DYNAMICS

- Vector Mechanics for Engineers: Dynamics, 10th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Dynamics, 7th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Dynamics"

Kinetics of Particles: Energy Method

By: Kaveh Karami
Associate Prof. of Structural Engineering
https://prof.uok.ac.ir/Ka.Karami

Kinetics of Particles: Energy Method

❑ Energy and Momentum Methods

The pogo stick allows the boy to change between kinetic energy, potential energy from gravity, and potential energy in the spring.

Accidents are often analyzed by using momentum methods.

Kinetics of Particles: Energy Method

❑ Introduction

- **Previously, problems dealing with the motion of particles were solved through the fundamental equation of motion,**

$$\vec{F} = m\vec{a}$$

- **The current chapter introduces two additional methods of analysis.**

- ***Method of work and energy*****:** directly relates ***force***, ***mass***, ***velocity*** and ***displacement***.

- ***Method of impulse and momentum*****:** directly relates ***force***, ***mass***, ***velocity***, and ***time***.

Kinetics of Particles: Energy Method

❑ Introduction

Approaches to Kinetics Problems

Forces and Accelerations → **Newton's Second Law (last chapter)**

$$\sum \vec{F} = m\vec{a}_G$$

Velocities and Displacements → **Work-Energy**

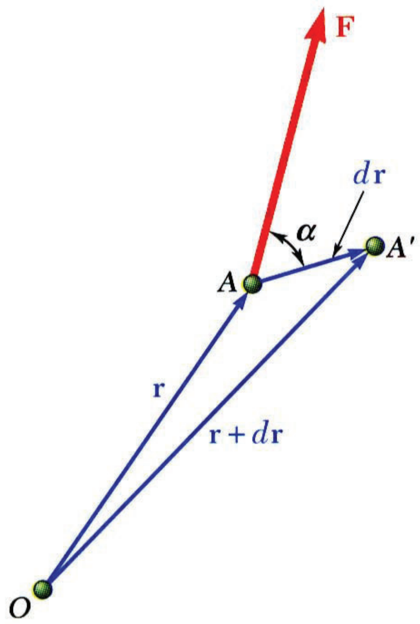
$$T_1 + U_{1\to 2} = T_2$$

Velocities and Time → **Impulse-Momentum**

$$m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}dt = m\vec{v}_2$$

Kinetics of Particles: Energy Method

❑ Work of a Force



- Differential vector $d\vec{r}$ is the *particle displacement.*
- *Work of the force* is $$dU = \vec{F} \bullet d\vec{r}$$

$$|d\vec{r}| = ds \quad \Rightarrow \quad dU = F ds \cos\alpha$$

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$
$$d\vec{r} = dx \vec{i} + dy \vec{j} + dz \vec{k} \quad \Rightarrow \quad dU = F_x dx + F_y dy + F_z dz$$

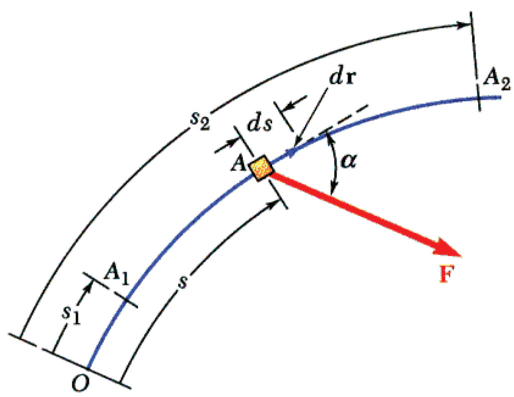
- Work is a ***scalar*** quantity, i.e., it has magnitude and sign but not direction.
- Dimensions of work are length × force. Units are

$$1\,\text{J}\,(\textit{joule}) = (1\,\text{N})(1\,\text{m}) \qquad 1\,\text{ft}\cdot\text{lb} = 1.356\,\text{J}$$

Kinetics of Particles: Energy Method

❑ Work of a Force

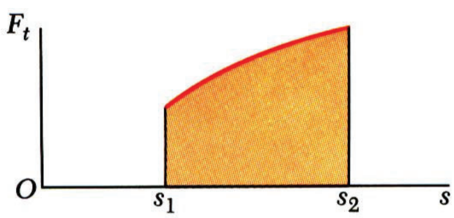
- Work of a force during a finite displacement,



$$U_{1\to2} = \int_{A_1}^{A_2} dU = \int_{A_1}^{A_2} \vec{F} \bullet d\vec{r}$$
$$\Rightarrow \quad U_{1\to2} = \int_{A_1}^{A_2} (F_x dx + F_y dy + F_z dz)$$

$$U_{1\to2} = \int_{A_1}^{A_2} dU = \int_{A_1}^{A_2} \vec{F} \bullet d\vec{r} = \int_{s_1}^{s_2} (F\cos\alpha) ds$$
$$\Rightarrow \quad U_{1\to2} = \int_{s_1}^{s_2} F_t\, ds$$

- F_t is the force in the direction of the displacement *ds*

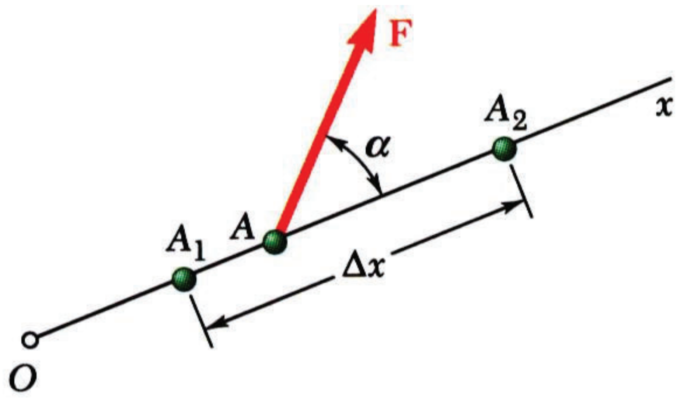


- Work is represented by the ***area under the curve of F_t plotted against s****.*

Kinetics of Particles: Energy Method

❑ Work of a Force

What is the work of a constant force in rectilinear motion?



a) $U_{1\to2} = F\,\Delta x$

b) $U_{1\to2} = (F\cos\alpha)\,\Delta x$

c) $U_{1\to2} = (F\sin\alpha)\,\Delta x$

d) $U_{1\to2} = 0$

Kinetics of Particles: Energy Method

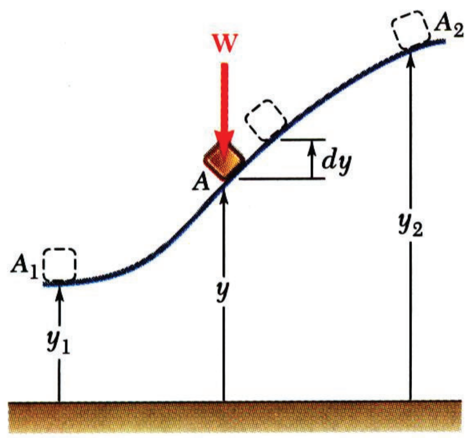
❑ Work of a Force

- Work of the force of gravity,

$$dU = F_x dx + F_y dy + F_z dz \quad \Rightarrow \quad dU = -W\,dy$$

$$U_{1\to2} = \int_{A_1}^{A_2} dU = -\int_{y_1}^{y_2} W\,dy = -W(y_2 - y_1)$$

$$\Rightarrow \boxed{U_{1\to2} = -W\,\Delta y}$$



- Work *of the weight* is equal to product of weight W and vertical displacement Δy.

- **In the figure above, when is the work done by the weight positive?**

a) Moving from y_1 to y_2 | b) Moving from y_2 to y_1 | c) Never

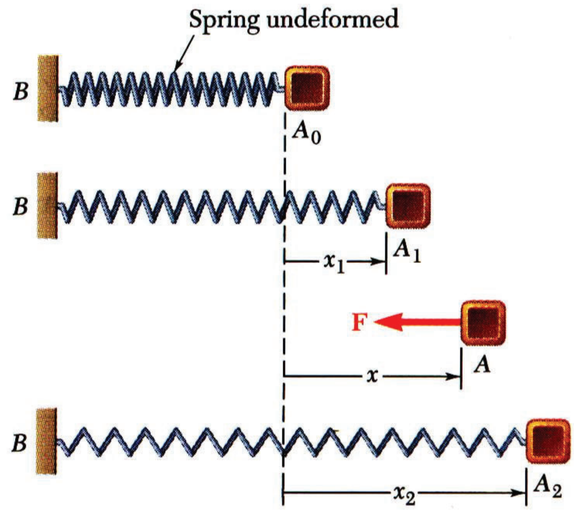
Kinetics of Particles: Energy Method

❑ Work of a Force

- Magnitude of the force exerted by a spring is proportional to deflection,

$$F = k\,x$$

$k =$ Spring Constant (N/m or lb/in.)



