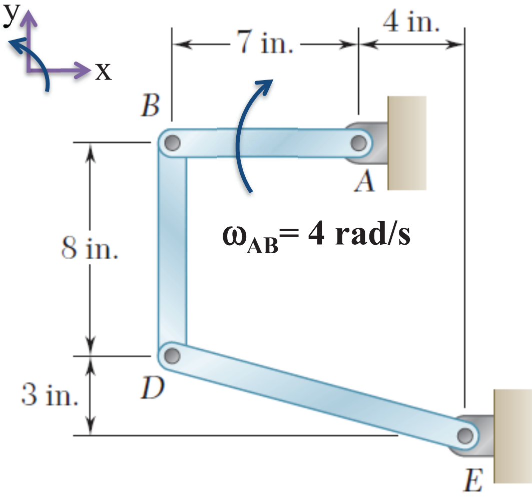
❑ Sample Problem 04

In the position shown, bar *AB* has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars *BD* and *DE*.



❑ Sample Problem 04

SOLUTION:



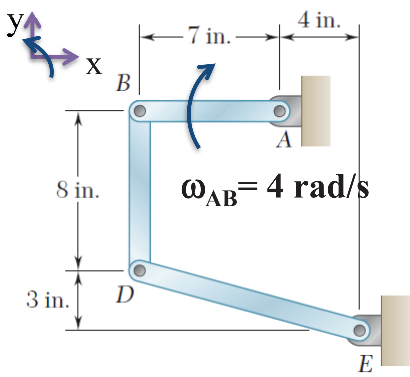
Write v_B in terms of point A, calculate v_B.

Determine v_D with respect to B.

Kinematics of Rigid Bodies

❑ Sample Problem 04

SOLUTION:

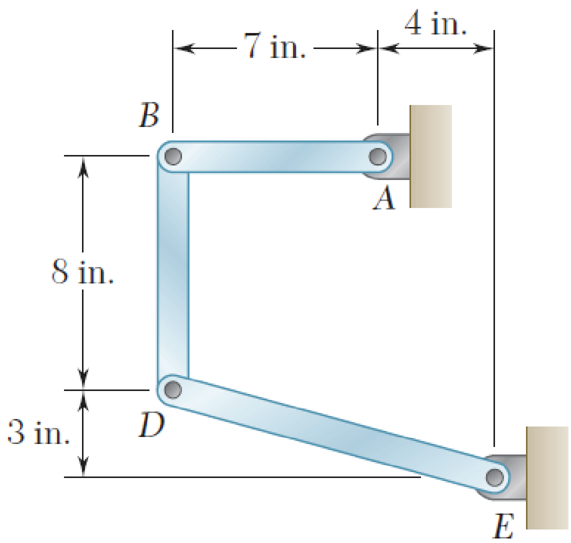


Determine v_D with respect to E, then equate it to equation above.

Equating components of the two expressions for v_D

Kinematics of Rigid Bodies

❑ Group Problem Solving



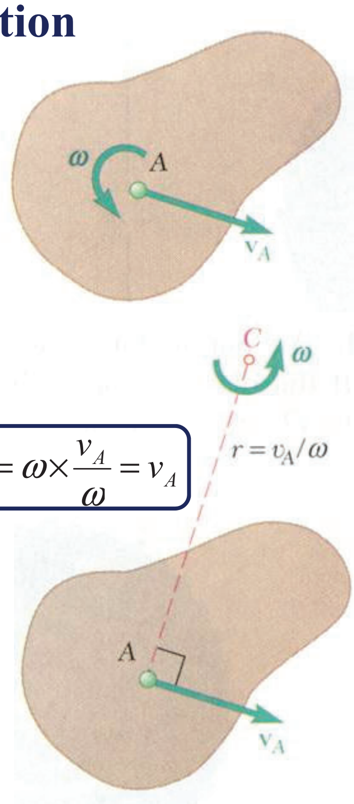
Which of the following is true?

a) The direction of v_B is ↑
b) The direction of v_D is →
c) Both a) and b) are correct

Kinematics of Rigid Bodies

❑ Instantaneous Center of Rotation in Plane Motion

- Plane motion of all particles in a slab can always be replaced by the ***translation of an arbitrary point A and a rotation about A*** with an angular velocity that is independent of the choice of *A*.
- The same translational and rotational velocities at *A* are obtained by allowing the slab to ***rotate with the same angular velocity about the point C on a perpendicular to the velocity at A.***
- ***The velocity of all other particles in the slab are the same as originally defined*** since the angular velocity and translational velocity at *A* are equivalent.
- As far as the velocities are concerned, the slab seems to rotate about the ***instantaneous center of rotation C.***



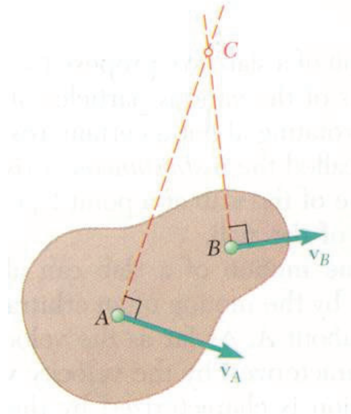
$$|v_A| = \omega \times \frac{v_A}{\omega} = v_A$$

Kinematics of Rigid Bodies

❑ Instantaneous Center of Rotation in Plane Motion

- If the velocity at two points *A* and *B* are known, the instantaneous center of rotation lies at ***the intersection of the perpendiculars to the velocity vectors through A and B .***

- If the velocity vectors are parallel, ***the instantaneous center of rotation is at infinity and the angular velocity is zero.***



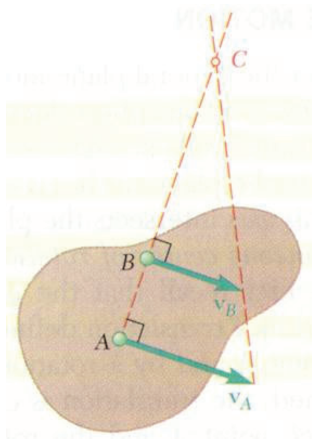
$$r = \frac{v_A}{\omega} \to \infty \quad \Rightarrow \quad \omega \to 0$$

Kinematics of Rigid Bodies

❑ Instantaneous Center of Rotation in Plane Motion

- If the velocity vectors at *A* and *B* are perpendicular to the line *AB*, ***the instantaneous center of rotation lies at the intersection of the line AB with the line joining the extremities of the velocity vectors at A and B.***

- If the velocity magnitudes are equal, ***the instantaneous center of rotation is at infinity and the angular velocity is zero.***



$$r = \frac{v_A}{\omega} \to \infty \quad \Rightarrow \quad \omega \to 0$$

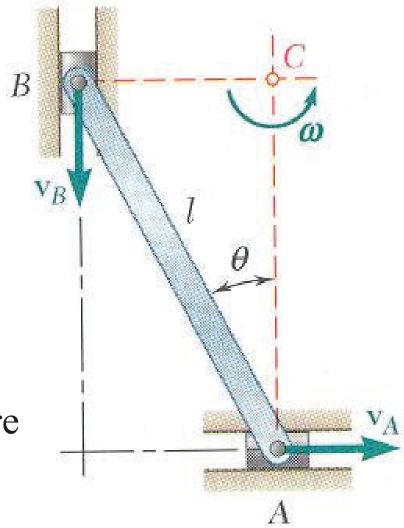
Kinematics of Rigid Bodies

❑ Instantaneous Center of Rotation in Plane Motion

- The instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through A and B.

