DYNAMICS

- Vector Mechanics for Engineers: Dynamics, 10th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Dynamics, 7th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Dynamics"

Kinetics of Particles: Newton's Second Law

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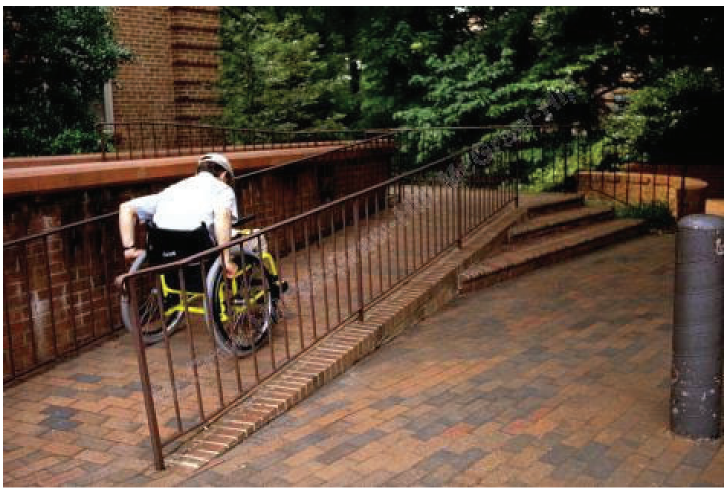
Kinetics of Particles: Newton's Second Law

Introduction

We must analyze all of the forces acting on the wheelchair in order to design a good ramp

High swing velocities can result in large forces on a swing chain or rope, causing it to break.



Kinetics of Particles: Newton's Second Law

❑ Introduction

$$\Sigma \text{F} = ma$$

- **Newton's Second Law of Motion**
  - If the *resultant force* acting on a particle is not zero, the particle will have an acceleration *proportional to the magnitude of resultant* and in the *direction of the resultant.*
  - Must be expressed with respect to a *Newtonian (or inertial) frame of reference*, i.e., one that is not accelerating or rotating.
  - This form of the equation is for a constant mass system(not be used to solve problems involving the motion of bodies, such as rockets, which gain or lose mass)

Kinetics of Particles: Newton's Second Law

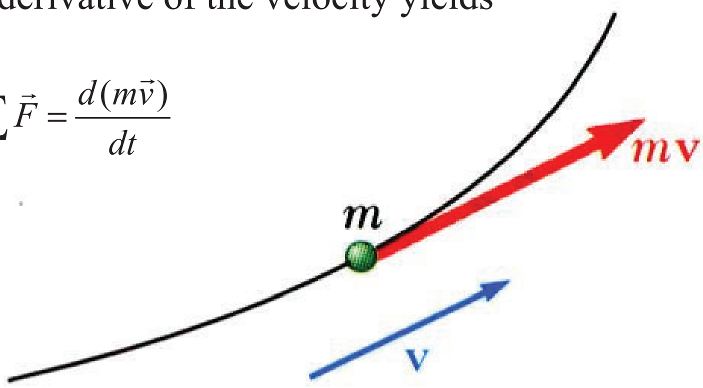
❑ Linear Momentum of a Particle

- Replacing the acceleration by the derivative of the velocity yields

$$\left.\begin{array}{l}\sum \vec{F} = m\vec{a} \\ \vec{a} = \dfrac{d\vec{v}}{dt}\end{array}\right\} \Rightarrow \sum \vec{F} = m\frac{d\vec{v}}{dt} \overset{m=cte}{\Rightarrow} \sum \vec{F} = \frac{d(m\vec{v})}{dt}$$

$$\Rightarrow \boxed{\sum \vec{F} = \frac{d(\vec{L})}{dt}}$$

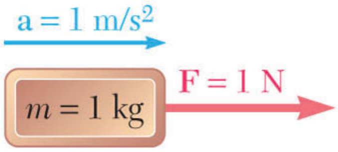
$\vec{L}$ = linear momentum of the particle



- *Linear Momentum Conservation Principle* :
  If the resultant force on a particle is zero, the linear momentum of the particle remains constant in both magnitude and direction (Newton's First Law).

Kinetics of Particles: Newton's Second Law

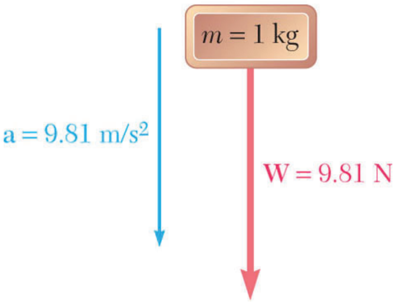
❑ Systems of Units



Length | **Time** | **Mass** | **Force**

Three base units may be chosen arbitrarily

The fourth must be compatible with Newton's 2nd Law.



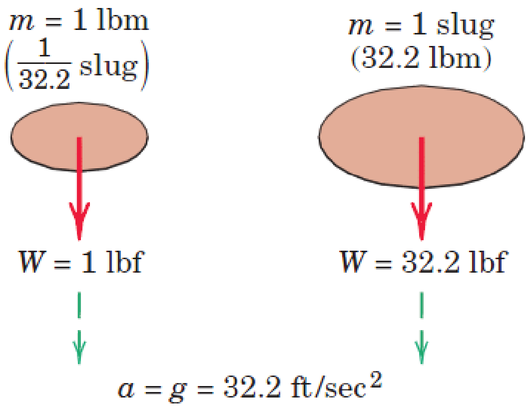
- ***International System of Units (SI Units):*** In this system, the base units are the units of length, mass, and time, and are called, respectively, the meter (m), the kilogram (kg), and the second (s). The unit of force is called the newton (N) and is defined as the force which gives an acceleration of 1 m/ s^2 to a mass of 1 kg.

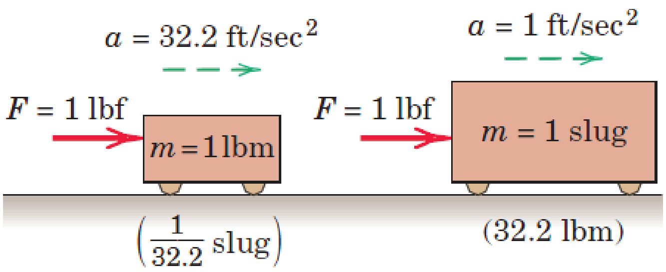
$$1\,\text{N} = (1\,\text{kg})\left(1\frac{\text{m}}{\text{s}^2}\right) = 1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

Kinetics of Particles: Newton's Second Law

❑ Systems of Units

U.S. Customary Units: The base units are the units of length, force, and time. These units are, respectively, the foot (ft), the pound (lbf), and the second (s). The pound is defined as the weight of a platinum standard, called the standard pound, which is kept at the National Institute of Standards and Technology outside Washington and the mass of which is 0.45359243 kg.





$$1\,\text{slug} = \frac{1\,\text{lbf}}{1\,\text{ft}/\text{s}^2} = 1\frac{\text{lbf}\cdot\text{s}^2}{\text{ft}}$$

Kinetics of Particles: Newton's Second Law

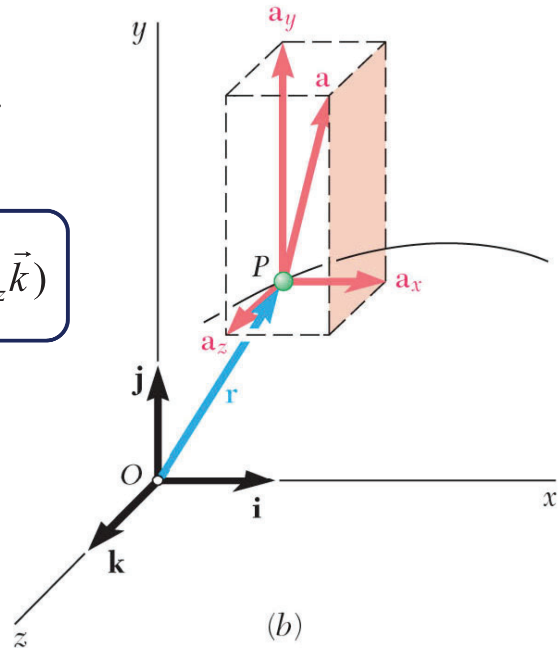
❑ Equations of Motion

- Newton's second law $\sum \vec{F} = m\vec{a}$

- ***Rectangular Components.*** Resolving each force F and the acceleration a into rectangular components

$$\sum \vec{F} = \sum (F_x\vec{i} + F_y\vec{j} + F_z\vec{k}) = m(a_x\vec{i} + a_y\vec{j} + a_z\vec{k})$$

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$
$$\sum F_x = m\ddot{x} \quad \sum F_y = m\ddot{y} \quad \sum F_z = m\ddot{z}$$



Kinetics of Particles: Newton's Second Law

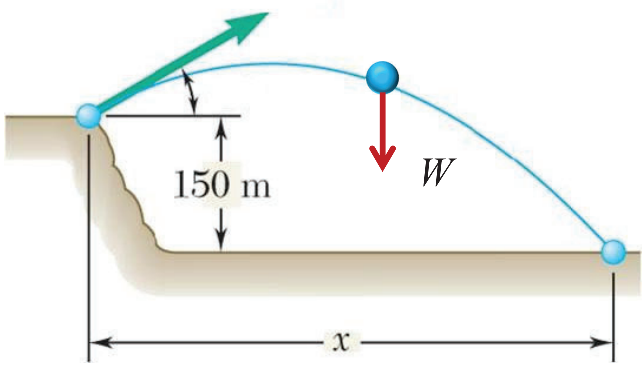
❑ Equations of Motion

Consider, as an example, the motion of a projectile. If the resistance of the air is neglected, the only force acting on the projectile after it has been fired is its weight W=-Wj. The equations defining the motion of the projectile are therefore

$$m\ddot{x} = 0 \quad m\ddot{y} = -W \quad m\ddot{z} = 0$$

and the components of the acceleration of the projectile are

$$\ddot{x} = 0 \quad \ddot{y} = -\frac{W}{m} = -g \quad \ddot{z} = 0$$



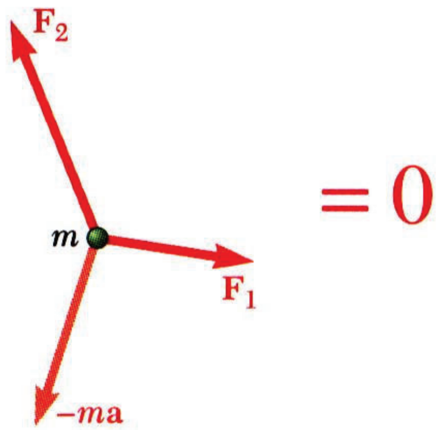
Kinetics of Particles: Newton's Second Law

❑ Dynamic Equilibrium

- Alternate expression of Newton's second law,

$$\sum \vec{F} - m\vec{a} = 0$$
$$-m\vec{a} \equiv \textit{inertial vector}$$

- With the inclusion of the inertial vector, the system of forces acting on the particle is equivalent to zero. The particle is in *dynamic equilibrium*.
- Methods developed for particles in static equilibrium may be applied, e.g., coplanar forces may be represented with a closed vector polygon.
- Inertia vectors are often called ***inertial forces*** as they measure the resistance that particles offer to changes in motion, i.e., changes in speed or direction.
- Inertial forces may be conceptually useful but are not like the contact and gravitational forces found in statics.

