DYNAMICS

دانشگاه کردستان
University of Kurdistan
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- Vector Mechanics for Engineers: Dynamics, 10th edition. Ferdinand Beer- E. Russell Johnston Jr. - Phillip Cornwell.
- Engineering Mechanics-Dynamics, 7th Edition. J. L. Meriam, L. G. Kraige.
- Other Reference: Brain P.Self "Lectures notes on Dynamics"

Kinematics of Particles

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Kinematics of Particles

❑ Introduction

Kinematic relationships are used to help us determine the trajectory of a **golf ball**, the **orbital speed of a satellite**, and the **accelerations during acrobatic flying**.

Kinematics of Particles

❑ Introduction

- **Dynamics includes:**

[***Kinematics***]: study of the geometry of motion.
Relates displacement, velocity, acceleration, and time ***without reference*** to the cause of motion.

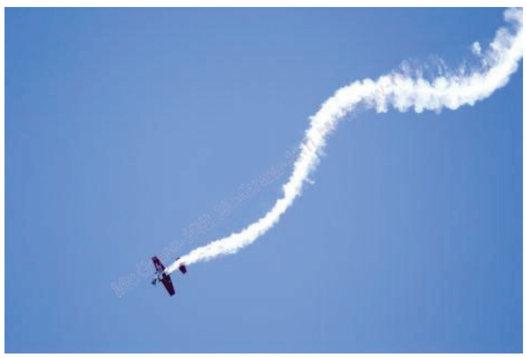
[***Kinetics***]: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Kinematics of Particles

❑ Introduction

- **Particle kinematics includes:**

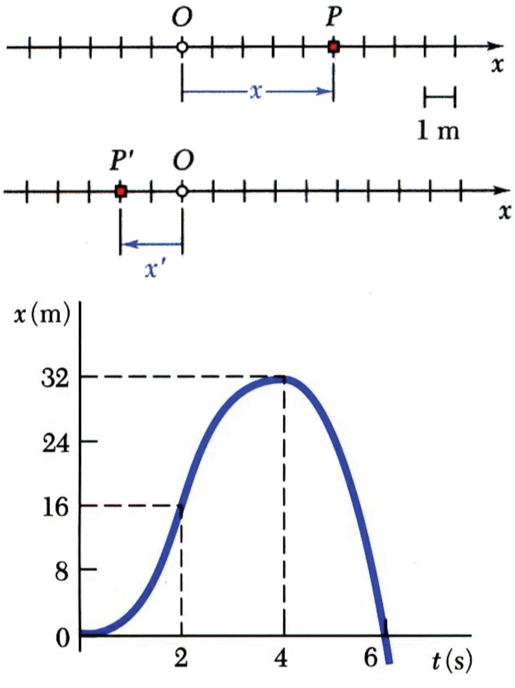
  - ***Rectilinear motion***: position, velocity, and acceleration of a particle as it moves along a straight line.



- ***Curvilinear motion***: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

Kinematics of Particles

❑ Rectilinear Motion: Position, Velocity & Acceleration



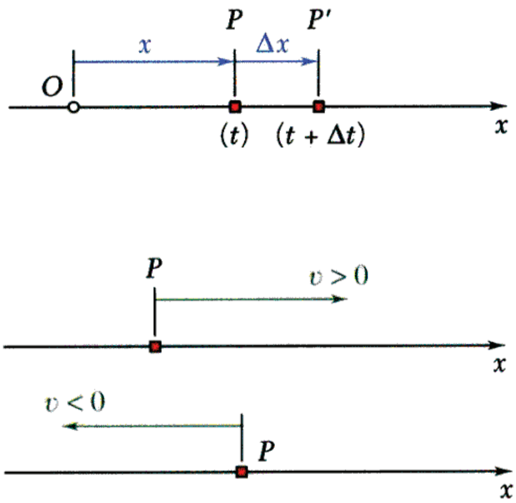
- ***Rectilinear motion:*** particle moving along a straight line
- ***Position coordinate:*** defined by positive or negative distance from a fixed origin on the line.
- The ***motion*** of a particle is known if the position coordinate for particle is known for every value of time *t*.
- May be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$

or in the form of a graph *x* vs. *t*.

Kinematics of Particles

❑ Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle which occupies position *P* at time *t* and *P'* at *t+Δt*,

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

$$\text{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

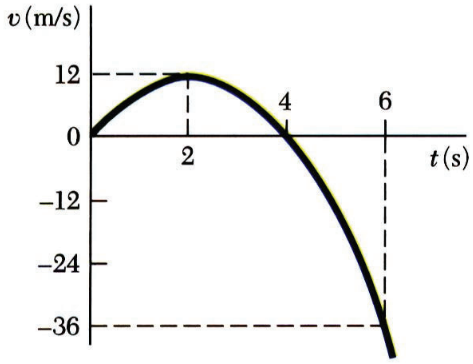
- Instantaneous velocity may be positive (Increasing x) or **negative** (Decreasing x). Magnitude of velocity is referred to as ***particle speed.***

Kinematics of Particles

❑ Rectilinear Motion: Position, Velocity & Acceleration

- From the definition of a derivative,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

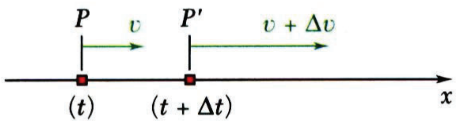


e.g., $x = 6t^2 - t^3$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

Kinematics of Particles

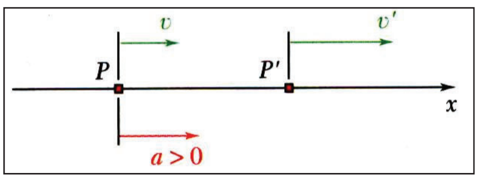
❑ Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle with velocity v at time t and $v+\Delta v$ at $t+\Delta t$,

$$Average\ acceleration = \frac{\Delta v}{\Delta t}$$

$$Instantaneous\ acceleration = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

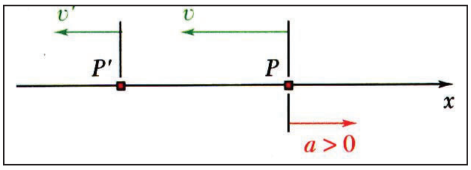


- Instantaneous acceleration may be:
  - **Positive** ($\Delta v > 0$) : increasing positive velocity

An object going right (+) and speeding up (+) has positive acceleration $\Rightarrow (+)\times(+) = (+)$

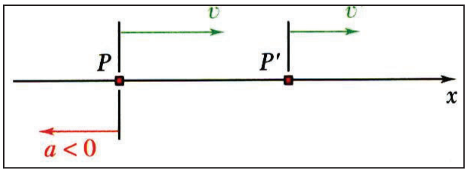
Kinematics of Particles

❑ Rectilinear Motion: Position, Velocity & Acceleration



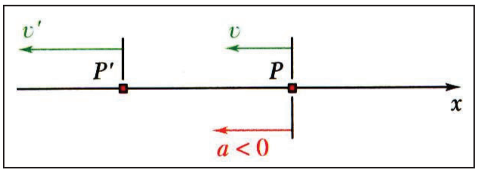
- Instantaneous acceleration may be:
  - **Positive** ($\Delta v > 0$) : decreasing negative velocity

An object going left (-) and slowing down (-) has positive acceleration $\Rightarrow (-)\times(-) = (+)$



- Instantaneous acceleration may be:
  - **Negative** ($\Delta v < 0$) : decreasing positive velocity

An object moving right (+) and slowing down (-) has negative acceleration $\Rightarrow (+)\times(-) = (-)$



- Instantaneous acceleration may be:
  - **Negative** ($\Delta v < 0$) : increasing negative velocity

An object going left (-) and speeding up (+) has negative acceleration $\Rightarrow (-)\times(+) = (-)$

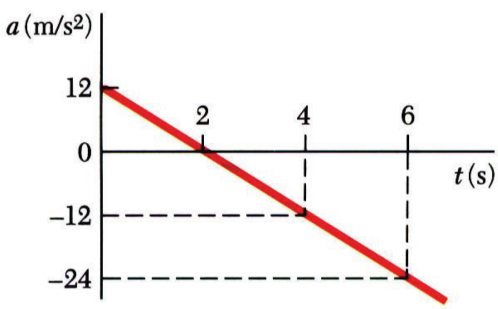
Kinematics of Particles

❑ Rectilinear Motion: Position, Velocity & Acceleration

- From the definition of a derivative,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$v = \frac{dx}{dt} \quad \Rightarrow \quad a = \frac{d^2 x}{dt^2}$$



e.g. $v = 12t - 3t^2$

$$a = \frac{dv}{dt} = 12 - 6t$$

Kinematics of Particles

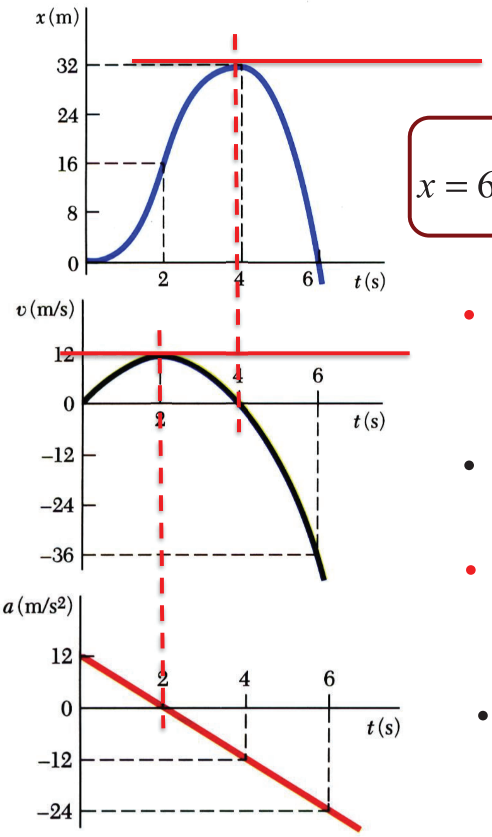
❑ Concept Quiz

What is true about the kinematics of a particle?

a) The velocity of a particle is always positive
b) The velocity of a particle is equal to the slope of the position-time graph
c) If the position of a particle is zero, then the velocity must zero
d) If the velocity of a particle is zero, then its acceleration must be zero

Kinematics of Particles

❑ Rectilinear Motion: Position, Velocity & Acceleration



- From our example,

$x = 6t^2 - t^3$ $\quad$ $v = \frac{dx}{dt} = 12t - 3t^2$ $\quad$ $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$

- What are x, v, and a at $t = 2$ s ?

  - at $t = 2$ s, $x = 16$ m, $v = v_{max} = 12$ m/s, $a = 0$

- Note that v_{max} occurs when $a = 0$, and that the slope of the velocity curve is zero at this point.
- What are x, v, and a at $t = 4$ s ?

  - at $t = 4$ s, $x = x_{max} = 32$ m, $v = 0$, $a = -12$ m/s^2

- Note that x_{max} occurs when $v = 0$, and that the slope of the position curve is zero at this point.

Kinematics of Particles

❑ Determination of the Motion of a Particle

g

Fall Free

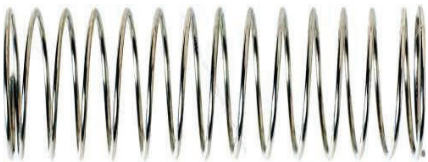
- **We often describe motion based on accelerations**

- **Generally have three classes of motion**
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = f(x)$
  - acceleration given as a function of *velocity*, $a = f(v)$

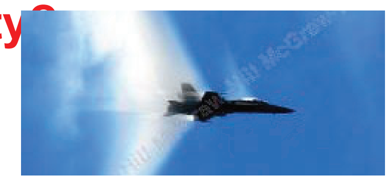
$x = f(t)$?

- Can you think of a physical example of when force is a function of position?

What force is a function of velocity?



a spring



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Kinematics of Particles

❑ Acceleration as a function of time, position, or velocity

I. Acceleration as a function of time:

$$a = \frac{dv}{dt} \Rightarrow \left.\begin{aligned} dv &= a\,dt \\ a &= a(t) \end{aligned}\right\} \Rightarrow dv = a(t)\,dt \Rightarrow \int_{v_0}^{v} dv = \int_0^t a(t)\,dt$$

$$\Rightarrow \boxed{v - v_0 = \int_0^t a(t)\,dt}$$

$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt \Rightarrow dx = \left(v_0 + \int_0^t a(t)\,dt\right)dt$$

$$\Rightarrow \int_{x_0}^{x} dx = \int_0^t \left(v_0 + \int_0^t a(t)\,dt\right)dt \Rightarrow \boxed{x - x_0 = v_0 t + \int_0^t \int_0^t a(t)\,dt\,dt}$$

The motion of a particle is known for every value of time t.

Kinematics of Particles

❑ Acceleration as a function of time, position, or velocity

II. Acceleration as a function of position:

$$\left.\begin{aligned} v = \frac{dx}{dt} \quad &\Rightarrow \quad dt = \frac{dx}{v} \\ a = \frac{dv}{dt} \quad &\Rightarrow \quad dt = \frac{dv}{a} \end{aligned}\right\} \Rightarrow \frac{dx}{v} = \frac{dv}{a} \Rightarrow \left.\begin{aligned} v\,dv = a\,dx \\ a = a(x) \end{aligned}\right\} \Rightarrow v\,dv = a(x)\,dx$$

$$\Rightarrow \int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx \Rightarrow \boxed{\frac{1}{2}v^2 - \frac{1}{2}v_0^{\,2} = \int_{x_0}^{x} a(x)\,dx}$$

$$v = \frac{dx}{dt} \Rightarrow dt = \frac{dx}{v} \Rightarrow dt = \frac{dx}{\sqrt{v_0^{\,2} + 2\int_{x_0}^{x} a(x)\,dx}} \Rightarrow \int_0^t dt = \int_{x_0}^{x} \frac{dx}{\sqrt{v_0^{\,2} + 2\int_{x_0}^{x} a(x)\,dx}}$$

$$\Rightarrow \boxed{t = \int_{x_0}^{x} \frac{dx}{\sqrt{v_0^{\,2} + 2\int_{x_0}^{x} a(x)\,dx}}}$$

The motion of a particle is known for every value of time t.

Kinematics of Particles

❑ Acceleration as a function of time, position, or velocity

III. Acceleration as a function of velocity:

a)

$$\left.\begin{aligned} a = \frac{dv}{dt} \quad \Rightarrow \quad dt &= \frac{dv}{a} \\ a &= a(v) \end{aligned}\right\} \Rightarrow dt = \frac{dv}{a(v)} \Rightarrow \int_0^t dt = \int_{v_0}^{v} \frac{dv}{a(v)} \Rightarrow \boxed{t = \int_{v_0}^{v} \frac{dv}{a(v)}}$$

$$\Rightarrow \boxed{v = v(t)}$$

$$\left.\begin{aligned} v = \frac{dx}{dt} \quad \Rightarrow \quad dx &= v\,dt \\ v &= v(t) \end{aligned}\right\} \Rightarrow dx = v(t)\,dt \Rightarrow \int_{x_0}^{x} dx = \int_{t_0}^{t} v(t)\,dt$$

$$\Rightarrow \boxed{x - x_0 = \int_{t_0}^{t} v(t)\,dt}$$

The motion of a particle is known for every value of time t.

Kinematics of Particles

❑ Acceleration as a function of time, position, or velocity

III. Acceleration as a function of velocity:

b)

$$\left.\begin{array}{l} v = \dfrac{dx}{dt} \;\Rightarrow\; dt = \dfrac{dx}{v} \\ a = \dfrac{dv}{dt} \;\Rightarrow\; dt = \dfrac{dv}{a} \end{array}\right\} \Rightarrow \frac{dx}{v} = \frac{dv}{a} \Rightarrow \left.\begin{array}{l} dx = \dfrac{v}{a} dv \\ a = a(v) \end{array}\right\} \Rightarrow dx = \frac{v}{a(v)} dv$$

$$\Rightarrow \int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v}{a(v)} dv \Rightarrow \boxed{x - x_0 = \int_{v_0}^{v} \frac{v}{a(v)} dv} \Rightarrow \boxed{v = v(x)}$$

$$\left.\begin{array}{l} v = \dfrac{dx}{dt} \;\Rightarrow\; dt = \dfrac{dx}{v} \\ v = v(x) \end{array}\right\} \Rightarrow dt = \frac{dx}{v(x)} \Rightarrow \int_0^t dt = \int_{x_0}^{x} \frac{dx}{v(x)} \Rightarrow \boxed{t = \int_{x_0}^{x} \frac{dx}{v(x)}}$$

The motion of a particle is known for every value of time t.

Kinematics of Particles

❑ Acceleration as a function of time, position, or velocity

If….	Kinematic relationship	Integrate
$a = a(t)$	$dv = a(t)\,dt$ $dx = v\,dt$	$v - v_0 = \int_0^t a(t)\,dt$ $x - x_0 = v_0 t + \int_0^t \int_0^t a(t)\,dt dt$
$a = a(x)$	$v\,dv = a(x)\,dx$ $dt = \dfrac{dx}{v}$	$\frac{1}{2}v^2 - \frac{1}{2}{v_0}^2 = \int_{x_0}^{x} a(x)\,dx$ $t = \int_{x_0}^{x} \dfrac{dx}{\sqrt{{v_0}^2 + 2\int_{x_0}^{x} a(x)\,dx}}$
$a = a(v)$	$dt = \dfrac{dv}{a(v)}$ $dx = v(t)\,dt$	$t = \int_{v_0}^{v} \dfrac{dv}{a(v)} \Rightarrow v = v(t)$ $x - x_0 = \int_{t_0}^{t} v(t)\,dt$
	$dx = \dfrac{v}{a(v)} dv$ $dt = \dfrac{dx}{v(x)}$	$x - x_0 = \int_{v_0}^{v} \dfrac{v}{a(v)} dv \Rightarrow v = v(x)$ $t = \int_{x_0}^{x} \dfrac{dx}{v(x)}$

Kinematics of Particles

❑ Sample Problem 01

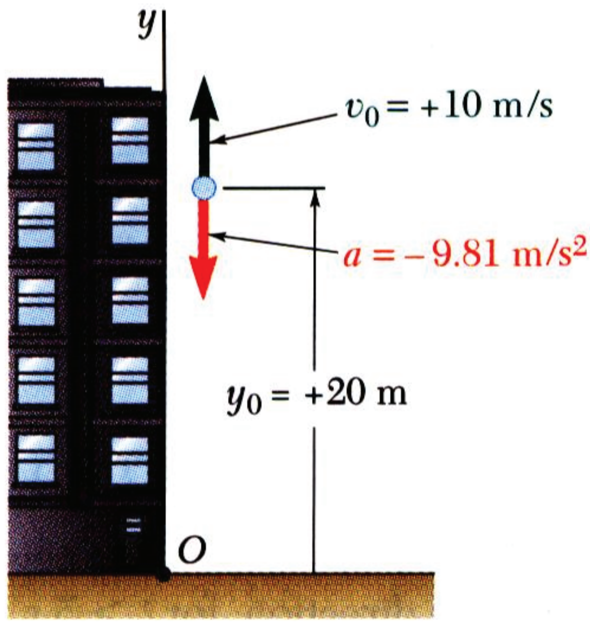
Ball tossed with 10 m/s vertical velocity from window 20 m above ground.

Determine:

a) velocity and elevation above ground at time *t*,

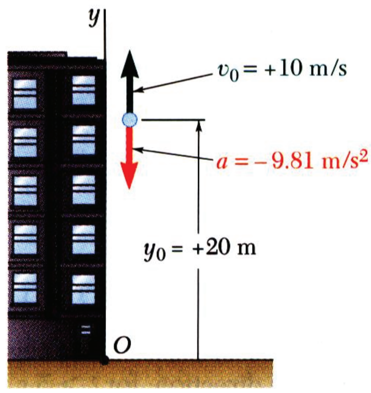
b) highest elevation reached by ball and corresponding time, and

c) time when ball will hit the ground and corresponding velocity.