$$\omega = \frac{v_A}{l_{AC}} = \frac{v_A}{l\cos\theta} \quad \Rightarrow \quad \boxed{\omega = \frac{v_A}{l\cos\theta}}$$

$$v_B = l_{BC}\omega = (l\sin\theta)\frac{v_A}{l\cos\theta} \quad \Rightarrow \quad \boxed{v_B = v_A\tan\theta}$$

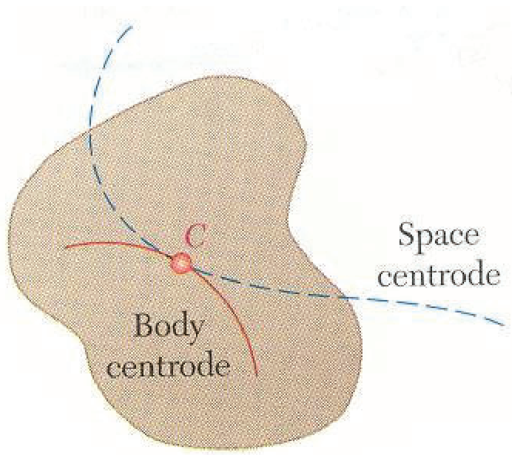


- The velocities of all particles on the rod are as if they were rotated about C.

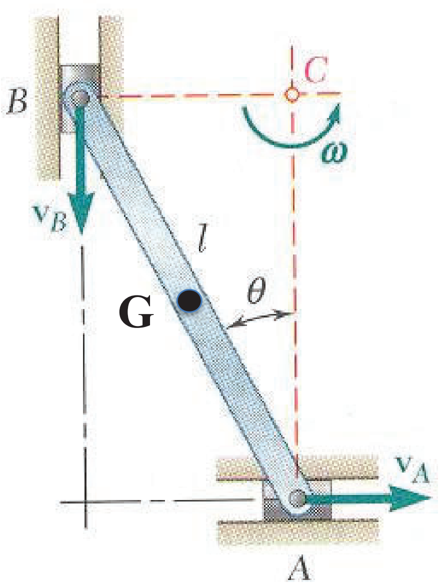
Kinematics of Rigid Bodies

❑ Instantaneous Center of Rotation in Plane Motion

- The particle at the center of rotation has zero velocity.
- The particle coinciding with the center of rotation changes with time and ***the acceleration of the particle at the instantaneous center of rotation is not zero.***
- The acceleration of the particles in the slab cannot be determined as if the slab were simply rotating about C.
- The trace of the locus of the center of rotation on the body is the ***body centrode*** and in space is the ***space centrode.***



❑ Group Problem Solving



At the instant shown, what is the approximate direction of the velocity of point G, the center of bar AB?

a) 

b) 

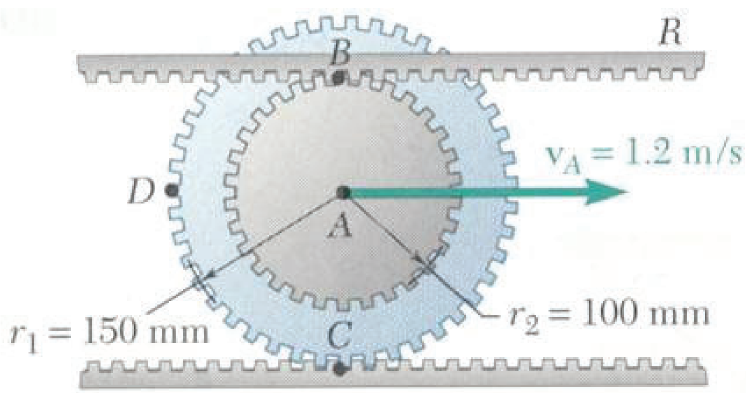
c) 

d) 

❑ Sample Problem 05

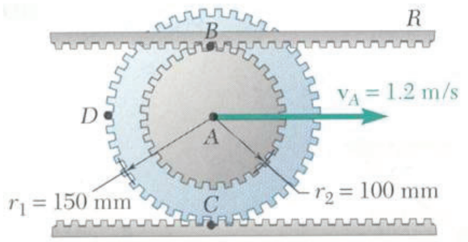
The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s.

Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack *R* and point *D* of the gear.



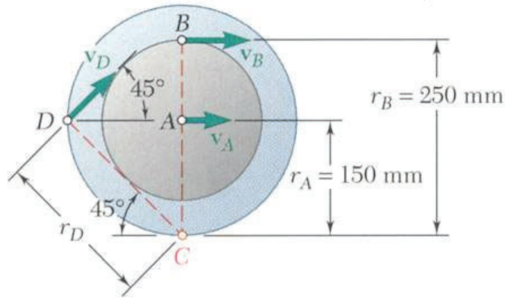
Kinematics of Rigid Bodies

❑ Sample Problem 05



SOLUTION:

- The point C is in contact with the stationary lower rack and, instantaneously, has zero velocity. ***It must be the location of the instantaneous center of rotation.***



- Determine the angular velocity about C based on the given velocity at A.

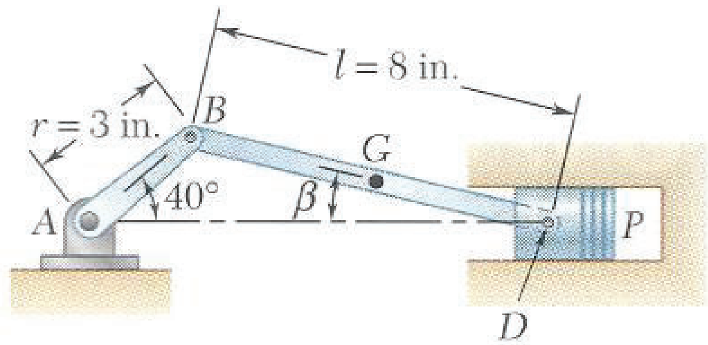
- Evaluate the velocities at B and D based on their rotation about C.

Kinematics of Rigid Bodies

❑ Sample Problem 06

The crank AB has a constant clockwise angular velocity of 2000 rpm.

For the crank position indicated, determine *(a)* the angular velocity of the connecting rod BD, and *(b)* the velocity of the piston P.