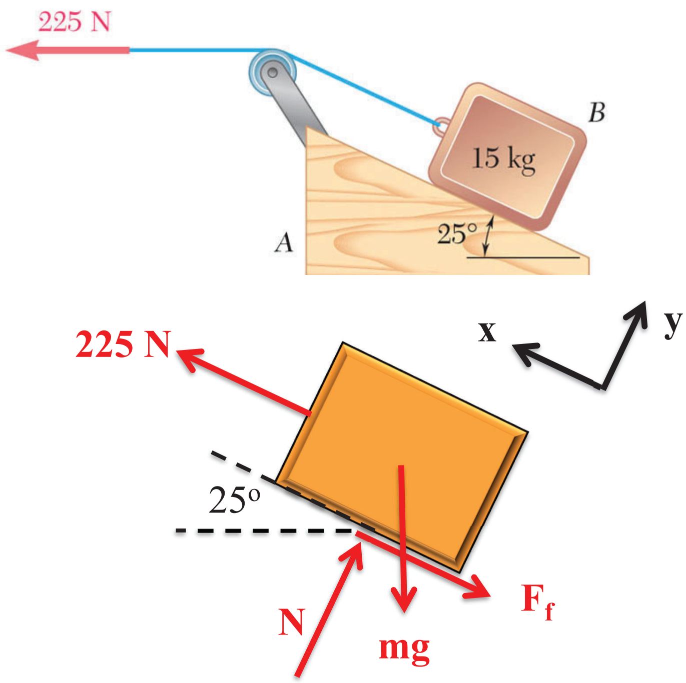


Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

The free body diagram is the same as you have done in statics; we will add the kinetic diagram in our dynamic analysis.

1. Isolate the body of interest (free body)
2. Draw your axis system (e.g., Cartesian, polar, path)
3. Add in applied forces (e.g., weight, 225 N pulling force)
4. Replace supports with forces (e.g., normal force)
5. Draw appropriate dimensions (usually angles for particles)

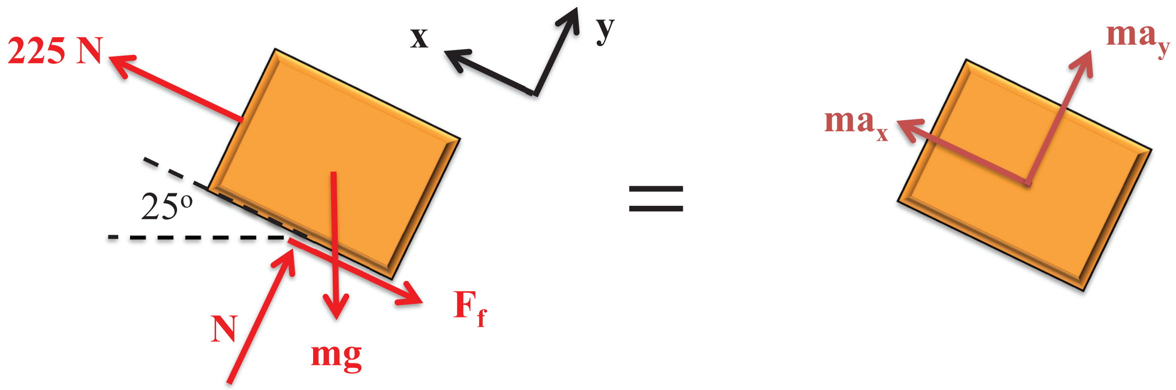


Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

Put the inertial terms for the body of interest on the kinetic diagram.

1. Isolate the body of interest (free body)
2. Draw in the mass times acceleration of the particle; if unknown, do this in the positive direction according to your chosen axes

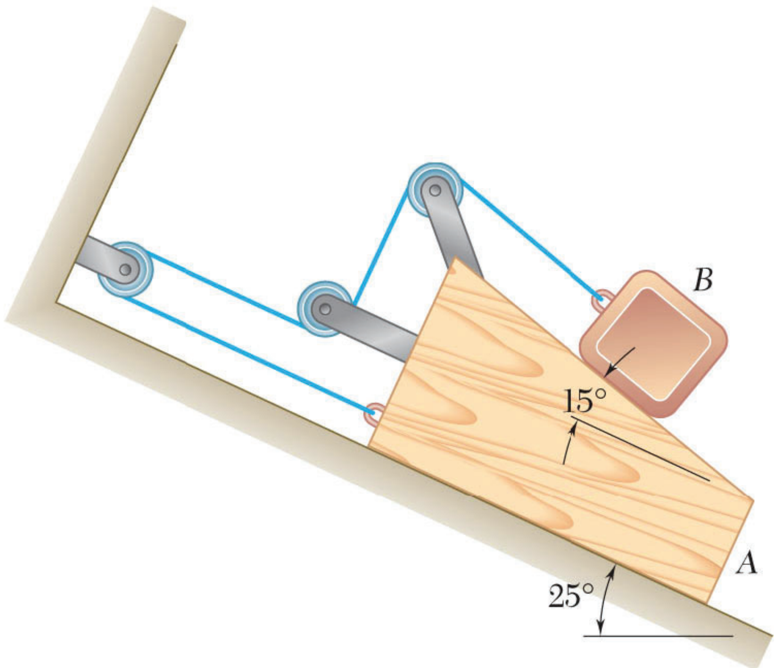


$$\Sigma \mathbf{F} = m\mathbf{a}$$

Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

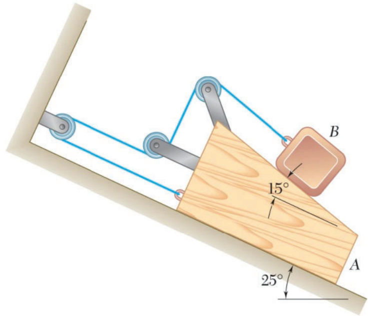
Draw the FBD and KD for block A (note that the massless, frictionless pulleys are attached to block A and should be included in the system).

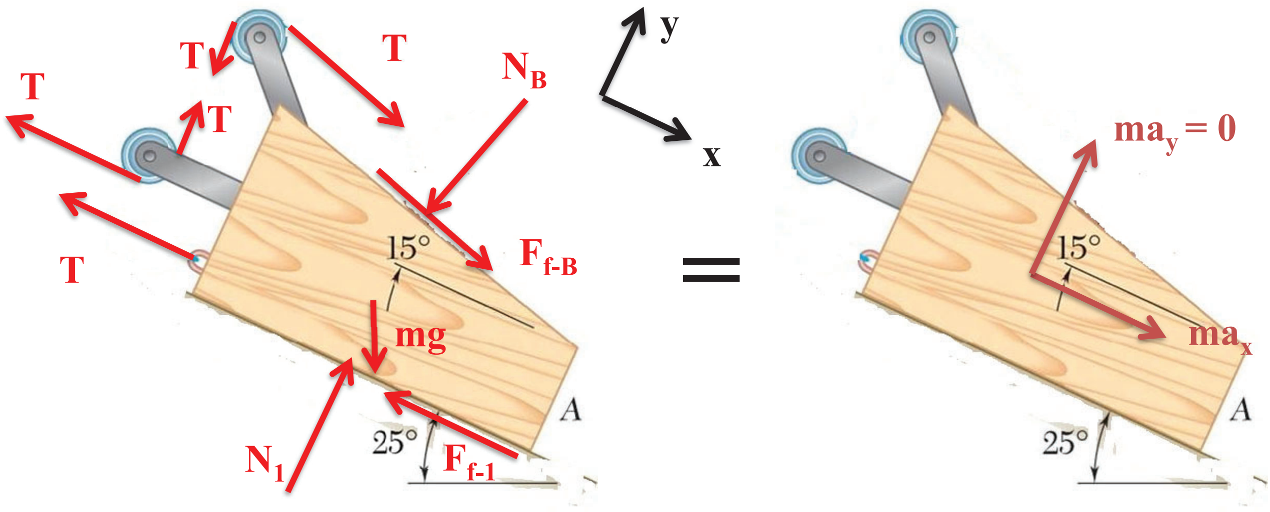


Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

1. Isolate body
2. Axes
3. Applied forces
4. Replace supports with forces
5. Dimensions (already drawn)
6. Kinetic diagram