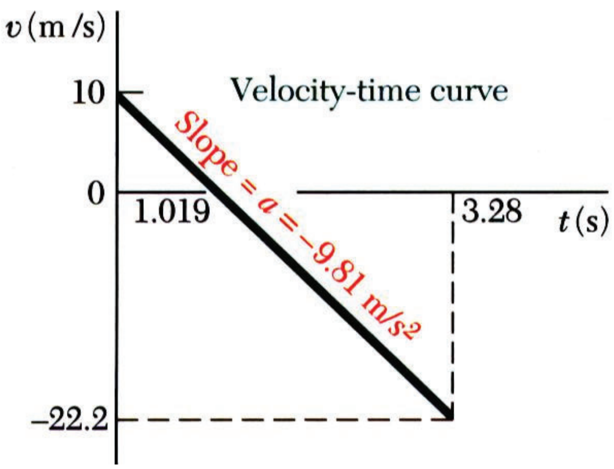
Kinematics of Particles

❑ Sample Problem 01



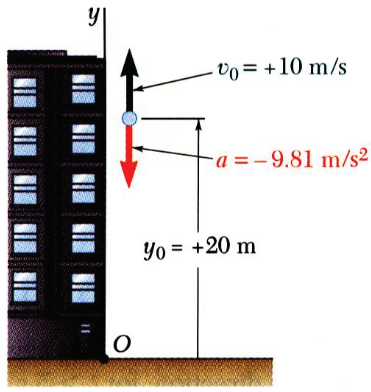
SOLUTION:

(a):



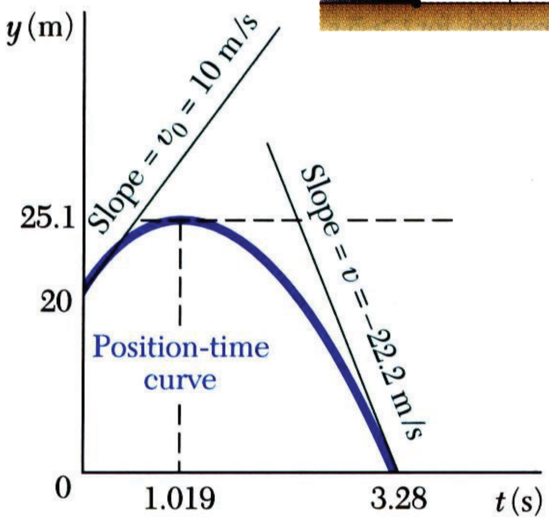
Kinematics of Particles

❑ Sample Problem 01



SOLUTION:

(a):



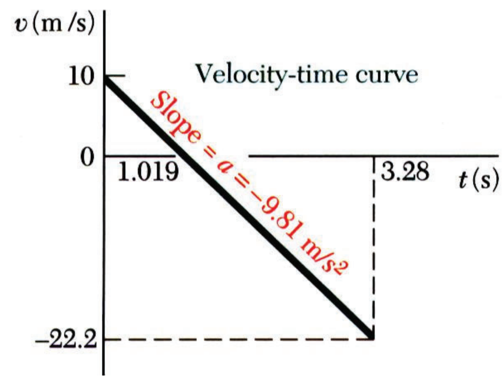
Kinematics of Particles

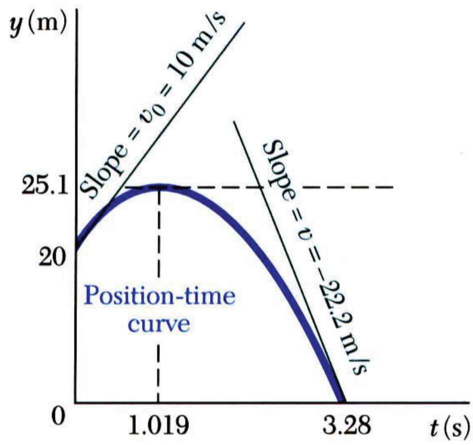
❑ Sample Problem 01

SOLUTION:

(b):

y_{max} occurs when v=0, and that the slope of the position curve is zero at this point.





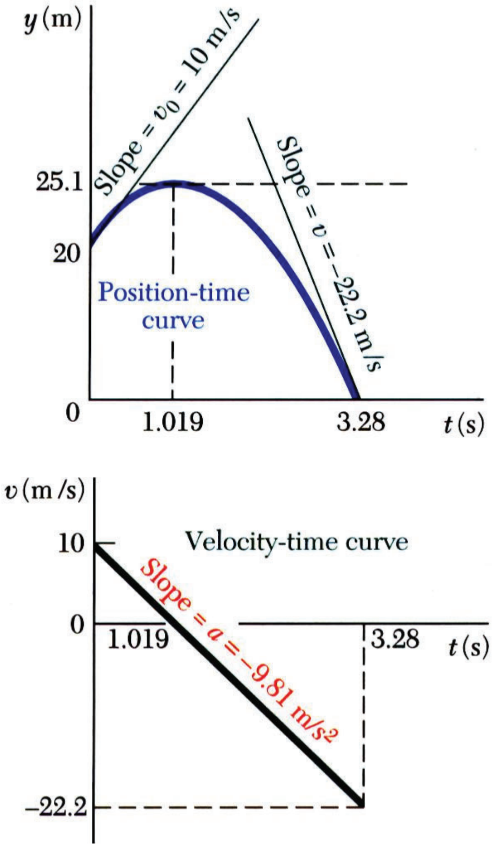
Kinematics of Particles

❑ Sample Problem 01

SOLUTION:

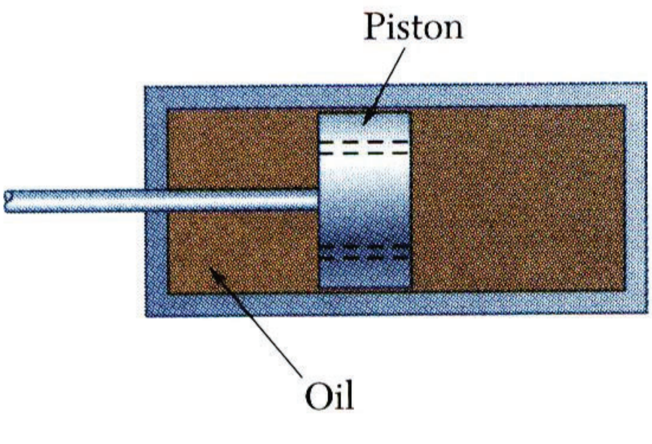
(c):

- Solve for *t* when altitude equals zero and evaluate corresponding velocity.



Kinematics of Particles

❑ Sample Problem 02



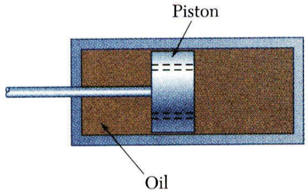
Brake mechanism used to reduce gun recoil consists of piston attached to barrel moving in fixed cylinder filled with oil. As barrel recoils with initial velocity v_0, piston moves and oil is forced through orifices in piston, causing piston and cylinder to decelerate at rate proportional to their velocity. $a = -kv$

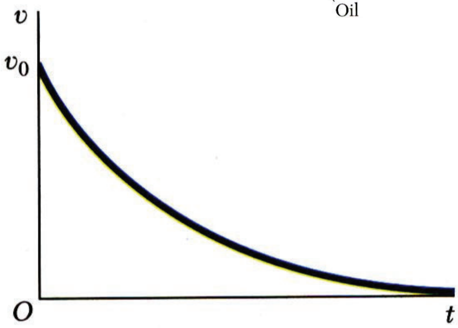
Determine $v(t)$, $x(t)$, and $v(x)$.

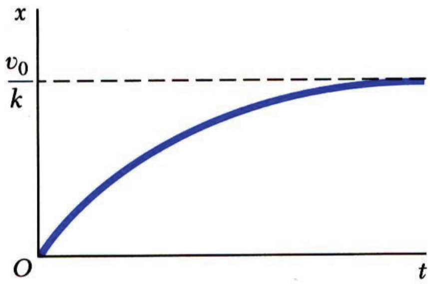
Kinematics of Particles

❑ Sample Problem 02

SOLUTION:



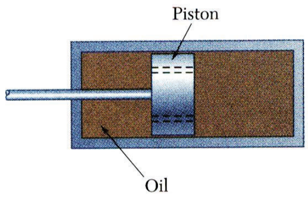


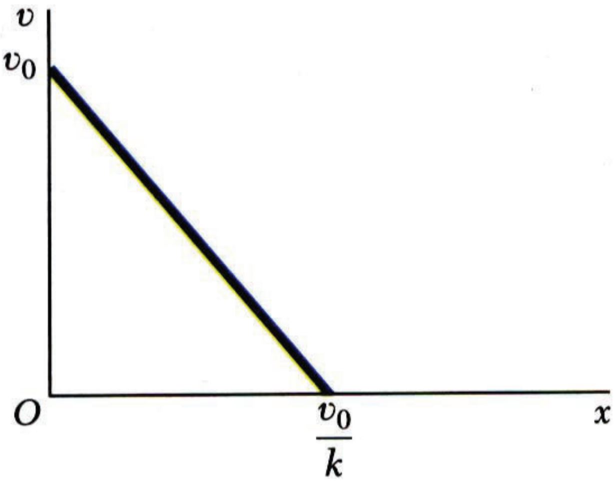


Kinematics of Particles

❑ Sample Problem 02

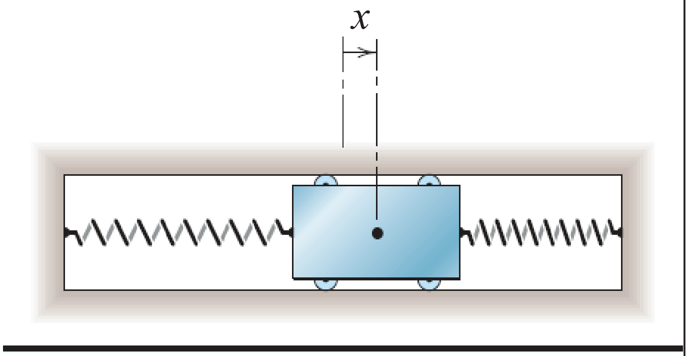
SOLUTION:





Kinematics of Particles

❑ Sample Problem 03



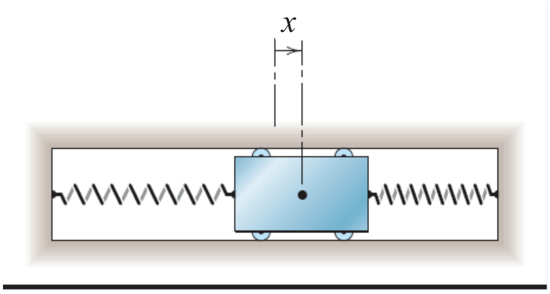
The spring-mounted slider moves in the horizontal guide with negligible friction and has a velocity v_0 in the s-direction as it crosses the mid-position where x=0 and t=0. The two springs together exert a retarding force to the motion of the slider, which gives it an acceleration proportional to the displacement but oppositely directed and equal to $a = -k^2 x$, where k is constant.

Determine the expressions for the displacement and velocity as functions of the time.

Kinematics of Particles

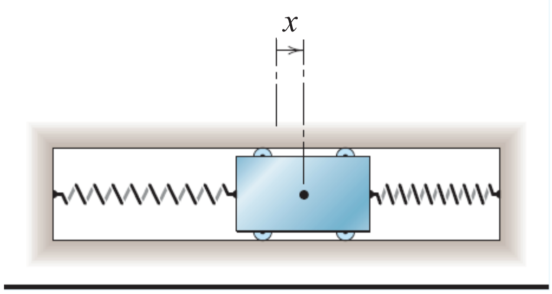
❑ Sample Problem 03

SOLUTION:

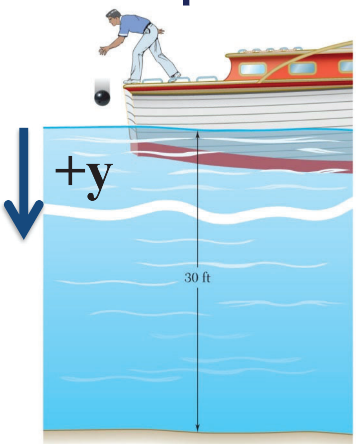


❑ Sample Problem 03

SOLUTION:



❑ Group Problem Solving



A bowling ball is dropped from a boat so that it strikes the surface of a lake with a speed of 15 ft/s. Assuming the ball experiences a downward acceleration of $a = 10 - 0.01v^2$ when in the water, determine the velocity of the ball when it strikes the bottom of the lake.

Which integral should you choose?

(a) $\int_{v_0}^{v} dv = \int_{0}^{t} a(t)\,dt$

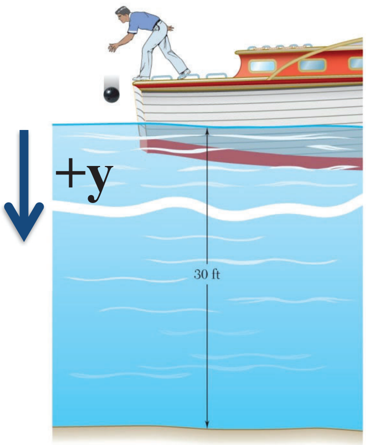
(c) $\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$

(b) $\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{a(v)}$

(d) $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_{0}^{t} dt$

❑ Concept Question

When will the bowling ball start slowing down?



A bowling ball is dropped from a boat so that it strikes the surface of a lake with a speed of 15 ft/s. Assuming the ball experiences a downward acceleration of $a = 10 - 0.01v^2$ when in the water, determine the velocity of the ball when it strikes the bottom of the lake.

❑ Sample Problem 04

The car starts from rest and accelerates according to the relationship

$$a = 3 - 0.001v^2$$

It travels around a circular track that has a radius of 200 meters. Calculate the velocity of the car after it has travelled halfway around the track. What is the car's maximum possible speed?

Kinematics of Particles

❑ Sample Problem 04

SOLUTION:

Given: $a = 3 - 0.001v^2$

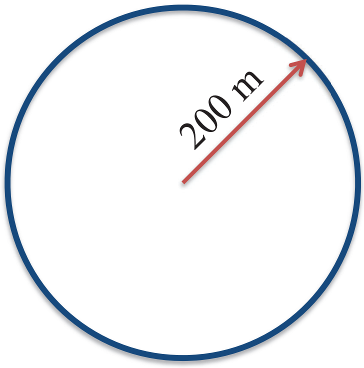
$v_o = 0,\ r = 200$ m

Find: v after ½ lap

Maximum speed

Choose the proper kinematic relationship

Acceleration is a function of velocity, and we also can determine distance. Time is not involved in the problem, so we choose:



Determine total distance travelled

Kinematics of Particles

❑ Sample Problem 04

SOLUTION:

Determine the full integral, including limits

Take the exponential of each side

Kinematics of Particles

❑ Sample Problem 04

SOLUTION:

Solve for v $$3 - 0.001v^2 = e^{-0.15802}$$

How do you determine the maximum speed the car can reach?

Velocity is a maximum when acceleration is zero

Kinematics of Particles

❑ Uniform Rectilinear Motion

During free-fall, a parachutist reaches terminal velocity when her weight equals the drag force. If motion is in a straight line, this is uniform rectilinear motion.

For a particle in uniform rectilinear motion, the acceleration is zero and the velocity is constant.

$$\frac{dx}{dt} = v = \text{constant}$$

$$\Rightarrow \int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$\Rightarrow \quad x - x_0 = vt$$

$$\Rightarrow \quad \boxed{x = x_0 + vt}$$

Careful – these only apply to uniform rectilinear motion!

Kinematics of Particles

❑ Uniformly Accelerated Rectilinear Motion

If forces applied to a body are constant (and in a constant direction), then you have uniformly accelerated rectilinear motion.

Another example is free-fall when drag is negligible

Kinematics of Particles

❑ Uniformly Accelerated Rectilinear Motion

For a particle in uniformly accelerated rectilinear motion, the acceleration of the particle is constant.

$$\frac{dv}{dt} = a = cte \quad \Rightarrow \quad dv = a\,dt \quad \Rightarrow \quad \int_{v_0}^{v} dv = a\int_0^t dt \quad \Rightarrow \quad v - v_0 = at \quad \Rightarrow \quad \boxed{v = v_0 + at}$$

$$\frac{dx}{dt} = v \quad \Rightarrow \quad \frac{dx}{dt} = v_0 + at \quad \Rightarrow \quad dx = (v_0 + at)dt \quad \Rightarrow \quad \int_{x_0}^{x} dx = \int_0^t (v_0 + at)dt$$

$$\Rightarrow \quad x - x_0 = v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad \boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

$$v\frac{dv}{dx} = a = cte \quad \Rightarrow \quad v\,dv = a\,dx \quad \Rightarrow \quad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \quad \Rightarrow \quad \frac{1}{2}v^2 - \frac{1}{2}{v_0}^2 = a(x - x_0)$$

$$\Rightarrow \quad \boxed{v^2 = {v_0}^2 + 2a(x - x_0)}$$

Kinematics of Particles

❑ Uniformly Accelerated Rectilinear Motion

Careful – these only apply to uniformly accelerated rectilinear motion!

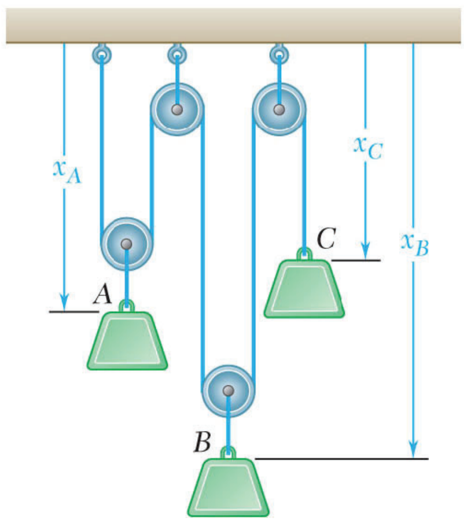
$$a = cte$$

Relate velocity to time	$v = v(t)$	$v = v_0 + at$
Relate position to time	$x = x(t)$	$x = x_0 + v_0 t + \frac{1}{2}at^2$
Relate velocity to Position	$v = v(x)$	$v^2 = v_0{}^2 + 2a(x - x_0)$

Kinematics of Particles

❑ Motion of Several Particles

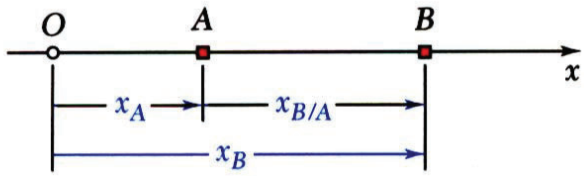
We may be interested in the motion of several different particles, whose motion may be independent or linked together.



Kinematics of Particles

❑ Motion of Several Particles: Relative Motion

- For particles moving along the same line, time should be recorded from the same starting instant and displacements should be measured from the same origin in the same direction.



$x_{B/A} = x_B - x_A =$ relative position of B with respect to A

$$x_B = x_A + x_{B/A}$$

$x_{B/A} > 0$

Particle B at right hand side of Particle A

$v_{B/A} = v_B - v_A =$ relative velocity of B with respect to A

$$v_B = v_A + v_{B/A}$$

$v_{B/A} > 0$

An observer at point A, see the particle B which increases distance from A.

$a_{B/A} = a_B - a_A =$ relative acceleration of B with respect to A

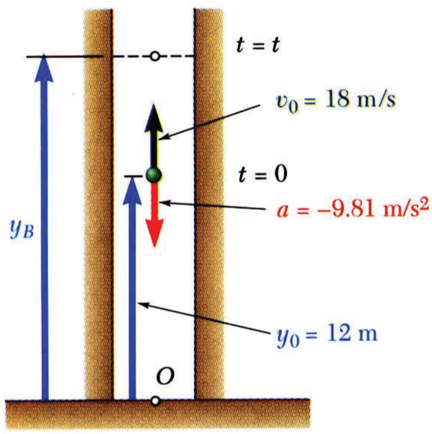
$$a_B = a_A + a_{B/A}$$

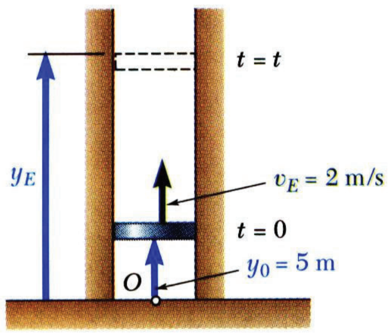
Kinematics of Particles

❑ Sample Problem 05

Ball thrown vertically from 12 m level in elevator shaft with initial velocity of 18 m/s. At same instant, open-platform elevator passes 5 m level moving upward at 2 m/s.

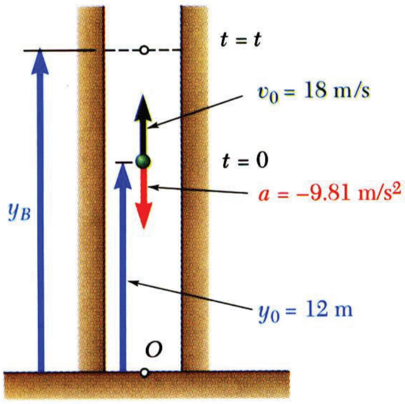
Determine *(a)* when and where ball hits elevator and *(b)* relative velocity of ball and elevator at contact.