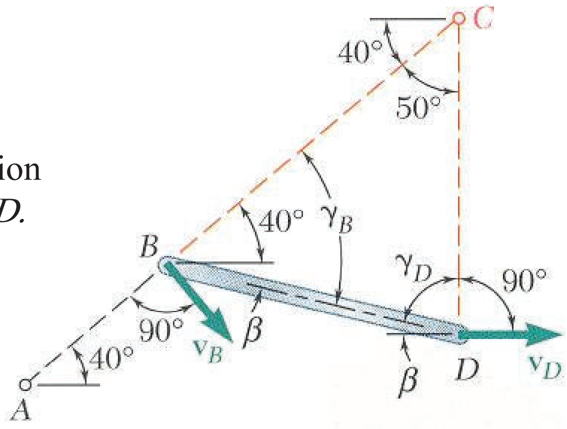


Kinematics of Rigid Bodies

❑ Sample Problem 06

SOLUTION:

- From Sample Problem 03,

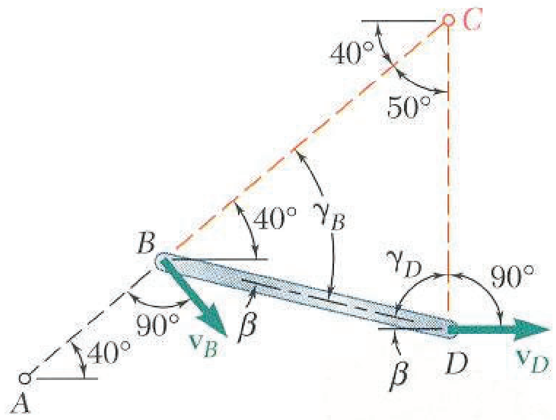


- The instantaneous center of rotation is at the intersection of the perpendiculars to the velocities through B and D.

Kinematics of Rigid Bodies

❑ Sample Problem 06

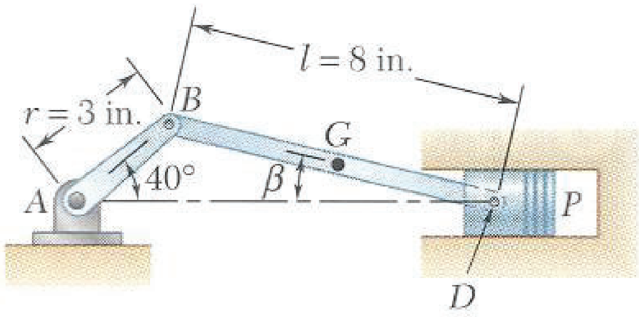
SOLUTION:

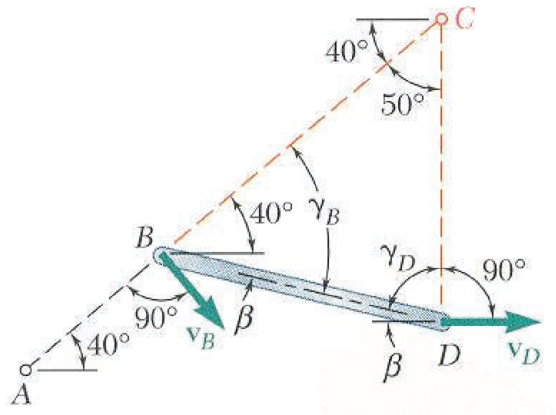


- Determine the angular velocity about the center of rotation based on the velocity at B.

- Calculate the velocity at D based on its rotation about the instantaneous center of rotation.

❑ Group Problem Solving



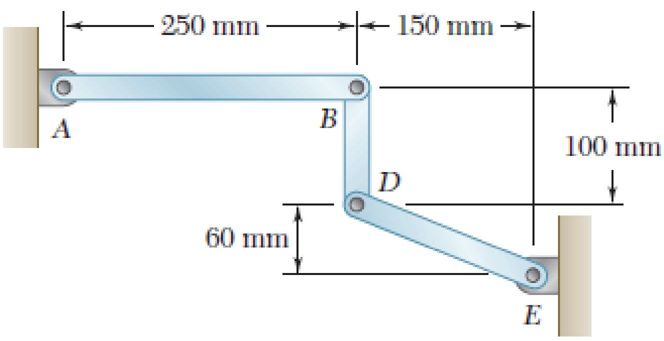


What happens to the location of the instantaneous center of velocity if the crankshaft angular velocity increases from 2000 rpm in the previous problem to 3000 rpm?

What happens to the location of the instantaneous center of velocity if the angle β is 0?

❑ Sample Problem 07

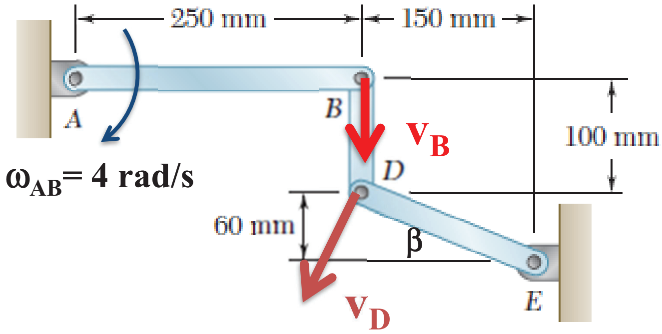
In the position shown, bar AB has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars BD and DE.



Kinematics of Rigid Bodies

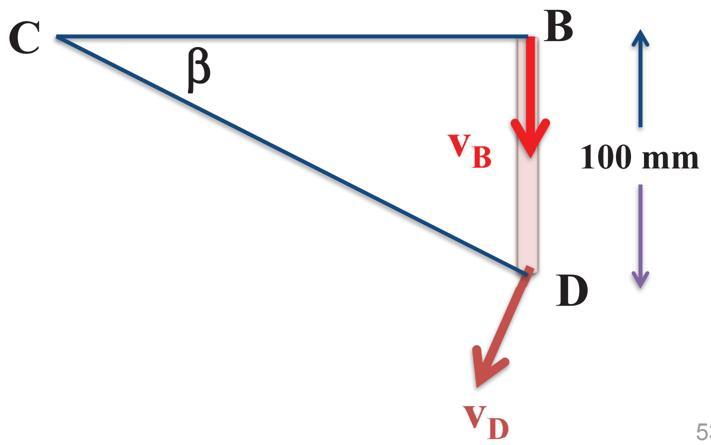
❑ Sample Problem 07

SOLUTION:



Locate instantaneous center *C* at intersection of lines drawn perpendicular to v_B and v_D.

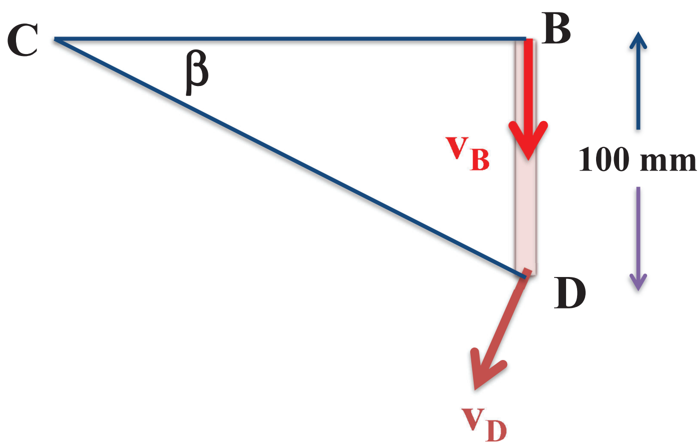
Find distances BC and DC



Kinematics of Rigid Bodies

❑ Sample Problem 07

SOLUTION:



Calculate ω_{BD}

Find ω_{DE}

Kinematics of Rigid Bodies

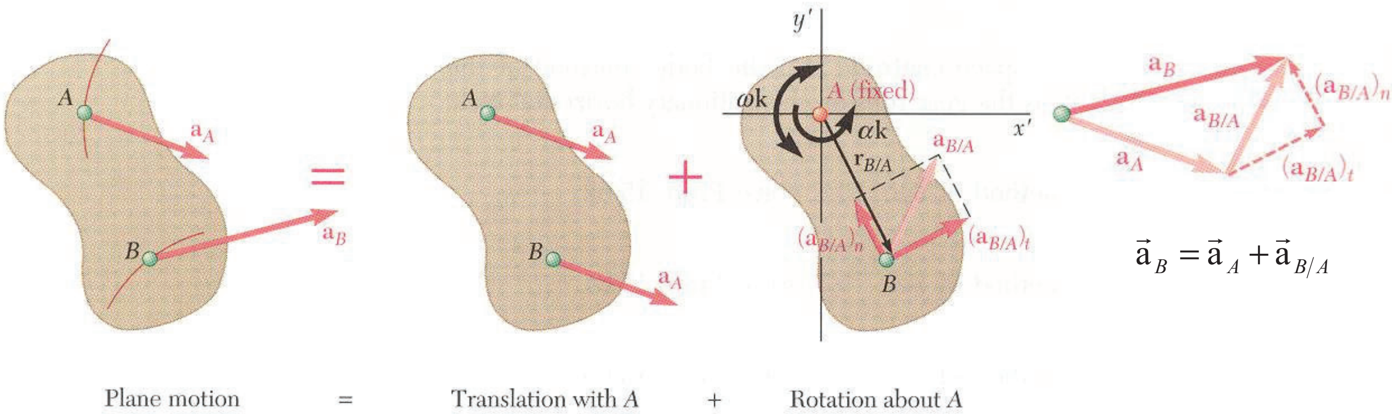
❑ Absolute and Relative Acceleration in Plane Motion

As the bicycle accelerates, a point on the top of the wheel will have acceleration due to the acceleration from the axle (the overall linear acceleration of the bike), the tangential acceleration of the wheel from the angular acceleration, and the normal acceleration due to the angular velocity.

Kinematics of Rigid Bodies

❑ Absolute and Relative Acceleration in Plane Motion

- Absolute acceleration of a particle of the slab,



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

- Relative acceleration $\vec{a}_{B/A}$ associated with rotation about A includes tangential and normal components,

$$(\vec{a}_{B/A})_t = \alpha\vec{k} \times \vec{r}_{B/A} \qquad (a_{B/A})_t = r\alpha$$

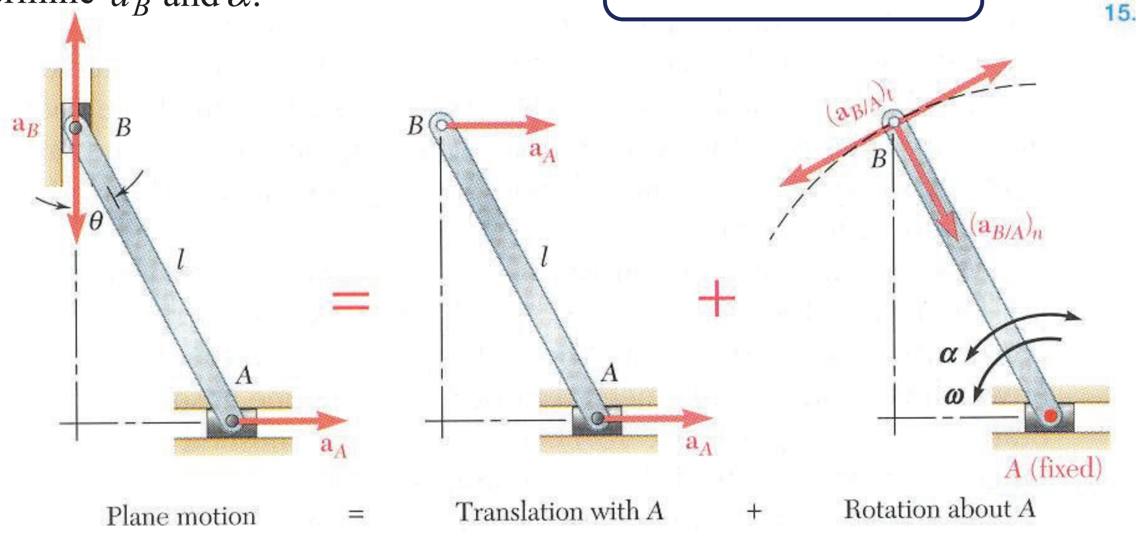
$$(\vec{a}_{B/A})_n = -\omega^2 \vec{r}_{B/A} \qquad (a_{B/A})_n = r\omega^2$$

Kinematics of Rigid Bodies

❑ Absolute and Relative Acceleration in Plane Motion

- Given $\vec{a}_A$ and $\vec{v}_A$, determine $\vec{a}_B$ and $\vec{\alpha}$.

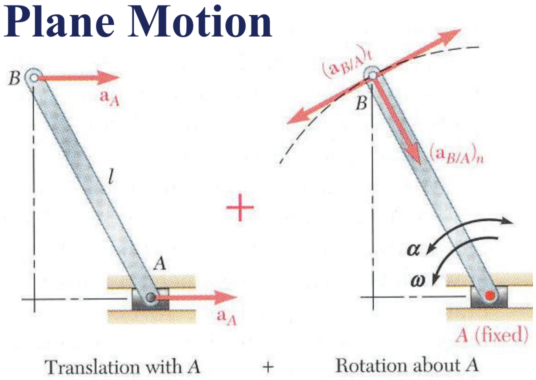
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \quad \Rightarrow \quad \boxed{\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_n + (\vec{a}_{B/A})_t}$$

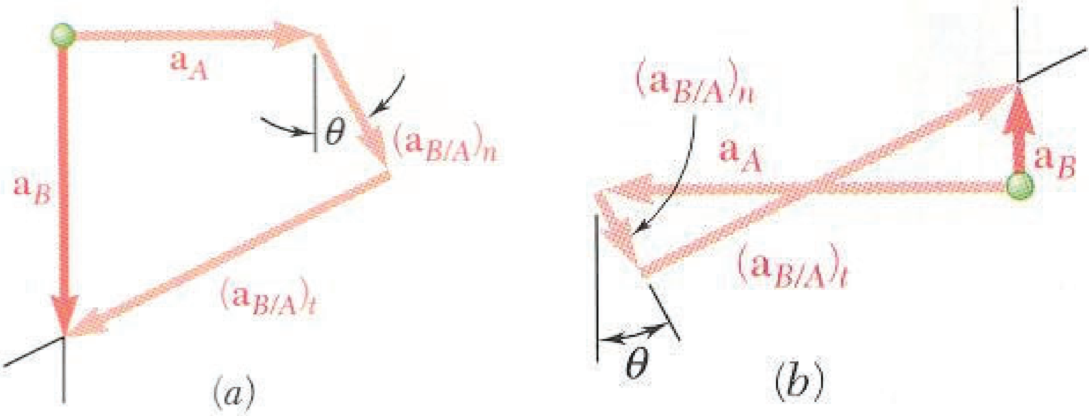


Kinematics of Rigid Bodies

❑ Absolute and Relative Acceleration in Plane Motion

- Vector result depends on sense of $\vec{a}_A$ and the relative magnitudes of a_A and $(a_{B/A})_n$
- Must also know angular velocity ω.