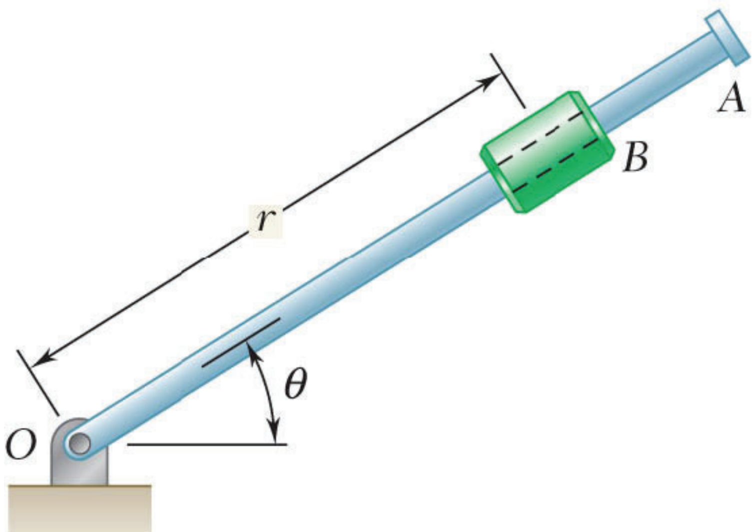




Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

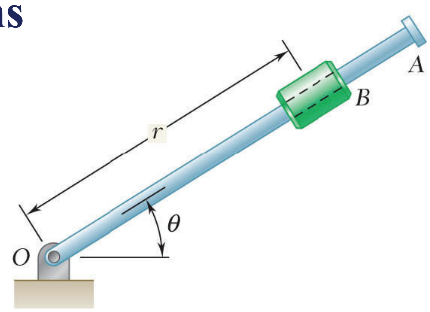
Draw the FBD and KD for the collar B. Assume there is friction acting between the rod and collar, motion is in the vertical plane, and θ is increasing

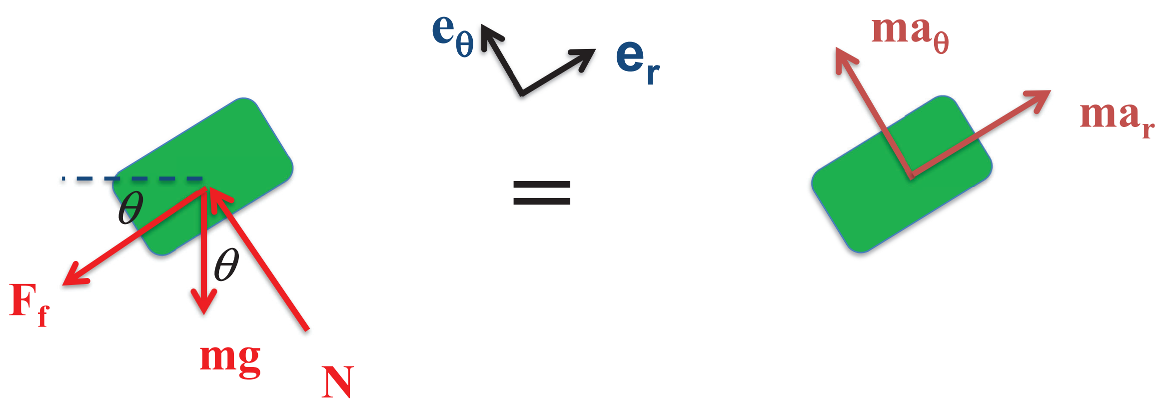


Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

1. Isolate body
2. Axes
3. Applied forces
4. Replace supports with forces
5. Dimensions
6. Kinetic diagram

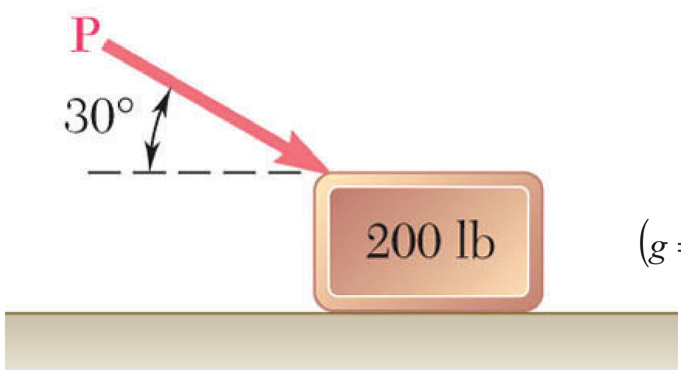




Kinetics of Particles: Newton's Second Law

❑ Sample Problem 01

A 200-lb block rests on a horizontal plane. Find the magnitude of the force ***P*** required to give the block an acceleration of 10 ft/s^2 to the right. The coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$.

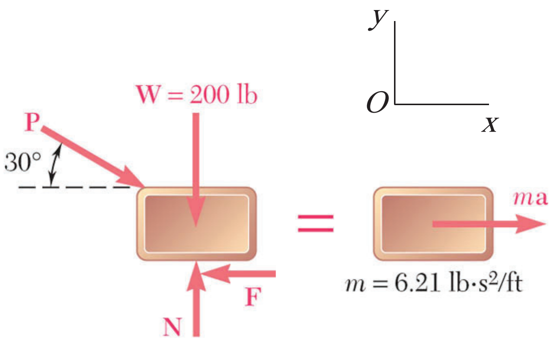


$(g = 32.2\ \text{ft/s}^2)$

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 01

SOLUTION:

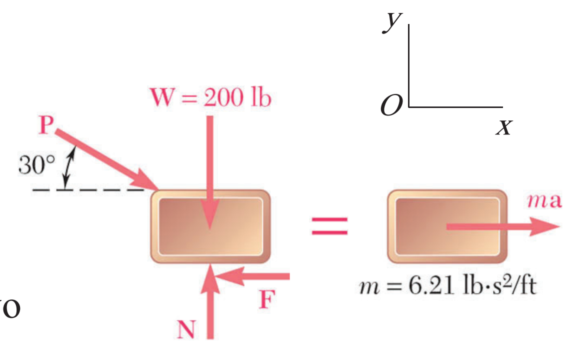


- Resolve the equation of motion for the block into two rectangular component equations.

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 01

SOLUTION:

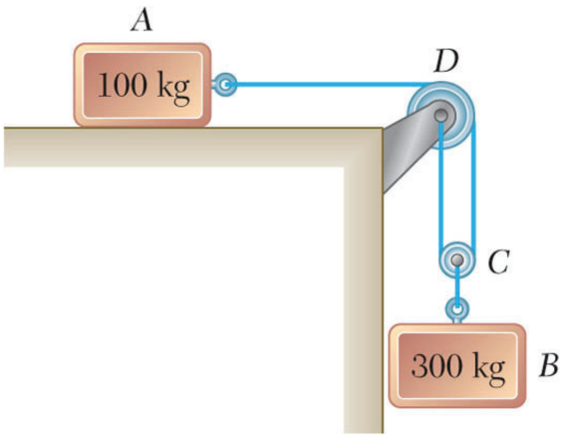


- Unknowns consist of the applied force ***P*** and the normal reaction ***N*** from the plane. The two equations may be solved for these unknowns.

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 02

The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in the cord.

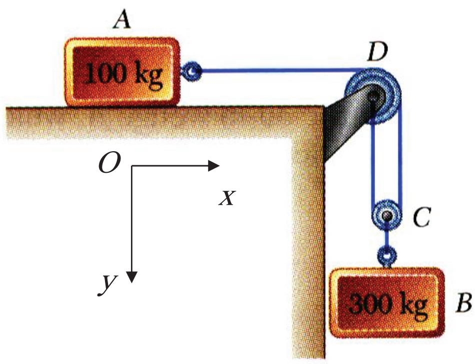


Kinetics of Particles: Newton's Second Law

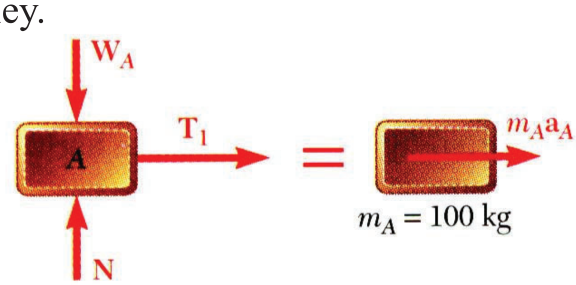
❑ Sample Problem 02

SOLUTION:

- Write the kinematic relationships for the dependent motions and accelerations of the blocks.

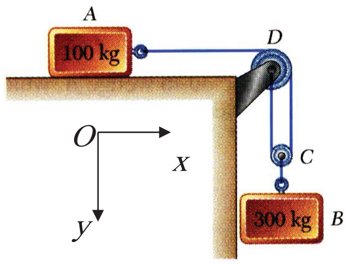


- Write equations of motion for blocks and pulley.



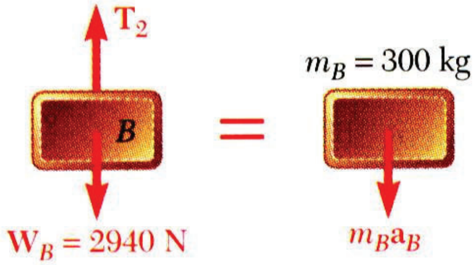
Kinetics of Particles: Newton’s Second Law

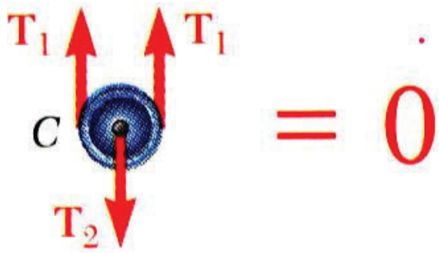
❑ Sample Problem 02



SOLUTION:

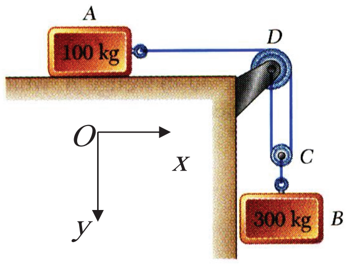
- Write equations of motion for blocks and pulley.