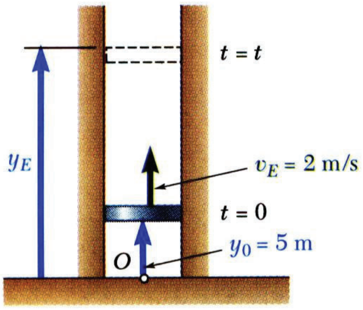




Kinematics of Particles

❑ Sample Problem 05





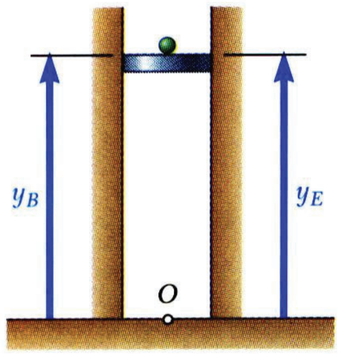
SOLUTION:

- **Uniformly accelerated rectilinear motion**: Substitute initial position and velocity and constant acceleration of ball into general equations

- **Uniform rectilinear motion** Substitute initial position and constant velocity of elevator into equation for.

Kinematics of Particles

❑ Sample Problem 05



SOLUTION:

(a):

- Write equation for relative position of ball with respect to elevator and solve for zero relative position, i.e., impact.

- Substitute impact time into equations for position of elevator

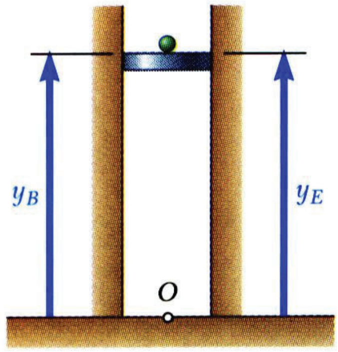
Kinematics of Particles

❑ Sample Problem 05

SOLUTION:

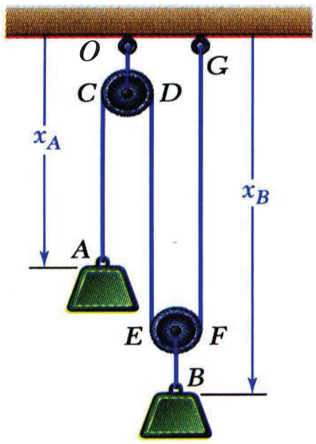
(b):

- Substitute impact time into equations for relative velocity of ball with respect to elevator.



Kinematics of Particles

❑ Motion of Several Particles: Dependent Motion



- Position of a particle may *depend* on position of one or more other particles.
- Position of block *B* depends on position of block *A*. Since rope is of constant length, it follows that sum of lengths of segments must be constant.

$$l_{AC} + l_{DE} + l_{FG} = l_{Total} = cte$$

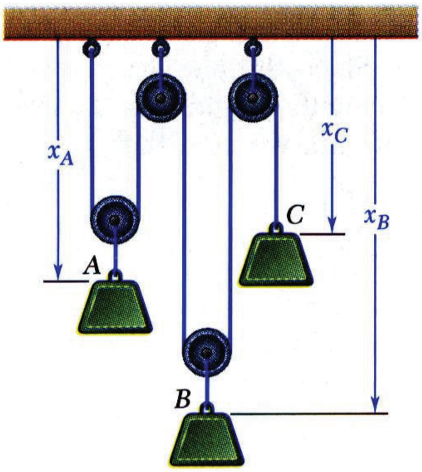
$$\Rightarrow \quad (x_A - OC) + (x_B - OC - FB) + (x_B - FB) = l_{Total}$$

$$\Rightarrow \quad x_A + 2x_B = l_{Total} + 2OC + 2FB = cte \;\Rightarrow\; \boxed{x_A + 2x_B = cte}$$ (one degree of freedom)

$$x_A + 2x_B = cte \quad \Rightarrow \quad \begin{cases} \dfrac{dx_A}{dt} + 2\dfrac{dx_B}{dt} = 0 \quad \Rightarrow \quad \boxed{v_A + 2v_B = 0} \\ \dfrac{dv_A}{dt} + 2\dfrac{dv_B}{dt} = 0 \quad \Rightarrow \quad \boxed{a_A + 2a_B = 0} \end{cases}$$

Kinematics of Particles

❑ Motion of Several Particles: Dependent Motion



- Positions of three blocks are dependent.

$$2x_A + 2x_B + x_C = cte$$

(two degrees of freedom)

- For linearly related positions, similar relations hold between velocities and accelerations.

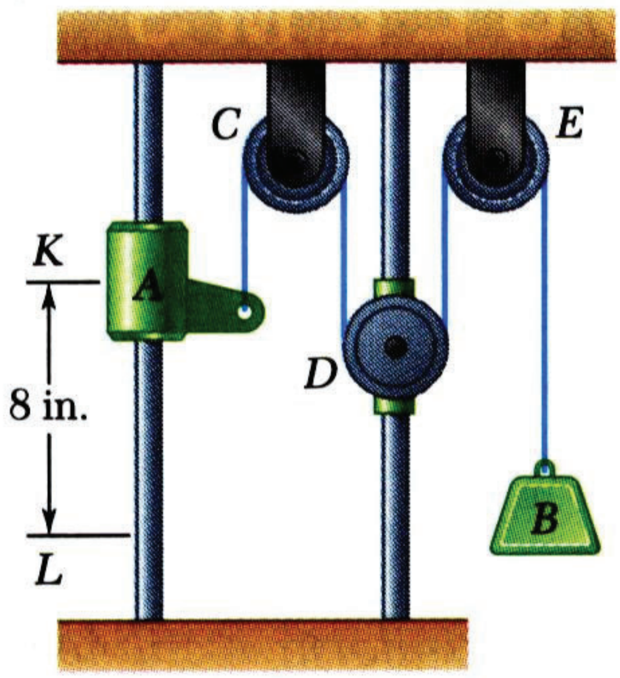
$$2\frac{dx_A}{dt} + 2\frac{dx_B}{dt} + \frac{dx_C}{dt} = 0 \quad \text{or} \quad 2v_A + 2v_B + v_C = 0$$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \quad \text{or} \quad 2a_A + 2a_B + a_C = 0$$

Kinematics of Particles

❑ Sample Problem 06

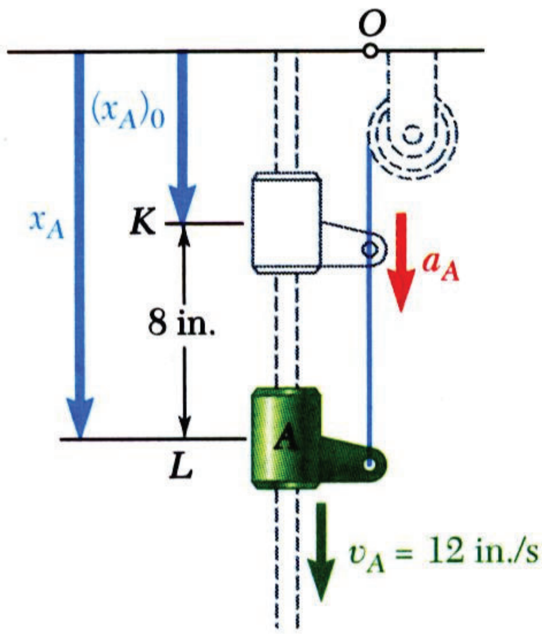
Pulley D is attached to a collar which is pulled down at 3 in./s. At $t = 0$, collar A starts moving down from K with constant acceleration and zero initial velocity. Knowing that velocity of collar A is 12 in./s as it passes L, determine the change in elevation, velocity, and acceleration of block B when block A is at L.



❑ Sample Problem 06

SOLUTION:

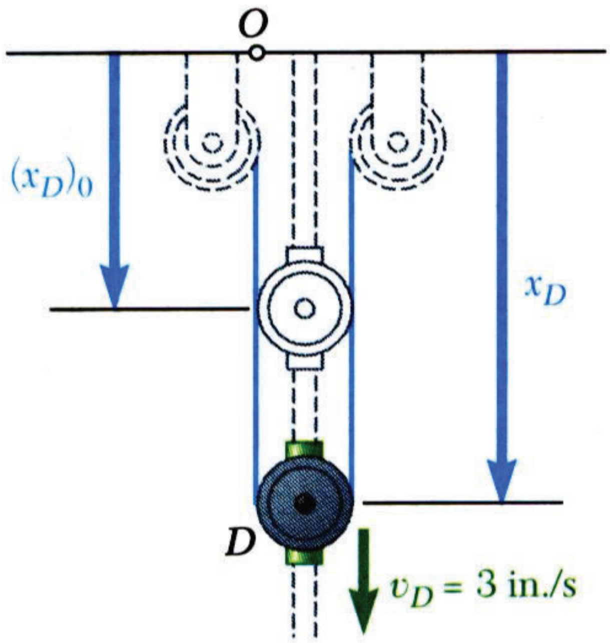
- Define origin at upper horizontal surface with positive displacement downward.
- Collar A has uniformly accelerated rectilinear motion. Solve for acceleration and time t to reach L.



❑ Sample Problem 06

SOLUTION:

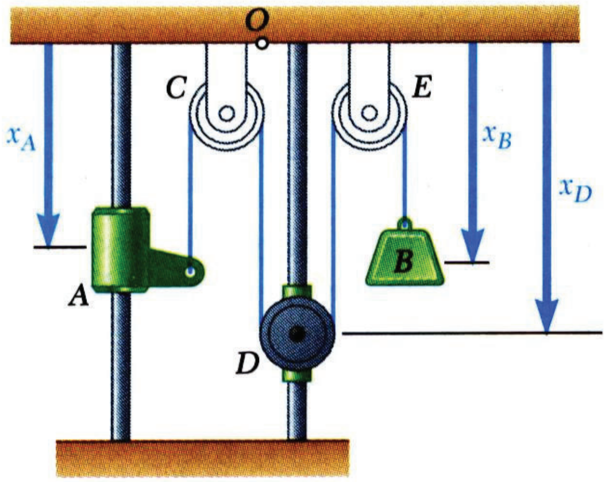
- Pulley D has uniform rectilinear motion. Calculate change of position at time t.



❑ Sample Problem 06

SOLUTION:

- Block *B* motion is dependent on motions of collar *A* and pulley *D*. Write motion relationship and solve for change of block *B* position at time *t*.

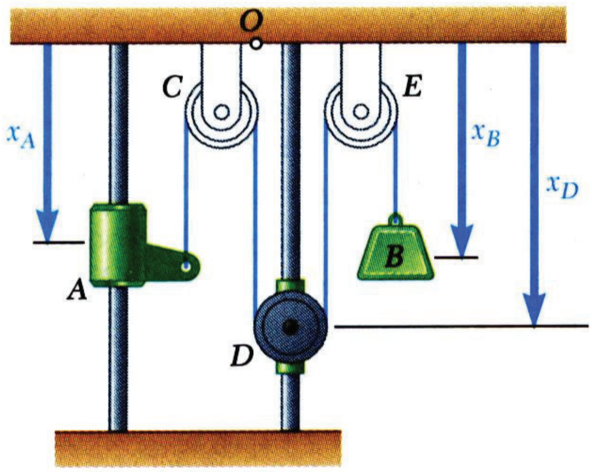


Total length of cable remains constant,

❑ Sample Problem 06

SOLUTION:

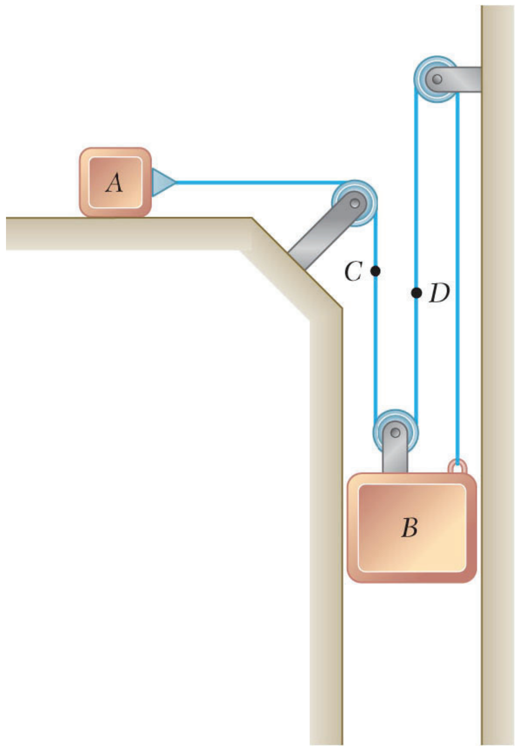
- Differentiate motion relation twice to develop equations for velocity and acceleration of block *B*.



Kinematics of Particles

❑ Sample Problem 07

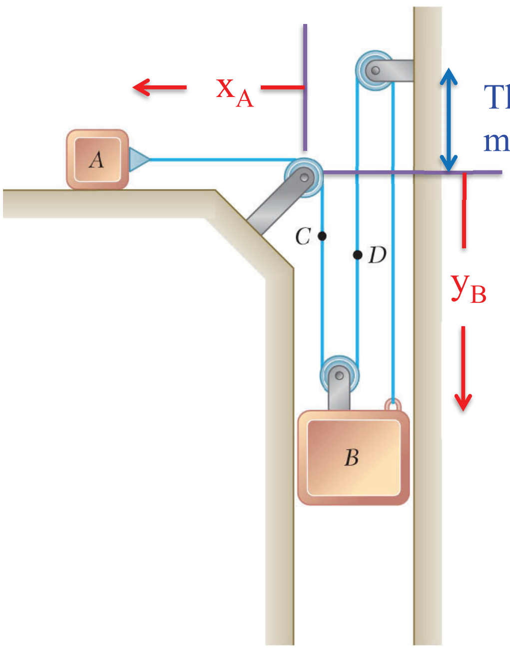
Slider block *A* moves to the left with a constant velocity of 6 m/s. Determine the velocity of block *B*.



Kinematics of Particles

❑ Sample Problem 07

SOLUTION:



Given: v_A = 6 m/s left Find: v_B

This length is constant no matter how the blocks move

Sketch your system and choose coordinates

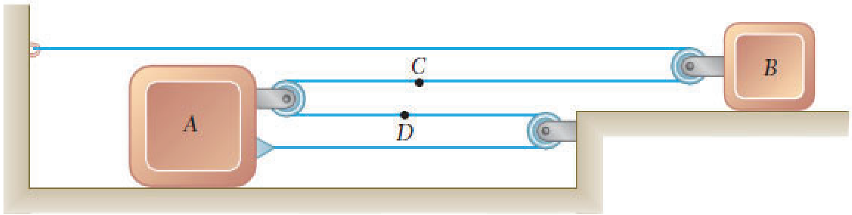
Define your constraint equation(s)

Differentiate the constraint equation to get velocity

Note that as x_A gets bigger, y_B gets smaller.

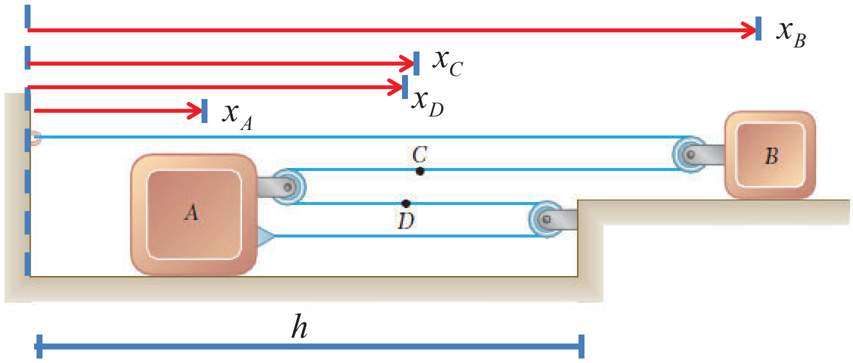
❑ Sample Problem 08

Slider block B moves to the right with a constant velocity of 300 mm/s. Determine (a) the velocity of slider block A, (b) the velocity of portion C of the cable, (c) the velocity of portion D of the cable, (d) the relative velocity of portion C of the cable with respect to slider block A.



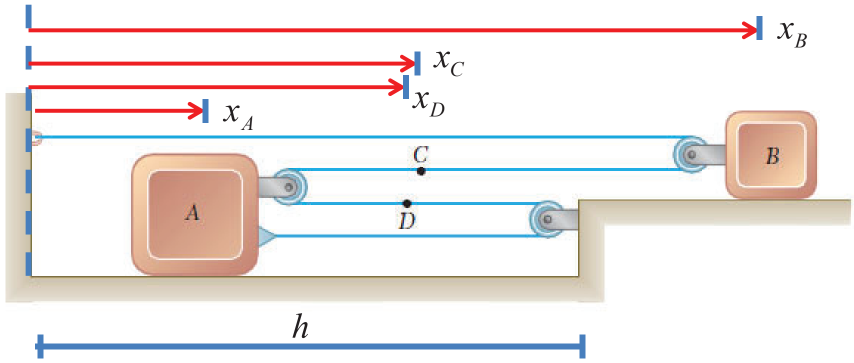
❑ Sample Problem 08

SOLUTION:

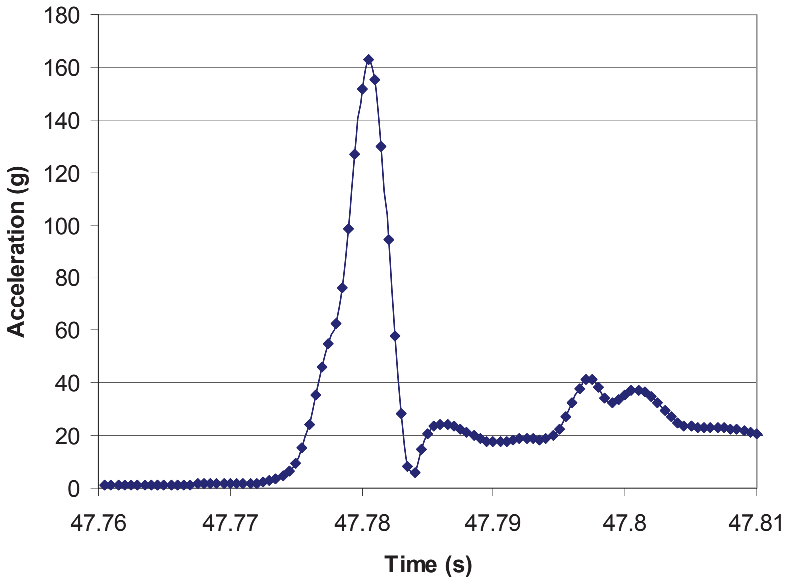


❑ Sample Problem 08

SOLUTION:



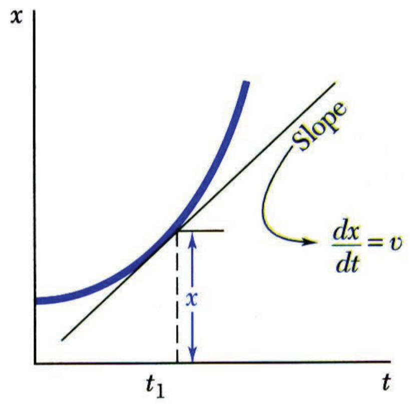
❑ Graphical Solution of Rectilinear-Motion Problems



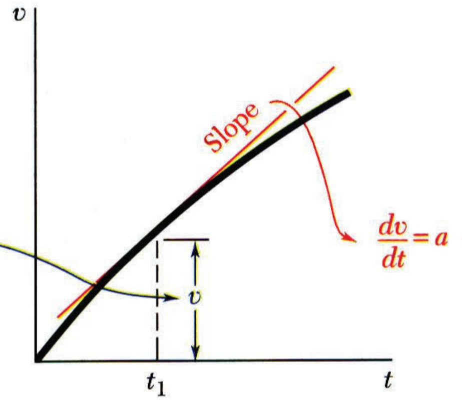
Acceleration data from a head impact during a round of boxing.

Kinematics of Particles

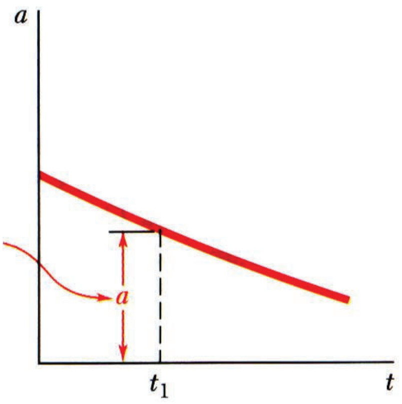
❑ Graphical Solution of Rectilinear-Motion Problems



The x-t curve



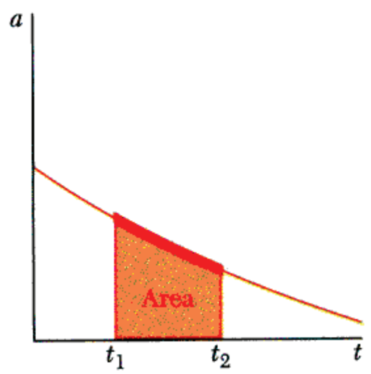
Given the x-t curve, the v-t curve is equal to the x-t curve slope.