- Work *of the force exerted by spring*,

$$dU = -F\,dx = -(kx)\,dx$$

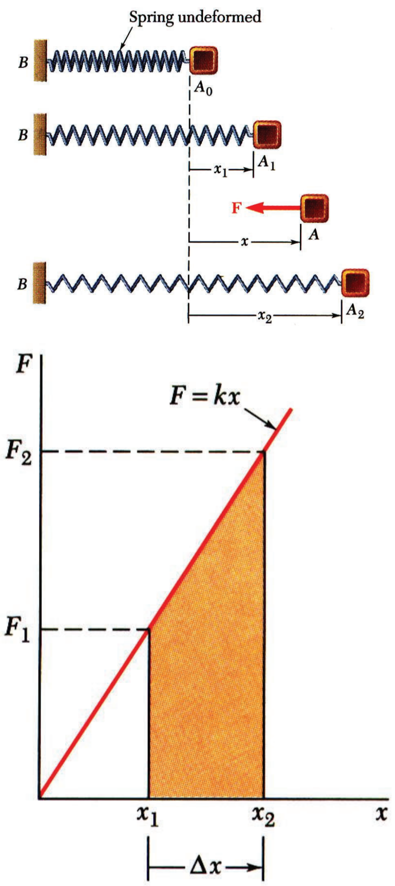
$$U_{1\to2} = -\int_{x_1}^{x_2} kx\,dx = \quad\Rightarrow\quad U_{1\to2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

Kinetics of Particles: Energy Method

❑ Work of a Force

$$U_{1\to2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

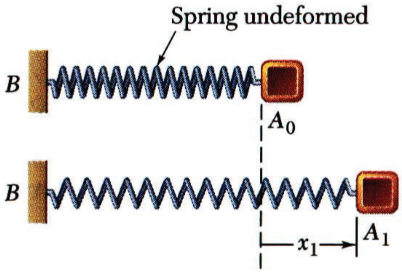
- Work *of the force exerted by spring* is positive when $x_2 < x_1$, i.e., when the spring is returning to its undeformed position.

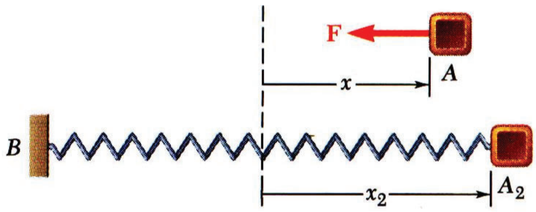


- Work of the force exerted by the spring is equal to ***negative of area under curve of F plotted against x***,

$$U_{1\to2} = -\frac{1}{2}(F_1 + F_2)\,\Delta x$$

❑ Work of a Force





As the block moves from A_0 to A_1, is the work positive or negative?

Positive **Negative**

As the block moves from A_2 to A_0, is the work positive or negative?

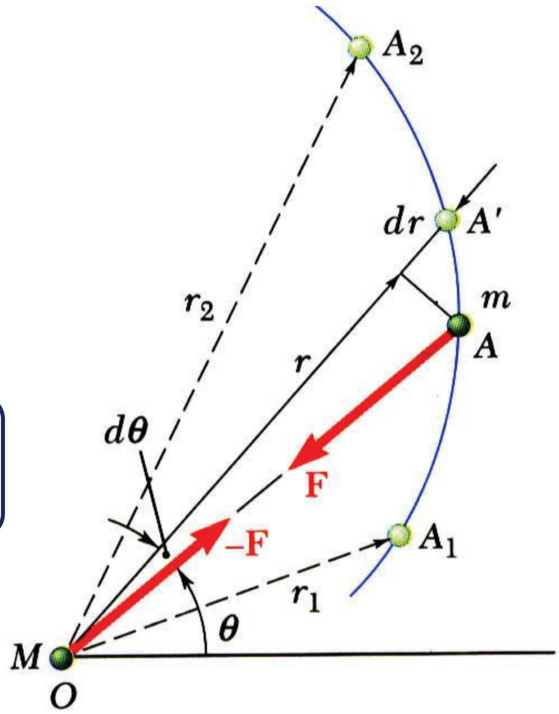
Positive **Negative**

❑ Work of a Force

Work of a gravitational force (assume particle M occupies fixed position O while particle m follows path shown),

$$dU = -F dr \quad \Rightarrow \quad dU = -G\frac{Mm}{r^2}dr$$

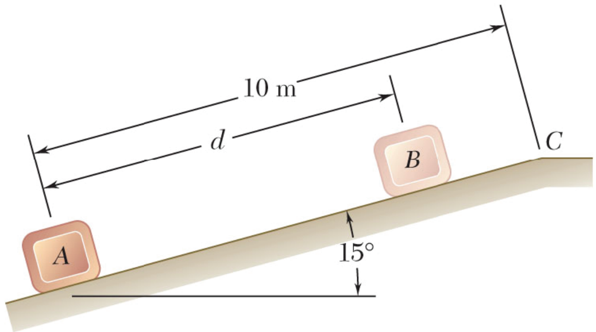
$$U_{1\to2} = \int_{A_1}^{A_2} dU = -\int_{r_1}^{r_2} G\frac{Mm}{r^2}dr \quad \Rightarrow \quad \boxed{U_{1\to2} = GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$



Kinetics of Particles: Energy Method

❑ Work of a Force

Does the normal force do work as the block slides from B to A?



YES **NO**

Does the weight do work as the block slides from B to A?

YES **NO**

Positive or

Negative work?

Kinetics of Particles: Energy Method

❑ Work of a Force

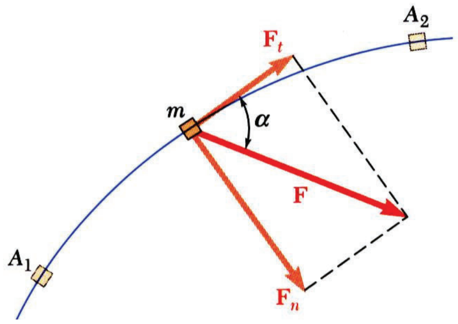
Forces which do not do work ($ds = 0$ or $\cos\alpha = 0$):

- Reaction at frictionless pin supporting rotating body,
- Reaction at frictionless surface when body in contact moves along surface,
- Reaction at a roller moving along its track, and
- Weight of a body when its center of gravity moves horizontally.

Kinetics of Particles: Energy Method

❑ Particle Kinetic Energy: Principle of Work & Energy

- Consider a particle of mass m acted upon by force $\vec{F}$



$$F_t = ma_t = m\frac{dv}{dt} = m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds} \quad \Rightarrow \quad \boxed{F_t\, ds = mv\, dv}$$

- Integrating from A_1 to A_2,

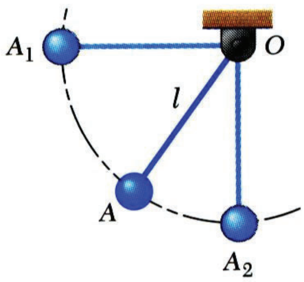
$$\int_{s_1}^{s_2} F_t ds = m\int_{v_1}^{v_2} v\, dv = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

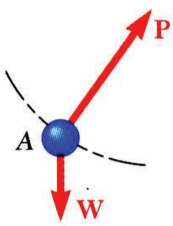
$$\boxed{Define: \quad T = \tfrac{1}{2}mv^2 = kinetic\ energy \quad \Rightarrow \quad U_{1\to 2} = T_2 - T_1}$$

- ***The work of the force $\vec{F}$ is equal to the change in kinetic energy of the particle.***
- Units of work and kinetic energy are the same: $T = \tfrac{1}{2}mv^2 = \mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)\mathrm{m} = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$

Kinetics of Particles: Energy Method

❑ Applications of the Principle of Work and Energy





- The bob is released from rest at position A_1 Determine the velocity of the pendulum bob at A_2 using work & kinetic energy.
- Force $\vec{P}$ acts normal to path and does no work.