Kinetics of Particles: Newton’s Second Law

❑ Sample Problem 02

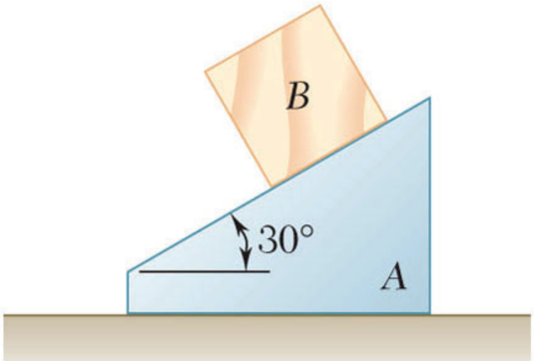


SOLUTION:

❑ Sample Problem 03

The 12-lb block *B* starts from rest and slides on the 30-lb wedge *A*, which is supported by a horizontal surface.

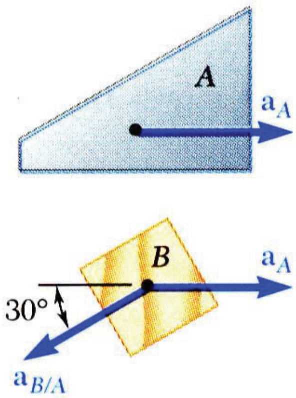
Neglecting friction, determine *(a)* the acceleration of the wedge A, and *(b)* the acceleration of the block B relative to the wedge A.



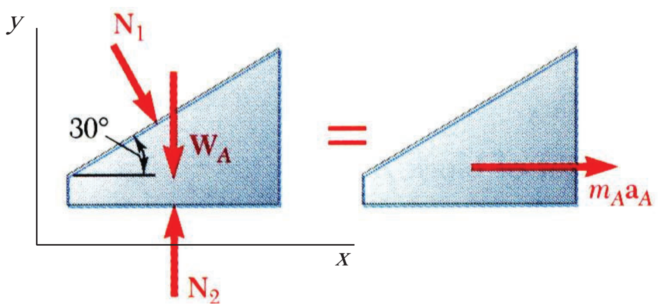
❑ Sample Problem 03

SOLUTION:

- The block is constrained to slide down the wedge. Therefore, their motions are dependent.



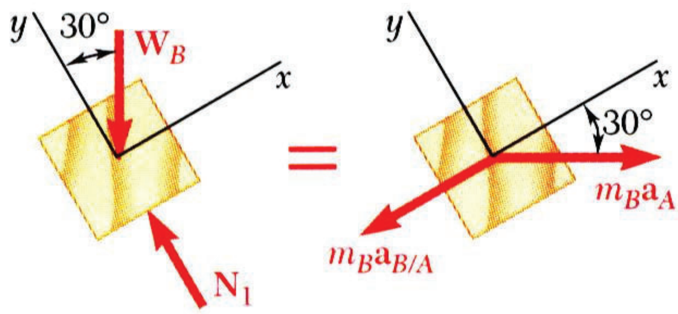
- Write equations of motion for wedge and block.



(*I*)

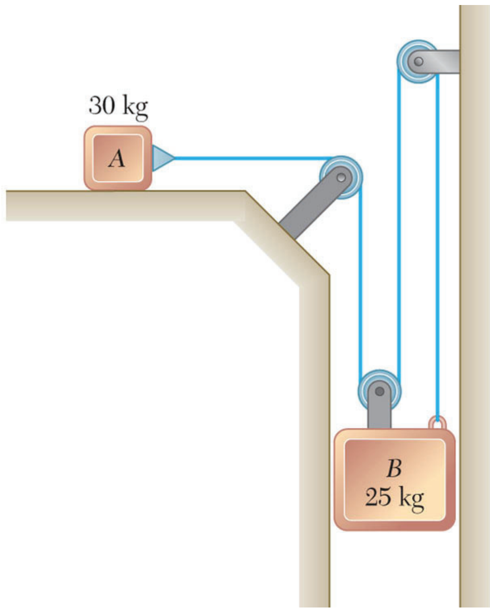
- **Sample Problem 03**

SOLUTION:



- **Sample Problem 04**

The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the horizontal surface, determine (a) the acceleration of each block, (b) the tension in the cable.

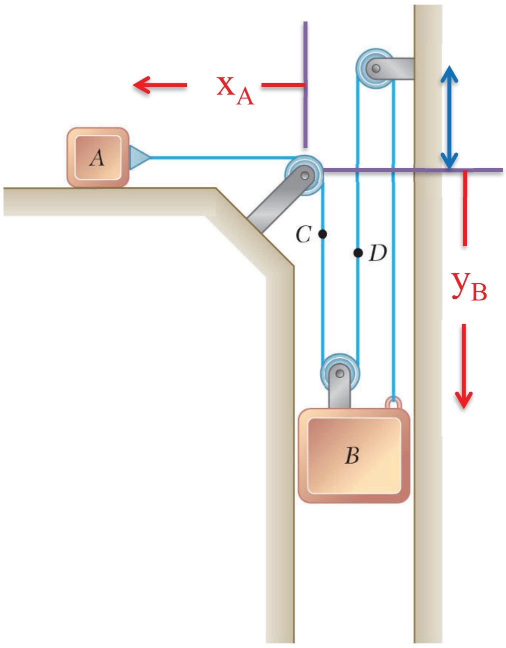


Kinetics of Particles: Newton's Second Law

❑ Sample Problem 04

SOLUTION:

- Write the kinematic relationships for the dependent motions and accelerations of the blocks.

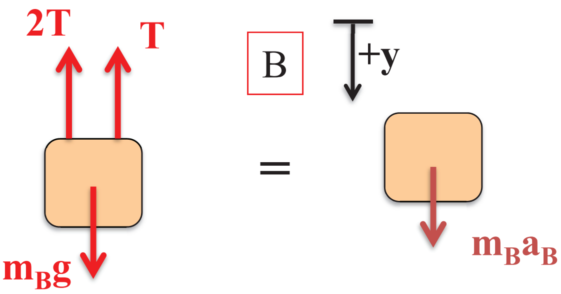


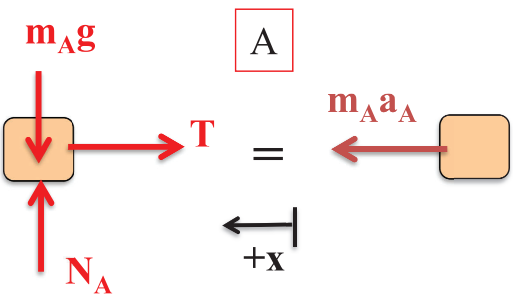
Kinetics of Particles: Newton's Second Law

❑ Sample Problem 04

SOLUTION:

- Draw the FBD and KD for each block





- Write the equation of motion for each block

Kinetics of Particles: Newton’s Second Law

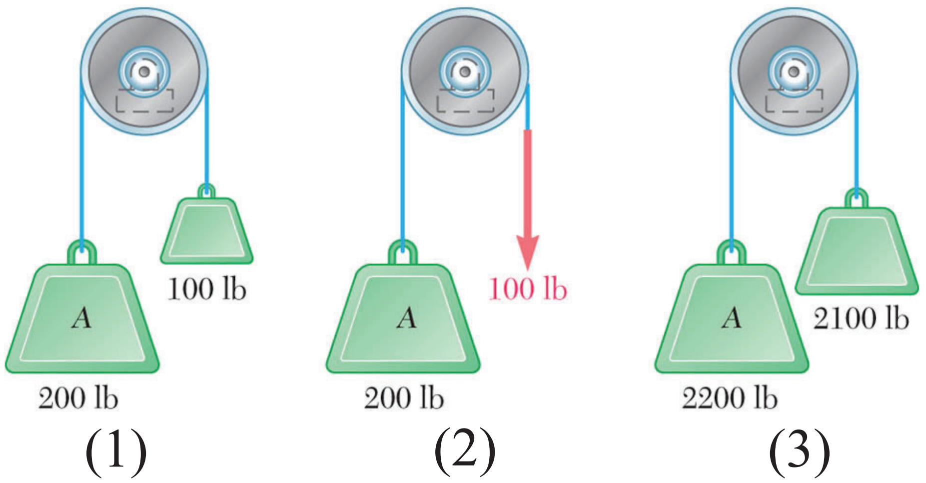
❑ Sample Problem 04

SOLUTION:

- Solve the three equations, 3 unknowns

Kinetics of Particles: Newton’s Second Law

❑ Concept Quiz



The three systems are released from rest. Rank the accelerations, from highest to lowest.

a) (1) > (2) > (3)

b) (1) = (2) > (3)

c) (2) > (1) > (3)

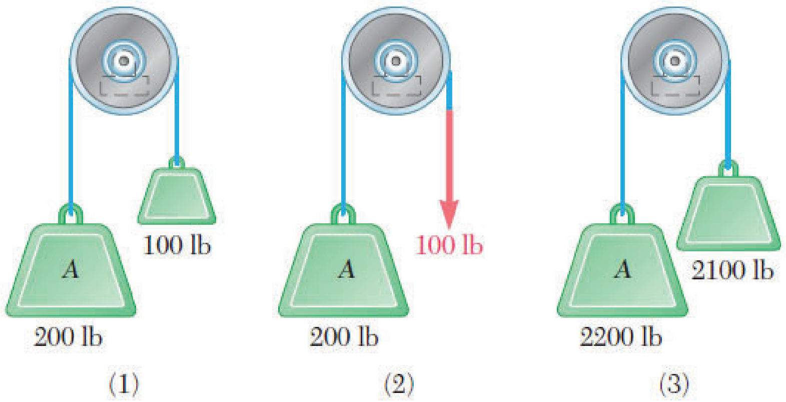
d) (1) = (2) = (3)

e) (1) = (2) < (3)

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 05

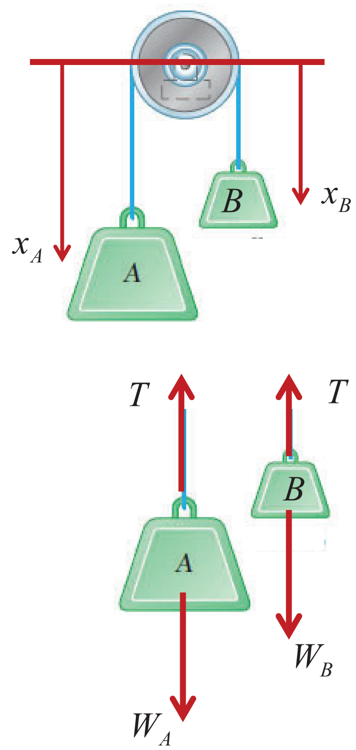
Each of the systems shown is initially at rest. Neglecting axle friction and the masses of the pulleys, determine for each system (a) the acceleration of block A, (b) the velocity of block A after it has moved through 10 ft, (c) the time required for block A to reach a velocity of 20 ft/s.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 05

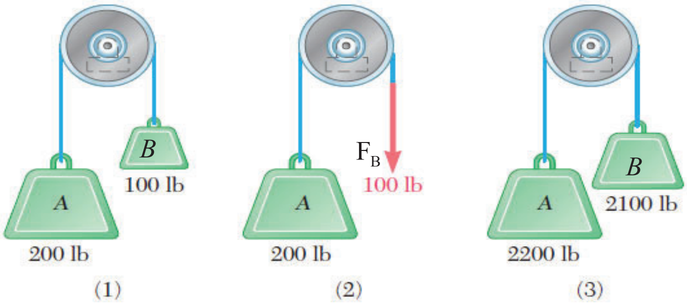
SOLUTION:



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 05

SOLUTION:



	(1)	(2)	(3)

❑ Kinetics: Normal and Tangential Coordinates

Aircraft and roller coasters can both experience large normal forces during turns.