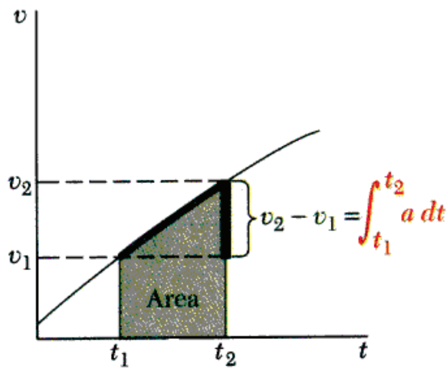
Given the v-t curve, the a-t curve is equal to the v-t curve slope.

Kinematics of Particles

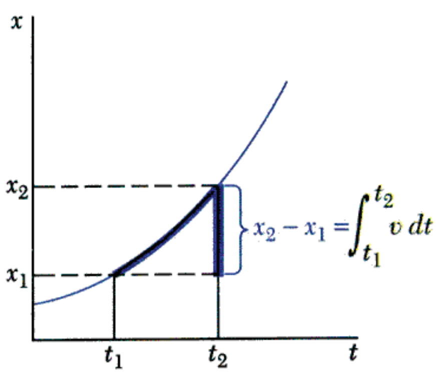
❑ Graphical Solution of Rectilinear-Motion Problems



The a-t curve



Given the a-t curve, the change in velocity between t_1 and t_2 is equal to the area under the a-t curve between t_1 and t_2.

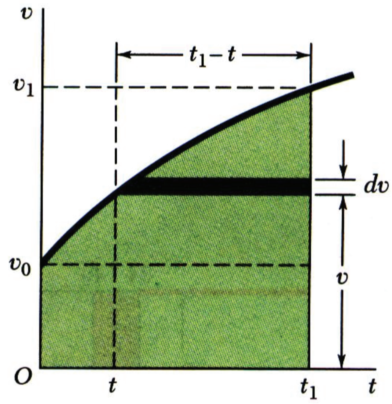


Given the v-t curve, the change in position between t_1 and t_2 is equal to the area under the v-t curve between t_1 and t_2.

Kinematics of Particles

❑ Other Graphical Methods

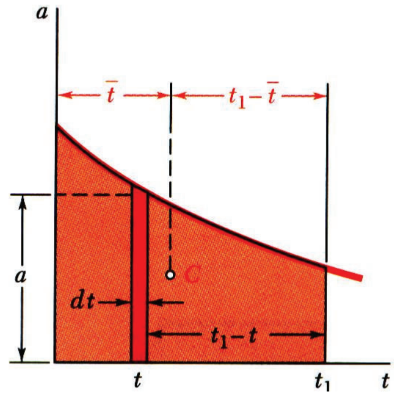
Moment-area method to determine particle position at time t directly from the a-t curve:



$$x_1 - x_0 = \text{area under } v-t \text{ curve} \Rightarrow x_1 - x_0 = v_0 t_1 + \int_{v_0}^{v_1} (t_1 - t) dv$$

$$\text{using } dv = a\,dt, \Rightarrow x_1 - x_0 = v_0 t_1 + \int_0^{t_1} (t_1 - t) a\,dt$$

$$\int_0^{t_1} (t_1 - t) a\,dt = \text{first moment of area under } a\text{-}t \text{ curve with respect to } t = t_1 \text{ line.}$$



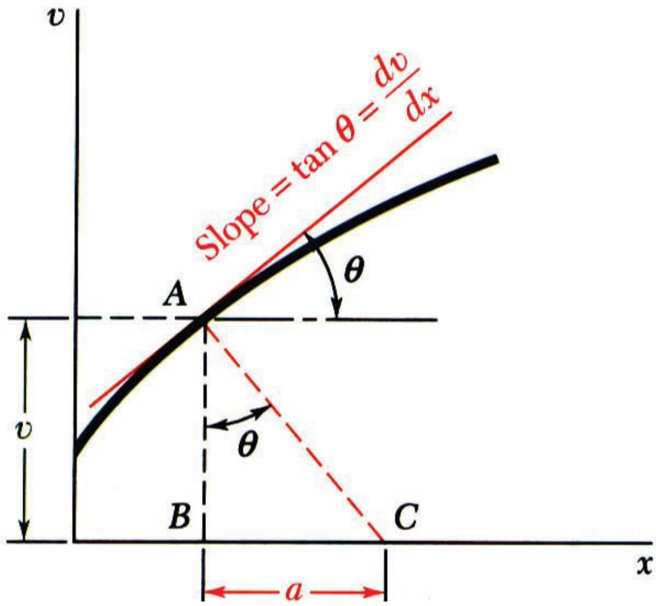
$$x_1 = x_0 + v_0 t_1 + (\text{area under } a\text{-}t \text{ curve})(t_1 - \bar{t})$$
$$\bar{t} = \text{abscissa of centroid } C$$

Kinematics of Particles

❑ Other Graphical Methods

- **Method to determine particle acceleration from *v-x* curve:**

$$a = v\frac{dv}{dx} = AB \tan\theta = BC$$



Kinematics of Particles

❑ Curvilinear Motion: Position, Velocity & Acceleration

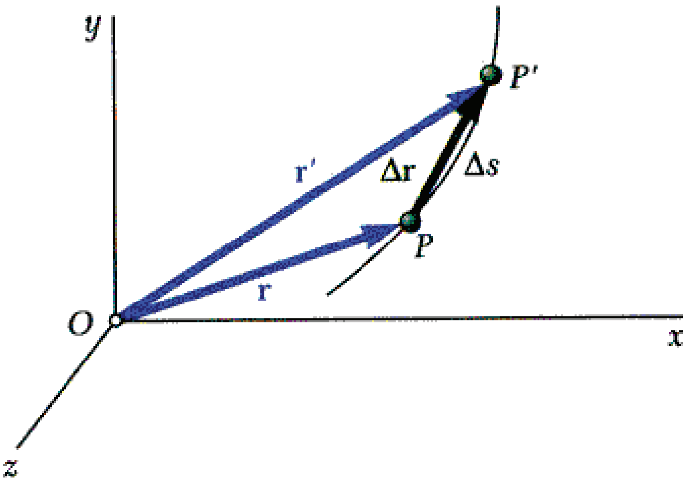
- A particle moving along a curve other than a straight line is in ***curvilinear motion***.

The softball and the car both undergo curvilinear motion.

Kinematics of Particles

❑ Curvilinear Motion: Position, Velocity & Acceleration

- The ***position vector*** of a particle at time t is defined by a vector between origin O of a fixed reference frame and the position occupied by particle.

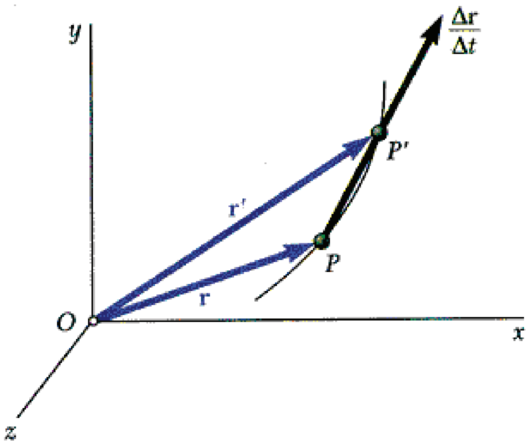


- Consider a particle which occupies position P defined by $\vec{r}$ at time t and P' defined by $\vec{r}\,'$ at $t + \Delta t$,

Kinematics of Particles

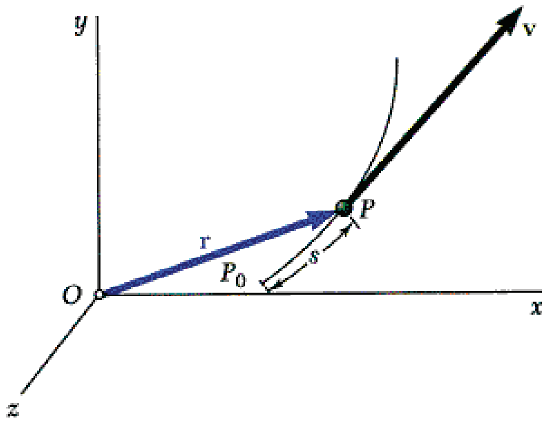
❑ Curvilinear Motion: Position, Velocity & Acceleration

Instantaneous velocity
(***vector***)



$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Instantaneous speed
(***scalar***)

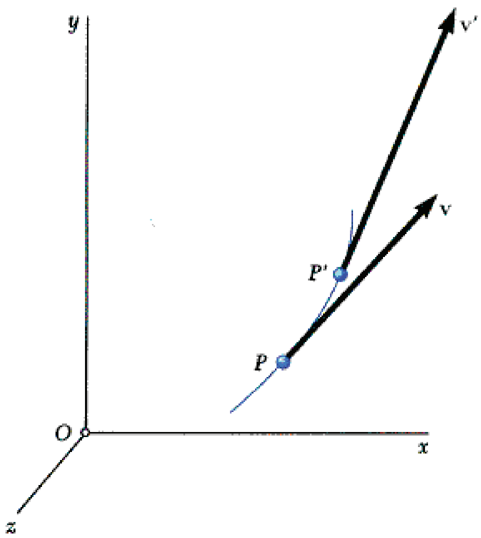


$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

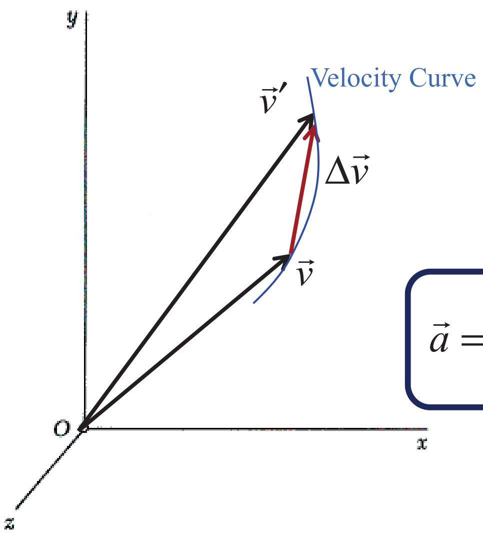
Kinematics of Particles

❑ Curvilinear Motion: Position, Velocity & Acceleration

- Consider velocity $\vec{v}$ of a particle at time t and velocity $\vec{v}'$ at $t + \Delta t$,



instantaneous acceleration (***vector***)

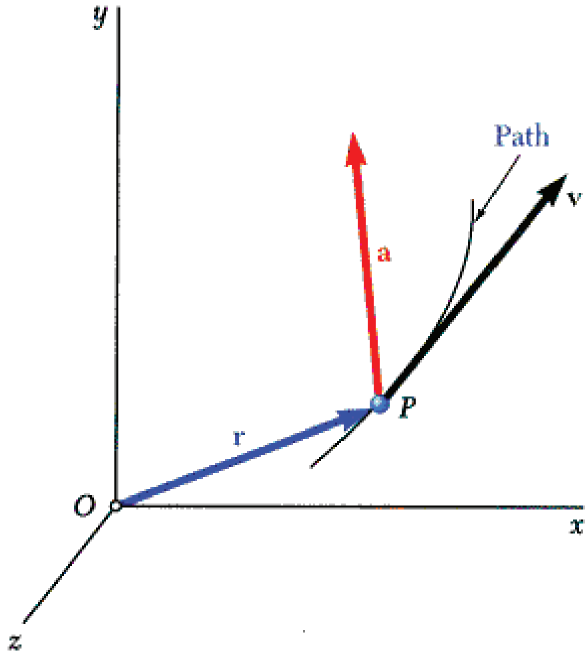


$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Kinematics of Particles

❑ Curvilinear Motion: Position, Velocity & Acceleration

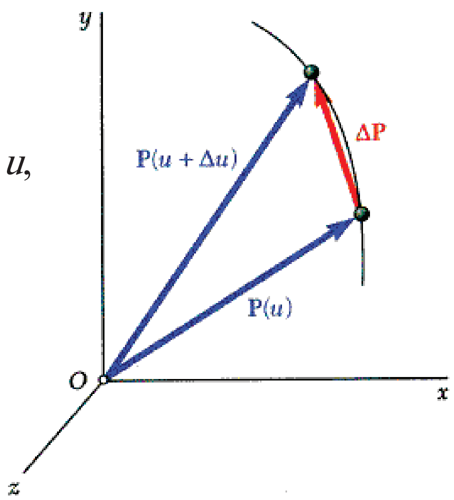
- In general, the acceleration vector is not tangent to the particle path and velocity vector.

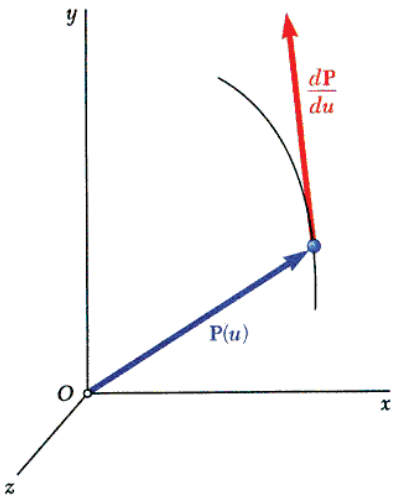


Kinematics of Particles

❑ Derivatives of Vector Functions

- Let $\vec{P}(u)$ be a vector function of scalar variable u,





$$\frac{d\vec{P}}{du} = \lim_{\Delta u \to 0} \frac{\Delta \vec{P}}{\Delta u} = \lim_{\Delta u \to 0} \frac{\vec{P}(u + \Delta u) - \vec{P}(u)}{\Delta u}$$

Kinematics of Particles

❑ Derivatives of Vector Functions

- Derivative of vector sum,

$$\frac{d(\vec{P}+\vec{Q})}{du}=\frac{d\vec{P}}{du}+\frac{d\vec{Q}}{du}$$

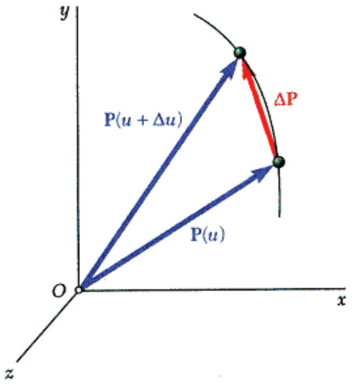
- Derivative of product of scalar and vector functions,

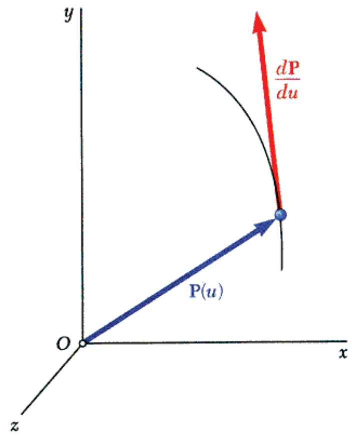
$$\frac{d(f\vec{P})}{du}=\frac{df}{du}\vec{P}+f\frac{d\vec{P}}{du}$$

- Derivative of *scalar product* and *vector product,*

$$\frac{d(\vec{P}\bullet\vec{Q})}{du}=\frac{d\vec{P}}{du}\bullet\vec{Q}+\vec{P}\bullet\frac{d\vec{Q}}{du}$$

$$\frac{d(\vec{P}\times\vec{Q})}{du}=\frac{d\vec{P}}{du}\times\vec{Q}+\vec{P}\times\frac{d\vec{Q}}{du}$$





Kinematics of Particles

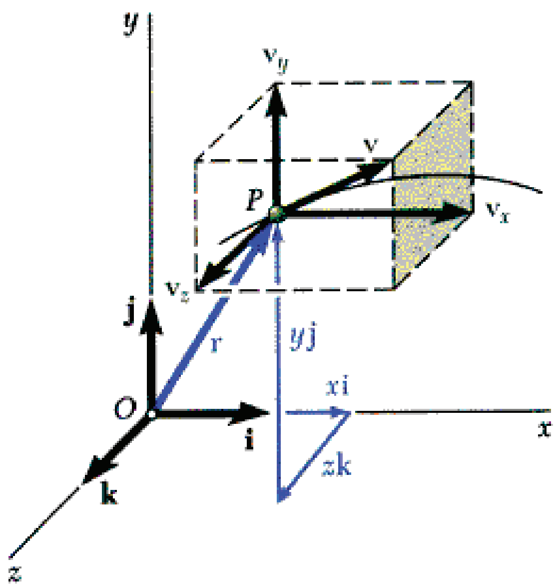
❑ Rectangular Components of Velocity & Acceleration

- When position vector of particle *P* is given by its rectangular components,

$$\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$$

- Velocity vector,

$$\vec{v}=\frac{dx}{dt}\vec{i}+\frac{dy}{dt}\vec{j}+\frac{dz}{dt}\vec{k}=\dot{x}\vec{i}+\dot{y}\vec{j}+\dot{z}\vec{k}$$
$$=v_x\vec{i}+v_y\vec{j}+v_z\vec{k}$$



Kinematics of Particles

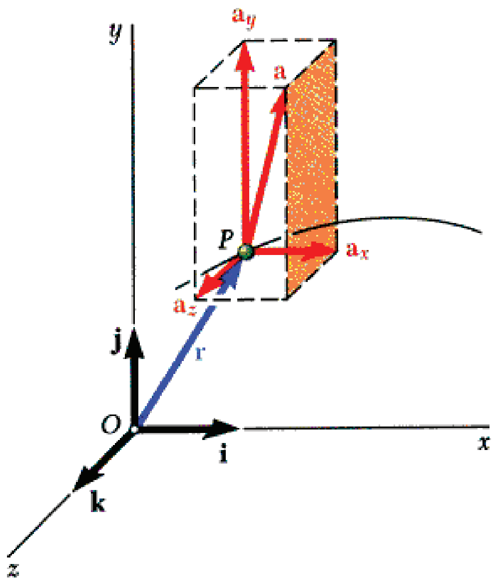
❑ Rectangular Components of Velocity & Acceleration

- When position vector of particle P is given by its rectangular components,

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

- Acceleration vector,

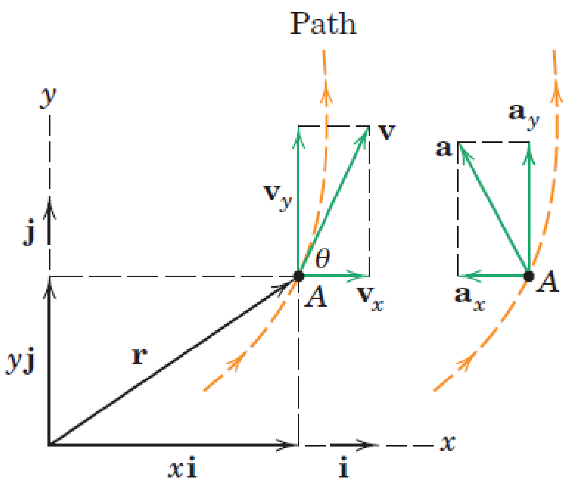
$$\vec{a} = \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j} + \frac{d^2z}{dt^2}\vec{k} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$
$$= a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$



Kinematics of Particles

❑ Sample Problem 09

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that x=0 when t=0.
Plot the path of the particle and determine its velocity and acceleration when the position y=0 is reached.