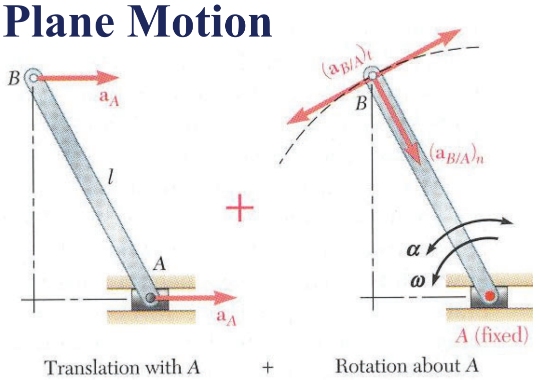


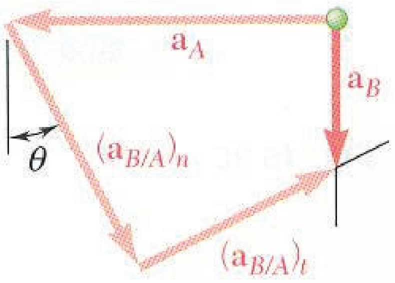


Kinematics of Rigid Bodies

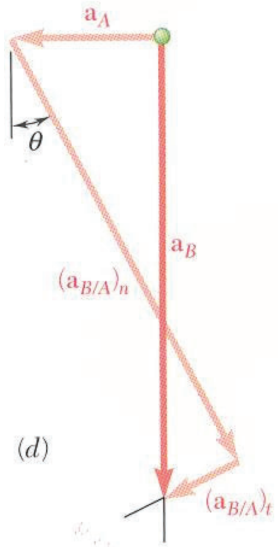
❑ Absolute and Relative Acceleration in Plane Motion

- Vector result depends on sense of $\vec{a}_A$ and the relative magnitudes of a_A and $(a_{B/A})_n$
- Must also know angular velocity ω.





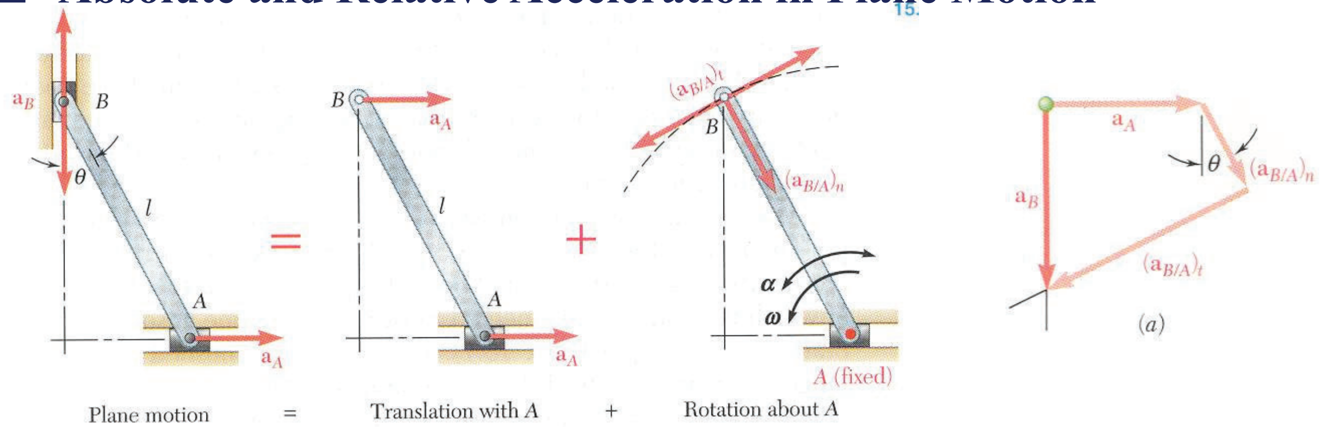
(c)



(d)

Kinematics of Rigid Bodies

❑ Absolute and Relative Acceleration in Plane Motion



- Write $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ in terms of the two component equations,

$\overset{+}{\rightarrow}$ x components: $0 = a_A + l\omega^2 \sin\theta - l\alpha\cos\theta$

$+\uparrow$ y components: $-a_B = -l\omega^2 \cos\theta - l\alpha\sin\theta$

- Solve for a_B and α.

Kinematics of Rigid Bodies

❑ Analysis of Plane Motion in Terms of a Parameter

- In some cases, it is advantageous to determine the absolute velocity and acceleration of a mechanism directly.

$$\boxed{x_A = l\sin\theta}$$

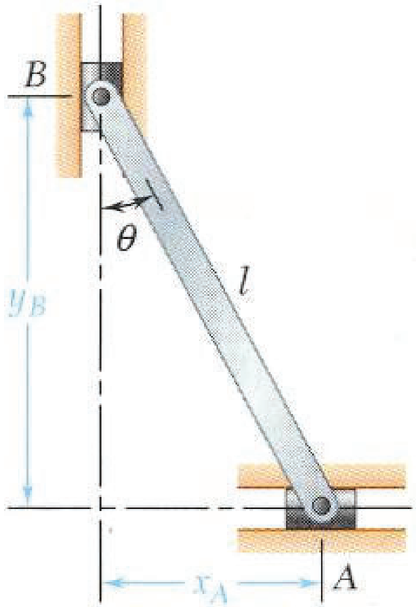
$$v_A = \dot{x}_A = l\dot{\theta}\cos\theta \quad \Rightarrow \quad \boxed{v_A = l\omega\cos\theta}$$

$$a_A = \ddot{x}_A = -l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta \quad \Rightarrow \quad \boxed{a_A = -l\omega^2\sin\theta + l\alpha\cos\theta}$$

$$\boxed{y_B = l\cos\theta}$$

$$v_B = \dot{y}_B = -l\dot{\theta}\sin\theta \quad \Rightarrow \quad \boxed{v_B = -l\omega\sin\theta}$$

$$a_B = \ddot{y}_B = -l\dot{\theta}^2\cos\theta - l\ddot{\theta}\sin\theta \quad \Rightarrow \quad \boxed{a_B = -l\omega^2\cos\theta - l\alpha\sin\theta}$$

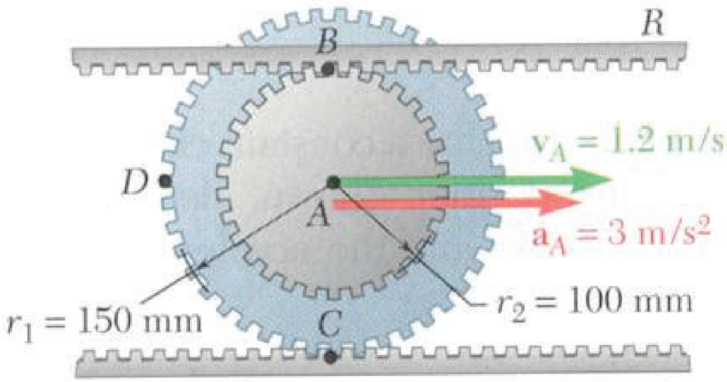


Kinematics of Rigid Bodies

❑ Sample Problem 08

The center of the double gear has a velocity and acceleration to the right of 1.2 m/s and 3 m/s^2, respectively. The lower rack is stationary.