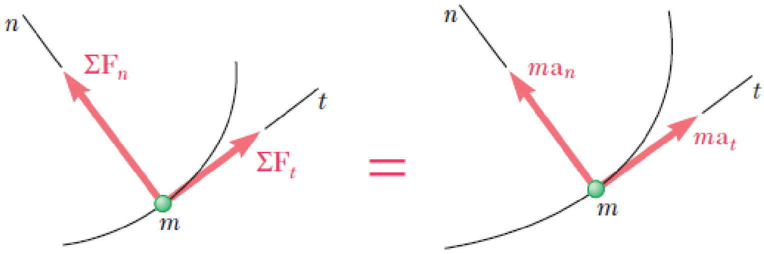
Kinetics of Particles: Newton's Second Law

❑ Equations of Motion

- ***Tangential and Normal Components.*** Resolving the forces and the acceleration of the particle into components along the tangent to the path (in the direction of motion) and the normal (toward the inside of the path)

$$\sum \vec{F} = \sum (F_t \vec{e}_t + F_n \vec{e}_n) = m(a_t \vec{e}_t + a_n \vec{e}_n)$$

$$\sum F_t = ma_t \qquad \sum F_n = ma_n$$
$$\sum F_t = m\frac{dv}{dt} \qquad \sum F_n = m\frac{v^2}{\rho}$$



Kinetics of Particles: Newton's Second Law

❑ Dynamic Equilibrium

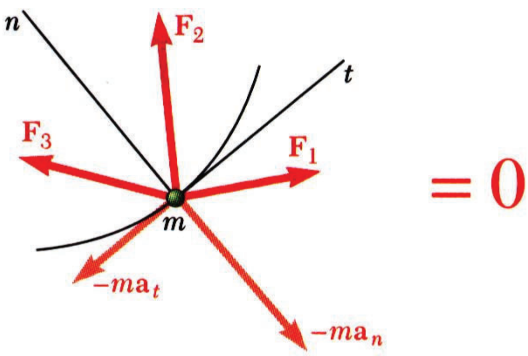
$$\sum \vec{F} - m\vec{a} = 0$$
$$-m\vec{a} \equiv inertial\ vector$$

The tangential component of the inertia vector provides a measure of the resistance the particle offers to a change in speed, while its normal component (also called centrifugal force) represents the tendency of the particle to leave its curved path.

We should note that either of these two components may be zero under special conditions:

(1) if the particle starts from rest, its initial velocity is zero and the normal component of the inertia vector is zero at t = 0;

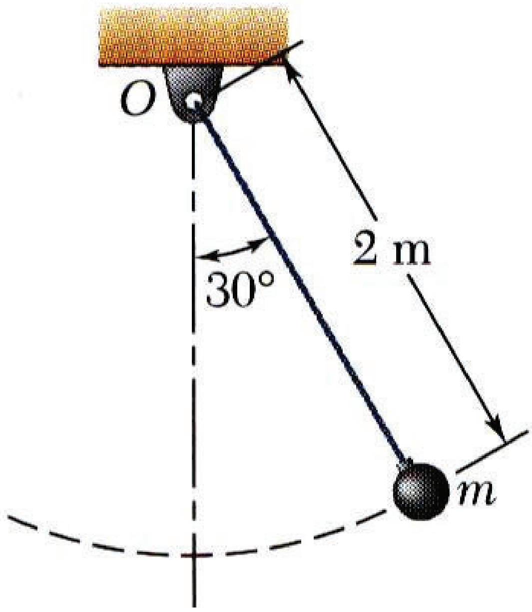
(2) if the particle moves at constant speed along its path, the tangential component of the inertia vector is zero and only its normal component needs to be considered.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 06

The bob of a 2-m pendulum describes an arc of a circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown, find the velocity and acceleration of the bob in that position.

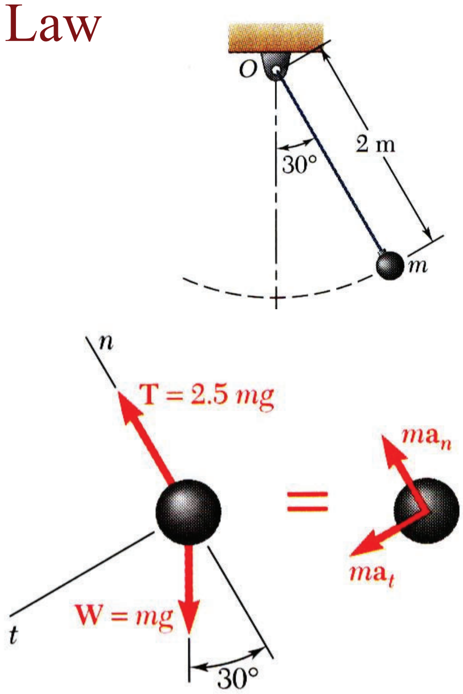


Kinetics of Particles: Newton's Second Law

❑ Sample Problem 06

SOLUTION:

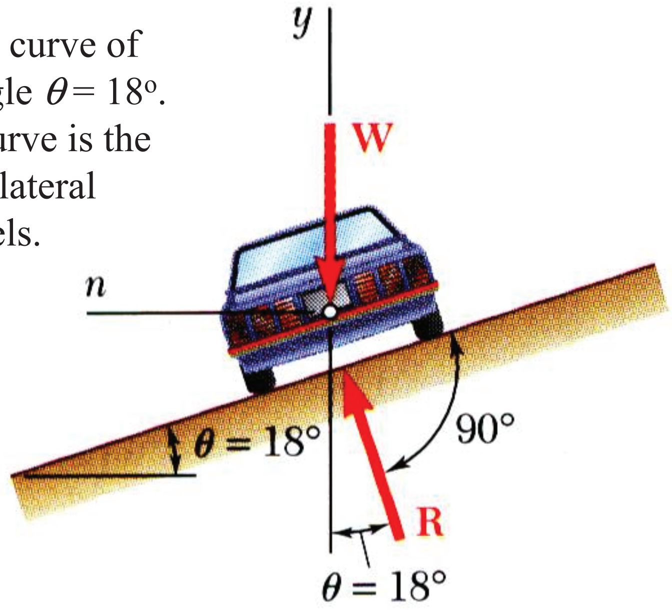
- Resolve the equation of motion for the bob into tangential and normal components.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 07

Determine the rated speed of a highway curve of radius $\rho = 400$ ft banked through an angle $\theta = 18^o$. The rated speed of a banked highway curve is the speed at which a car should travel if no lateral friction force is to be exerted at its wheels.

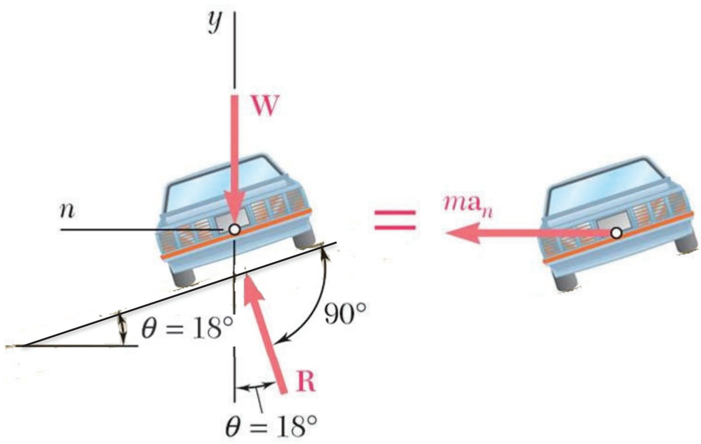


Kinetics of Particles: Newton's Second Law

❑ Sample Problem 07

SOLUTION:

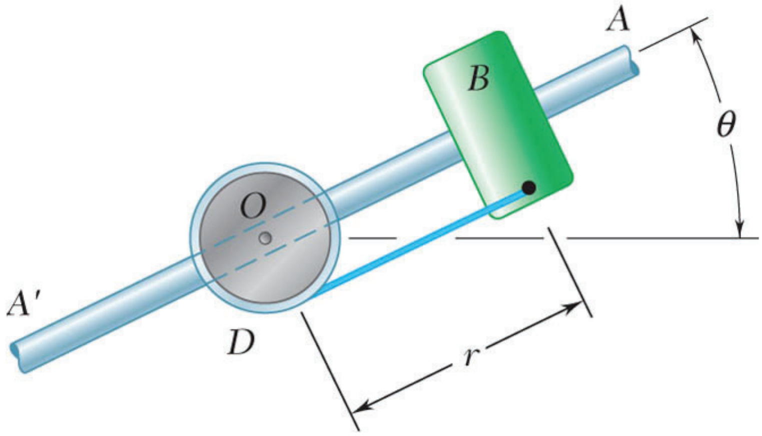
- The car travels in a horizontal circular path with a normal component of acceleration directed toward the center of the path. The forces acting on the car are its weight and a normal reaction from the road surface.