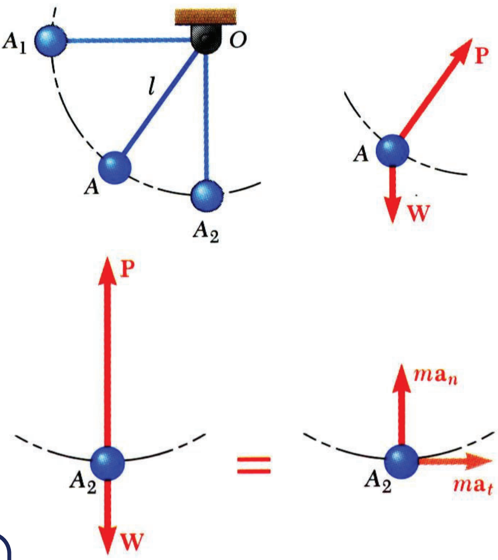
$$U_{1\to 2} = T_2 - T_1 \quad \Rightarrow \quad T_1 + U_{1\to 2} = T_2 \quad \Rightarrow \quad 0 + Wl = \frac{1}{2}\frac{W}{g}v_2^2 \quad \Rightarrow \quad \boxed{v_2 = \sqrt{2gl}}$$

Advantages using Energy method

- Velocity is found without determining expression for acceleration and integrating.
- All quantities are scalars and can be added directly.
- Forces which do no work are eliminated from the problem.

Kinetics of Particles: Energy Method

Applications of the Principle of Work and Energy



- Principle of work and energy cannot be applied to directly determine the acceleration of the pendulum bob.
- Calculating the tension in the cord requires supplementing the method of work and energy with an application of Newton's second law.
- As the bob passes through A_2,

$$\sum F_n = m\,a_n \quad \Rightarrow \quad P - W = \frac{W}{g}\frac{v_2^2}{l} \qquad v_2 = \sqrt{2gl}$$

$$\Rightarrow \quad \boxed{P = W + \frac{W}{g}\frac{2gl}{l} = 3W}$$

Kinetics of Particles: Energy Method

Power and Efficiency

- Power = rate at which work is done.

$$\boxed{Power = \frac{dU}{dt} = \frac{\vec{F}\bullet d\vec{r}}{dt} = \vec{F}\bullet\vec{v}}$$

- Dimensions of power are work/time or force*velocity. Units for power are

$$\boxed{1\ \mathrm{W\ (watt)} = 1\frac{\mathrm{J}}{\mathrm{s}} = 1\ \mathrm{N}\cdot\frac{\mathrm{m}}{\mathrm{s}}} \quad \text{or} \quad \boxed{1\ \mathrm{hp} = 550\frac{\mathrm{ft}\cdot\mathrm{lb}}{\mathrm{s}} = 746\ \mathrm{W}}$$

- Efficiency

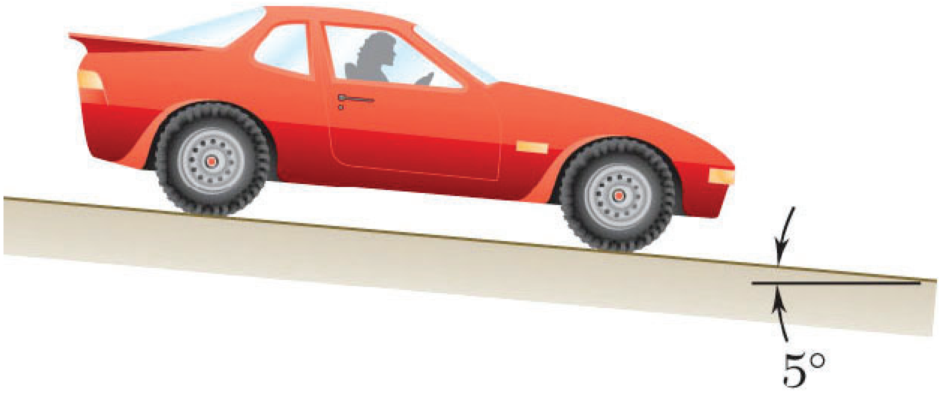
$$\boxed{\eta = \frac{\text{output work}}{\text{input work}} = \frac{\text{power output}}{\text{power input}}}$$

Kinetics of Particles: Energy Method

❑ Sample Problem 01

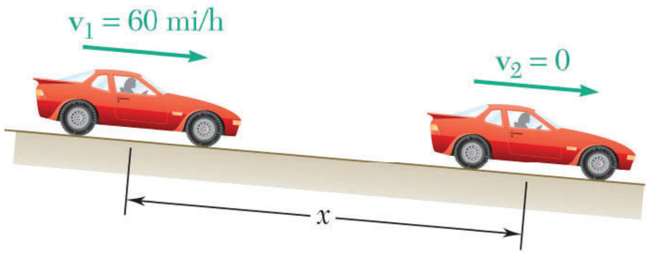
An automobile weighing 4000 lb is driven down a 5° incline at a speed of 60 mi/h when the brakes are applied causing a constant total breaking force of 1500 lb.

Determine the distance traveled by the automobile as it comes to a stop.



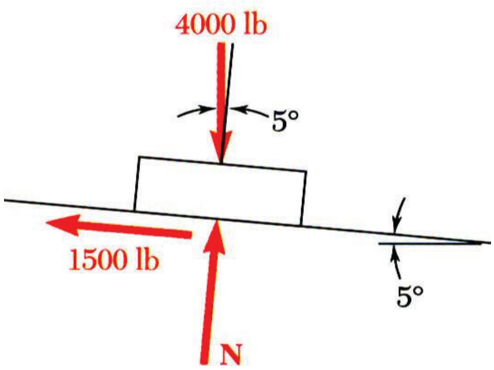
Kinetics of Particles: Energy Method

❑ Sample Problem 01



SOLUTION:

- Evaluate the change in kinetic energy.

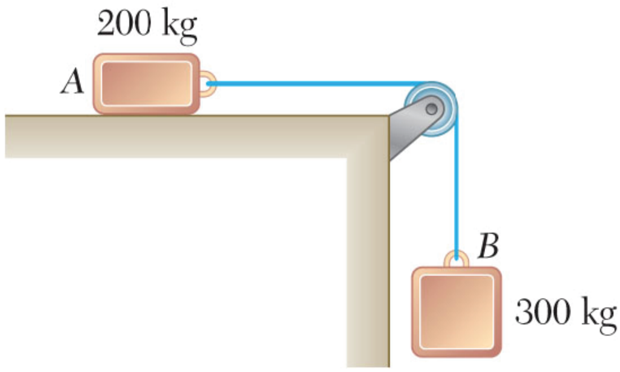


- Determine the distance required for the work to equal the kinetic energy change.

Kinetics of Particles: Energy Method

❑ Sample Problem 02

Two blocks are joined by an inextensible cable as shown. If the system is released from rest, determine the velocity of block *A* after it has moved 2 m. Assume that the coefficient of friction between block *A* and the plane is $\mu_k = 0.25$ and that the pulley is weightless and frictionless.

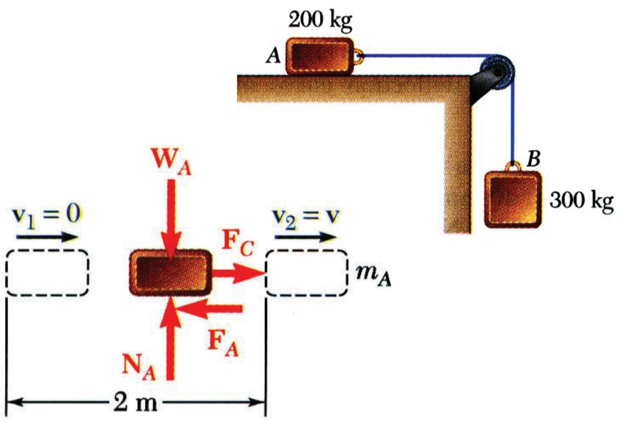


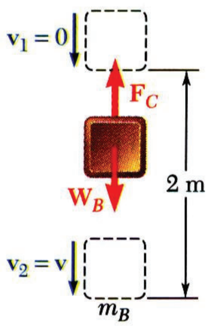
Kinetics of Particles: Energy Method

❑ Sample Problem 02

SOLUTION:

- Apply the principle of work and energy separately to blocks *A* and *B*.



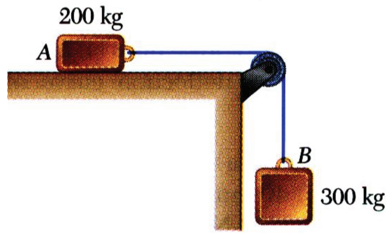


Kinetics of Particles: Energy Method

❑ Sample Problem 02

SOLUTION:

- When the two relations are combined, the work of the cable forces cancel. Solve for the velocity.