Determine *(a)* the angular acceleration of the gear, and *(b)* the acceleration of points *B*, *C*, and *D*.

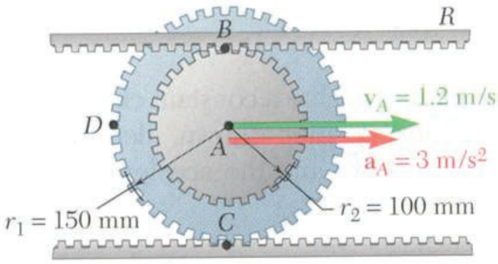


Kinematics of Rigid Bodies

❑ Sample Problem 08

SOLUTION:

- The expression of the gear position as a function of θ is differentiated twice to define the relationship between the translational and angular accelerations.

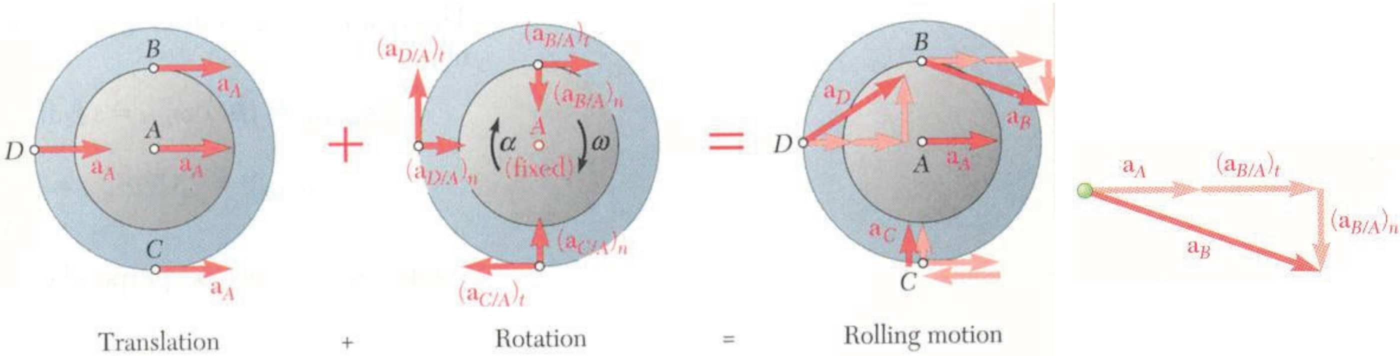


Kinematics of Rigid Bodies

❑ Sample Problem 08

SOLUTION:

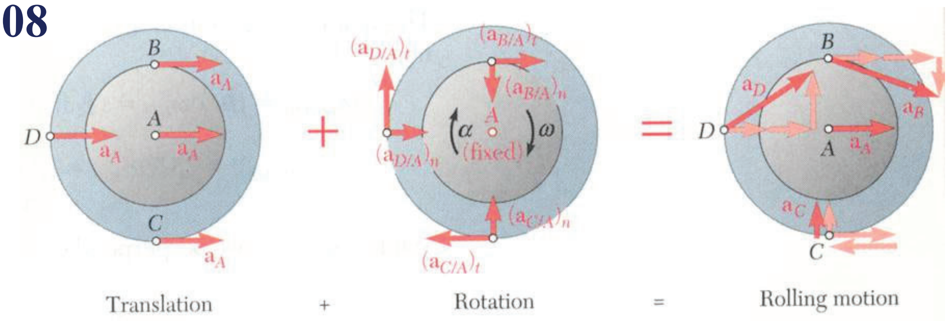
- The acceleration of each point is obtained by adding the acceleration of the gear center and the relative accelerations with respect to the center. The latter includes normal and tangential acceleration components.

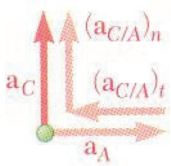


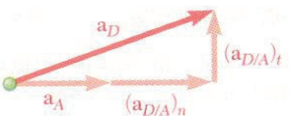
Kinematics of Rigid Bodies

❑ Sample Problem 08

SOLUTION:





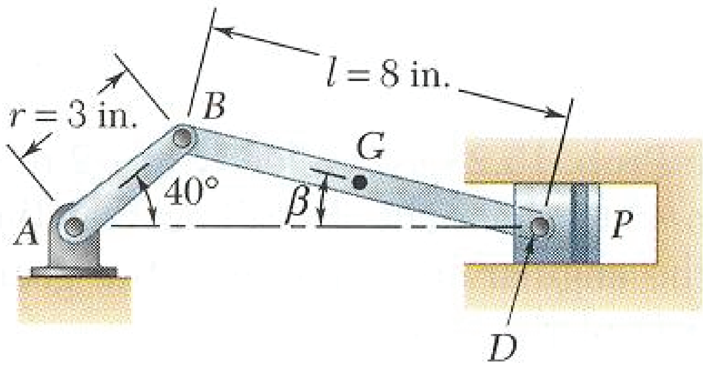


Kinematics of Rigid Bodies

❑ Sample Problem 09

Crank *AB* of the engine system has a constant clockwise angular velocity of 2000 rpm.

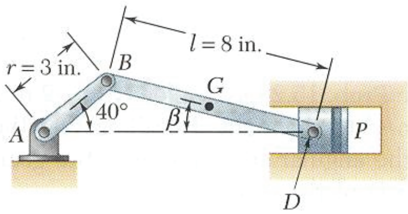
For the crank position shown, determine the angular acceleration of the connecting rod *BD* and the acceleration of point *D*.



Kinematics of Rigid Bodies

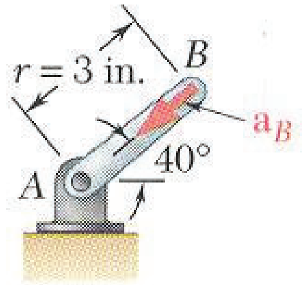
❑ Sample Problem 09

SOLUTION:



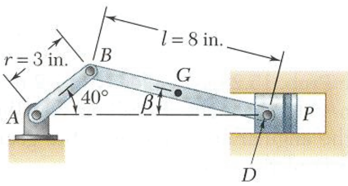
- The angular acceleration of the connecting rod *BD* and the acceleration of point *D* will be determined from

  - The acceleration of *B* is determined from the given rotation speed of *AB*.