Kinematics of Particles

❑ Sample Problem 09

SOLUTION:

Determine motion components in x direction

Determine motion components in y direction

Kinematics of Particles

❑ Sample Problem 09

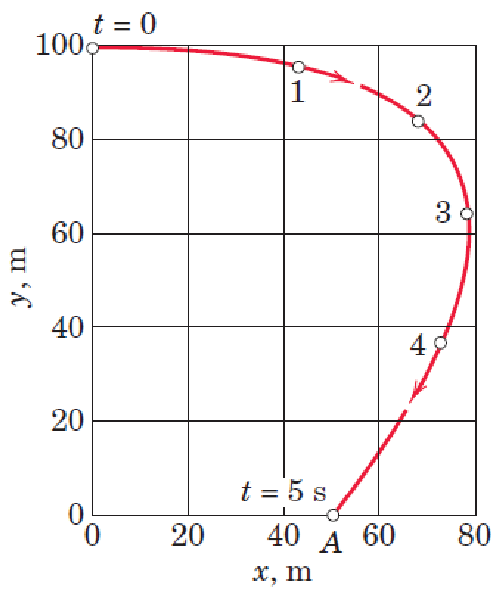
SOLUTION:

$$t = 2 \quad \Rightarrow \quad \vec{r} = 68\vec{i} + 84\vec{j}$$

$$t = 3 \quad \Rightarrow \quad \vec{r} = 78\vec{i} + 64\vec{j}$$

$$t = 4 \quad \Rightarrow \quad \vec{r} = 72\vec{i} + 36\vec{j}$$

$$t = 5 \quad \Rightarrow \quad \vec{r} = 50\vec{i} + 0\vec{j}$$

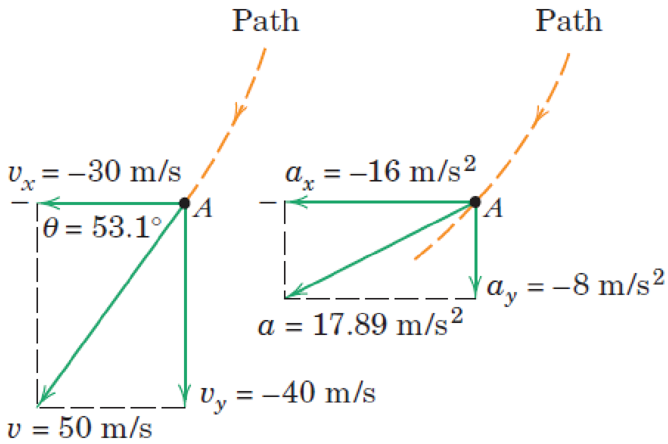


❑ Sample Problem 09

SOLUTION:

When y=0 a , v =?

$$y = 100 - 4t^2 = 0 \quad \Rightarrow \quad \boxed{t = 5\ s}$$



❑ Rectangular Components of Velocity & Acceleration

- Rectangular components particularly effective when component accelerations can be integrated independently, e.g., motion of a ***projectile***,

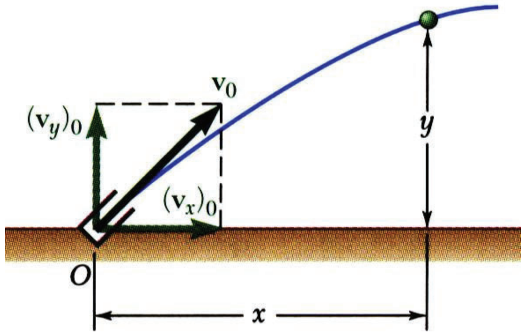
$$a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = -g \qquad a_z = \ddot{z} = 0$$

with initial conditions,

$$x_0 = y_0 = z_0 = 0 \quad (v_x)_0, (v_y)_0 \neq 0 \quad (v_z)_0 = 0$$

Integrating twice yields

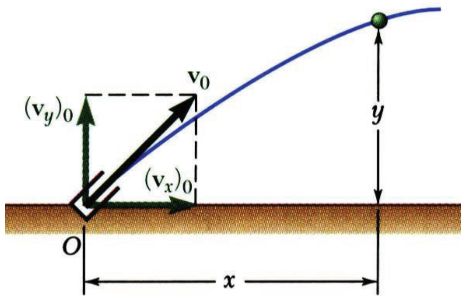
$$\begin{aligned} v_x &= (v_x)_0 & v_y &= (v_y)_0 - gt & v_z &= 0 \\ x &= (v_x)_0 t & y &= (v_y)_0 t - \tfrac{1}{2} g t^2 & z &= 0 \end{aligned}$$



Kinematics of Particles

Rectangular Components of Velocity & Acceleration

- Equation motion of projectile



$$x = (v_x)_0 t \quad \Rightarrow \quad \boxed{t = \frac{x}{(v_x)_0}}$$

$$y = (v_y)_0 t - \tfrac{1}{2} g t^2 \quad \Rightarrow \quad y = (v_y)_0 \left(\frac{x}{(v_x)_0} \right) - \tfrac{1}{2} g \left(\frac{x}{(v_x)_0} \right)^2$$

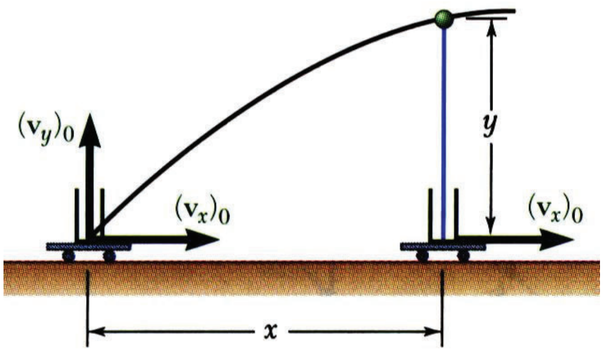
$$\Rightarrow \quad y = v_0 \sin\theta \left(\frac{x}{v_0 \cos\theta} \right) - \tfrac{1}{2} g \left(\frac{x}{v_0 \cos\theta} \right)^2 \quad \Rightarrow \quad \boxed{y = x \tan\theta - \tfrac{1}{2} \frac{g\, x^2}{v_0^{\ 2} \cos^2\theta}}$$

Kinematics of Particles

Rectangular Components of Velocity & Acceleration

Independently motion of a projectile

- Motion in horizontal direction is uniform.
- Motion in vertical direction is uniformly accelerated.
- Motion of projectile could be replaced by two independent rectilinear motions.

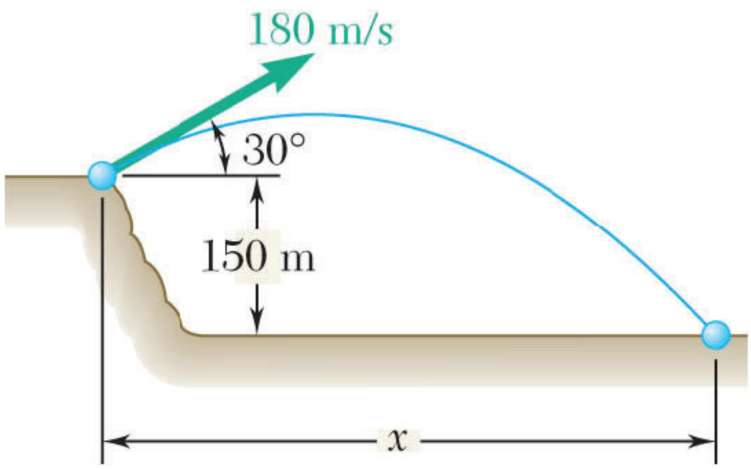


❑ Sample Problem 10

A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find

(a) the horizontal distance from the gun to the point where the projectile strikes the ground,

(b) the greatest elevation above the ground reached by the projectile.



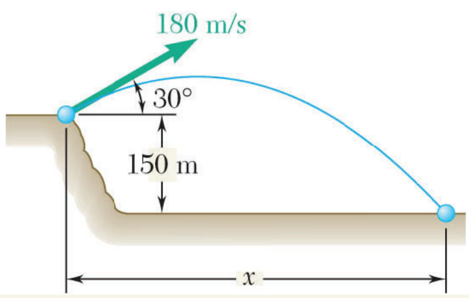
❑ Sample Problem 10

SOLUTION:

Given: $(v)_o$ =180 m/s $(y)_o$ =0

$(a)_y$ = - 9.81 m/s^2 $(a)_x$ = 0 m/s^2

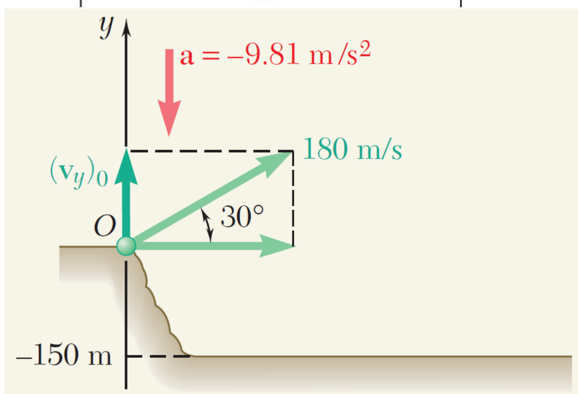
Vertical motion – uniformly accelerated:



(1)

(2)

(3)



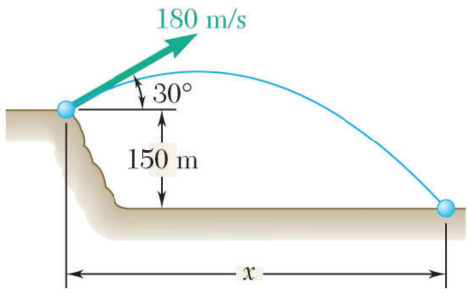
Given: $(v)_o$ =180 m/s $(y)_o$ =150 m

$(a)_y$ = - 9.81 m/s² $(a)_x$ = 0 m/s²

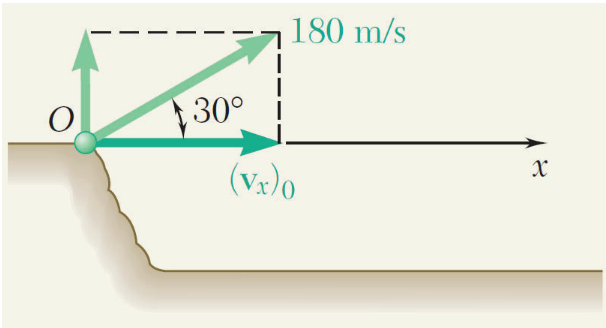
❑ Sample Problem 10

SOLUTION:

Horizontal motion – uniformly motion:

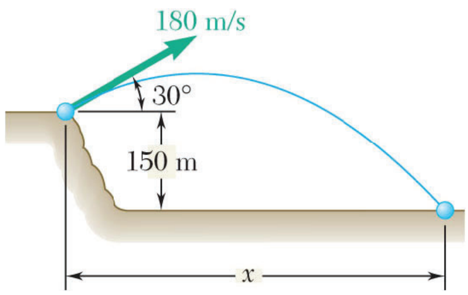


Choose positive x to the right as shown



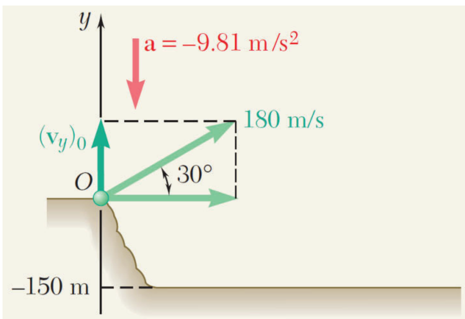
❑ Sample Problem 10

SOLUTION:



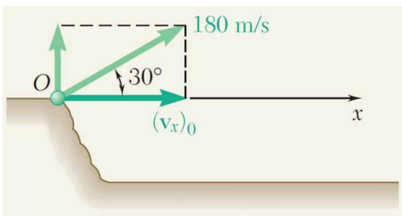
Projectile strikes the ground at: $y = -150\ m$

Substitute into equation (1) above

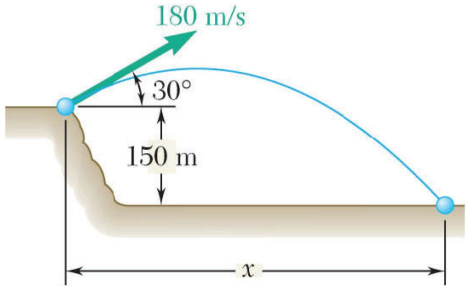


Solving for *t*, we take the positive root

Substitute t into equation (4)



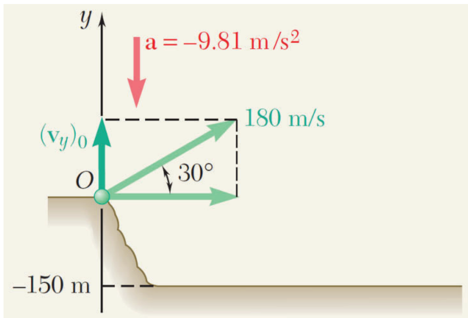
Kinematics of Particles



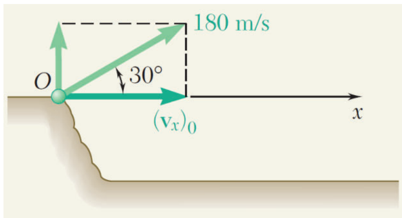
- **Sample Problem 10**

SOLUTION:

Maximum elevation occurs when v_y=0



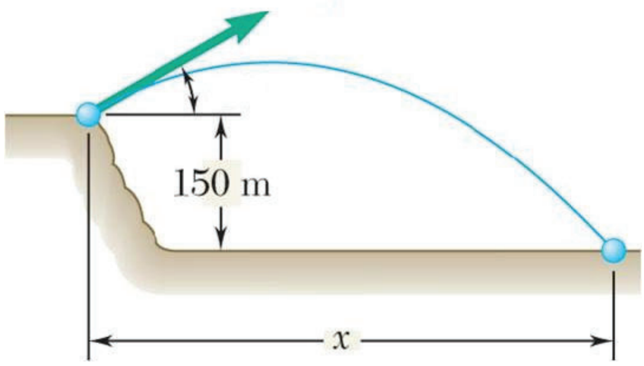
Maximum elevation above the ground



Kinematics of Particles

- **Concept Quiz**

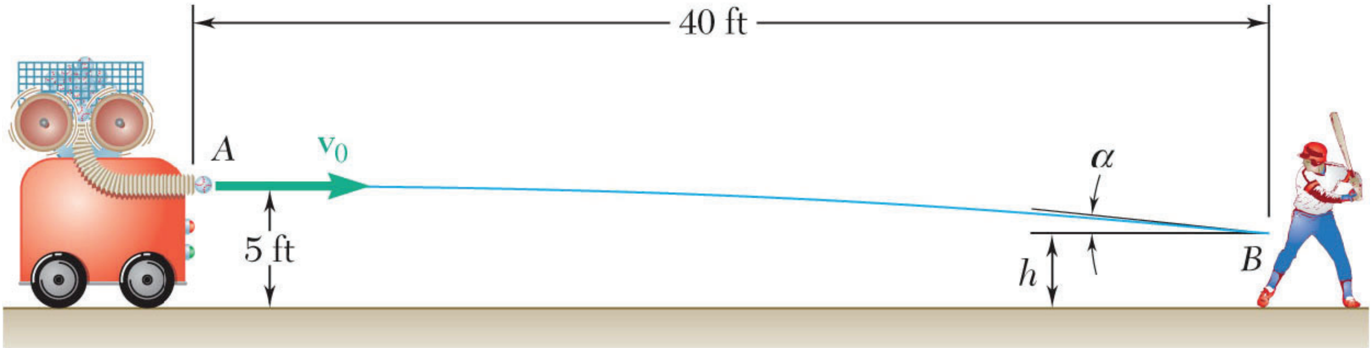
If you fire a projectile from 150 meters above the ground, what launch angle will give you the greatest horizontal distance *x*?



a) A launch angle of 45
b) A launch angle less than 45
c) A launch angle greater than 45
d) It depends on the launch velocity

❑ Sample Problem 11

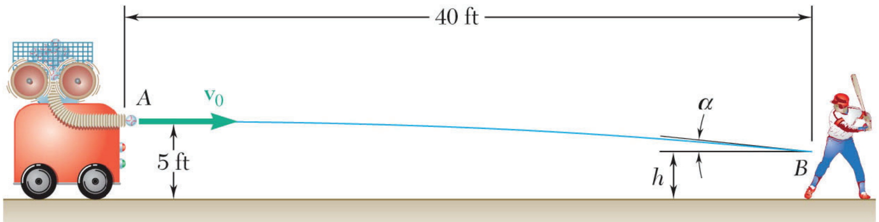
A baseball pitching machine "throws" baseballs with a horizontal velocity $\mathbf{v}_0$. If you want the height h to be 42 in., determine the value of v_0.



❑ Sample Problem 11

SOLUTION:

Given: x= 40 ft, y_o = 5 ft, y_f= 42 in.

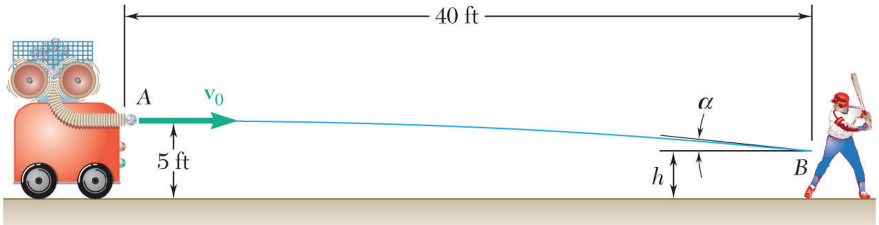


Analyze the motion in the y-direction

Analyze the motion in the x-direction

❑ Sample Problem 11

SOLUTION:



Given: x= 40 ft, y_o = 5 ft, y_f= 42 in.

Other Solution

❑ Motion Relative to a Frame in Translation

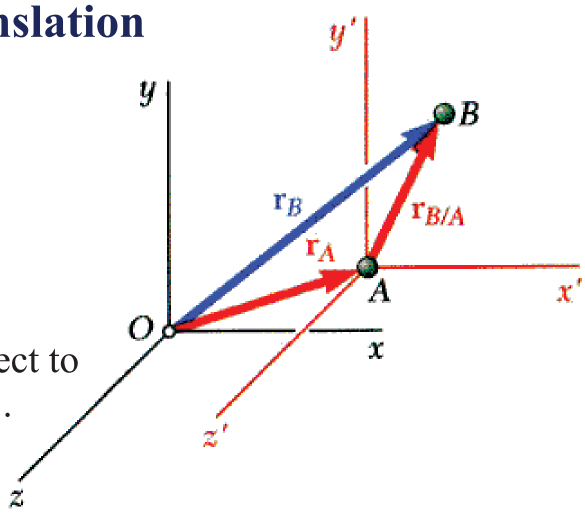
A soccer player must consider the relative motion of the ball and her teammates when making a pass.

It is critical for a pilot to know the relative motion of his aircraft with respect to the aircraft carrier to make a safe landing.

Kinematics of Particles

❑ Motion Relative to a Frame in Translation

- Designate one frame as the *fixed frame of reference*. All other frames not rigidly attached to the fixed reference frame are *moving frames of reference*.



- Position vectors for particles *A* and *B* with respect to the fixed frame of reference *Oxyz* are $\vec{r}_A$ and $\vec{r}_B$.

- Vector $\vec{r}_{B/A}$ joining *A* and *B* defines the position of *B* with respect to the moving frame $Ax'y'z'$ and

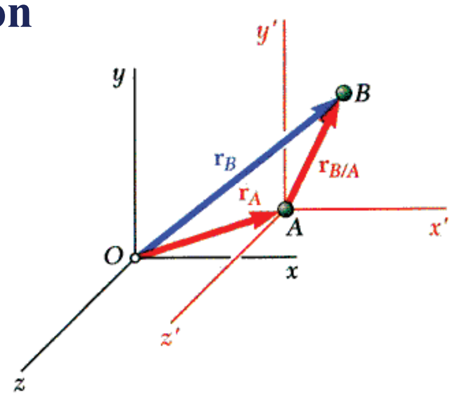
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

Kinematics of Particles

❑ Motion Relative to a Frame in Translation

- Absolute motion of *B* can be obtained by combining motion of *A* with relative motion of *B* with respect to moving reference frame attached to *A*.

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$



- **Differentiating twice,**

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$ $\vec{v}_{B/A}$ = velocity of *B* relative to *A*.

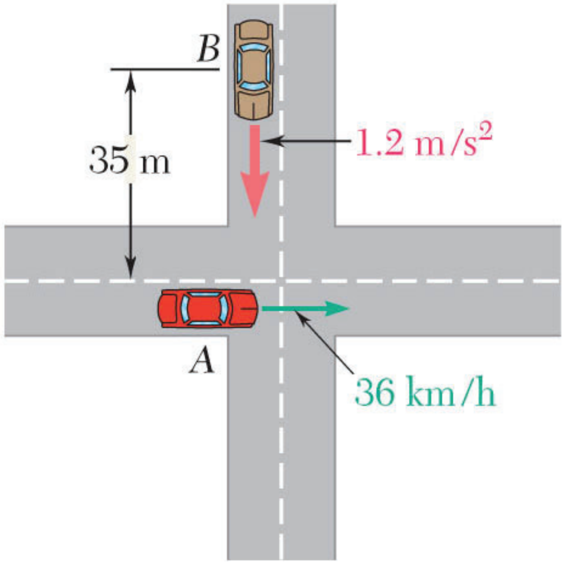
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$ $\vec{a}_{B/A}$ = acceleration of *B* relative to *A*.