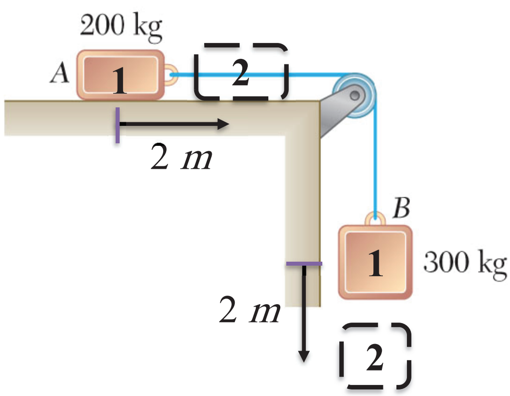


Kinetics of Particles: Energy Method

❑ Sample Problem 02

Could you apply work-energy to the combined system of blocks?

Alternate Solution, Group Problem Solving



What is T_1 of the system?

What is the total work done between points 1 and 2?

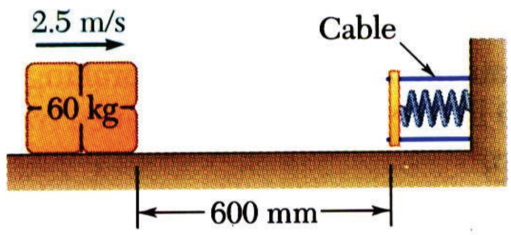
What is T_2 of the system?

Solve for v

Kinetics of Particles: Energy Method

❑ Sample Problem 03

A spring is used to stop a 60 kg package which is sliding on a horizontal surface. The spring has a constant $k = 20$ kN/m and is held by cables so that it is initially compressed 120 mm. The package has a velocity of 2.5 m/s in the position shown and the maximum deflection of the spring is 40 mm.



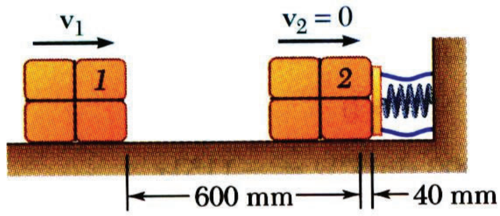
Determine *(a)* the coefficient of kinetic friction between the package and surface and *(b)* the velocity of the package as it passes again through the position shown.

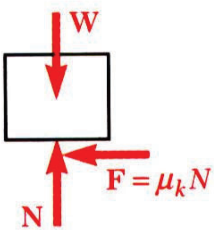
Kinetics of Particles: Energy Method

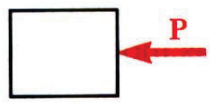
❑ Sample Problem 03

SOLUTION:

Apply principle of work and energy between initial position and the point at which spring is fully compressed.





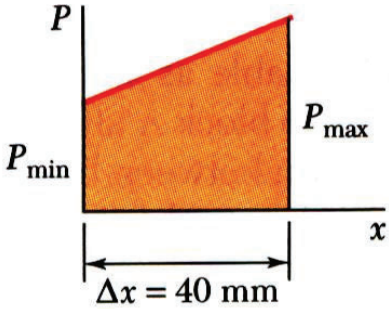


Kinetics of Particles: Energy Method

❑ Sample Problem 03

SOLUTION:

or

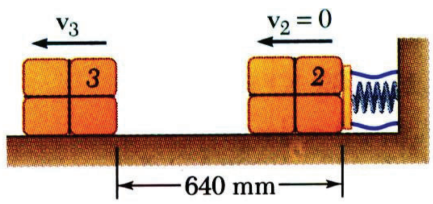


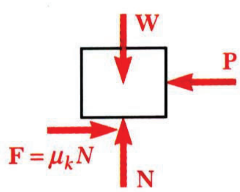
Kinetics of Particles: Energy Method

❑ Sample Problem 03

SOLUTION:

- Apply the principle of work and energy for the rebound of the package.





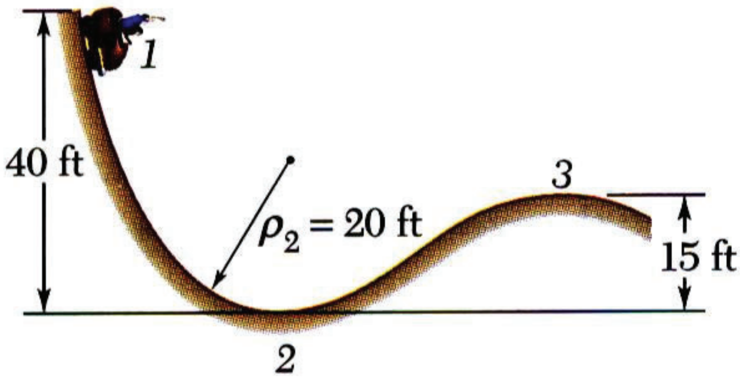
Kinetics of Particles: Energy Method

❑ Sample Problem 04

A 2000 lb car starts from rest at point 1 and moves without friction down the track shown. Determine:

a) the force exerted by the track on the car at point 2, and

b) the minimum safe value of the radius of curvature at point 3.

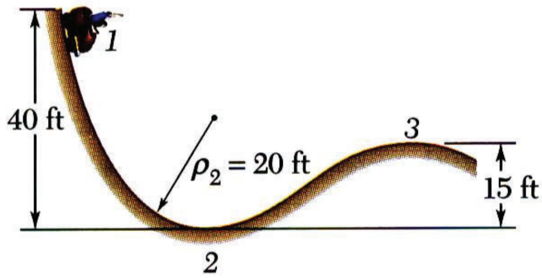


Kinetics of Particles: Energy Method

❑ Sample Problem 04

SOLUTION:

Apply principle of work and energy to determine velocity at point 2.

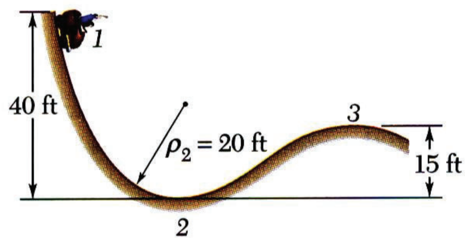


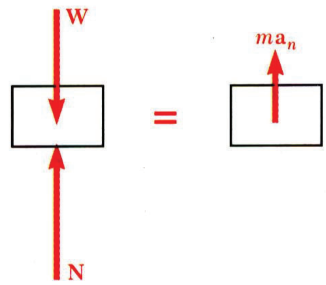
Kinetics of Particles: Energy Method

❑ Sample Problem 04

SOLUTION:

Apply Newton's second law to find normal force by the track at point 2.



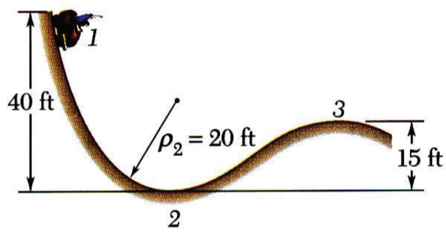


Kinetics of Particles: Energy Method

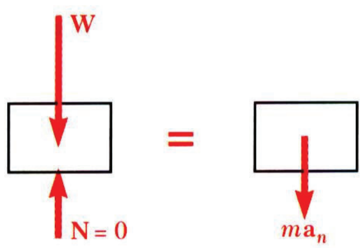
❑ Sample Problem 04

SOLUTION:

Apply principle of work and energy to determine velocity at point 3.



- Apply Newton's second law to find minimum radius of curvature at point 3 such that a positive normal force is exerted by the track.

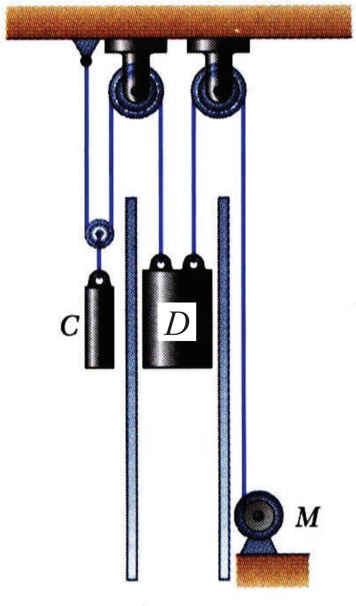


Kinetics of Particles: Energy Method

❑ Sample Problem 05

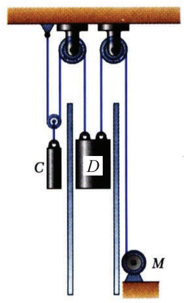
The dumbwaiter *D* and its load have a combined weight of 600 lb, while the counterweight *C* weighs 800 lb.

Determine the power delivered by the electric motor *M* when the dumbwaiter *(a)* is moving up at a constant speed of 8 ft/s and *(b)* has an instantaneous velocity of 8 ft/s and an acceleration of 2.5 ft/s^2, both directed upwards.