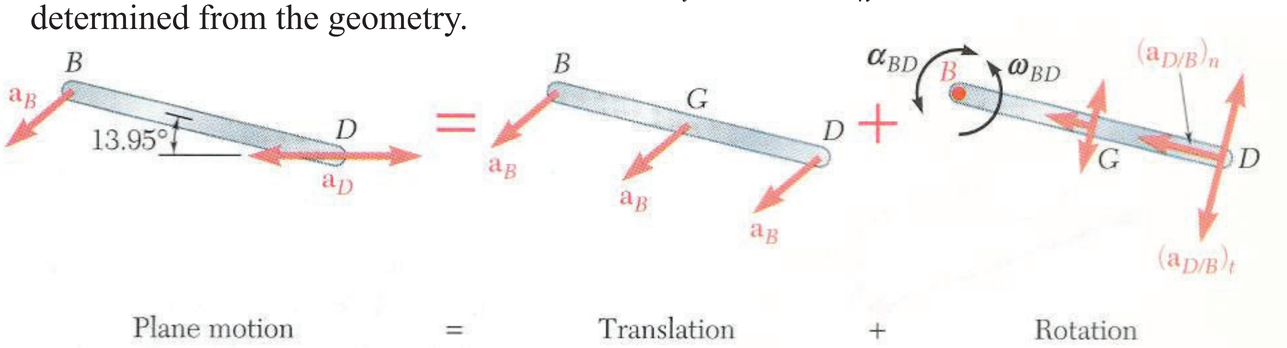
Kinematics of Rigid Bodies

❑ Sample Problem 09



SOLUTION:

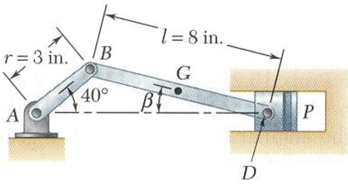
- The directions of the accelerations $\vec{a}_D$, $\left(\vec{a}_{D/B}\right)_t$, and $\left(\vec{a}_{D/B}\right)_n$ are determined from the geometry.



From Sample Problem 3, $\omega_{BD} = 62.0$ rad/s, $\beta = 13.95^{\circ}$.

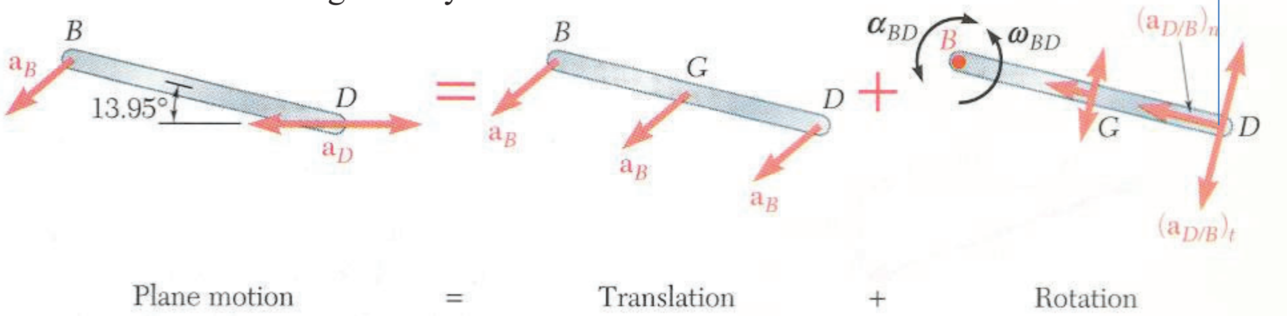
Kinematics of Rigid Bodies

❑ Sample Problem 09



SOLUTION:

- The directions of the accelerations $\vec{a}_D, (\vec{a}_{D/B})_t$, and $(\vec{a}_{D/B})_n$ are determined from the geometry.

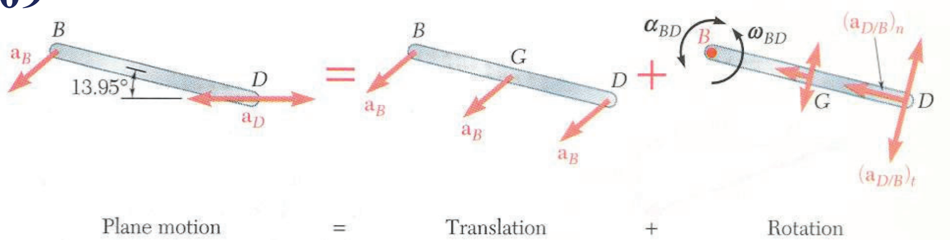


The direction of $(a_{D/B})_t$ is known but the sense is not known,

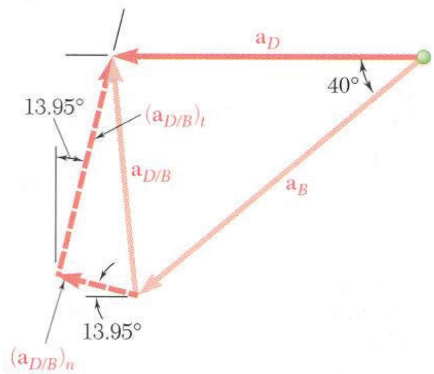
Kinematics of Rigid Bodies

❑ Sample Problem 09

SOLUTION:



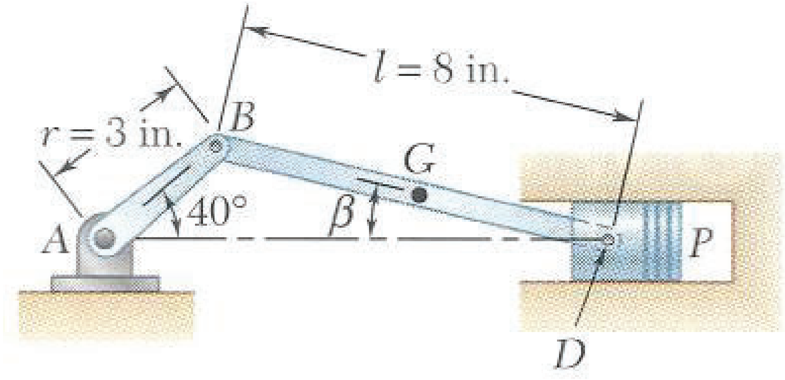
- Component equations for acceleration of point D are solved simultaneously.



x components:

y components:

❑ Concept Question



If the clockwise angular velocity of crankshaft AB is constant, which of the following statement is true?

a) The angular velocity of BD is constant

b) The linear acceleration of point B is zero

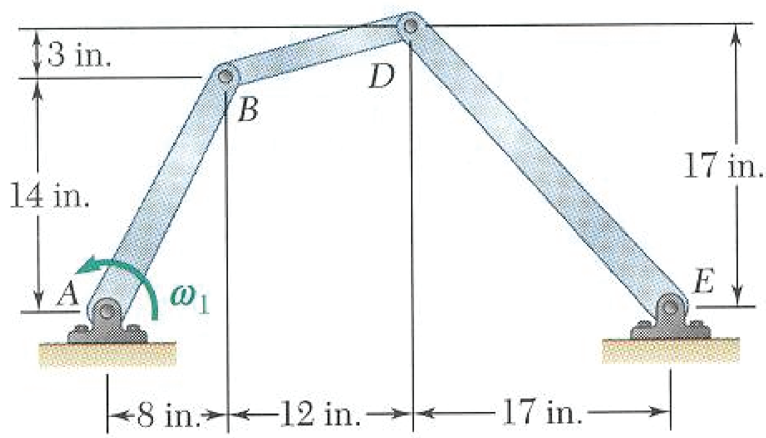
c) The angular velocity of BD is counterclockwise

d) The linear acceleration of point B is tangent to the path

❑ Sample Problem 10

In the position shown, crank *AB* has a constant angular velocity ω_1 = 20 rad/s counterclockwise.