Kinematics of Particles

❑ Sample Problem 12

Automobile *A* is traveling east at the constant speed of 36 km/h. As automobile *A* crosses the intersection shown, automobile *B* starts from rest 35 m north of the intersection and moves south with a constant acceleration of 1.2 m/s^2.

Determine the position, velocity, and acceleration of *B* relative to *A*, 5s after *A* crosses the intersection.



Kinematics of Particles

❑ Sample Problem 12

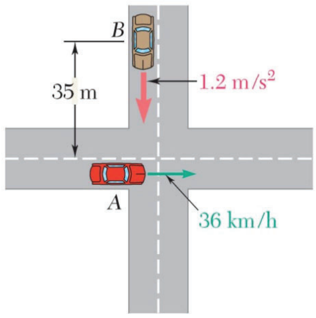
SOLUTION:

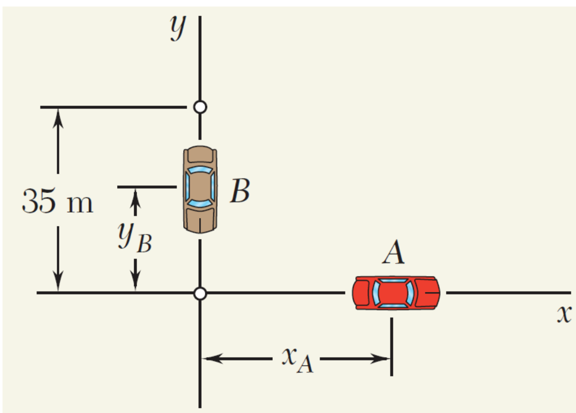
Given:

$v_A = 36$ km/h, $a_A = 0$, $(x_A)_0 = 0$

$(v_B)_0 = 0$, $a_B = -1.2$ m/s^2, $(y_B)_0 = 35$ m

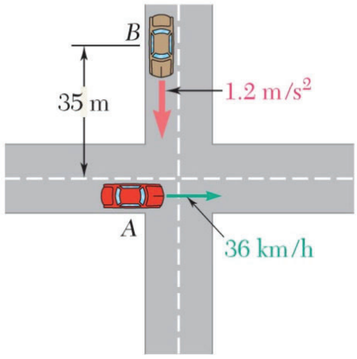
- Define axes along the road





Determine motion of Automobile A:

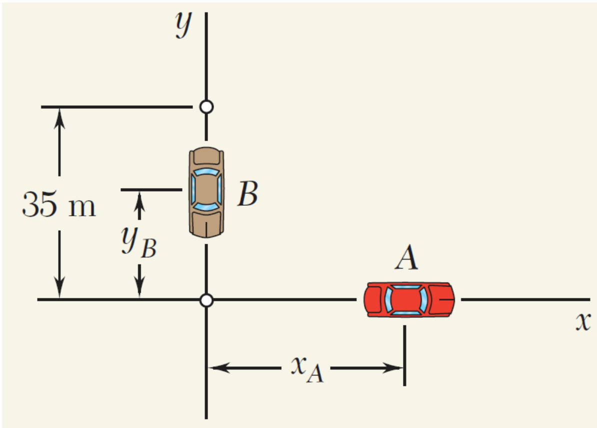
Kinematics of Particles



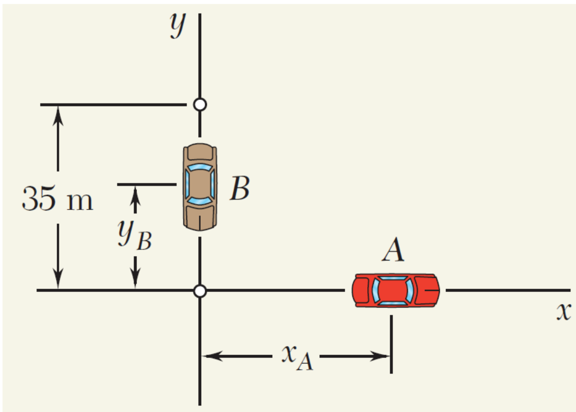
❑ Sample Problem 12

SOLUTION:

We have uniform motion for A so:



At $t = 5$ s



❑ Sample Problem 12

SOLUTION:

Determine motion of Automobile B:

We have uniform acceleration for B so:

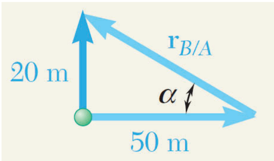
❑ Sample Problem 12

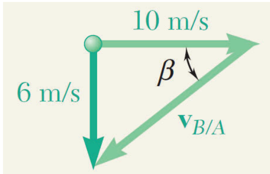
$\mathbf{a}_A = 0$
$\mathbf{v}_A = 10\,(m/s) \rightarrow$
$\mathbf{r}_A = 50\,m \rightarrow$

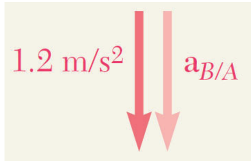
$\mathbf{a}_B = 1.2\,(m/s^2) \downarrow$
$\mathbf{v}_B = 6\,(m/s) \downarrow$
$\mathbf{r}_B = 20\,m \uparrow$

SOLUTION:

We can solve the problems geometrically, and apply the arctangent relationship:







$\mathbf{r}_{B/A} = 53.9\,m\,,\ \alpha = 21.8^\circ$

$\mathbf{v}_{B/A} = 11.66\,m/s\,,\ \beta = 31.0^\circ$

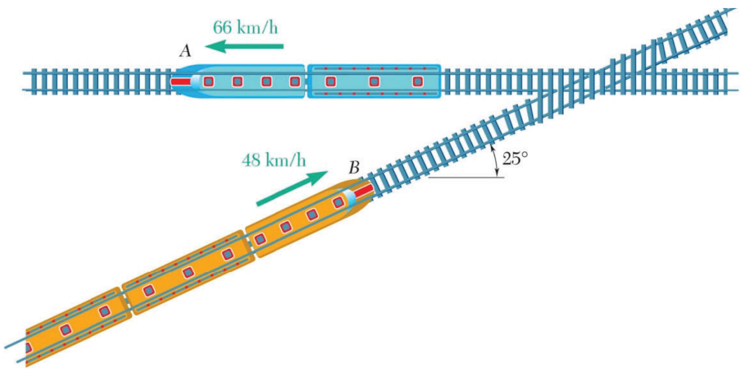
$\mathbf{a}_{B/A} = 1.2\,m/s^2 \downarrow$

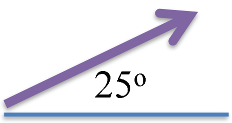
Or we can solve the problems using vectors to obtain equivalent results:

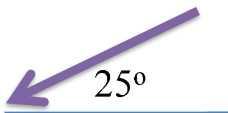
Physically, a rider in car A would "see" car B travelling south and west.

❑ Concept Quiz

If you are sitting in train B looking out the window, it which direction does it appear that train A is moving?



a)


b)


c)


d)


Kinematics of Particles

❑ Tangential and Normal Components

If we have an idea of the path of a vehicle, it is often convenient to analyze the motion using tangential and normal components (sometimes called *path* coordinates).

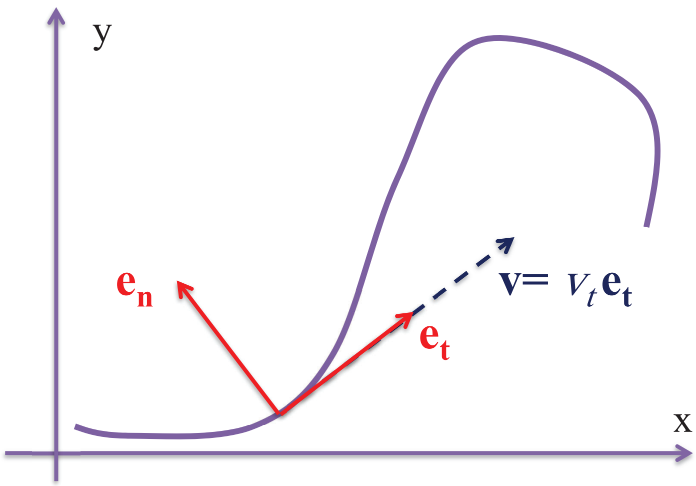
Kinematics of Particles

❑ Tangential and Normal Components

- The tangential direction ($\mathbf{e_t}$) is tangent to the path of the particle.
- This velocity vector of a particle is in this direction

$$\mathbf{v} = v\,\mathbf{e}_t$$

- The normal direction ($\mathbf{e_n}$) is perpendicular to $\mathbf{e_t}$ and points towards the inside of the curve.



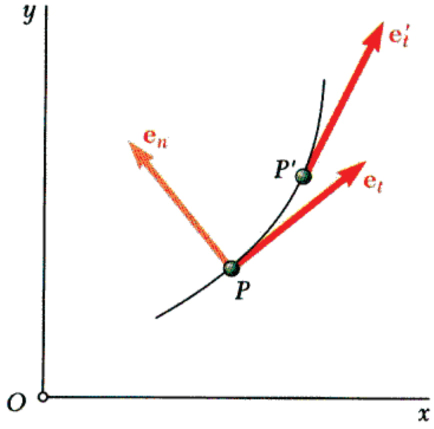
- The acceleration can have components in both the $\mathbf{e_n}$ and $\mathbf{e_t}$ directions

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n$$

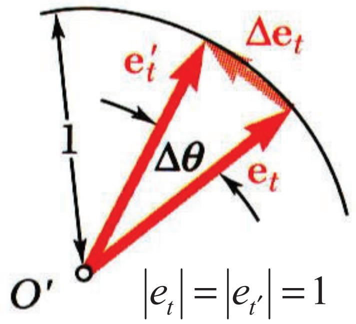
ρ= the instantaneous radius of curvature

Kinematics of Particles

❑ Tangential and Normal Components



- To derive the acceleration vector in tangential and normal components, define the motion of a particle as shown in the figure.
- $\vec{e}_t$ and $\vec{e}_t'$ are tangential unit vectors for the particle path at P and P'.
- When $\vec{e}_t$ and $\vec{e}_t'$ are drawn with respect to the same origin, $\Delta\vec{e}_t = \vec{e}_t' - \vec{e}_t$ and $\Delta\theta$ is the angle between them.



$|e_t| = |e_{t'}| = 1$

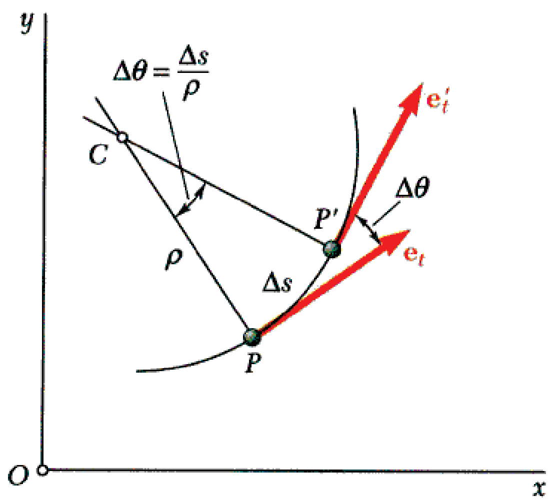
$$\Delta e_t = 2|e_t|\sin(\Delta\theta/2) \quad \Rightarrow \quad \Delta e_t = 2\sin(\Delta\theta/2)$$

$$\lim_{\Delta\theta\to 0}\frac{\Delta\vec{e}_t}{\Delta\theta} = \lim_{\Delta\theta\to 0}\frac{\sin(\Delta\theta/2)}{\Delta\theta/2}\vec{e}_n = \vec{e}_n \quad \Rightarrow \quad \boxed{\frac{d\vec{e}_t}{d\theta} = \vec{e}_n}$$

Kinematics of Particles

❑ Tangential and Normal Components

- With the velocity vector expressed as $\vec{v} = v\vec{e}_t$ the particle acceleration may be written as



$$\boxed{\vec{v} = v\vec{e}_t} \quad \Rightarrow$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\vec{e}_t + v\frac{d\vec{e}_t}{dt} = \frac{dv}{dt}\vec{e}_t + v\left(\frac{d\vec{e}_t}{d\theta}\right)\left(\frac{d\theta}{ds}\right)\left(\frac{ds}{dt}\right)$$

but

$$\boxed{\frac{d\vec{e}_t}{d\theta} = \vec{e}_n} \qquad \boxed{\rho\, d\theta = ds} \qquad \boxed{\frac{ds}{dt} = v}$$

After substituting,

$$\Rightarrow \quad \vec{a} = \frac{dv}{dt}\vec{e}_t + v(\vec{e}_n)\left(\frac{1}{\rho}\right)(v) \quad \Rightarrow \quad \boxed{\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n}$$

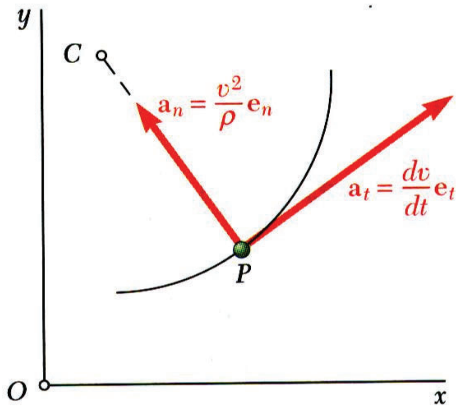
Kinematics of Particles

❑ Tangential and Normal Components

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$$

- The tangential component of acceleration reflects change of speed and the normal component reflects change of direction.

- The tangential component may be positive or negative. Normal component always points toward center of path curvature.



$$\rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} \right|$$

Kinematics of Particles

❑ Tangential and Normal Components

- Relations for tangential and normal acceleration also apply for particle moving along a space curve.

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n \qquad a_t = \frac{dv}{dt} \quad , \quad a_n = \frac{v^2}{\rho}$$

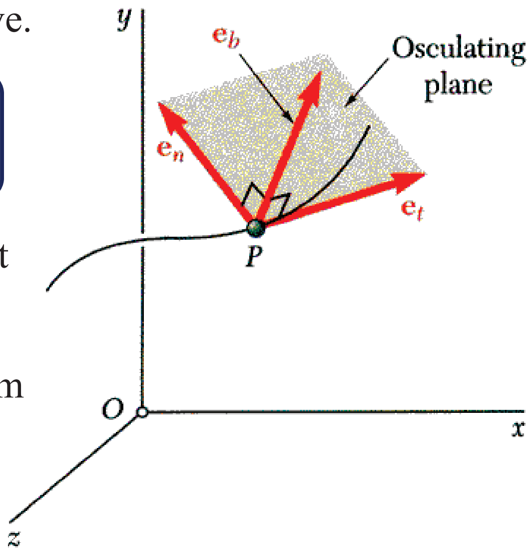
- The plane containing tangential and normal unit vectors is called the ***osculating plane***.

- The normal to the osculating plane is found from

$$\vec{e}_b = \vec{e}_t \times \vec{e}_n$$

$$\vec{e}_n = principal\ normal$$

$$\vec{e}_b = binormal$$

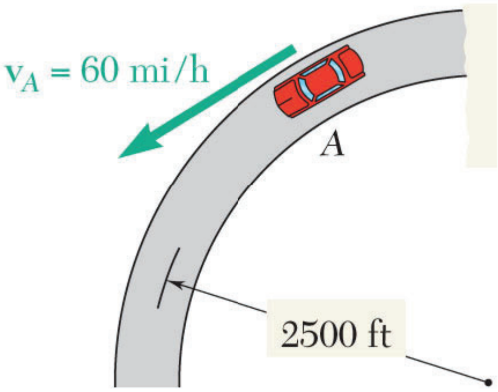


- Acceleration has no component along the binormal.

Kinematics of Particles

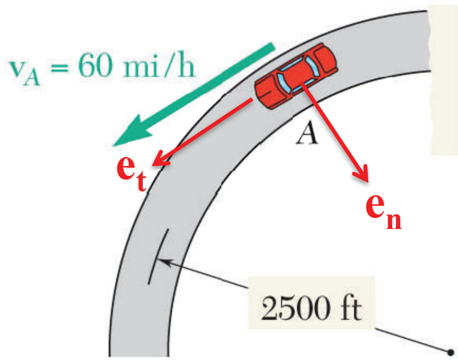
❑ Sample Problem 13

A motorist is traveling on a curved section of highway of radius 2500 ft at the speed of 60 mi/h. The motorist suddenly applies the brakes, causing the automobile to slow down at a constant rate. Knowing that after 8 s the speed has been reduced to 45 mi/h, determine the acceleration of the automobile immediately after the brakes have been applied.



Kinematics of Particles

❑ Sample Problem 13



SOLUTION:

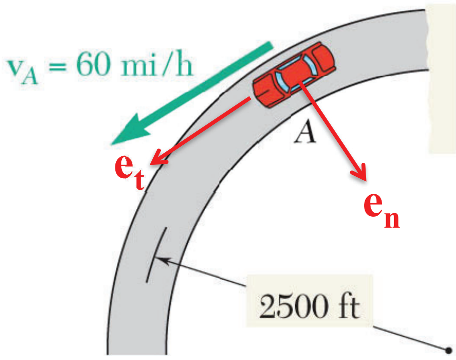
- Define your coordinate system
- Determine velocity and acceleration in the tangential direction

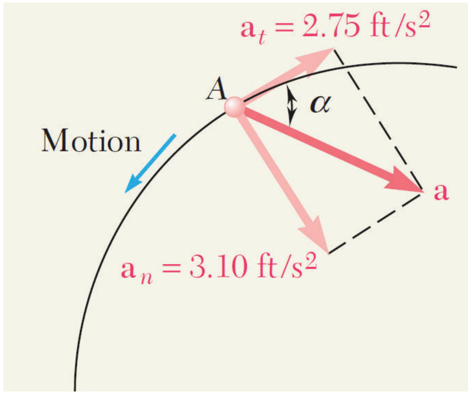
- The deceleration constant, therefore

❑ Sample Problem 13

SOLUTION:

- Immediately after the brakes are applied, the speed is still 88 ft/s





❑ Tangential and Normal Components

In 2001, a race scheduled at the Texas Motor Speedway was cancelled because the normal accelerations were too high and caused some drivers to experience excessive g-loads (similar to fighter pilots) and possibly pass out. What are some things that could be done to solve this problem?

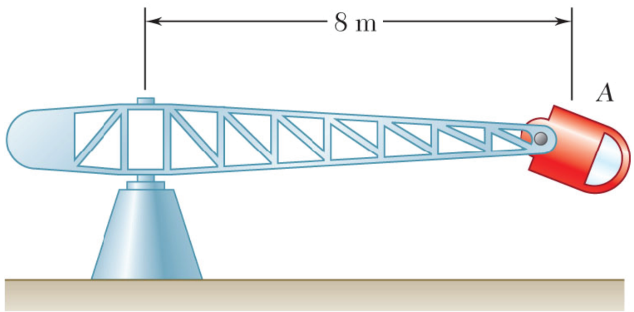
Kinematics of Particles

❑ Sample Problem 14

The tangential acceleration of the centrifuge cab is given by

$$a_t = 0.5t \quad (\text{m/s}^2)$$

where t is in seconds and a_t is in m/s^2. If the centrifuge starts from rest, determine the total acceleration magnitude of the cab after 10 seconds.