Kinetics of Particles: Energy Method

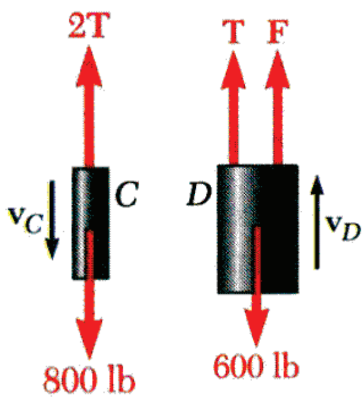
❑ Sample Problem 05



SOLUTION:

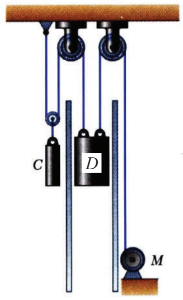
- In the first case, bodies are in ***uniform motion***. Determine force exerted by motor cable from conditions for static equilibrium.

Free-body C:



Kinetics of Particles: Energy Method

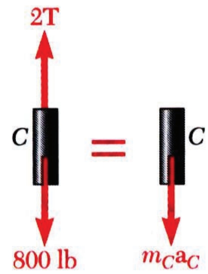
❑ Sample Problem 05



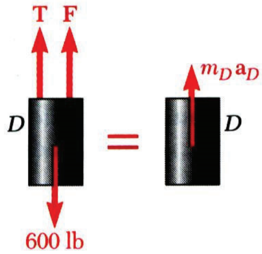
SOLUTION:

- In the second case, both bodies are accelerating. Apply Newton's second law to each body to determine the required motor cable force.

Free-body C:



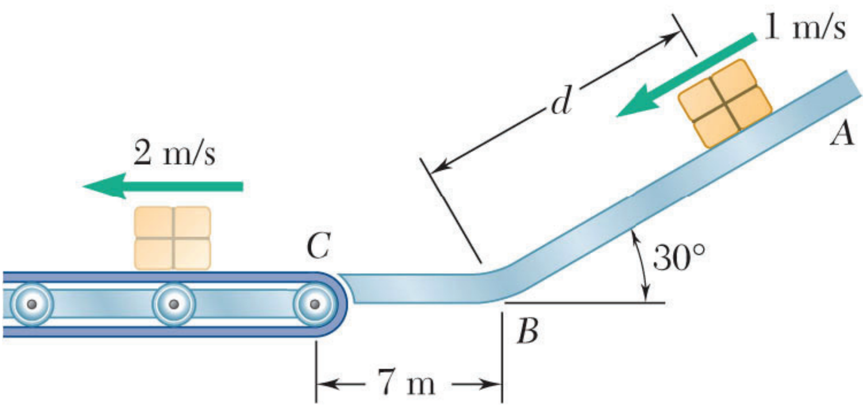
Free-body D:



Kinetics of Particles: Energy Method

❑ Sample Problem 06

Packages are thrown down an incline at *A* with a velocity of 1 m/s. The packages slide along the surface *ABC* to a conveyor belt which moves with a velocity of 2 m/s. Knowing that $\mu_k = 0.25$ between the packages and the surface *ABC*, determine the distance *d* if the packages are to arrive at *C* with a velocity of 2 m/s.

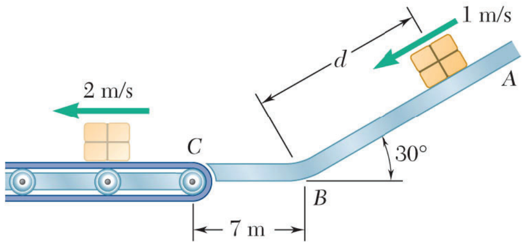


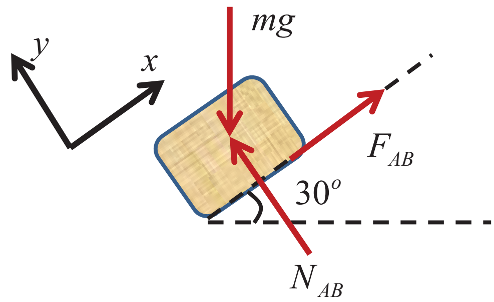
Kinetics of Particles: Energy Method

- **Sample Problem 06**

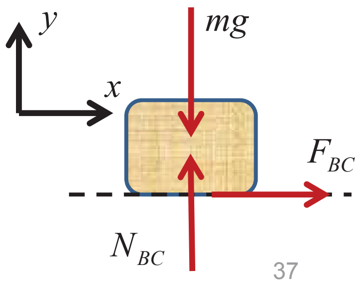
SOLUTION:

Determine work done A → B





Determine work done B → C

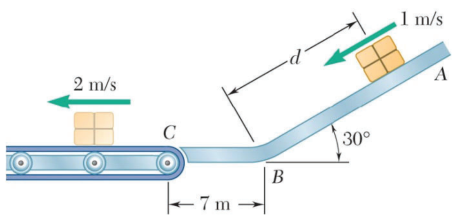


Kinetics of Particles: Energy Method

- **Sample Problem 06**

SOLUTION:

Determine kinetic energy at A and at C



Apply principle of work and energy to determine d

Kinetics of Particles: Energy Method

❑ Potential Energy

The potential energy stored at the top of the roller coaster is transferred to kinetic energy as the cars descend.

The elastic potential energy stored in the trampoline is transferred to kinetic energy and gravitational potential energy as the girl flies upwards.

Kinetics of Particles: Energy Method

❑ Potential Energy

"If the work of a force only depends on differences in position, we can express this work as potential energy."

Can the work done by the following forces be expressed as potential energy?

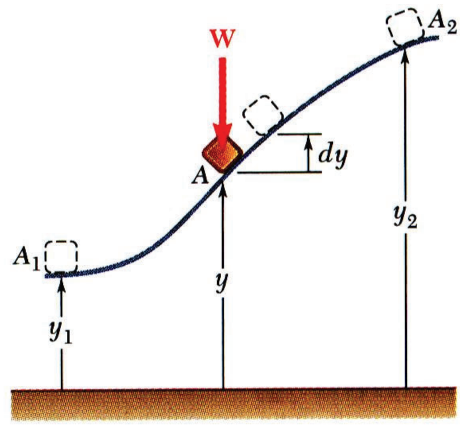
Weight	Yes	No
Friction	Yes	No
Normal force	Yes	No
Spring force	Yes	No

Kinetics of Particles: Energy Method

❑ Potential Energy

- Work of the force of gravity $\vec{W}$,

$$U_{1\to 2} = W\,y_1 - W\,y_2$$



Work is independent of path followed; depends only on the initial and final values of Wy.

$V_g = Wy =$ ***potential energy* of the body with respect to *force of gravity.***

$$U_{1\to 2} = (V_g)_1 - (V_g)_2$$

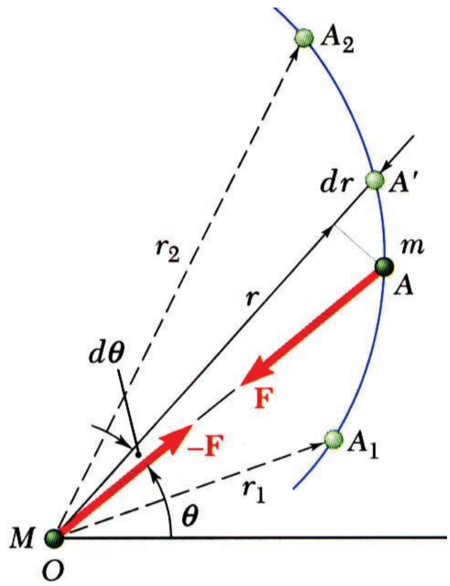
- Units of work and potential energy are the same: $V_g = Wy = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$

Kinetics of Particles: Energy Method

❑ Potential Energy

- Previous expression for potential energy of a body with respect to gravity is only valid when the weight of the body can be assumed constant.
- For a space vehicle, the variation of the force of gravity with distance from the center of the earth should be considered.
- Work of a gravitational force,

$$U_{1\to 2} = \frac{GMm}{r_2} - \frac{GMm}{r_1} \Rightarrow V_g = -\frac{GMm}{r}$$



- Potential energy V_g when the variation in the force of gravity can not be neglected,

$$V_g = -\frac{GMm}{r} \quad W = \frac{GMm}{R^2} \Rightarrow V_g = -\frac{WR^2}{r}$$

Kinetics of Particles: Energy Method

❑ Potential Energy

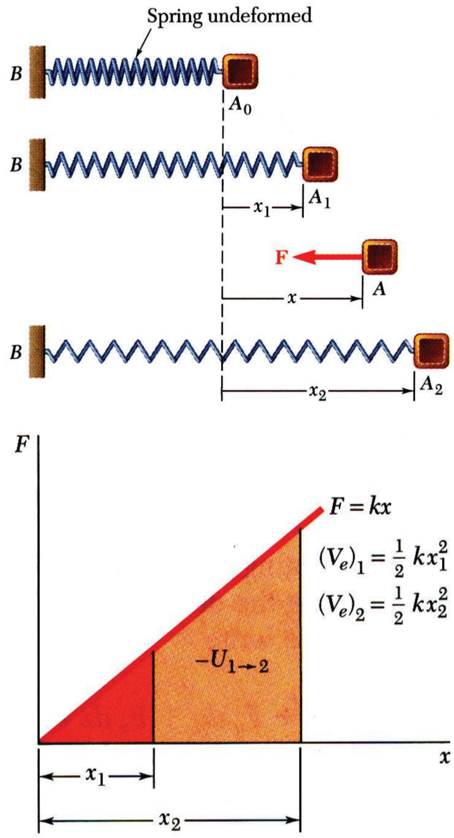
- Work of the force exerted by a spring ***depends only on the initial and final deflections of the spring***,

$$U_{1\to 2} = \frac{1}{2}k\,x_1^2 - \frac{1}{2}k\,x_2^2$$

- The potential energy of the body with respect to the elastic force,

$$V_e = \frac{1}{2}kx^2 \quad \Rightarrow \quad U_{1\to 2} = (V_e)_1 - (V_e)_2$$

- Note that the preceding expression for V_e is valid only if the deflection of the spring is measured from its undeformed position.