Kinetics of Particles: Newton’s Second Law

❑ Sample Problem 08

The 3-kg collar *B* rests on the frictionless arm *AA′*. The collar is held in place by the rope attached to drum *D* and rotates about *O* in a horizontal plane. The linear velocity of the collar B is increasing according to $v = 0.2\ t^2$ where v is in m/s and t is in sec. Find the tension in the rope and the force of the bar on the collar after 5 seconds if $r = 0.4$ m.



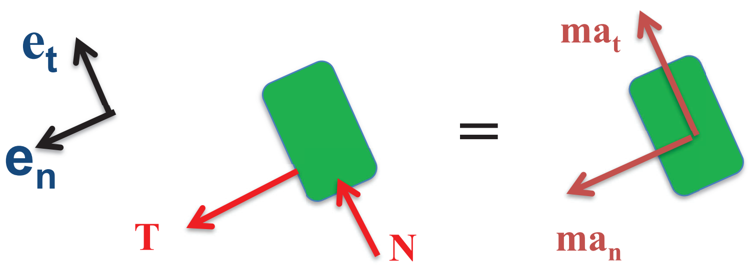
Kinetics of Particles: Newton’s Second Law

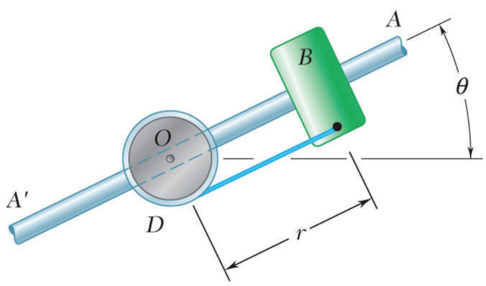
❑ Sample Problem 08

SOLUTION:

- Given: $v = 0.2\ t^2$, $r = 0.4$ m
- Find: T and N at $t = 5$ sec

Draw the FBD and KD of the collar





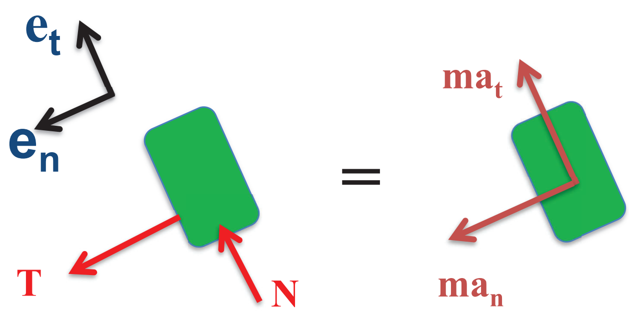
Write the equations of motion

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 08

SOLUTION:

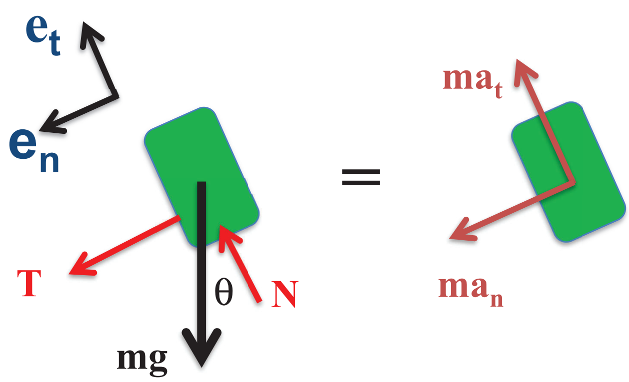
Kinematics : find v_t, a_n, a_t



Kinetics of Particles: Newton's Second Law

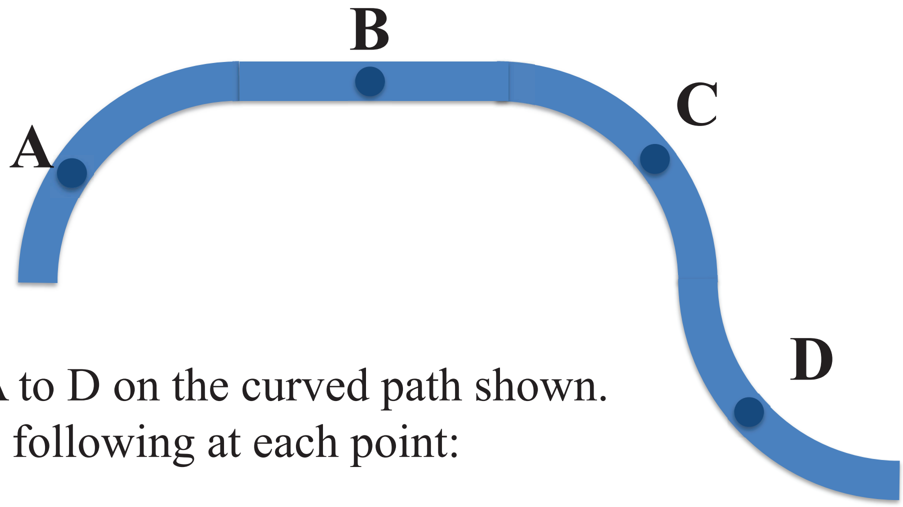
❑ Group Problem Solving

How would the problem change if motion was in the vertical plane?



Kinetics of Particles: Newton's Second Law

❑ Concept Question



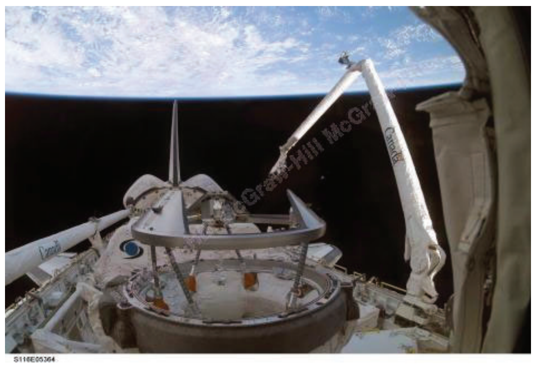
A car is driving from A to D on the curved path shown.
The driver is doing the following at each point:

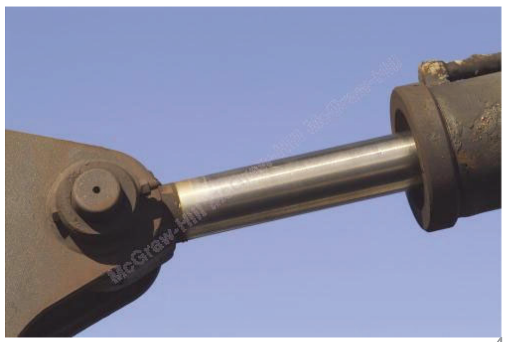
A – going at a constant speed
B – stepping on the accelerator
C – stepping on the brake
D – stepping on the accelerator

Kinetics of Particles: Newton's Second Law

❑ Kinetics: Radial and Transverse Coordinates

Hydraulic actuators and extending robotic arms are often analyzed using radial and transverse coordinates.



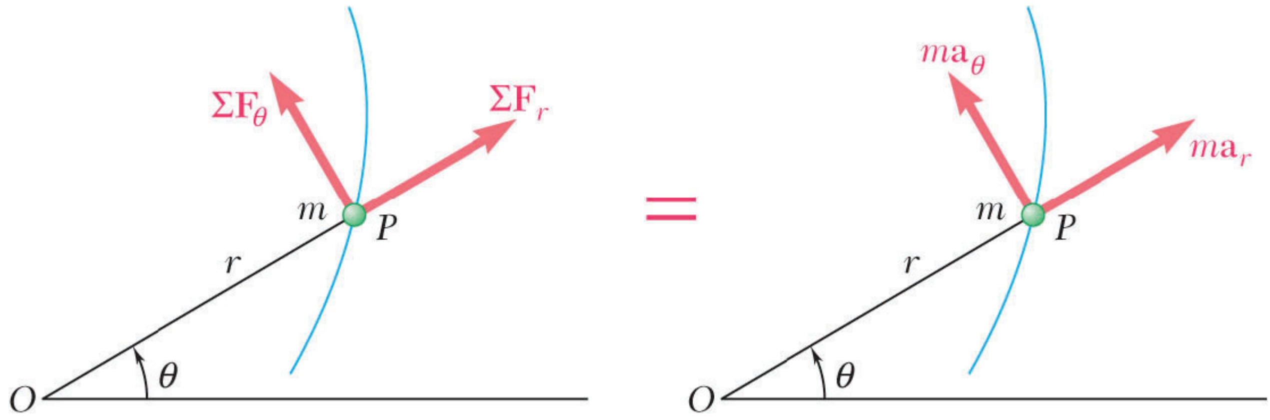


Kinetics of Particles: Newton's Second Law

❑ Equations of Motion in Radial & Transverse Components

- Consider particle at r and θ, in polar coordinates,

$$\left.\begin{array}{l}\sum F_r = ma_r \\ a_r = (\ddot{r} - r\dot{\theta}^2)\end{array}\right\} \Rightarrow \boxed{\sum F_r = m(\ddot{r} - r\dot{\theta}^2)} \qquad \left.\begin{array}{l}\sum F_\theta = ma_\theta \\ a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta})\end{array}\right\} \Rightarrow \boxed{\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})}$$



Kinetics of Particles: Newton's Second Law

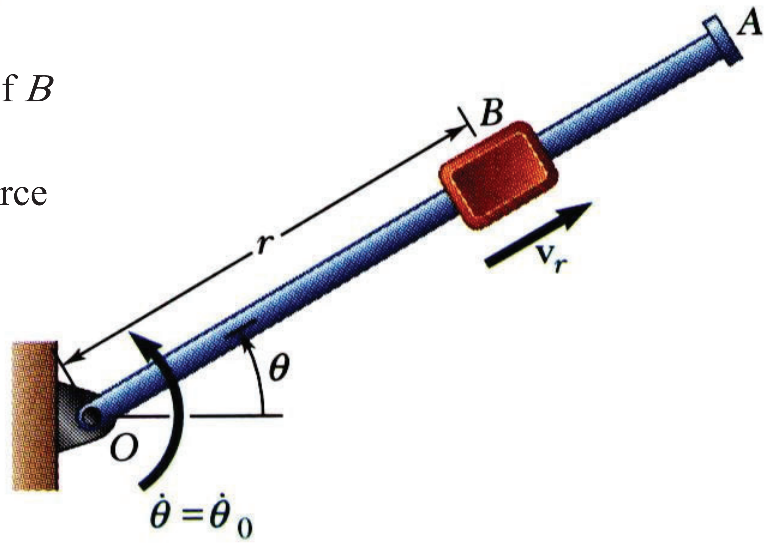
❑ Sample Problem 09

A block B of mass m can slide freely on a frictionless arm OA which rotates in a horizontal plane at a constant $\dot{\theta}_0$ rate

Knowing that B is released at a distance r_0 from O, express as a function of r

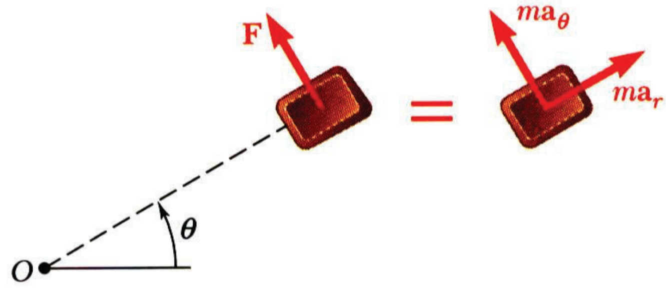
a) the component v_r of the velocity of B along OA, and

b) the magnitude of the horizontal force exerted on B by the arm OA.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 09



SOLUTION:

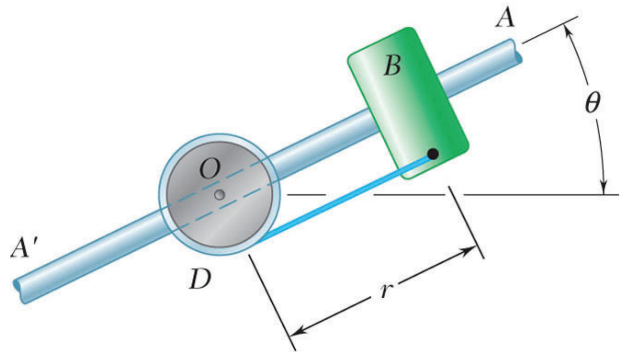
- Write the radial and transverse equations of motion for the block.

- Substitute known information into the transverse equation to find an expression for the force on the block.

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 10

The 3-kg collar B slides on the frictionless arm AA'. The arm is attached to drum D and rotates about O in a horizontal plane at the rate $\dot{\theta} = 0.75t$ where $\dot{\theta}$ and t are expressed in rad/s and seconds, respectively. As the arm-drum assembly rotates, a mechanism within the drum releases the cord so that the collar moves outward from O with a constant speed of 0.5 m/s. Knowing that at $t = 0$, $r = 0$, determine the time at which the tension in the cord is equal to the magnitude of the horizontal force exerted on B by arm AA'.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 10

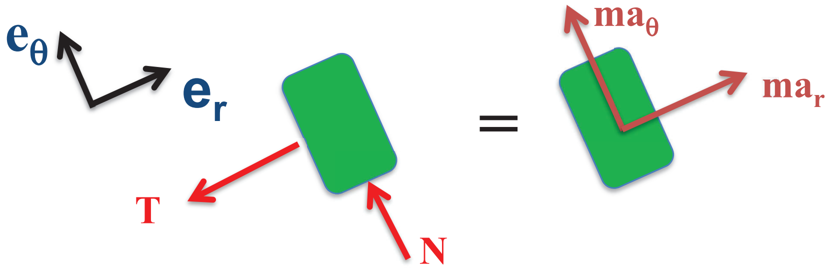
SOLUTION:

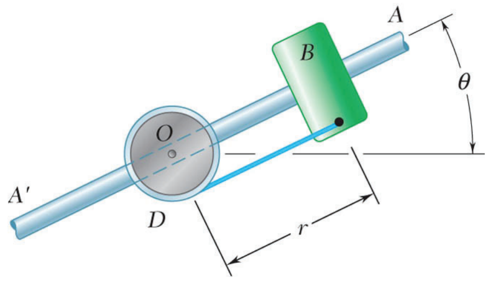
- Given: $\dot{\theta} = 0.75t \qquad r_{(0)} = 0$

  $\dot{r} = 0.5\,(m/s)$

- Find: time when T = N

Draw the FBD and KD of the collar





Write the equations of motion

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 10

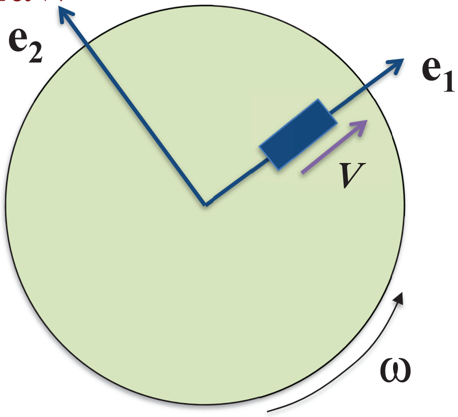
SOLUTION:

Kinematics : find expressions for r and θ

Kinetics of Particles: Newton's Second Law

❑ Concept Quiz

Top View



The girl starts walking towards the outside of the spinning platform, as shown in the figure. She is walking at a constant rate with respect to the platform, and the platform rotates at a constant rate. In which direction(s) will the forces act on her?

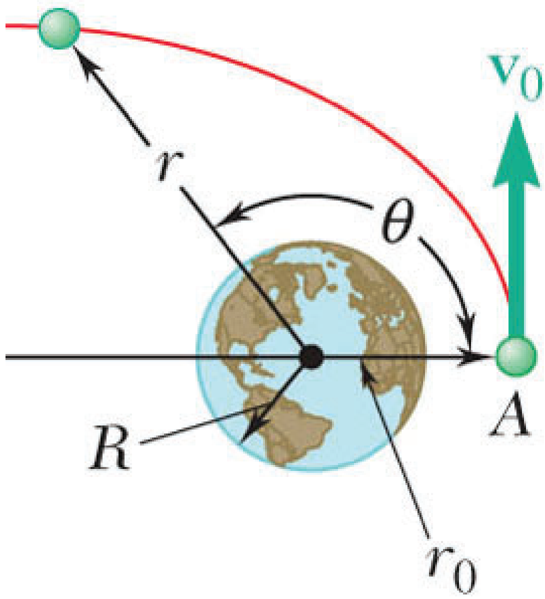
a) $+e_1$ b) $-e_1$ c) $+e_2$ d) $-e_2$

e) The forces are zero in the e_1 and e_2 directions

Kinetics of Particles: Newton's Second Law

❑ Angular Momentum of a Particle

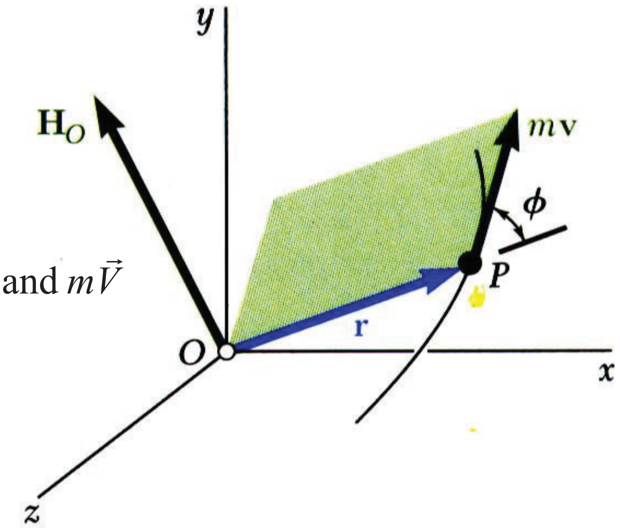
Satellite orbits are analyzed using conservation of angular momentum.



Angular Momentum of a Particle

- The moment about O of the vector mv is called the ***moment of momentum***, or the ***angular momentum***, of the particle about O at that instant and is denoted by **Ho**.

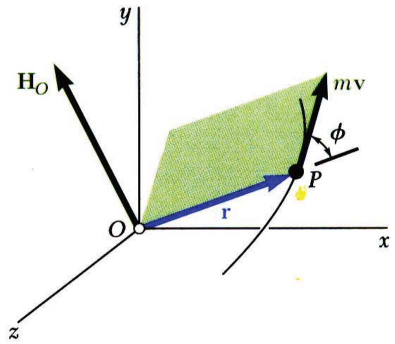
$$\vec{H}o = \vec{r} \times m\vec{v}$$



- $\vec{H}_O$ is ***perpendicular*** to plane containing $\vec{r}$ and $m\vec{V}$

$$Ho = r\, m\, v \sin\phi \quad (kg \cdot m^2 / s)$$

Angular Momentum of a Particle



The components of **Ho** in ***Cartesian coordinates***

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$m\vec{v} = mv_x\vec{i} + mv_y\vec{j} + mv_z\vec{k}$$

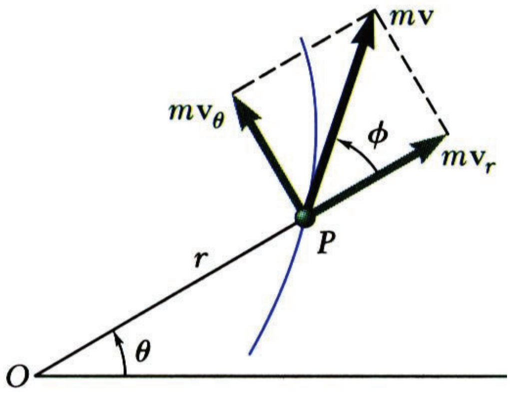
$$\Rightarrow \quad Ho = r \times mv = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix} \Rightarrow Ho = H_x\vec{i} + H_y\vec{j} + H_z\vec{k} \Rightarrow \begin{cases} H_x = m(yv_z - zv_y) \\ H_y = m(zv_x - xv_z) \\ H_z = m(xv_y - yv_x) \end{cases}$$