Kinematics of Particles

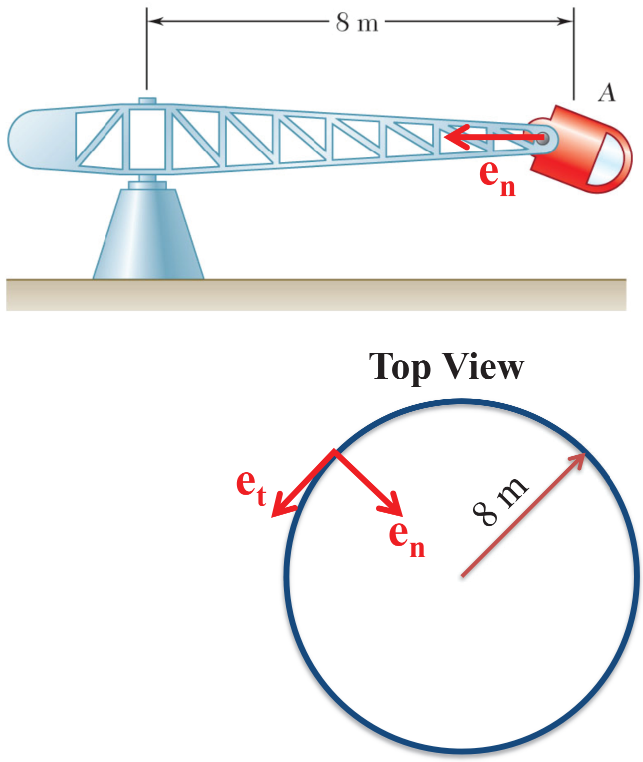
❑ Sample Problem 14

SOLUTION:

Define your coordinate system

In the side view, the tangential direction points into the "page"

Determine the tangential velocity

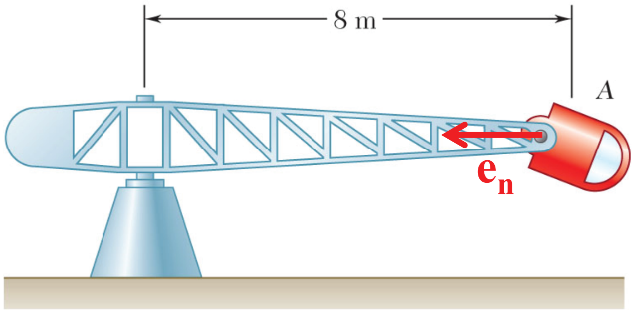


Kinematics of Particles

- **Sample Problem 14**

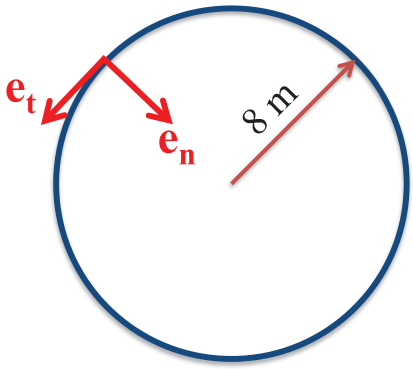
SOLUTION:

Determine the normal acceleration



Determine the total acceleration magnitude

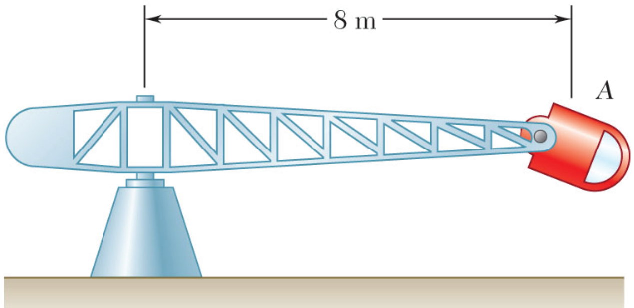
Top View



Kinematics of Particles

- **Group Problem Solving**

Notice that the normal acceleration is much higher than the tangential acceleration. What would happen if, for a given tangential velocity and acceleration, the arm radius was doubled?

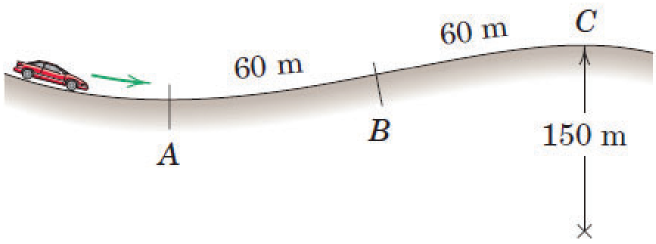


a) The accelerations would remain the same
b) The a_n would increase and the a_t would decrease
c) The a_n and a_t would both increase
d) The a_n would decrease and the a_t would increase

❑ Sample Problem 15

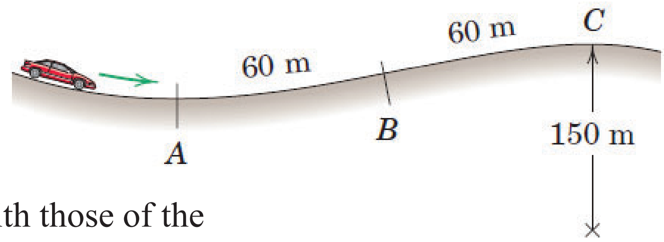
To anticipate the dip and hump in the road, the driver of a car applies her brakes to produce a **uniform deceleration**. Her speed is 100 km/h at the bottom A of the dip and 50 km/h at the top C of the hump, which is 120 m along the road from A. If the passengers experience a total acceleration of $3\,m/s^2$ at A and if the radius of curvature of the hump at C is 150m,

Calculate
(a) the radius of curvature at A,
(b) the acceleration at the inflection point B
(c) the total acceleration at C.



Kinematics of Particles

❑ Sample Problem 15



SOLUTION:

The dimensions of the car are small compared with those of the path, so we will treat the car as a particle. The velocities are

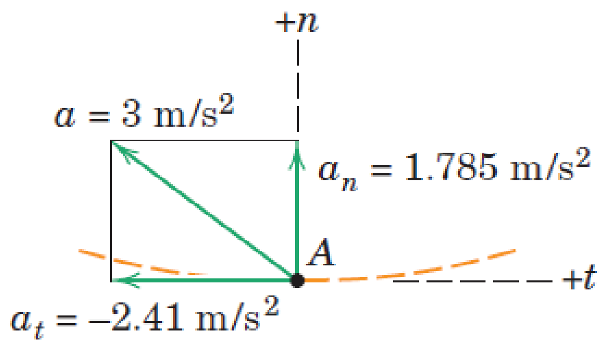
We find the constant deceleration along the path from

Is constant during the total path.

❑ Sample Problem 15

SOLUTION:

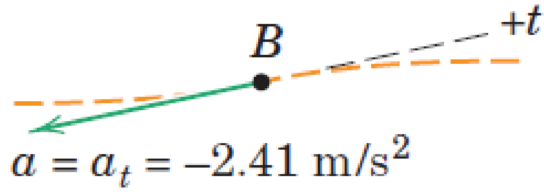
(a) Condition at A. With the total acceleration given and a_t determined, we can easily compute a_n and hence from



❑ Sample Problem 15

SOLUTION:

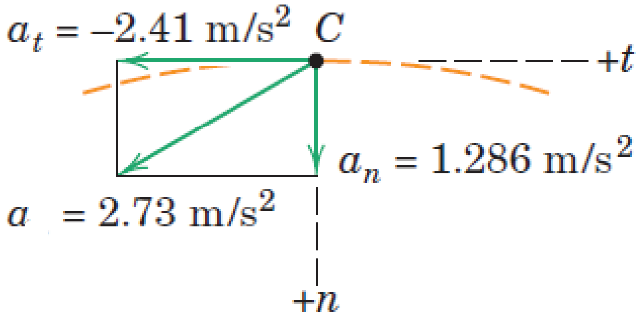
(b) Condition at B. Since the radius of curvature is infinite at the inflection point, an = 0 and



❑ Sample Problem 15

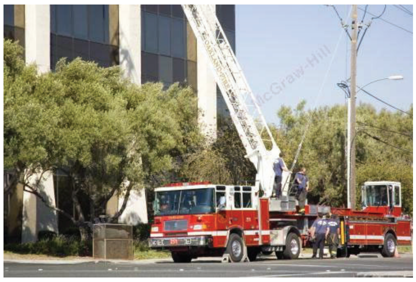
SOLUTION:

(c) Condition at C. **The normal acceleration becomes**



❑ Radial and Transverse Components

By knowing the distance to the aircraft and the angle of the radar, air traffic controllers can track aircraft.

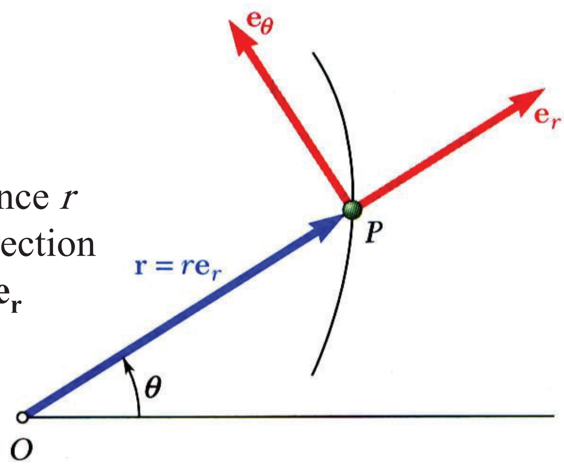


Fire truck ladders can rotate as well as extend; the motion of the end of the ladder can be analyzed using radial and transverse components.

Kinematics of Particles

❑ Radial and Transverse Components

- The position of a particle P is expressed as a distance r from the origin O to P – this defines the radial direction $\mathbf{e_r}$. The transverse direction $\mathbf{e_\theta}$ is perpendicular to $\mathbf{e_r}$



$$\vec{r} = r\vec{e}_r$$

- The particle velocity vector is

$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$v_r = \dot{r} \quad \& \quad v_\theta = r\dot{\theta}$$

- The particle acceleration vector is

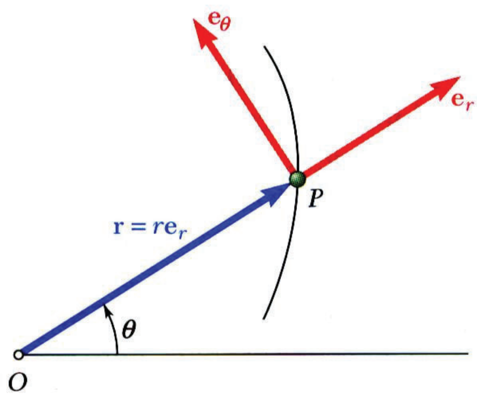
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad \& \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Kinematics of Particles

❑ Radial and Transverse Components

- We can derive the velocity and acceleration relationships by recognizing that the unit vectors change direction.



$$\vec{r} = r\vec{e}_r$$

$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt} = \vec{e}_\theta \frac{d\theta}{dt}$$

$$\frac{d\vec{e}_\theta}{dt} = \frac{d\vec{e}_\theta}{d\theta}\frac{d\theta}{dt} = -\vec{e}_r \frac{d\theta}{dt}$$

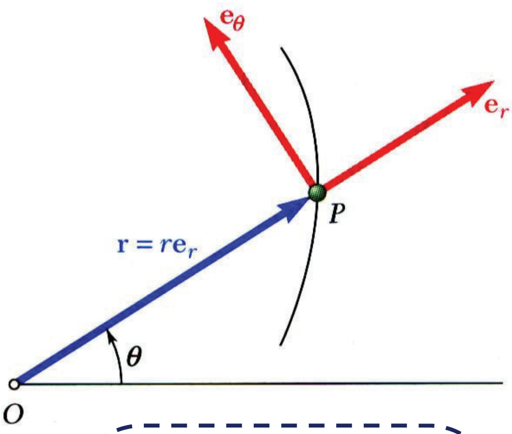
- The particle velocity vector is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\vec{e}_r + r\frac{d\vec{e}_r}{dt} = \dot{r}\vec{e}_r + r\left(\frac{d\theta}{dt}\right)\left(\frac{d\vec{e}_r}{d\theta}\right)$$

$$\Rightarrow \vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

Kinematics of Particles

❑ Radial and Transverse Components



$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

- Similarly, the particle acceleration vector is

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\frac{d\dot{r}}{dt}\vec{e}_r + \dot{r}\frac{d\vec{e}_r}{dt}\right) + \left(\frac{dr}{dt}\dot{\theta}\vec{e}_\theta + r\frac{d\dot{\theta}}{dt}\vec{e}_\theta + r\dot{\theta}\frac{d\vec{e}_\theta}{dt}\right)$$

$$\Rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} = \left(\ddot{r}\vec{e}_r + \dot{r}\dot{\theta}\vec{e}_\theta\right) + \left(\dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}(-\dot{\theta}\vec{e}_r)\right)$$

$$\Rightarrow \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

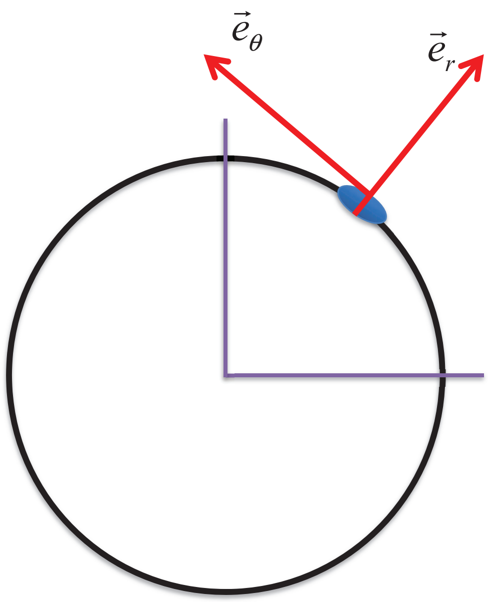
$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt} = \vec{e}_\theta\frac{d\theta}{dt}$$

$$\frac{d\vec{e}_\theta}{dt} = \frac{d\vec{e}_\theta}{d\theta}\frac{d\theta}{dt} = -\vec{e}_r\frac{d\theta}{dt}$$

Kinematics of Particles

❑ Radial and Transverse Components



The components of velocity and acceleration in circle motion

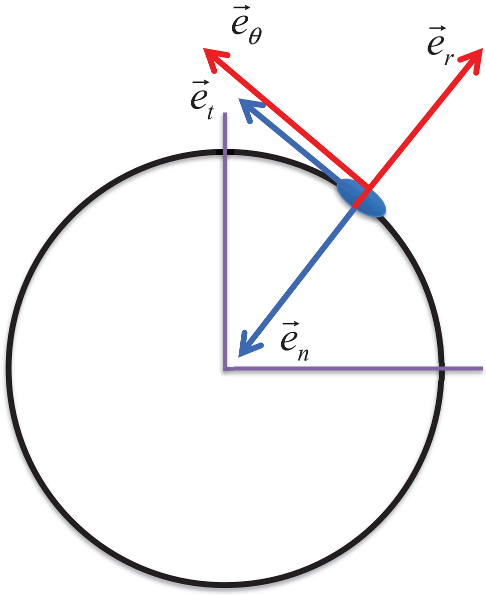
$$r = cte \quad \Rightarrow \quad \dot{r} = \ddot{r} = 0$$

$$\Rightarrow \quad \begin{aligned} \vec{v} &= \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta \\ \vec{a} &= (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta \end{aligned}$$

$$\Rightarrow \quad \vec{v} = r\dot{\theta}\vec{e}_\theta$$

$$\vec{a} = -r\dot{\theta}^2\vec{e}_r + r\ddot{\theta}\vec{e}_\theta$$

Concept Quiz



If you are travelling in a perfect circle, what is always true about radial/transverse coordinates and normal/tangential coordinates?

a) The $\mathbf{e_r}$ direction is identical to the $\mathbf{e_n}$ direction.
b) The $\mathbf{e_\theta}$ direction is perpendicular to the $\mathbf{e_n}$ direction.
c) The $\mathbf{e_\theta}$ direction is parallel to the $\mathbf{e_r}$ direction.

Radial and Transverse Components

- When particle position is given in ***cylindrical coordinates***, it is convenient to express the velocity and acceleration vectors using the unit vectors $\vec{e}_R, \vec{e}_\theta$, and $\vec{k}$.
- Position vector,

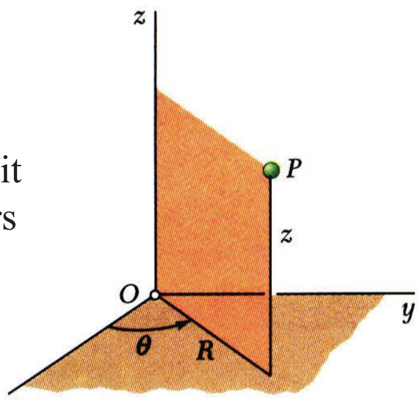
$$\vec{r} = R\vec{e}_R + z\vec{k}$$

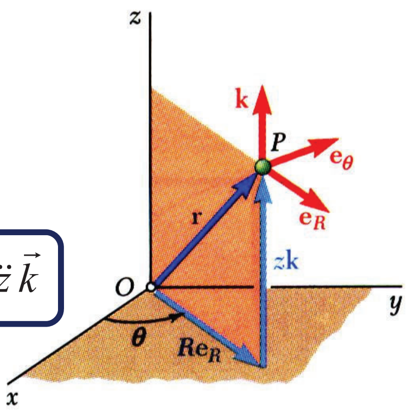
- Velocity vector,

$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{v} = \dot{R}\vec{e}_R + R\dot{\theta}\vec{e}_\theta + \dot{z}\vec{k}$$

- Acceleration vector,

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \vec{a} = (\ddot{R} - R\dot{\theta}^2)\vec{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\vec{e}_\theta + \ddot{z}\vec{k}$$



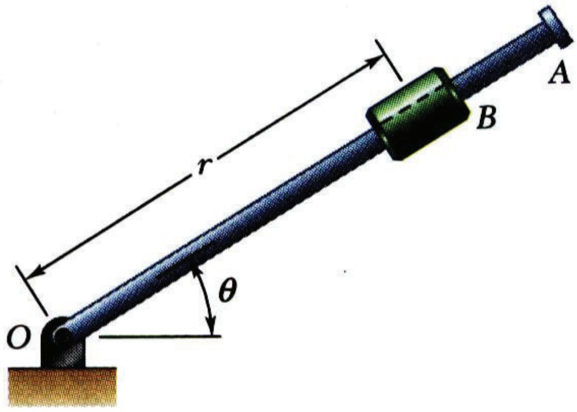


Kinematics of Particles

❑ Sample Problem 16

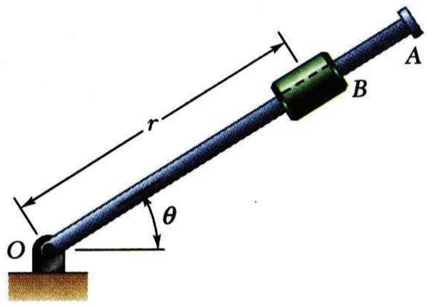
Rotation of the arm about O is defined by $\theta = 0.15t^2$ where θ is in radians and t in seconds. Collar B slides along the arm such that $r = 0.9 - 0.12t^2$ where r is in meters.

After the arm has rotated through 30°, determine *(a)* the total velocity of the collar, *(b)* the total acceleration of the collar, and *(c)* the relative acceleration of the collar with respect to the arm.



Kinematics of Particles

❑ Sample Problem 16



SOLUTION:

- Evaluate time t for $\theta = 30°$.

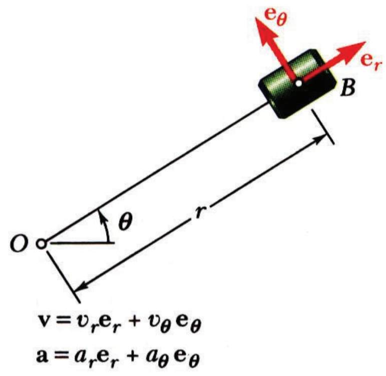
- Evaluate radial and angular positions, and first and second derivatives at time t.

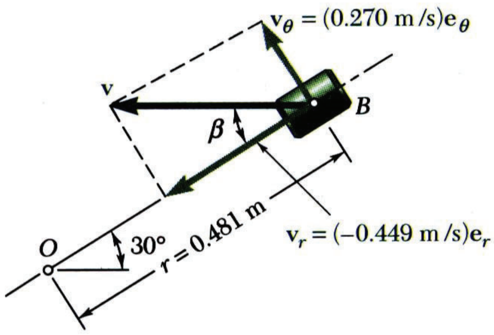
Kinematics of Particles

❑ Sample Problem 16

SOLUTION:

- Calculate velocity



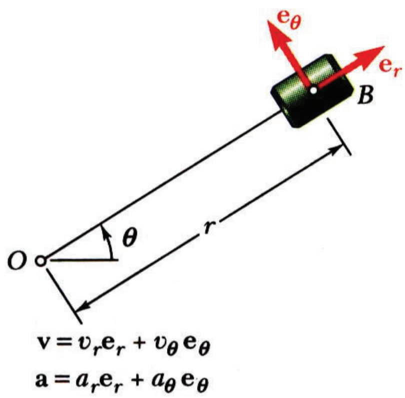


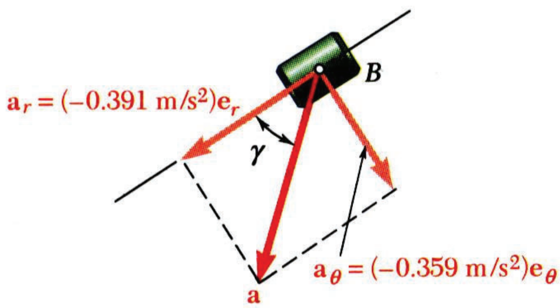
Kinematics of Particles

❑ Sample Problem 16

SOLUTION:

- Calculate acceleration.