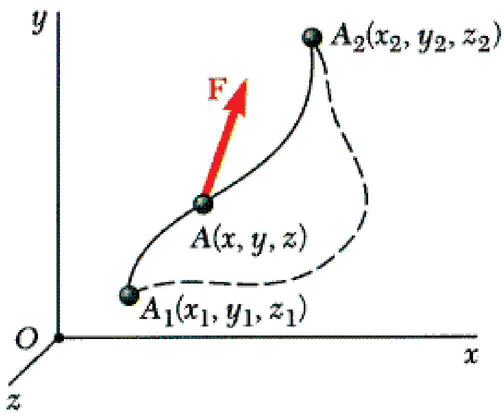
Kinetics of Particles: Energy Method

❑ Conservative Forces

- Concept of potential energy can be applied if the work of the force is ***independent of the path*** followed by its point of application.

$$U_{1\to 2} = V(x_1, y_1, z_1) - V(x_2, y_2, z_2)$$

Such forces are described as ***conservative forces.***



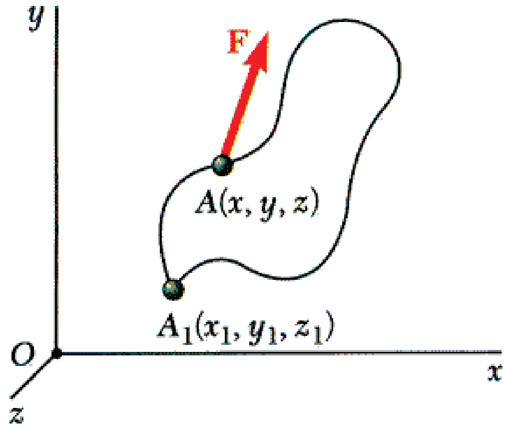
- For any conservative force applied on a closed path,

$$\oint \vec{F} \bullet d\vec{r} = 0$$

Kinetics of Particles: Energy Method

❑ Conservative Forces

- Elementary work corresponding to displacement between two neighboring points,



$$dU = V(x,y,z) - V(x+dx, y+dy, z+dz)$$

$$\Rightarrow \boxed{dU = -dV(x,y,z)}$$

$$dU = -\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)$$

$$dU = F_x dx + F_y dy + F_z dz$$

$$\Rightarrow \boxed{\vec{F} = -\left(\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\right) = -\mathbf{grad}\, V}$$

Kinetics of Particles: Energy Method

❑ Conservative Forces

- Concept of work and energy,

$$\boxed{U_{1\to 2} = T_2 - T_1}$$

- Work of a conservative force,

$$\boxed{U_{1\to 2} = V_1 - V_2}$$

$$\Rightarrow \quad T_1 + V_1 = T_2 + V_2 \quad \Rightarrow \quad \boxed{E = T + V = \text{cte}}$$

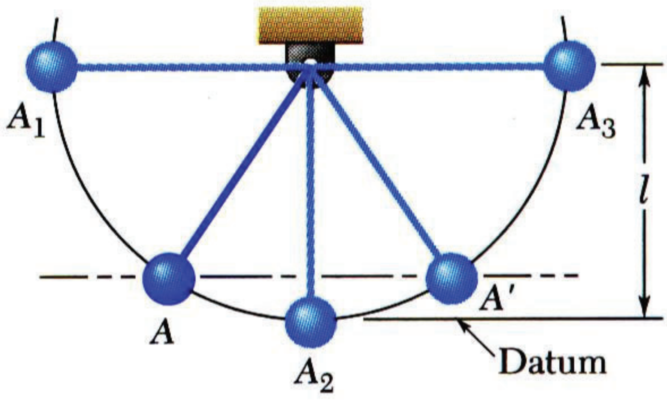
- When a particle moves under the action of conservative forces, ***the total mechanical energy is constant***.

- ***Friction forces are not conservative.*** Total mechanical energy of a system involving friction decreases.

- Mechanical energy is dissipated by friction into thermal energy. Total energy is constant.

Kinetics of Particles: Energy Method

❑ Conservative Forces



$$T_1 = 0, \quad V_1 = W\ell \quad \Rightarrow \quad \boxed{T_1 + V_1 = W\ell}$$

$$v_2 = \sqrt{2gl}, \quad T_2 = \frac{1}{2} m v_2^2 \quad \Rightarrow \quad T_2 = \frac{1}{2}\frac{W}{g}(2g\ell) \quad \Rightarrow \quad \left.\begin{array}{l} T_2 = W\ell \\ V_2 = 0 \end{array}\right\} \Rightarrow \boxed{T_2 + V_2 = W\ell}$$

$$\Rightarrow \quad T_1 + V_1 = T_2 + V_2 \quad \Rightarrow \quad \boxed{E = T + V = \text{cte}}$$

Kinetics of Particles: Energy Method

❑ Motion Under a Conservative Central Force

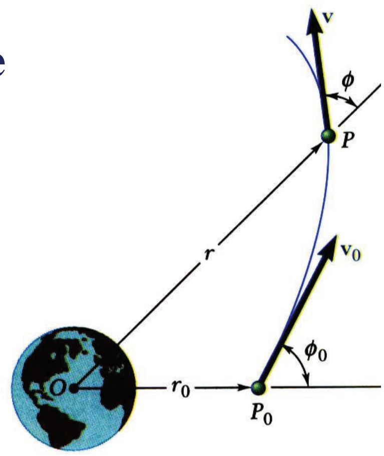
- When a particle moves under a conservative central force, both the ***principle of conservation of angular momentum***

$$\boxed{r_0 m v_0 \sin\phi_0 = r m v \sin\phi}$$

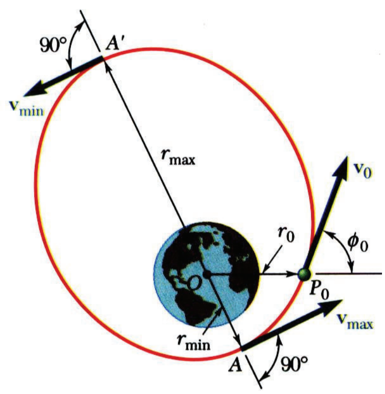
- and ***the principle of conservation of energy***

$$T_0 + V_0 = T + V \quad \Rightarrow \quad \boxed{\frac{1}{2} m v_0^2 - \frac{GMm}{r_0} = \frac{1}{2} m v^2 - \frac{GMm}{r}}$$

- may be applied.



- Given r, the equations may be solved for v and φ.
- At minimum and maximum r, $\varphi = 90°$. Given the launch conditions, the equations may be solved for r_{min}, r_{max}, v_{min}, and v_{max}

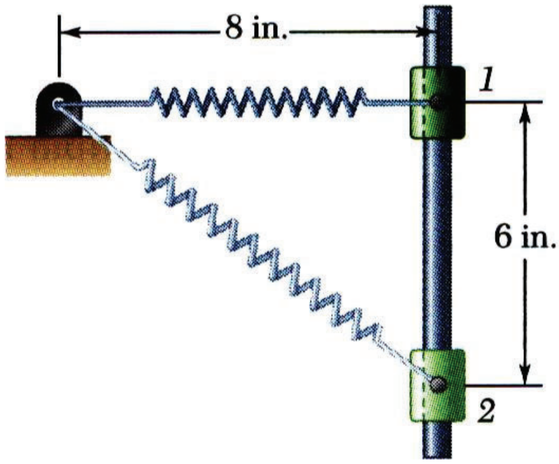


Kinetics of Particles: Energy Method

❑ Sample Problem 07

A 20 lb collar slides without friction along a vertical rod as shown. The spring attached to the collar has an undeflected length of 4 in. and a constant of 3 lb/in.