Determine the angular velocities and angular accelerations of the connecting rod *BD* and crank *DE*.

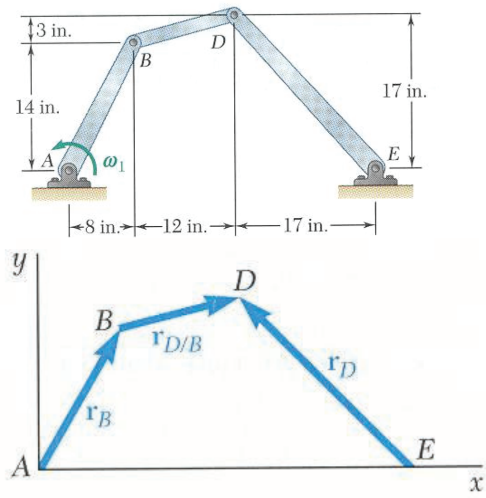


Kinematics of Rigid Bodies

❑ Sample Problem 10

SOLUTION:

- The angular velocities are determined by simultaneously solving the component equations for $\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$



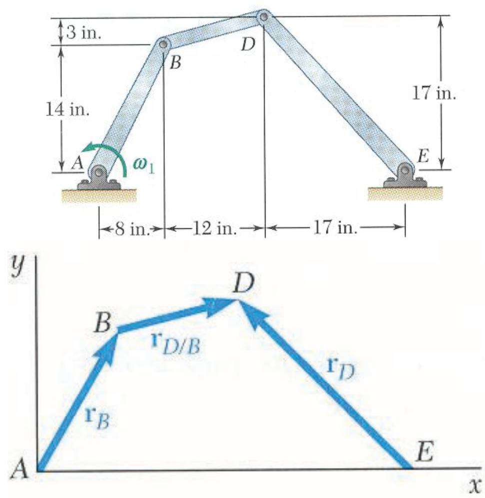
$$r_B = 8i + 14j$$
$$r_D = -17i + 17j$$
$$r_{D/B} = 12i + 3j$$

Kinematics of Rigid Bodies

❑ Sample Problem 10

SOLUTION:

- The angular accelerations are determined by simultaneously solving the component equations for



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Kinematics of Rigid Bodies

❑ Sample Problem 10

SOLUTION:

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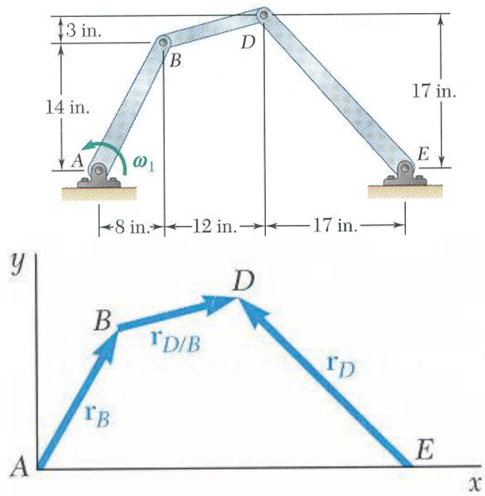


$\mathbf{r}_B = 8\mathbf{i} + 14\mathbf{j}$

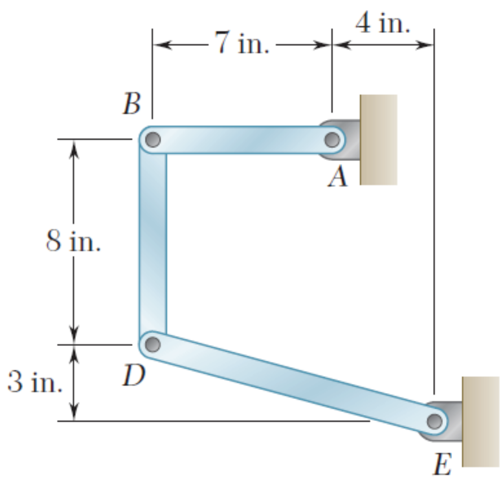
$\mathbf{r}_D = -17\mathbf{i} + 17\mathbf{j}$

$\mathbf{r}_{D/B} = 12\mathbf{i} + 3\mathbf{j}$

Kinematics of Rigid Bodies

❑ Sample Problem 11

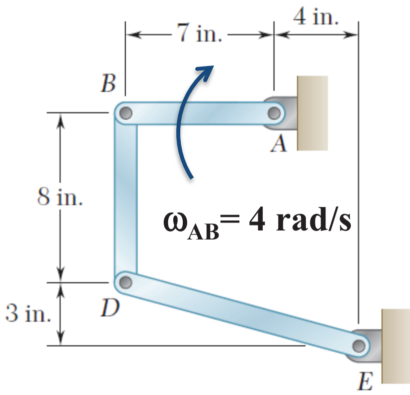
Knowing that at the instant shown bar *AB* has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration of bars *BD* and *DE*.



Kinematics of Rigid Bodies

❑ Sample Problem 11

SOLUTION:



From our sample problem 04, we used the relative velocity equations to find that:

We can now apply the relative acceleration equation with

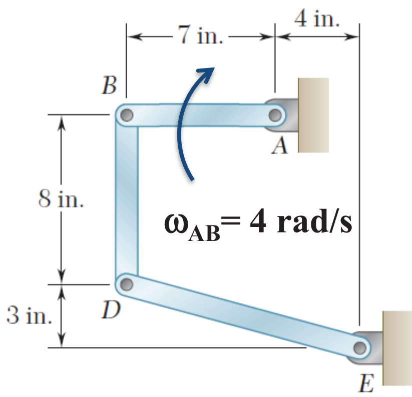
Analyze Bar AB

Analyze Bar BD

Kinematics of Rigid Bodies

❑ Sample Problem 11

SOLUTION:



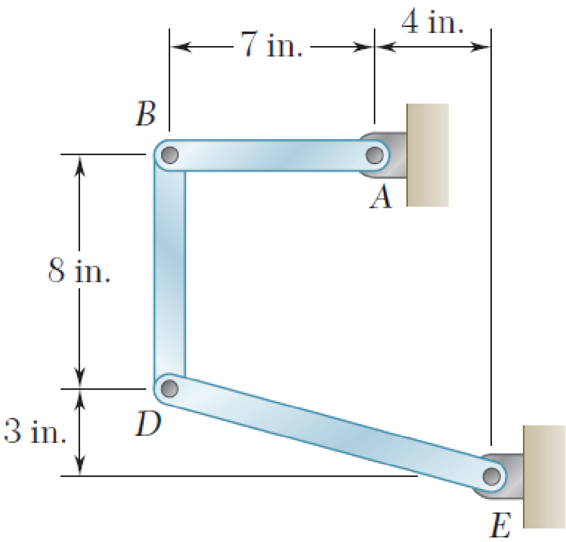
Analyze Bar DE

From previous page, we had:

Equate like components of a_D

Kinematics of Rigid Bodies

❑ Group Problem Solving



Knowing that at the instant shown bar *AB* has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration of bars *BD* and *DE*.

Which of the following is true?

a) The direction of a_D is ↗
b) The angular acceleration of BD must also be the same as AB
c) The direction of the linear acceleration of B is →