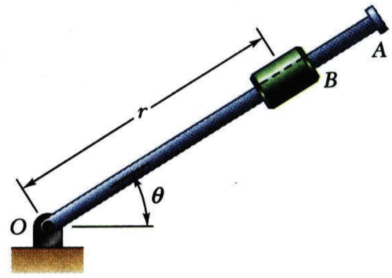


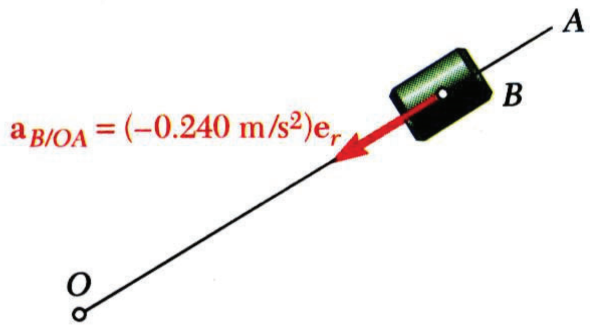
❑ Sample Problem 16

SOLUTION:

- Evaluate acceleration with respect to arm.

  Motion of collar with respect to arm is rectilinear and defined by coordinate *r*.



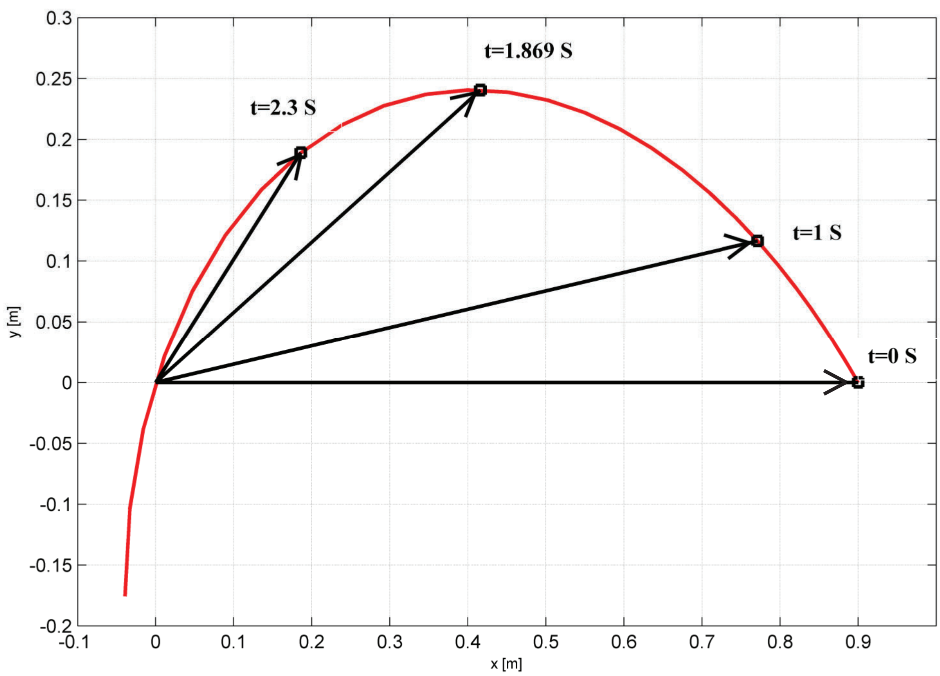


❑ Sample Problem 16

SOLUTION:

- Plotted in the final figure is the path of the slider B over the time interval $0 \le t \le 3\,\mathrm{s}$. This plot is generated by varying t in the given expressions for r and θ. Conversion from polar to rectangular coordinates is given by

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$



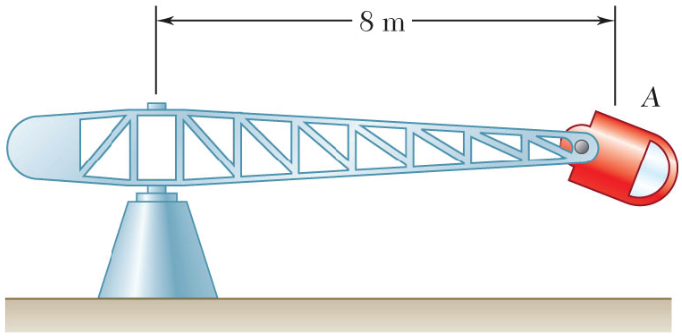
Kinematics of Particles

❑ Sample Problem 17

The angular acceleration of the centrifuge arm varies according to

$$\ddot{\theta} = 0.05\theta \quad (\text{rad/s}^2)$$

where θ is measured in radians. If the centrifuge starts from rest, determine the acceleration magnitude after the gondola has travelled two full rotations.



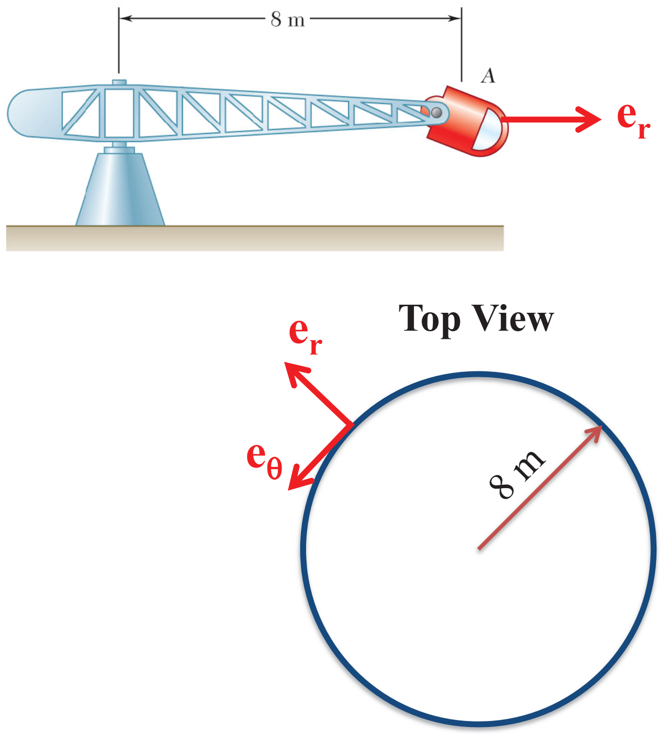
Kinematics of Particles

❑ Sample Problem 17

SOLUTION:

Define your coordinate system

In the side view, the transverse direction points into the "page"



Determine the angular velocity

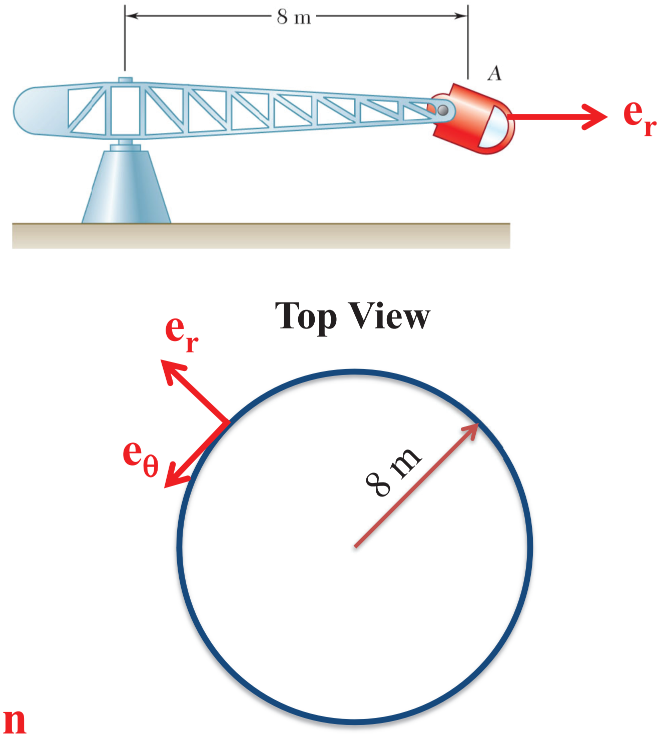
Acceleration is a function of position, so use:

Kinematics of Particles

- Sample Problem 17

SOLUTION:

Evaluate the integral



Determine the angular velocity

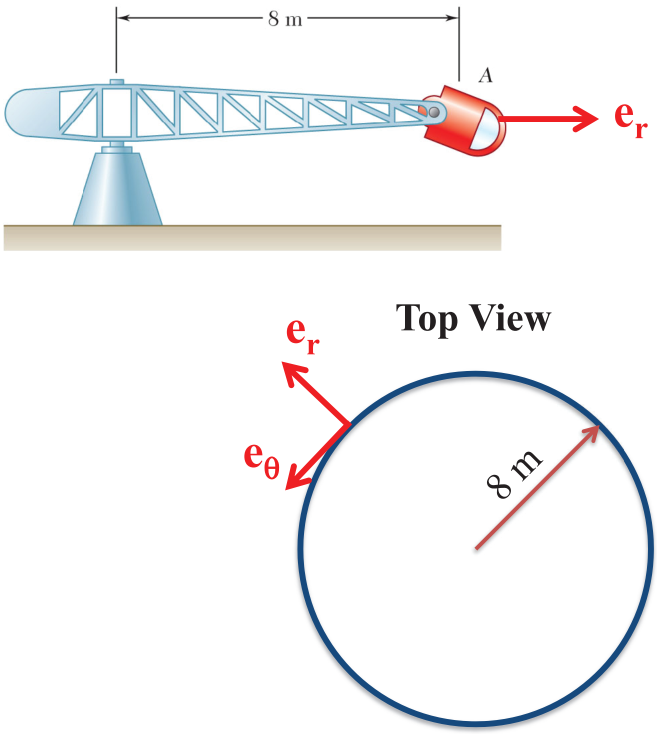
Determine the angular acceleration

Kinematics of Particles

- Sample Problem 17

SOLUTION:

Find the radial and transverse accelerations

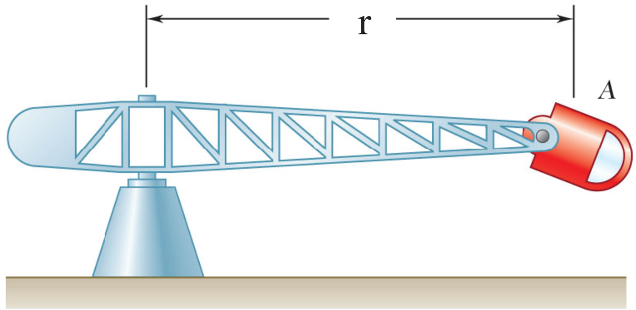


Magnitude:

Kinematics of Particles

❑ Group Problem Solving

What would happen if you designed the centrifuge so that the arm could extend from 6 to 10 meters?



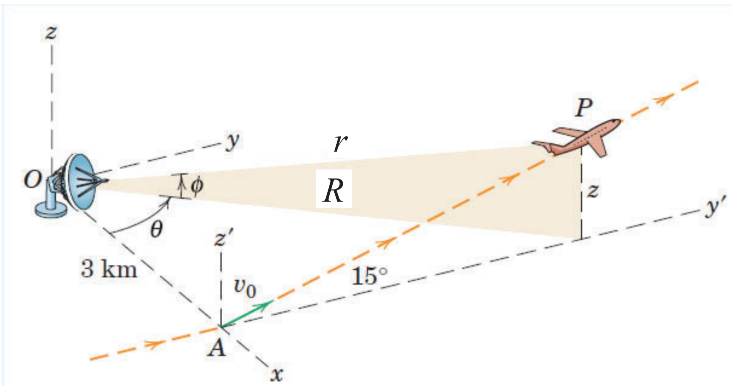
You could now have additional acceleration terms. This might give you more control over how quickly the acceleration of the gondola changes (this is known as the G-onset rate).

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

Kinematics of Particles

❑ Sample Problem 18

An aircraft P takes off at A with a velocity v_0 of 250 km/h and climbs in the vertical $y' - z'$ plane at the constant 15° angle with an acceleration along its flight path of 0.8 m/s^2. Flight progress is monitored by radar at point O. Resolve the velocity of P into cylindrical-coordinate components 60 seconds after takeoff and find $\dot{R}$, $\dot{\theta}$ *and* $\dot{z}$ for that instant.

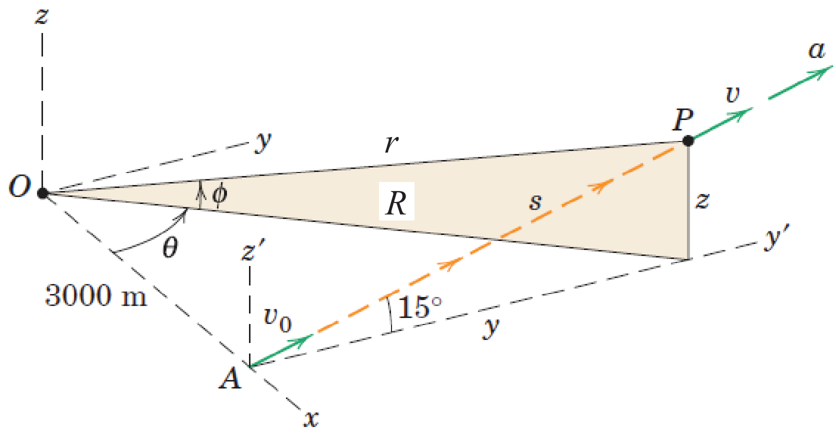


Kinematics of Particles

❑ Sample Problem 18

SOLUTION:

The takeoff speed is



the speed after 60 seconds is

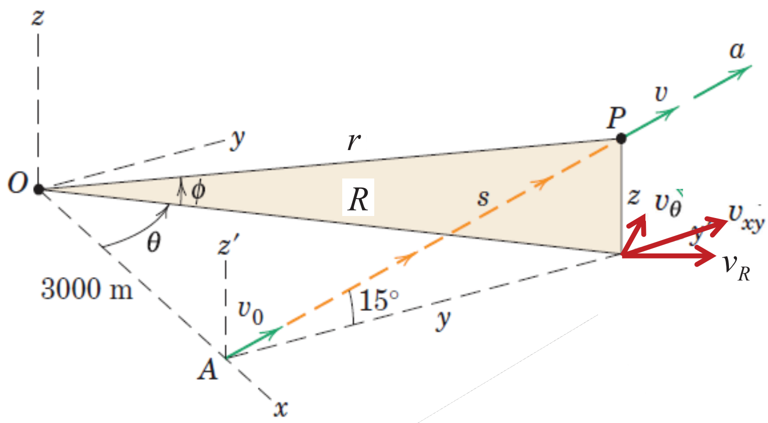
The distance s traveled after takeoff is

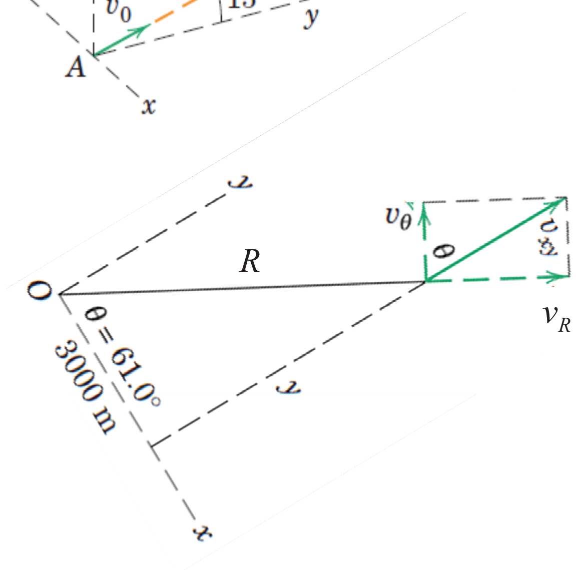
The y-coordinate and associated angle θ are

Kinematics of Particles

❑ Sample Problem 18

SOLUTION:





Kinematics of Particles

❑ UNITS CONVERSION TABLES

Table 1: Multiples and Submultiples of SI units

Prefix	Symbol	Multiplying Factor	
exa	E	10^{18}	1 000 000 000 000 000 000
peta	P	10^{15}	1 000 000 000 000 000
tera	T	10^{12}	1 000 000 000 000
giga	G	10^{9}	1 000 000 000
mega	M	10^{6}	1 000 000
kilo	k	10^{3}	1 000
hecto*	h	10^{2}	100
deca*	da	10	10
deci*	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	u	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001
femto	f	10^{-15}	0.000 000 000 000 001
atto	a	10^{-18}	0.000 000 000 000 000 001

* these prefixes are not normally used

Kinematics of Particles

❑ UNITS CONVERSION TABLES

Table 2: Length Units

Millimeters	Centimeters	Meters	Kilometers	Inches	Feet	Yards	Miles
mm	cm	m	km	in	ft	yd	mi
1	0.1	0.001	0.000001	0.03937	0.003281	0.001094	6.21e-07
10	1	0.01	0.00001	0.393701	0.032808	0.010936	0.000006
1000	100	1	0.001	39.37008	3.28084	1.093613	0.000621
1000000	100000	1000	1	39370.08	3280.84	1093.613	0.621371
25.4	2.54	0.0254	0.000025	1	0.083333	0.027778	0.000016
304.8	30.48	0.3048	0.000305	12	1	0.333333	0.000189
914.4	91.44	0.9144	0.000914	36	3	1	0.000568
1609344	160934.4	1609.344	1.609344	63360	5280	1760	1

Table 3: Area Units

Millimeter square	Centimeter square	Meter square	Inch square	Foot square	Yard square
mm^2	cm^2	m^2	in^2	ft^2	yd^2
1	0.01	0.000001	0.00155	0.000011	0.000001
100	1	0.0001	0.155	0.001076	0.00012
1000000	10000	1	1550.003	10.76391	1.19599
645.16	6.4516	0.000645	1	0.006944	0.000772
92903	929.0304	0.092903	144	1	0.111111
836127	8361.274	0.836127	1296	9	1

❑ UNITS CONVERSION TABLES

Table 4: Volume Units

Centimeter cube	Meter cube	Liter	Inch cube	Foot cube	US gallons	Imperial gallons	US barrel (oil)
cm^3	m^3	ltr	in^3	ft^3	US gal	Imp. gal	US brl
1	0.000001	0.001	0.061024	0.000035	0.000264	0.00022	0.000006
1000000	1	1000	61024	35	264	220	6.29
1000	0.001	1	61	0.035	0.264201	0.22	0.00629
16.4	0.000016	0.016387	1	0.000579	0.004329	0.003605	0.000103
28317	0.028317	28.31685	1728	1	7.481333	6.229712	0.178127
3785	0.003785	3.79	231	0.13	1	0.832701	0.02381
4545	0.004545	4.55	277	0.16	1.20	1	0.028593
158970	0.15897	159	9701	6	42	35	1

Table 5: Mass Units

Grams	Kilograms	Metric tonnes	Short ton	Long ton	Pounds	Ounces
g	kg	tonne	shton	Lton	lb	oz
1	0.001	0.000001	0.000001	9.84e-07	0.002205	0.035273
1000	1	0.001	0.001102	0.000984	2.204586	35.27337
1000000	1000	1	1.102293	0.984252	2204.586	35273.37
907200	907.2	0.9072	1	0.892913	2000	32000
1016000	1016	1.016	1.119929	1	2239.859	35837.74
453.6	0.4536	0.000454	0.0005	0.000446	1	16
28	0.02835	0.000028	0.000031	0.000028	0.0625	1

Kinematics of Particles

❑ UNITS CONVERSION TABLES

Table 10: High Pressure Units

Bar	Pound/square inch	Kilopascal	Megapascal	Kilogram force/ centimeter square	Millimeter of mercury	Atmospheres
bar	psi	kPa	MPa	kgf/cm^2	mm Hg	atm
1	14.50326	100	0.1	1.01968	750.0188	0.987167
0.06895	1	6.895	0.006895	0.070307	51.71379	0.068065
0.01	0.1450	1	0.001	0.01020	7.5002	0.00987
10	145.03	1000	1	10.197	7500.2	9.8717
0.9807	14.22335	98.07	0.09807	1	735.5434	0.968115
0.001333	0.019337	0.13333	0.000133	0.00136	1	0.001316
1.013	14.69181	101.3	0.1013	1.032936	759.769	1

Table 16: Temperature Conversion Formulas

Degree Celsius (°C)	(°F - 32) x 5/9
	(K - 273.15)
Degree Fahrenheit (°F)	(°C x 9/5) + 32
	(1.8 x K) - 459.67
Kelvin (K)	(°C + 273.15)
	(°F + 459.67) ÷ 1.8