In the case of a particle moving in the ***xy plane***

$$\begin{matrix} z = 0 \\ v_z = 0 \end{matrix} \Rightarrow \begin{matrix} H_x = H_y = 0 \\ H_O = H_z = m(xv_y - yv_x) \end{matrix}$$

Kinetics of Particles: Newton's Second Law

❑ Angular Momentum of a Particle



The components of **Ho** in ***polar coordinates***

$$Ho = r\,m\,v\sin\phi = r\,m\,v_\theta \qquad v_\theta = r\dot{\theta} \qquad \Rightarrow \boxed{Ho = mr^2\dot{\theta}}$$

- Derivative of angular momentum with respect to time,

$$\vec{H}o = \vec{r}\times m\vec{v} \quad \Rightarrow \quad \frac{d(\vec{H}o)}{dt} = \dot{\vec{H}}_O = \dot{\vec{r}}\times m\vec{V} + \vec{r}\times m\dot{\vec{V}} = \underbrace{\vec{V}\times m\vec{V}}_{0} + \vec{r}\times m\vec{a}$$

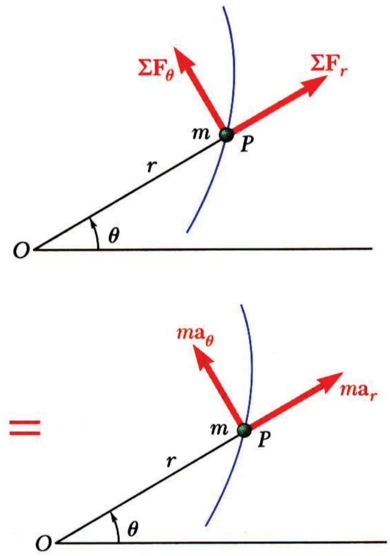
$$\Rightarrow \quad \dot{\vec{H}}_O = \vec{r}\times m\vec{a} = \vec{r}\times\sum\vec{F} \quad \Rightarrow \boxed{\dot{\vec{H}}_O = \sum\vec{M}_O}$$

- It follows from Newton's second law that ***the sum of the moments about O of the forces acting on the particle is equal to the rate of change of the angular momentum of the particle about O.***

Kinetics of Particles: Newton's Second Law

❑ Equations of Motion in Radial & Transverse Components

- Consider particle at r and θ, in polar coordinates,



- Consider particle at r and θ, in polar coordinates,

$$\boxed{\sum F_r = m(\ddot{r} - r\dot{\theta}^2)} \qquad \boxed{\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})} \ (*)$$

- Equation (*) could have been derived from

$$\sum\vec{M}_O = \dot{\vec{H}}_O \quad \Rightarrow \quad r\sum F_\theta = \frac{d}{dt}(mr^2\dot{\theta}) = m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta})$$

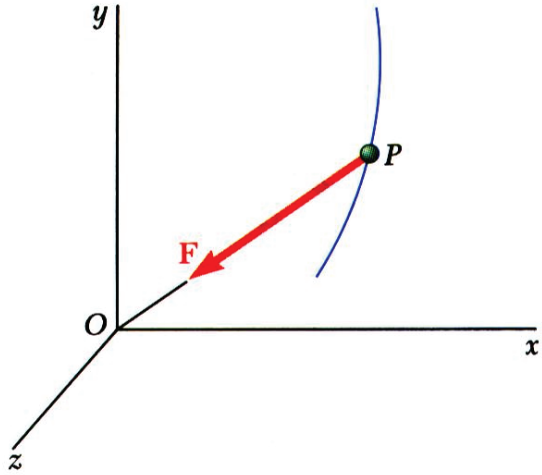
$$\Rightarrow \boxed{\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})}$$

Kinetics of Particles: Newton's Second Law

❑ Conservation of Angular Momentum

- When only force acting on particle is directed toward or away from a fixed point *O*, the particle is said to be ***moving under a central force***.

- Since the line of action of the central force passes through *O*,



$$\sum \vec{M}_O = \dot{\vec{H}}_O = 0 \quad \Rightarrow \quad \boxed{\vec{H}_O = \vec{r} \times m\vec{V} = cte}$$

moving under a central force

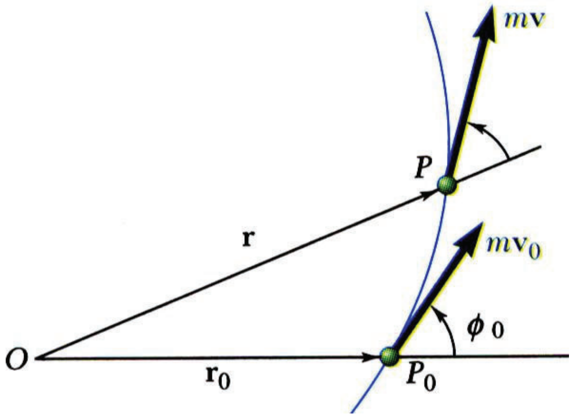
Kinetics of Particles: Newton's Second Law

❑ Conservation of Angular Momentum

moving under a central force

- Position vector and motion of particle are in a plane perpendicular to $\vec{H}_O$.

- Magnitude of angular momentum,



$$\boxed{H_O = r_0 m V_0 \sin\phi_0 = rmV\sin\phi = \text{cte}}$$

or

$$H_O = mr^2\dot{\theta} = \text{cte} \quad \Rightarrow \quad \boxed{\frac{H_O}{m} = r^2\dot{\theta} = h = \frac{\text{angular momentum}}{\text{unit mass}}}$$

Kinetics of Particles: Newton's Second Law

❑ Conservation of Angular Momentum

- Radius vector *OP* sweeps infinitesimal area

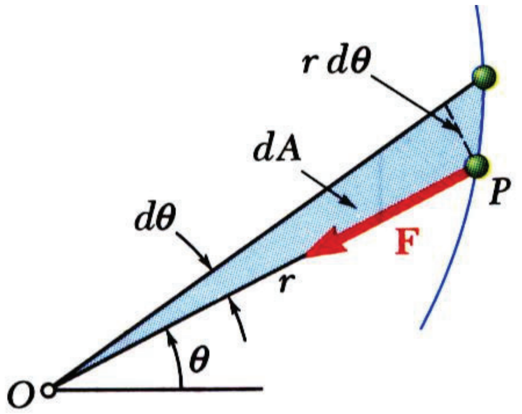
  $$dA = \tfrac{1}{2} r^2 d\theta$$

- Define $\dfrac{dA}{dt} = \tfrac{1}{2} r^2 \dfrac{d\theta}{dt} = \tfrac{1}{2} r^2 \dot{\theta} =$ ***areal velocity***

- Recall, for a body *moving under a central force*,

  $$h = r^2 \dot{\theta} = \text{constant}$$

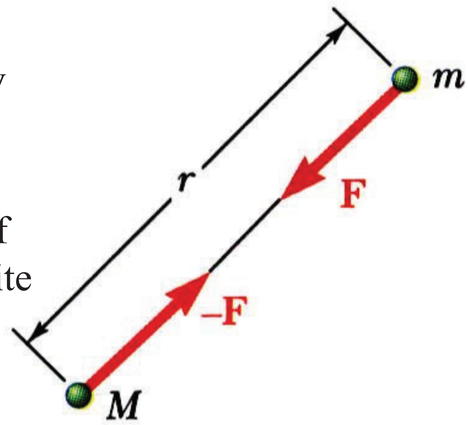
- ***When a particle moves under a central force, its areal velocity is constant.***



Kinetics of Particles: Newton's Second Law

❑ Newton's Law of Gravitation

- Gravitational force exerted by the sun on a planet or by the earth on a satellite is an important example of gravitational force.
- *Newton's law of universal gravitation* - two particles of mass *M* and *m* attract each other with equal and opposite force directed along the line connecting the particles,



$$F = G\frac{Mm}{r^2}$$

$$G: \text{Constant of gravitation} \quad = 66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} = 34.4 \times 10^{-9} \frac{\text{ft}^4}{\text{lb} \cdot \text{s}^4}$$

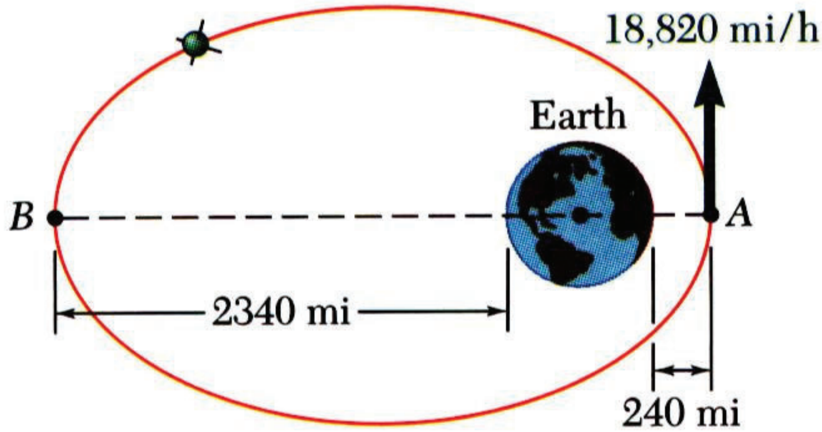
- For particle of mass *m* on the earth's surface,

$$W = m\frac{MG}{R^2} = mg \qquad g = 9.81\frac{\text{m}}{\text{s}^2} = 32.2\frac{\text{ft}}{\text{s}^2}$$

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 11

A satellite is launched in a direction parallel to the surface of the earth with a velocity of 18820 mi/h from an altitude of 240 mi. Determine the velocity of the satellite as it reaches it maximum altitude of 2340 mi. The radius of the earth is 3960 mi.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 11

SOLUTION:

- Since the satellite is moving under a central force, its angular momentum is constant. Equate the angular momentum at *A* and *B* and solve for the velocity at *B*.