If the collar is released from rest at position 1, determine its velocity after it has moved 6 in. to position 2.

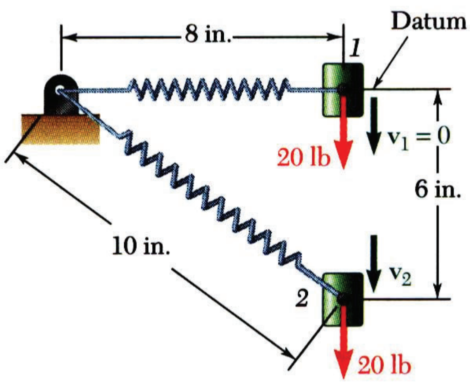


Kinetics of Particles: Energy Method

❑ Sample Problem 07

SOLUTION:

- Apply the principle of conservation of energy between positions 1 and 2.



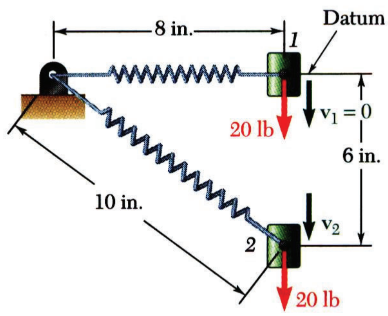
Position 1:

Kinetics of Particles: Energy Method

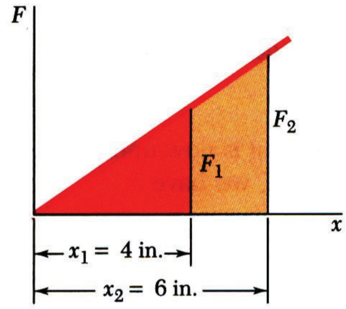
❑ Sample Problem 07

SOLUTION:

Position 2:



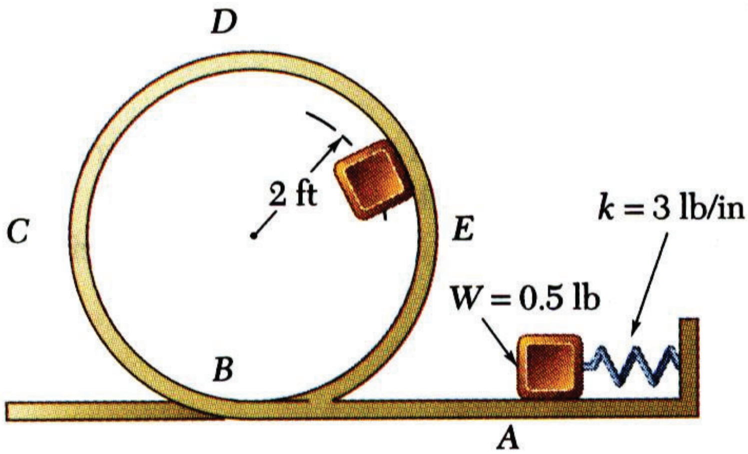
Conservation of Energy:



Kinetics of Particles: Energy Method

❑ Sample Problem 08

The 0.5 lb pellet is pushed against the spring and released from rest at *A*. Neglecting friction, determine the smallest deflection of the spring for which the pellet will travel around the loop and remain in contact with the loop at all times.

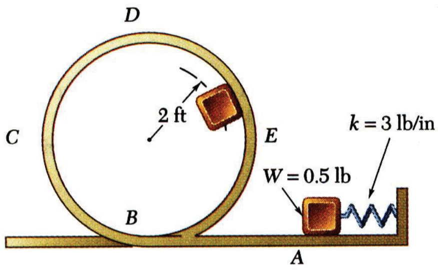


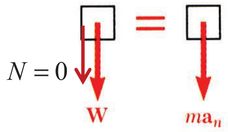
Kinetics of Particles: Energy Method

❑ Sample Problem 08

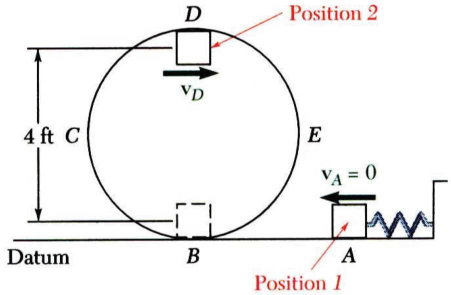
SOLUTION:

- Setting the force exerted by the loop to zero, solve for the minimum velocity at D.





- Apply the principle of conservation of energy between points A and D.

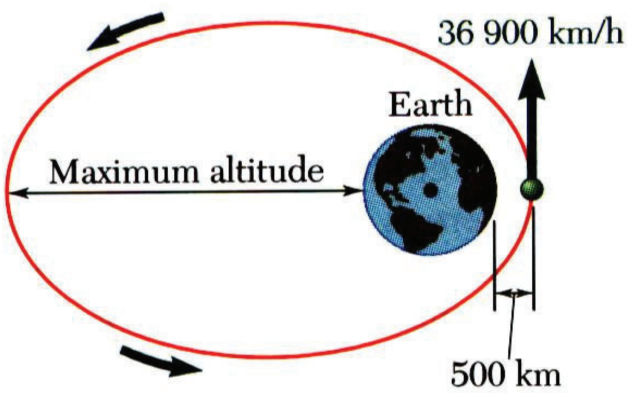


Kinetics of Particles: Energy Method

❑ Sample Problem 09

A satellite is launched in a direction parallel to the surface of the earth with a velocity of 36900 km/h from an altitude of 500 km.

Determine *(a)* the maximum altitude reached by the satellite, and *(b)* the maximum allowable error in the direction of launching if the satellite is to come no closer than 200 km to the surface of the earth

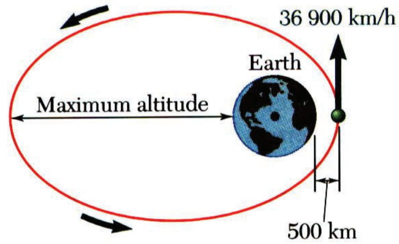


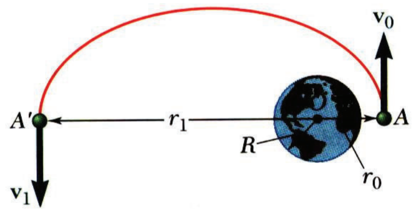
Kinetics of Particles: Energy Method

❑ Sample Problem 09

SOLUTION:

- Apply the principles of conservation of energy and conservation of angular momentum to the points of minimum and maximum altitude to determine the maximum altitude.

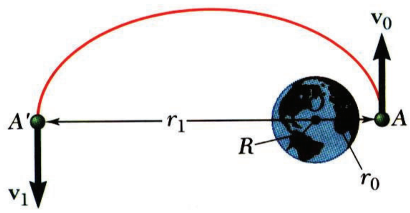




Kinetics of Particles: Energy Method

❑ Sample Problem 09

SOLUTION:

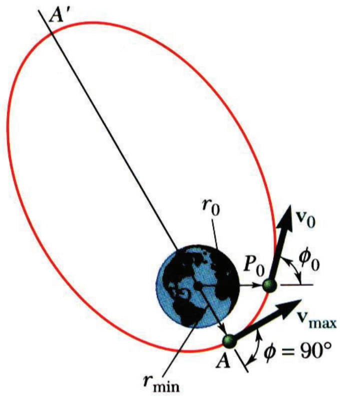


Kinetics of Particles: Energy Method

❑ Sample Problem 09

SOLUTION:

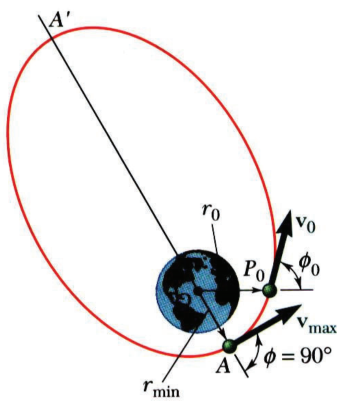
- Apply the principles to the orbit insertion point and the point of minimum altitude to determine maximum allowable orbit insertion angle error.



Kinetics of Particles: Energy Method

❑ Sample Problem 09

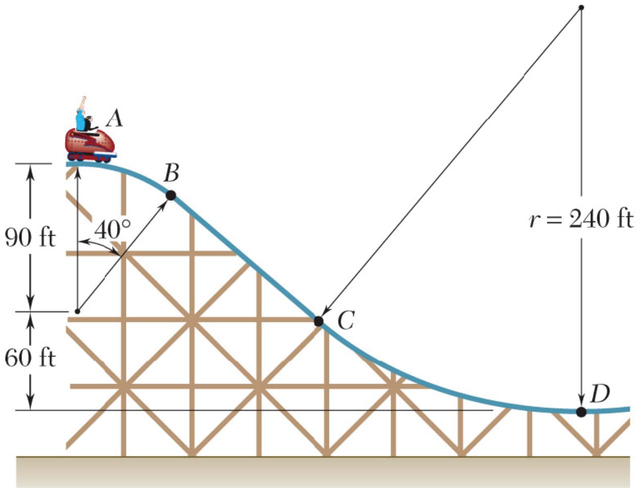
SOLUTION:



Kinetics of Particles: Energy Method

❑ Sample Problem 10

A section of track for a roller coaster consists of two circular arcs *AB* and *CD* joined by a straight portion *BC*. The radius of *CD* is 240 ft. The car and its occupants, of total weight 560 lb, reach Point *A* with practically no velocity and then drop freely along the track. Determine the normal force exerted by the track on the car at point *D*. Neglect air resistance and rolling resistance.



Kinetics of Particles: Energy Method

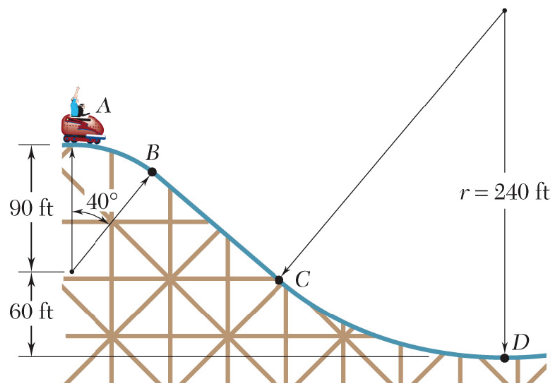
❑ Sample Problem 10

SOLUTION:

Given: v_A= 0 ft/s, r_{CD}= 240 ft, W=560 lbs

Find: N_D

Use conservation of energy to find v_D



Find T_A

Find V_A

Find T_D

Find V_D

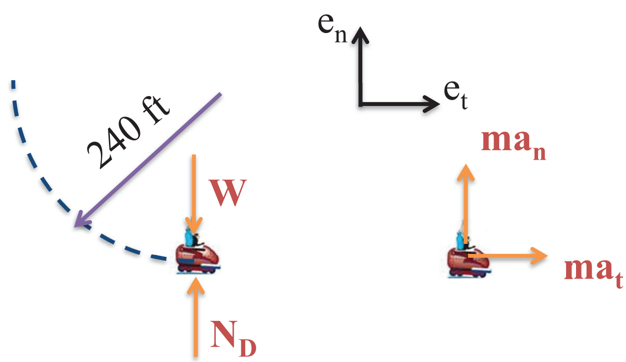
Solve for v_D

Kinetics of Particles: Energy Method

❑ Sample Problem 10

SOLUTION:

Draw FBD and KD at point D



Use Newton's second law in the normal direction

Kinetics of Particles: Energy Method

❑ Sample Problem 10

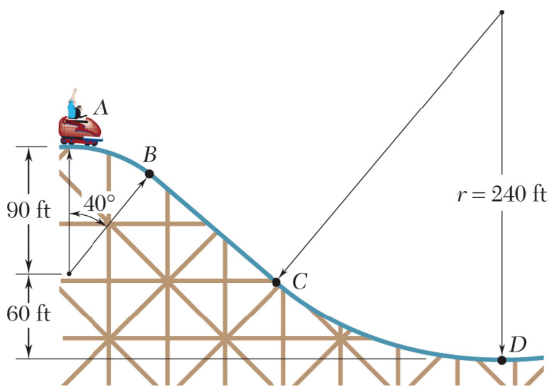
What happens to the normal force at D if….

…we include friction?

a) N_D gets larger

b) N_D gets smaller

c) N_D stays the same



…we move point A higher?

a) N_D gets larger

b) N_D gets smaller

c) N_D stays the same

…the radius is smaller?

a) N_D gets larger

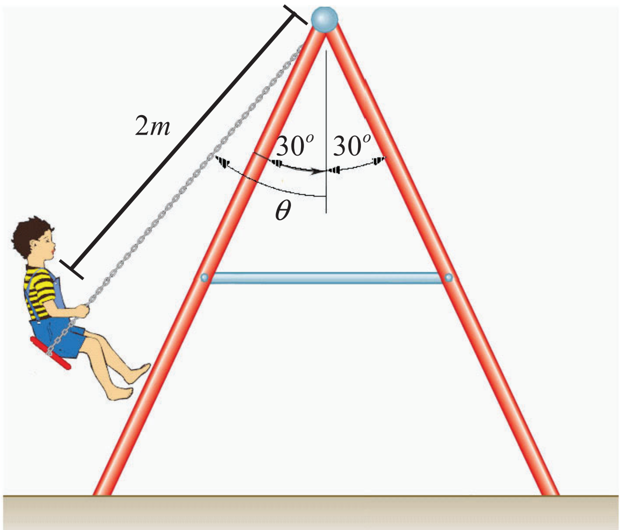
b) N_D gets smaller

c) N_D stays the same

Kinetics of Particles: Energy Method

❑ Sample Problem 11

A child having a mass of 20 kg sits on a swing and is playing. The maximum angle which the kids can moves up is $\theta_{max} = 60^o$. Neglecting the mass of the swing, determine the maximum axial force in each column of the swing device. The swing is in a symmetric position relative to the four columns.

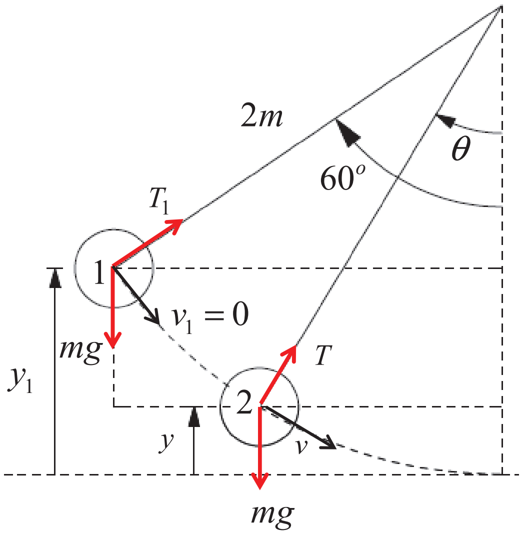


Kinetics of Particles: Energy Method

❑ Sample Problem 11

SOLUTION:

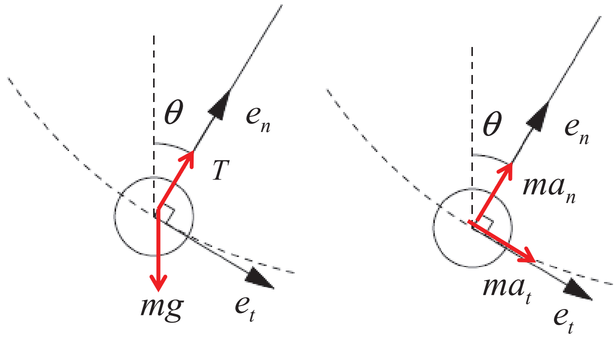
Use conservation of energy to find *v*



Kinetics of Particles: Energy Method

❑ Sample Problem 11

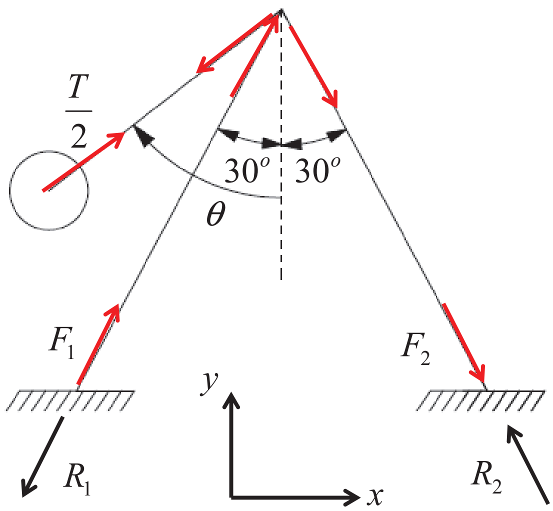
SOLUTION:



Kinetics of Particles: Energy Method

❑ Sample Problem 11

SOLUTION:



Kinetics of Particles: Energy Method

❑ Sample Problem 11

SOLUTION: